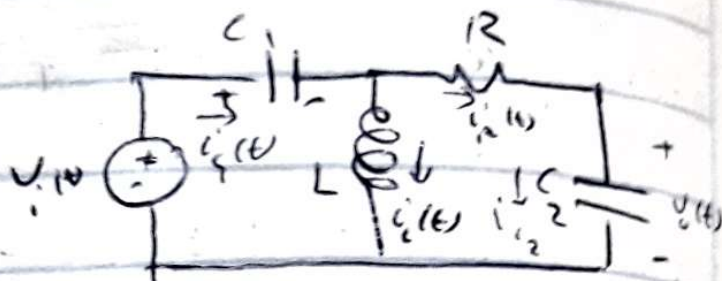


# Chapter 300 Modeling in the time domain

Exo:

Find the SS ??



3-storage element  $\Rightarrow$  3 state variable.

$$V_1(t) = V_{C_1}(t) + L \frac{di_L(t)}{dt} \Rightarrow$$

$$V_{C_2}(t) = -R i_R(t) + L \frac{di_L(t)}{dt}$$

$$i_{C_1}(t) = i_L(t) + i_{C_2}(t)$$

$$i_{C_1} = C_1 \frac{dV_{C_1}}{dt}$$

$$i_{C_2} = C_2 \frac{dV_{C_2}}{dt}$$

$$i_{C_1} = C_1 \frac{dV_{C_1}}{dt}, \quad i_{C_2} = C_2 \frac{dV_{C_2}}{dt}, \quad V_L = L \frac{di_L}{dt}$$

$(i_L, V_{C_1}, V_{C_2})$  this is the state variable.

$$i_{C_1} = i_L + i_{C_2} \quad \text{but} \quad i_{C_2} = i_R = \frac{V_L - V_{C_2}}{R}$$

$$\Rightarrow i_{C_1} = i_L + \frac{V_L}{R} - \frac{V_{C_2}}{R}$$

$$V_L = V_i - V_{C_1}$$

$$\Rightarrow i_{C_1} = C_1 \frac{dV_{C_1}}{dt} = i_L + \frac{V_L}{R} - \frac{V_{C_2}}{R}$$

$$\Rightarrow \frac{dV_{C_1}}{dt} = \frac{1}{C_1} i_L + \frac{1}{RC_1} (V_L - V_{C_1}) - \frac{V_{C_2}}{RC_1}$$

$$\Rightarrow \left| \frac{dV_{C_1}}{dt} = \frac{1}{C_1} i_L - \frac{1}{RC_1} V_{C_1} - \frac{1}{RC_1} V_{C_2} + \frac{1}{RC_1} V_L \right|$$

$$i_{C_2} = C_2 \frac{dV_{C_2}}{dt} = \frac{V_L - V_{C_1}}{R} - \frac{V_{C_2}}{R}$$

$$\Rightarrow \left| \frac{dV_{C_2}}{dt} = -\frac{1}{RC_2} V_{C_1} - \frac{1}{RC_2} V_{C_2} + \frac{1}{RC_2} V_L \right|$$

$$V_L = L \frac{di_L}{dt} = V_L - V_{C_1}$$

$$\Rightarrow \left| \frac{di_L}{dt} = -\frac{1}{L} V_{C_1} + \frac{1}{L} V_L \right|$$

$$\begin{bmatrix} i_L' \\ V_{C_1}' \\ V_{C_2}' \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} & 0 \\ \frac{1}{C_1} & -\frac{1}{RC_1} & -\frac{1}{RC_1} \\ 0 & -\frac{1}{RC_2} & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} i_L \\ V_{C_1} \\ V_{C_2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ \frac{1}{RC_1} \\ \frac{1}{RC_2} \end{bmatrix} V_L$$

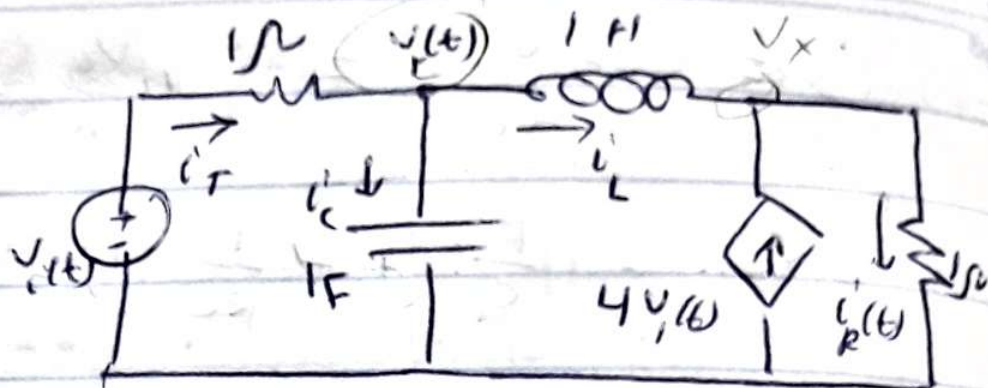
$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ V_{C_1} \\ V_{C_2} \end{bmatrix} + 0$$

$\rightarrow V_o = V_{C_2}$

P 3.2

Find SS??

$i_R$  is output.



2 - Storage element  $\Rightarrow$  2 - state eqn ( $i_L, v_C$ )

$$v_L = L \frac{di_L}{dt}, \quad i_C = C \frac{dv_C}{dt}$$

$$\begin{bmatrix} i_L \\ v_C \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} \quad \\ \quad \end{bmatrix} v_i(t) \quad \text{المصفوفة}$$

$$\frac{v_i(t) - v_C(t)}{1} + C \frac{dv_C}{dt} + i_L(t) = 0$$

$$-i_L - 4v_C(t) + v_X = 0$$

$$v_X = i_R(t) = i_L + 4v_C(t)$$

$\Rightarrow$

$$v_i(t) = (i_L + i_C) + L \frac{di_L}{dt} + (i_L + 4v_C(t))$$

$$L \frac{di_L}{dt} = v_i(t) - 2i_L - (i_C) - 4v_C(t)$$

$$i_C = v_i - v_C - i_L$$

$$\Rightarrow L \frac{di_L}{dt} = -i_L - 3v_C$$

$$\Rightarrow \boxed{\frac{di_L}{dt} = -i_L - 3v_C}$$

$$\frac{dV_c}{dt} = i_c' = V_i - V_c - i_L$$

$$\therefore \left[ \frac{dV_c}{dt} = -i_L' - V_c + V_i \right]$$

$$\begin{bmatrix} i_L' \\ V_c' \end{bmatrix} = \begin{bmatrix} -1 & -3 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} i_L' \\ V_c' \end{bmatrix} + \begin{bmatrix} 0 \\ +1 \end{bmatrix} V_i(t)$$

$$i_R' = i_L' + 4V_c(t)$$

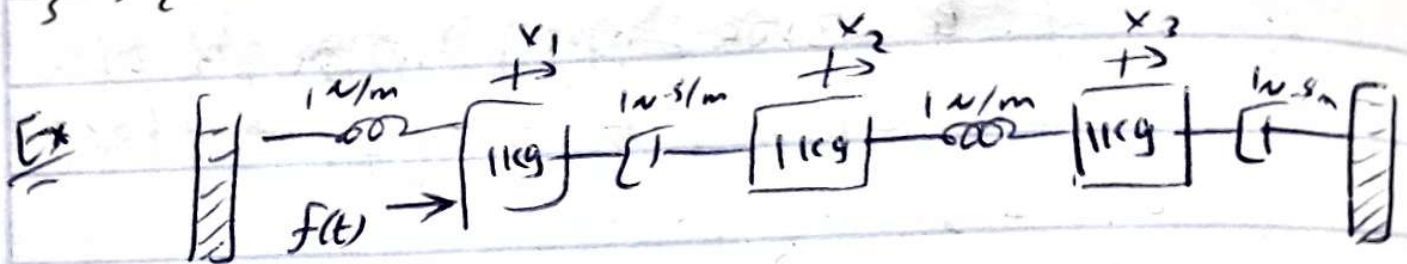
$$i_R' = \begin{bmatrix} 1 & 4 \end{bmatrix} \begin{bmatrix} i_L' \\ V_c' \end{bmatrix}$$

#

× Mechanical system :-

× اذا تم كسر معادلات الحركة بشيخه كذا

$x_1 = y_1 \Rightarrow x_2 = \dot{y}_1$  ← 4 state variables  
 $x_3 = y_2 \Rightarrow x_4 = \dot{y}_2$



Find S.S where  $x_3(t)$  is the output ?!

لنحسب في  $s$  (في كسور) time

$$F(s) = (s^2 + s + 1) X_1(s) - s X_2(s)$$

$$0 = -s X_1(s) + (s^2 + s + 1) X_2(s) - X_3(s)$$

$$0 = -X_2(s) + (s^2 + s + 1) X_3(s)$$

time

$$f(t) = x_1'' + x_1' + x_1 - x_2'$$

$$0 = -x_1' + x_2'' + x_2' + x_2 - x_3$$

$$0 = -x_2 + x_3'' + x_3' + x_3$$

3-state equation . 6 assumption

$$y_1 = x_1 \Rightarrow y_2 = \dot{x}_1$$

$$y_3 = x_2 \Rightarrow y_4 = \dot{x}_2$$

$$y_5 = x_3 \Rightarrow y_6 = \dot{x}_3$$

تعريف بالمصفوفة

$$\dot{y}_2 + y_2 + y_1 - y_4 = f(t) \Rightarrow \dot{y}_2 = -y_1 - y_2 + y_4 + f(t)$$

$$-y_2 + \dot{y}_4 + y_4 + y_3 - y_5 = 0 \Rightarrow \dot{y}_4 = y_2 - y_3 - y_4 + y_5$$

$$-y_3 + \dot{y}_6 + y_6 + y_5 = 0 \Rightarrow \dot{y}_6 = y_3 - y_5 - y_6$$

$$\boxed{y_1' = y_2}, \boxed{y_3' = y_4}, \boxed{y_5' = y_6}$$

$$\begin{bmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \\ y_5' \\ y_6' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} f(t)$$

$$x_3 = y_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix}$$



$$G(s) = \frac{5(s+1)}{s^3 + 12s^2 + 20s}$$

3<sup>rd</sup> order

So need 3-State variable.

$$G(s) = \frac{5s^{-2} + 5s^{-3}}{1 + 12s^{-1} + 20s^{-2}}$$

$\frac{X(s)}{X(s)}$  dummy variable.

المخرج ←  $1 + 12s^{-1} + 20s^{-2}$  كإزاحة في الزمن  
المفتوح 1

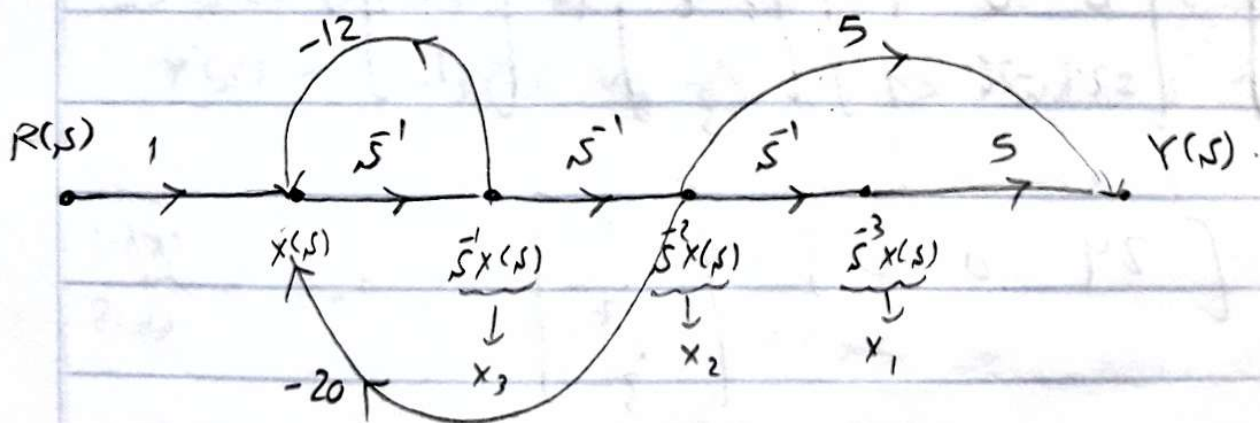
$$\Rightarrow Y(s) = (5s^{-2} + 5s^{-3}) X(s)$$

$$R(s) = (1 + 12s^{-1} + 20s^{-2}) X(s)$$

$$\Rightarrow Y(s) = 5s^{-2} X(s) + 5s^{-3} X(s)$$

$$R(s) = X(s) + 12s^{-1} X(s) + 20s^{-2} X(s)$$

$$\Rightarrow X(s) = R(s) - 12s^{-1} X(s) - 20s^{-2} X(s)$$



$$X_1 = s^{-3} X(s) \Rightarrow \dot{X}_2 = X_1$$

$$X_2 = s^{-2} X(s) \Rightarrow \dot{X}_3 = X_2$$

$$X_3 = s^{-1} X(s) \Rightarrow \dot{X}_3 = X(s) = R(s) - 12X_3 - 20X_2$$

$$Y = 5X_2 + 5X_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -20 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(t)$$

$$y = [5 \ 5 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

General form

$$\underline{\underline{Ex}} \quad G(s) = \frac{24}{s^3 + 9s^2 + 26s + 24}$$

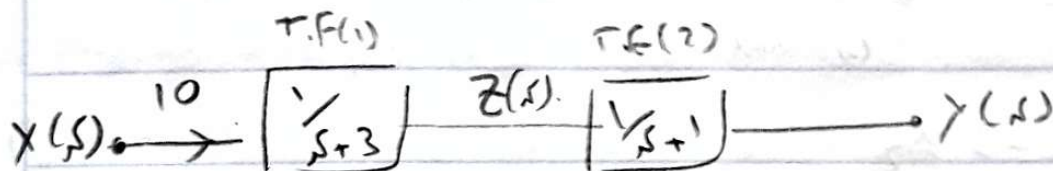
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(t)$$

$$y = [24 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \#$$

## 2) Cascade Method :

Ex:  $H(s) = \frac{10}{s^2 + 4s + 3}$

$$H(s) = \frac{10}{(s+3)(s+1)} = \frac{10}{s+3} \cdot \frac{1}{s+1} = \frac{Y(s)}{X(s)}$$



Now repeat the Direct Decomposition Steps.

$$\frac{Z(s)}{X(s)} = \frac{10}{s+3} = \frac{10s^{-1}}{1+3s^{-1}} \cdot \left( \frac{U_1}{V_1} \right)$$

$$\Rightarrow Z(s) = 10s^{-1} U_1 \quad \text{--- (1)}$$

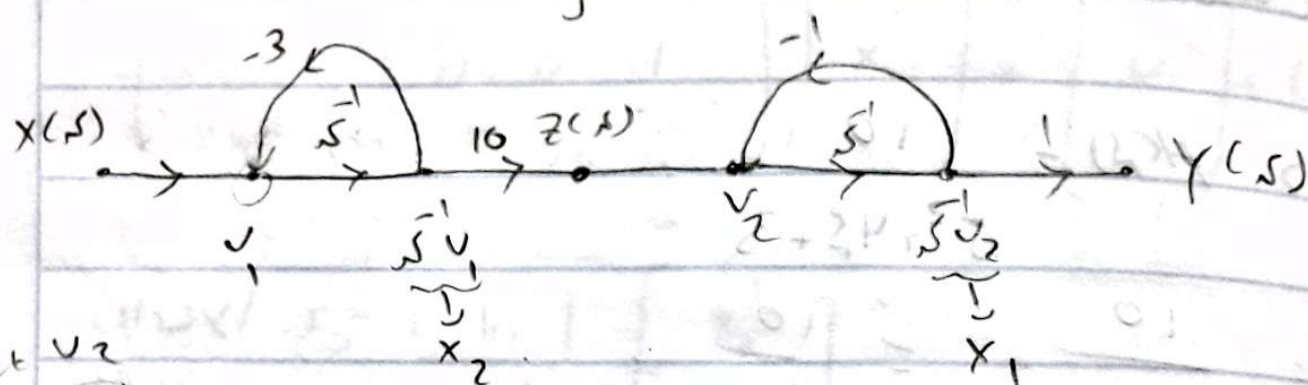
$$Y(s) = (1+3s^{-1}) U_1 \Rightarrow U_1 = Y(s) - 3s^{-1} U_1 \quad \text{--- (2)}$$

$$\frac{Y(s)}{Z(s)} = \frac{1}{s+1} = \frac{s^{-1}}{1+s^{-1}} \cdot \frac{U_2}{V_2}$$

$$Y(s) = s^{-1} U_2 \quad \text{--- (3)}$$

$$Z(s) = (1+3s^{-1}) U_2 \Rightarrow U_2 = Z(s) - 3s^{-1} U_2 \quad \text{--- (4)}$$

\* we need 2-integrator



at  $u_2$

$$\dot{X}_1 = 10X_2 - X_1$$

at  $u_1$

$$\dot{X}_2 = Y(s) - 3X_2$$

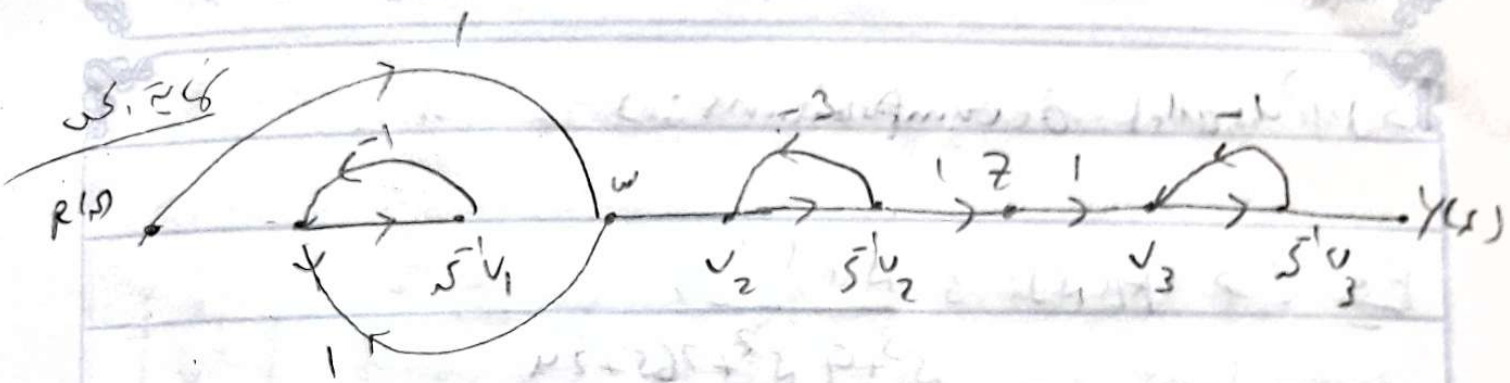
$$Y = X_1$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -1 & 10 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} X(t)$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

For  $x_2 \rightarrow$   $H(s) = \frac{s+1}{(s+3)(s+1)}$

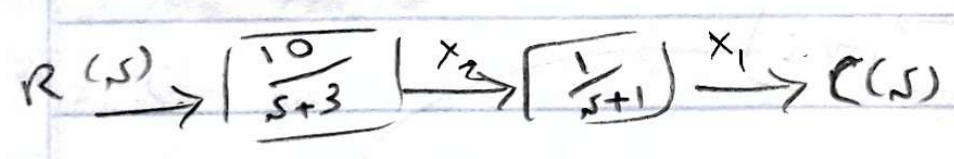
This T.F can't convert it to s.s. By using cascade because can't represent the state eqn as linearly independent



9

$$H(s) = \frac{C(s)}{R(s)} = \frac{10}{s^2 + 4s + 3}$$

① By cascade.  $\frac{10}{s^2 + 4s + 3} = \frac{10}{(s+3)(s+1)}$



$$\frac{x_2}{R} = \frac{10}{s+3} \Rightarrow \dot{x}_2 = 10r - 3x_2 \quad \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

$$\frac{x_1}{x_2} = \frac{1}{s+1} \Rightarrow \dot{x}_1 = x_2 - x_1 \quad C = [1 \ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$C = x_1$

تأكد من (T.F)

② By direct.

$$C(s^2 + 4s + 3) = 10R \Rightarrow C'' + 4C' + 3C = 10r$$

$$\therefore (x_1 = C) \Rightarrow \begin{bmatrix} x_2 = \dot{x}_1 \end{bmatrix} = C' \Rightarrow \dot{x}_2 + 4x_2 + 3x_1 = 10r$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} \Rightarrow \dot{x}_2 = -3x_1 - 4x_2 + 10r$$

$$C = [1 \ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

تأكد من (T.F)

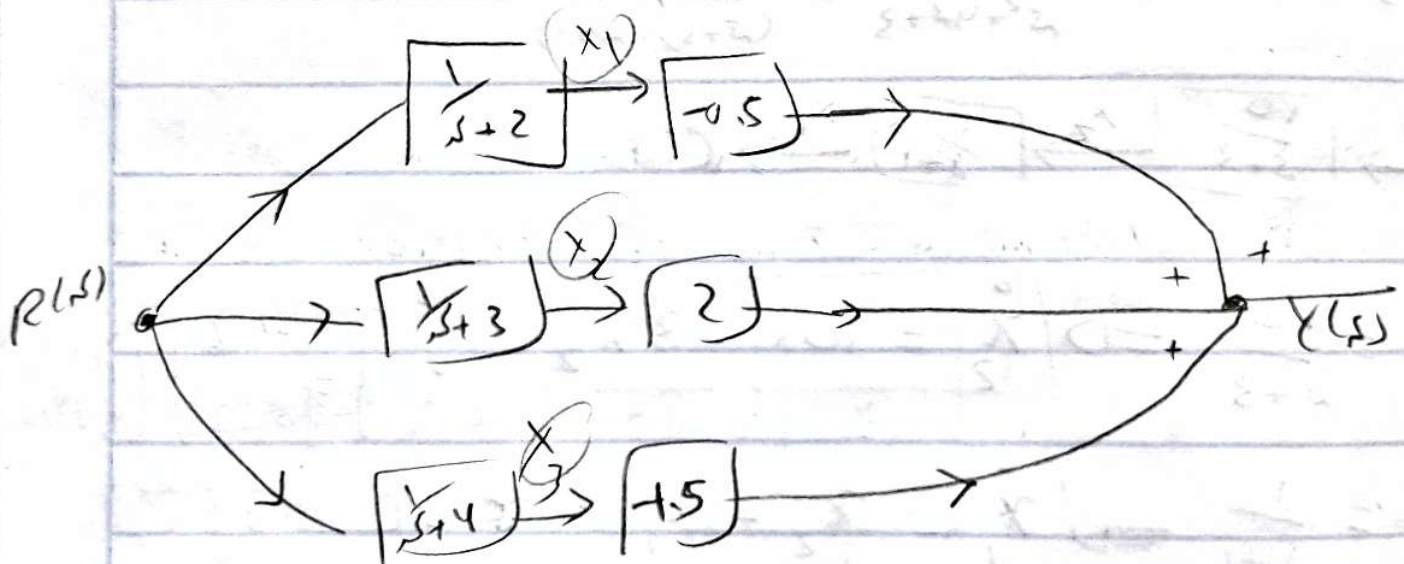
### B) Parallel decompositions: .)

Ex:  $\Rightarrow H(s) = \frac{s+1}{s^3+9s^2+26s+24}$

by Partial fraction

$$H(s) = \frac{-0.5}{s+2} \oplus \frac{2}{s+3} \oplus \frac{-1.5}{s+4}$$

This sign must be positive to make the sum equal



State variable.

$$\frac{x_1}{R(s)} = \frac{1}{s+2} \Rightarrow \dot{x}_1 = -2x_1 + r(t)$$

$$\frac{x_2}{R(s)} = \frac{1}{s+3} \Rightarrow \dot{x}_2 = -3x_2 + r(t)$$

$$\frac{x_3}{R(s)} = \frac{1}{s+4} \Rightarrow \dot{x}_3 = -4x_3 + r(t)$$

$$y = -0.5x_1 + 2x_2 - 1.5x_3$$

S.S Representation

always one

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$y = [-0.5 \quad +2 \quad -1.5] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Coefficient for "Partial Fraction"

- So in this representation we can know if the system is stable or not from the diagonal matrix A.

General form is

$$G(s) = \frac{k_1}{(s+a_0)} + \frac{k_2}{(s+a_1)} + \dots + \frac{k_n}{(s+a_{n-1})}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} -a_0 & 0 & 0 & \dots & 0 \\ 0 & -a_1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [k_1 \quad k_2 \quad \dots \quad k_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

### 3.6) Converting From S.S to T.F.

$$\begin{array}{ccc}
 \dot{X} = A X + B U & \text{convert} & Y(s) \\
 Y = C X + D U & \Rightarrow & U(s) \\
 \downarrow & \text{to} & \downarrow \\
 \text{time Domain} & & \text{freq Domain}
 \end{array}$$

- Assume all initial conditions are equal zero.

$$\frac{Y(s)}{U(s)} = C (sI - A)^{-1} B + D$$

identity matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Ex: Convert the state and output equation to transfer function.

$$\dot{X} = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix} X + \begin{bmatrix} 2 \\ 0 \end{bmatrix} U(t)$$

$$Y = [1.5 \quad 0.625] X$$

$$\frac{Y(s)}{U(s)} = C (sI - A)^{-1} B + D$$

by Complex:  $\frac{Y(s)}{U(s)} = \frac{3s+5}{s^2+4s+6}$

Without complex:

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} s+4 & 1.5 \\ -4 & s \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\det([sI - A])}{|sI - A|} = \frac{(\text{Cofactor } [sI - A])^T}{|sI - A|}$$

$$|sI - A| = (s+4)s - (-4)(1.5) = s^2 + 4s + 6$$

$$\text{Cofactor } A_{11} = s, A_{12} = 4, A_{21} = -1.5, A_{22} = s+4$$

$$\begin{bmatrix} s & 4 \\ -1.5 & s+4 \end{bmatrix} \xrightarrow{T} \begin{bmatrix} s & -1.5 \\ 4 & s+4 \end{bmatrix}$$

$$\Rightarrow [sI - A]^{-1} = \frac{\begin{bmatrix} s & -1.5 \\ 4 & s+4 \end{bmatrix}}{s^2 + 4s + 6} = \begin{bmatrix} \frac{s}{s^2 + 4s + 6} & \frac{-1.5}{s^2 + 4s + 6} \\ \frac{4}{s^2 + 4s + 6} & \frac{s+4}{s^2 + 4s + 6} \end{bmatrix}$$

$$C \times [sI - A]^{-1} = \begin{bmatrix} \frac{1.5s}{s^2 + 4s + 6} + \frac{2.5}{s^2 + 4s + 6} & \frac{-2.25}{s^2 + 4s + 6} + \frac{0.625(s+4)}{s^2 + 4s + 6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1.5s + 2.5}{s^2 + 4s + 6} & \frac{0.625s - 5}{s^2 + 4s + 6} \end{bmatrix}_{1 \times 2} \rightarrow E$$

$$E \times B = \frac{3s + 5}{s^2 + 4s + 6}$$

#

linearization :

$$Ex: \dot{x}_1 = x_1(t) + x_2^2(t)$$

$$\dot{x}_2 = x_1(t) + 1(t)$$

$$\dot{x}_3 = \cos x_2 x_1(t)$$

NON linear

$$Ex: \ddot{y}(t) + 0.1 \dot{y}(t) + \sin(y(t)) = u(t)$$

$$u_0 = \frac{1}{\sqrt{2}}, y_0 = \frac{\pi}{4}, \dot{y}_0 = 0 \text{ for linear s.s.}$$

قول المعادلة لـ s.s. / مبرج بشتية تحت حنا معادلية

$$x_1 = y$$

$$x_2 = \dot{y} = \dot{x}_1$$

$$\Rightarrow \dot{x}_1 = x_2 = f_1$$

$$\dot{x}_2 = \ddot{y} = -0.1 x_2 + \sin x_1 + u(t) = f_2$$

$$\frac{df_1}{dx_1} = 0, \frac{df_1}{dx_2} = 1, \frac{df_2}{dx_1} = \cos x_1, \frac{df_2}{dx_2} = -0.1$$

$$u_0 = \frac{1}{\sqrt{2}}, y_0 = \frac{\pi}{4}, \dot{y}_0 = 0$$

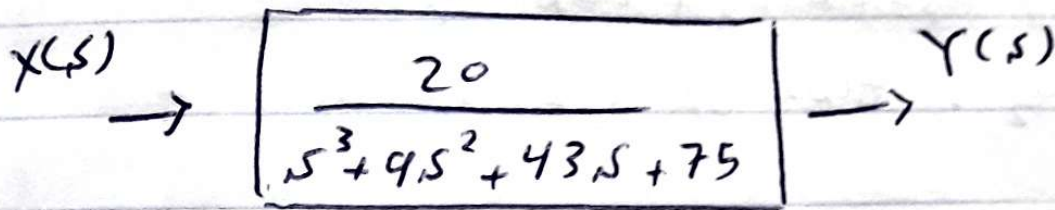
$$A^k = \begin{bmatrix} 0 & 1 \\ -\frac{1}{\sqrt{2}} & -0.1 \end{bmatrix}, B^k = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{\sqrt{2}} & -0.1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

#

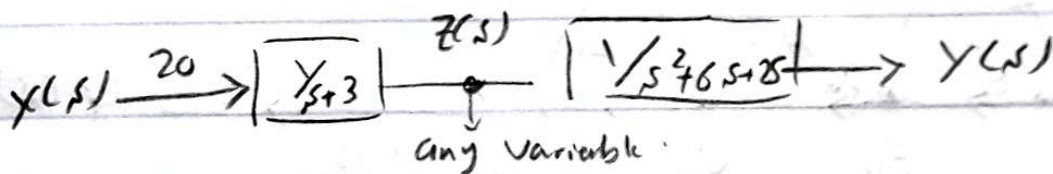
\* Additional Exo

1) Find S.S by Cascade decomposition.



$$\frac{Y(s)}{X(s)} = \frac{20}{(s+3)(s^2+6s+25)}$$

Real poles  $\leftarrow$   $\rightarrow$  Complex poles



Two subsystem are connected in series or cascade for

$$\frac{Z(s)}{X(s)} = \frac{20}{s+3}, \quad \frac{Y(s)}{Z(s)} = \frac{1}{s^2+6s+25}$$

Now, by using direct method :

$$\frac{Z(s)}{X(s)} = \frac{20s^{-1}}{1+3s^{-1}} \cdot \frac{U_1}{U_1} \Rightarrow \boxed{Z(s) = 20s^{-1}U_1} \quad \text{--- (x)}$$

$$X(s) = U_1 + 3s^{-1}U_1$$

$$\Rightarrow \boxed{U_1 = X(s) - 3s^{-1}U_1} \quad \text{--- (E)}$$

$$\frac{Y(s)}{Z(s)} = \frac{s^{-2}}{1 + 6s + 25s^2} \cdot \frac{U_2}{\sqrt{2}}$$

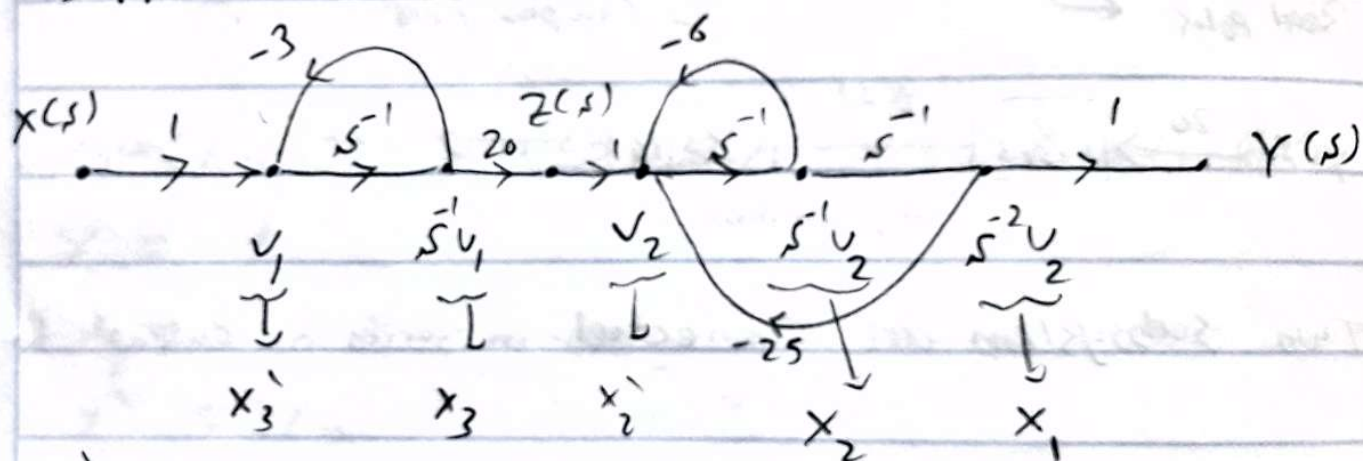
$$\Rightarrow |Y(s) = s^{-2} U_2| \quad \text{---} \quad \textcircled{x}$$

$$Z(s) = U_2 + 6s^{-1} U_2 + 25s^{-2} U_2$$

$$\Rightarrow |U_2 = Z(s) - 6s^{-1} U_2 - 25s^{-2} U_2| \quad \text{---} \quad \textcircled{x}$$

Now, we need to sketch state diagrams and

Suppose the state variables.



$$\dot{X}_1 = X_2$$

$$\dot{X}_2 = -25X_1 - 6X_2 + 20X_3$$

$$\dot{X}_3 = -3X_3 + X(t)$$

$$Y(t) = X_1$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -25 & -6 & 20 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} X(t)$$

$$Y = [1 \ 0 \ 0] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

2) Find S.S and output equation by Parallel:

$$G(s) = \frac{2s+3}{s^4 + 7s^3 + 19s^2 + 23s + 10}$$

~~Complex poles~~ ~~General form~~ ~~المركبة~~ ~~الشكل العام~~

By Partial fraction  $\Rightarrow G(s) = \frac{K_1}{s+1} + \frac{K_2}{s+2} + \frac{K_3}{s+2-j} + \frac{K_3^*}{s+2+j}$

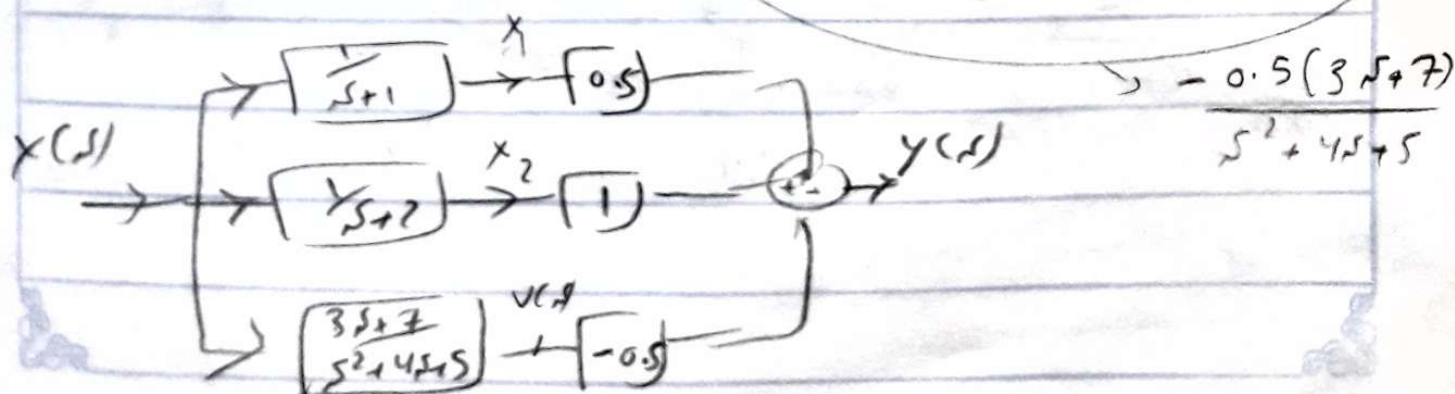
$$K_1 = G(s)(s+1) \Big|_{s=-1} = \frac{1}{2}$$

$$K_2 = G(s)(s+2) \Big|_{s=-2} = 1$$

$$K_3 = G(s)(s+2-j) \Big|_{s=-2+j} = -0.75 + j0.25 = 0.79 \angle 161.5^\circ$$

$$K_3^* = -0.75 - j0.25 = 0.79 \angle -161.5^\circ$$

$$G(s) = \frac{0.5}{s+1} + \frac{1}{s+2} + \frac{-0.75+j0.25}{(s+2-j)} + \frac{-0.75-j0.25}{(s+2+j)}$$



$$\frac{U(s)}{X(s)} = \frac{3s+7}{s^2+4s+5} = \frac{-3s^{-1}+7s^{-2}}{1+4s^{-1}+5s^{-2}} \cdot \frac{9}{9}$$

$$\Rightarrow U(s) = 3s^{-1}g + 7s^{-2}g \quad \text{--- } \textcircled{E}$$

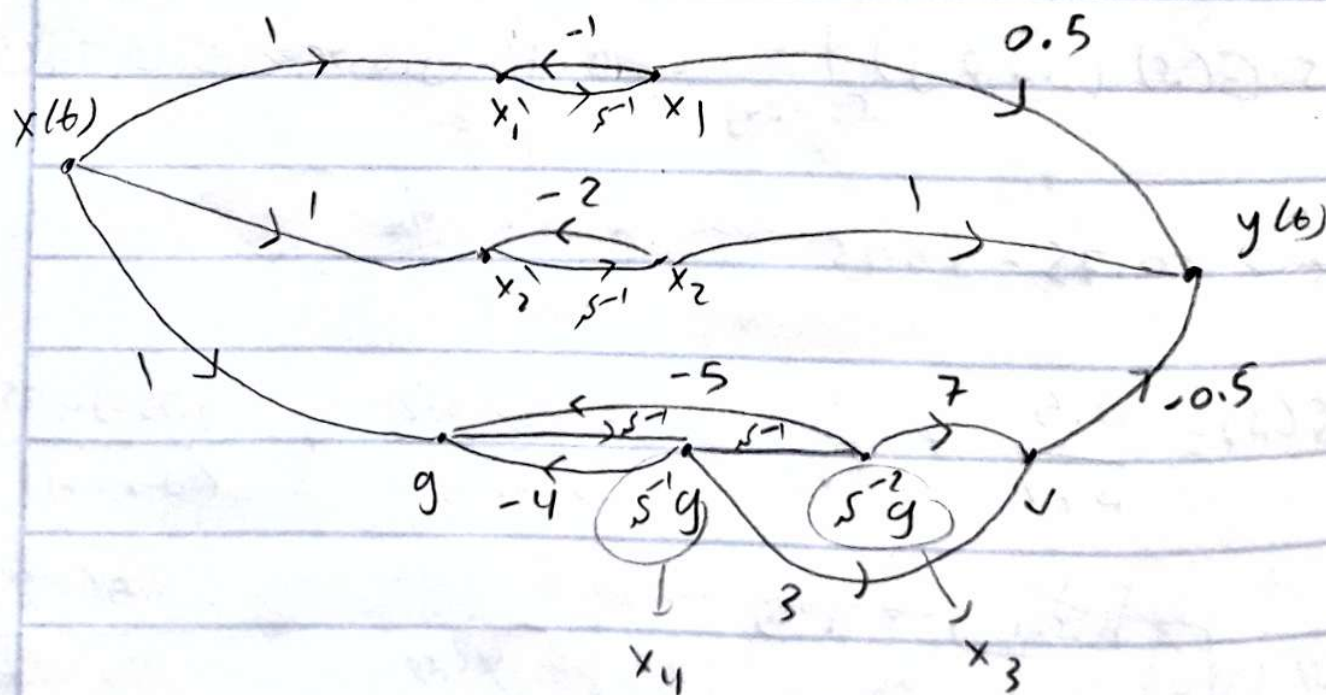
$$g = X(s) - 4s^{-1}g - 5s^{-2}g \quad \text{--- } \textcircled{E}$$

$$\frac{x_1}{X(s)} = \frac{1}{s+1} \Rightarrow s x_1 + x_1 = X(s)$$

$$x_1' = X(t) - x_1$$

$$\frac{x_2}{X(s)} = \frac{1}{s+2} \Rightarrow x_2' = X(t) - 2x_2$$

Now, sketch state diagram.



$$\dot{x}_1 = x(t) - x_1$$

$$\dot{x}_2 = x(t) - 2x_1$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = x(t) - 4x_4 - 5x_3$$

state equation.

$$y = 0.5x_1 + x_2 - 0.5(7x_3 + 3x_4) \rightarrow \text{output equation}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -5 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} x(t)$$

$$y = [0.5 \quad 1 \quad -3.5 \quad -1.5] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\frac{V(s)}{X(s)} = \frac{3s+7}{s^2+4s+5} \cdot \frac{y}{y}$$

$$\Rightarrow x(t) = y'' + 4y' + 5y$$

$$x_3 = y, \quad x_4 = \dot{x}_3 \Rightarrow \ddot{x}_4 + 4\dot{x}_4 + 5x_4 = x(t)$$

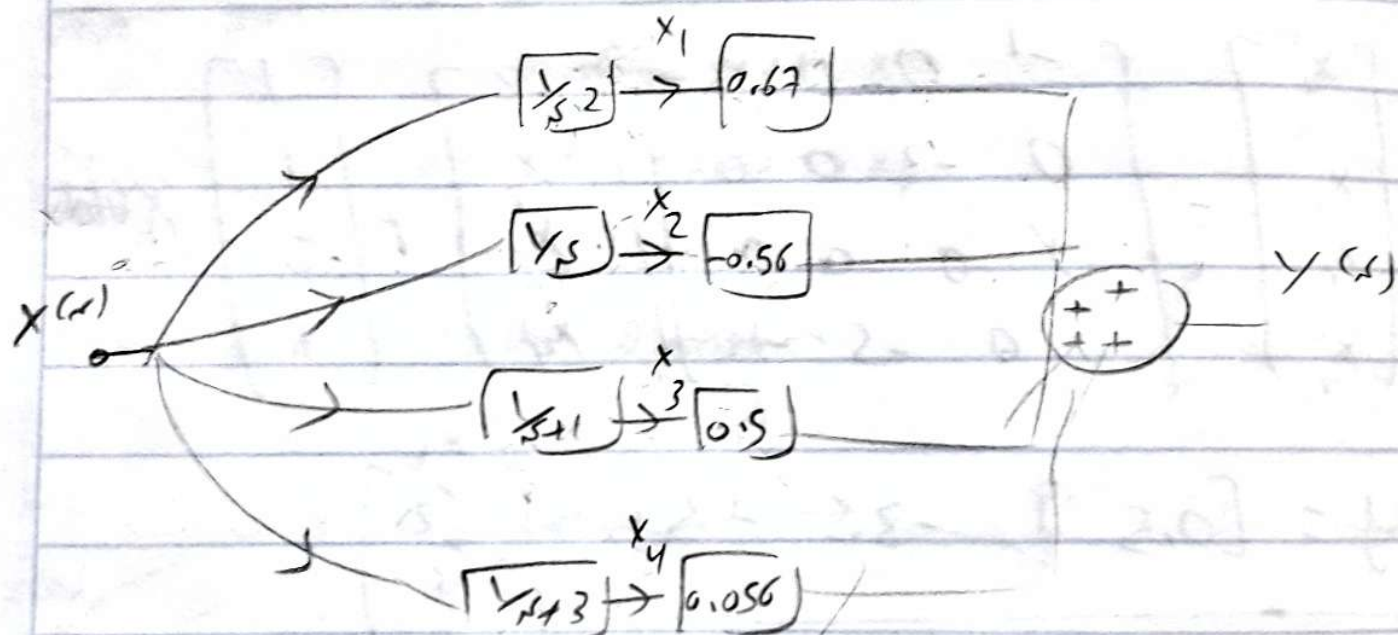
$$\Rightarrow \dot{x}_4 = -5x_3 - 4x_4 + x(t)$$

$$V(s)(3s+7)y \Rightarrow v(t) = 3y' + 7y$$

$$v(t) = 3x_4 + 7x_3 \quad \text{output } x = 0.5$$

3) Draw state diagram for G(s) By means of Partial fraction. By using minimum number of integrals the write state and output equations.  $G(s) = \frac{(s+2)}{s^2(s+1)(s+3)}$

By partial fraction  $G(s) = \frac{0.67}{s^2} + \frac{-0.56}{s} + \frac{0.5}{s+1} + \frac{0.056}{s+3}$



$$\frac{X_1}{X(s)} = \frac{1}{s^2} = \frac{s^{-2}}{1} \cdot \frac{V}{V} \Rightarrow X_1 = s^{-2} V$$

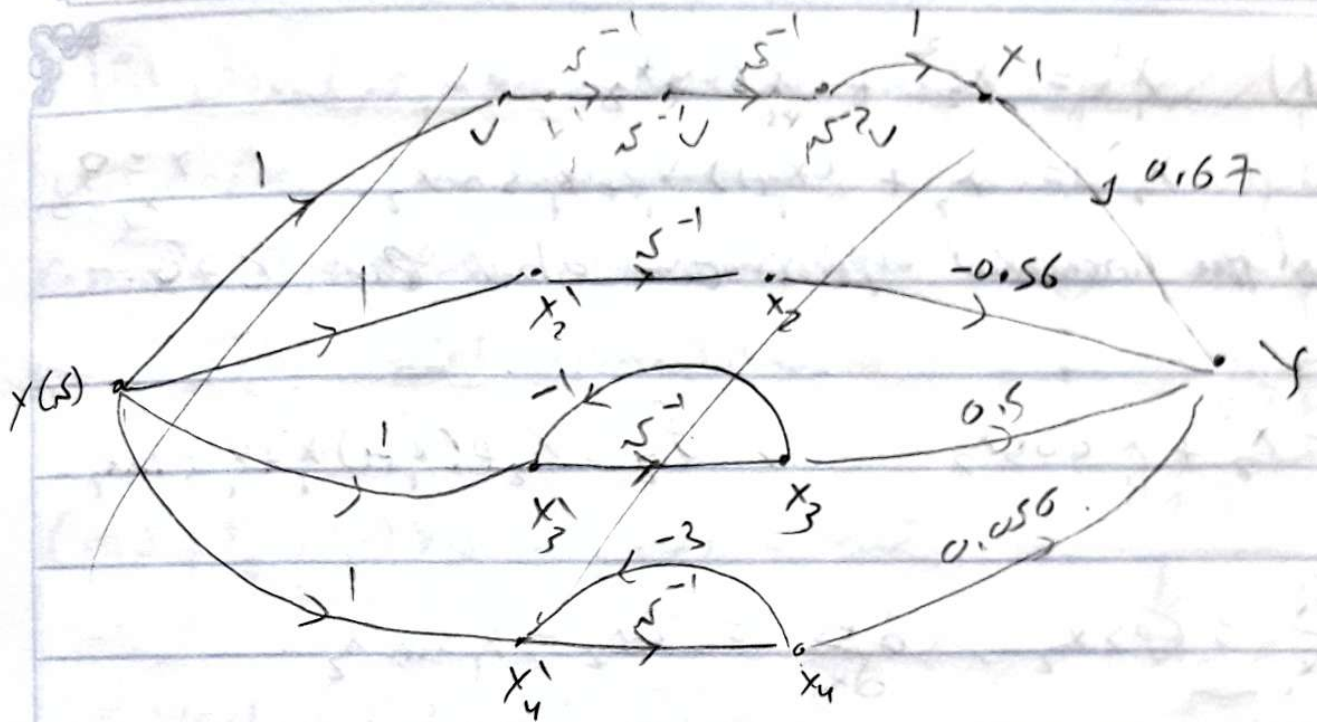
$$X(s) = V$$

$$\frac{X_2}{X(s)} = \frac{1}{s} \Rightarrow \dot{X}_2 = X(t)$$

$$X_1'' = Y(t)$$

$$\frac{X_3}{X(s)} = \frac{1}{s+1} \Rightarrow \dot{X}_3 = X(t) - X_3$$

$$\frac{X_4}{X(s)} = \frac{1}{s+3} \Rightarrow \dot{X}_4 = X(t) - 3X_4$$

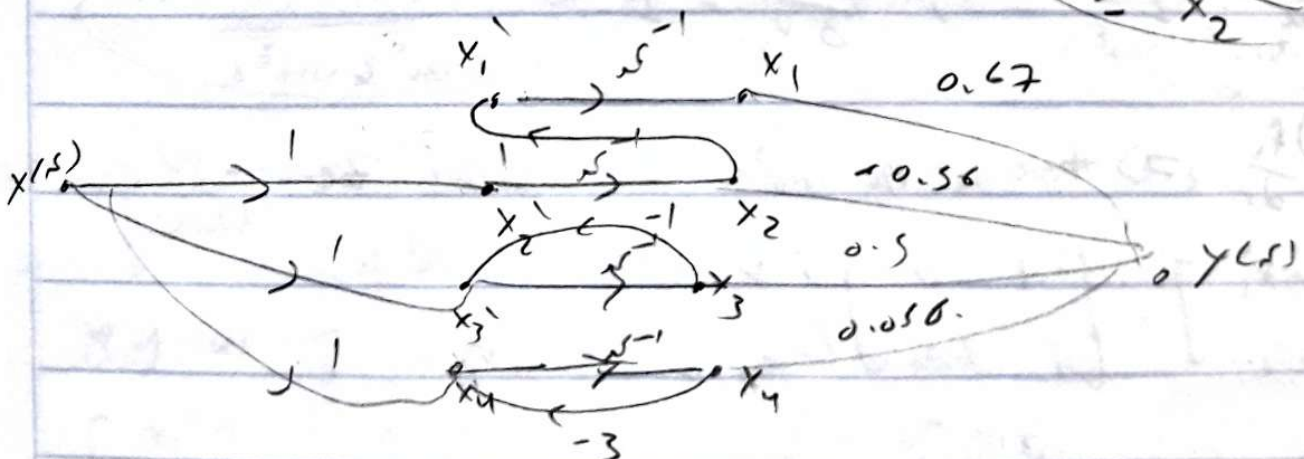


$$\frac{x_2}{x(s)} = \frac{1}{s} \Rightarrow x_2' = x(t) \quad / \quad x_2 = \frac{1}{s} x(s)$$

$$\frac{x_3}{x(s)} = \frac{1}{s+1} \Rightarrow x_3' = x(t) - x_3$$

$$\frac{x_4}{x(s)} = \frac{1}{s+3} \Rightarrow x_4' = x(t) - 3x_4$$

$$\frac{x_1}{x(s)} = \frac{1}{s^2} \Rightarrow x_1 = \frac{1}{s^2} x(s) \Rightarrow x_1' = \frac{1}{s} x(s) = x_2$$



$$y = 0.67x_1 - 0.56x_2 + 0.5x_3 + 0.056x_4$$

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} x(t)$$

$$y = [0.67 \quad -0.56 \quad 0.5 \quad 0.056] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$[4] \quad \dot{x}_1 = x_2^2 + x_1 \cos x_2$$

$$\dot{x}_2 = x_2 + (x_1 + 1)x_1 + x_1 \sin x_2 \quad \begin{matrix} x_1=0 \\ x_2=0 \end{matrix}$$

Find the linearized approximation about point  $x_0 = 0$ .

$$f_1 = x_2^2 + x_1 \cos x_2, \quad f_2 = x_2 + (x_1 + 1)x_1 + x_1 \sin x_2$$

$$\frac{df_1}{dx_1} = \cos x_2, \quad \frac{df_1}{dx_2} = 2x_2 - x_1 \sin x_2$$

$$\frac{df_2}{dx_1} = 2x_1 + 1 + \sin x_2, \quad \frac{df_2}{dx_2} = 1 + x_1 \cos x_2$$

المشتق

$$\frac{df_1}{dx_1} = 1, \quad \frac{df_1}{dx_2} = 0$$

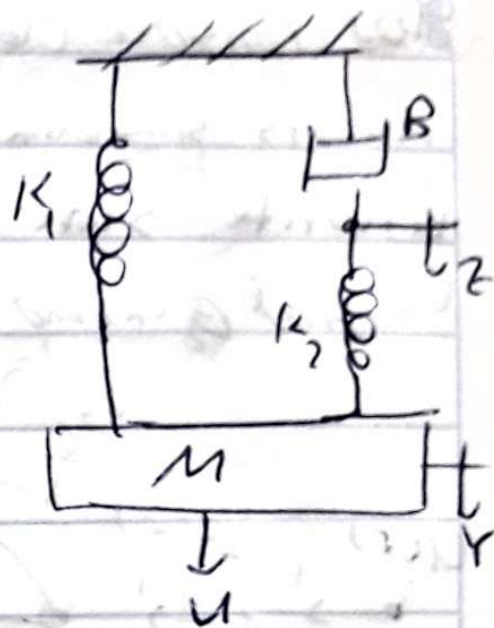
$$\frac{df_2}{dx_1} = 1, \quad \frac{df_2}{dx_2} = 1$$

$\frac{df_1}{dx_1} \Rightarrow$  there is no input so check zero.

$$\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

5)  $m=0.1, k_1=6, k_2=4, B=0.4$

$u$  is input,  $y$  and  $z$  displacement.  
consider  $y$  output, obtain T.F.  
and the state model representation.



$$(ms^2 + k_2 + k_1)y(s) - k_2 z(s) = u(s)$$

$$-k_2 y(s) + (Bs + k_2)z(s) = 0$$

$$(0.1s^2 + 10)y(s) - 4z(s) = u(s)$$

$$-4y(s) + (0.4s + 4)z(s) = 0$$

$$Y(s) = \frac{10(s+10)}{s^3 + 10s^2 + 100s + 600} u(s)$$

$$Z(s) = \frac{100}{s^3 + 10s^2 + 100s + 600} u(s)$$

$$\Rightarrow \frac{Y(s)}{u(s)} = \frac{10s + 100}{s^3 + 10s^2 + 100s + 600}$$

Convert s.s.

By using General direct method.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -600 & -100 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [100 \quad 10 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q6] Consider closed loop control system figure a<sub>3</sub> and its simulation diagram in figure b .

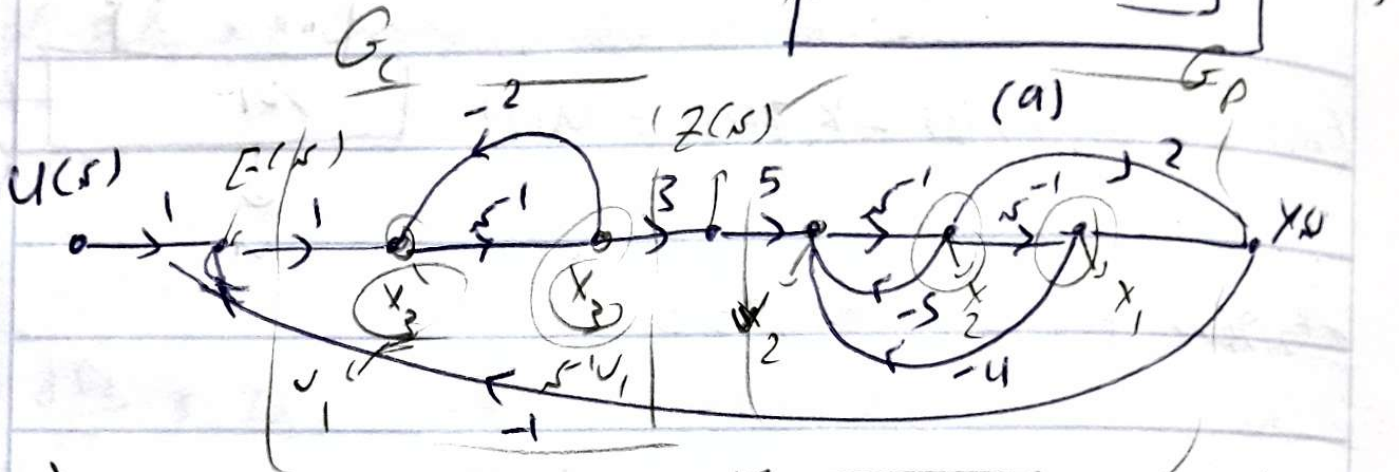
Q) Write state equation,

(b) Find  $G_c(s)$  and  $G_p(s)$ .

```

graph LR
    U["U(s)"] --> Sum(( ))
    Sum --> Gc["G_c(s)"]
    Gc --> Gp["G_p(s)"]
    Gp --> Y["Y(s)"]
    Y --> Sum
    style Sum fill:none,stroke:none

```



$x_1 = x_2$  — (د) مائیں کے ساتھ رکھ کر

$$x_2' = 15x_3 - 5x_2 - 4x_1 \quad \text{--- (2)}$$

$$x_3' = u(t) - 2x_3 - y(t).$$

$$y(t) = 2x_2 + x_1 \quad \text{--- (5)}$$

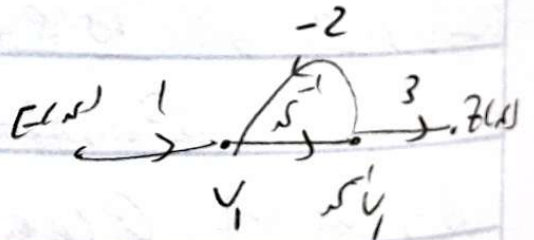
$$x_3' = u(t) - 2x_3 - 2x_2 - x_1 \quad \text{--- (8)}$$

$$Z(\mu) = 3.5^{-1} u_1$$

$$V_1 = E(s) - 2s^{-1}V_1$$

$$\frac{Z_{(n)}'}{E_{(n)}'} \leq \frac{3.5' U_1}{U_1 + 2.5' U_1}$$

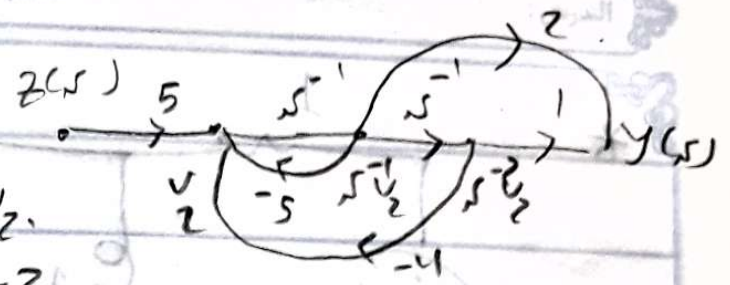
$$\therefore \frac{Q(s)}{E(s)} = \frac{3s^{-1}}{1+2s^{-1}} = \left( \frac{3}{s+2} \right)$$



$$Y(s) = 2s^{-1}U_2 + s^{-2}U_2$$

$$U_2 = 5Z(s) - 5s^{-1}U_2 - 4s^{-2}U_2$$

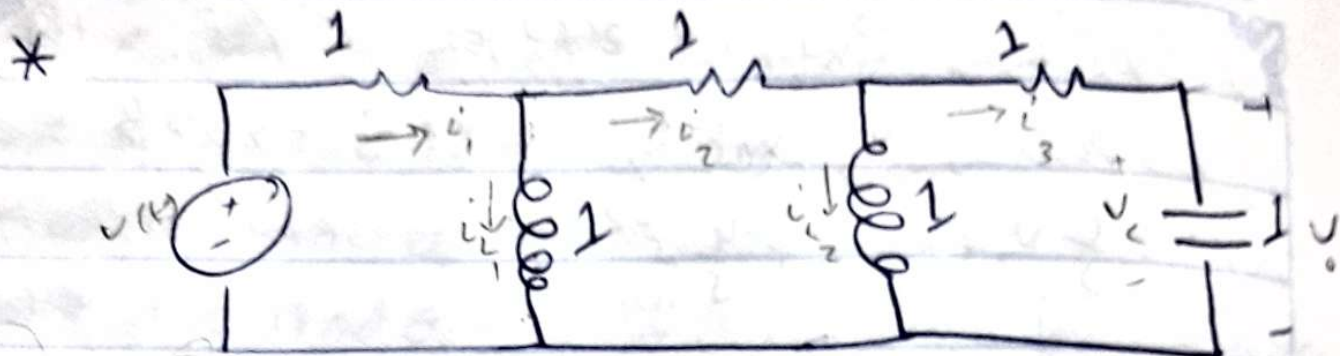
$$Z(s) = \frac{1}{5}U_2 + s^{-1}U_2 + \frac{4}{5}s^{-2}U_2$$



$$\therefore \frac{Y(s)}{Z(s)} = \frac{2s^{-1} + s^{-2}}{\frac{1}{5} + s^{-1} + \frac{4}{5}s^{-2}} = \frac{10s^{-1} + 5s^{-2}}{1 + 5s^{-1} + 4s^{-2}}$$

$$\frac{Y(s)}{Z(s)} = \frac{10s + 5}{s^2 + 5s + 4}$$

#



$$v_{L1} = L \frac{di_{L1}}{dt}$$

$$(i_1, i_{L1}, v_C)$$

$$v_{L2} = L \frac{di_{L2}}{dt}$$

$$i_C(t) = C \frac{dv_C}{dt}$$

$$\begin{bmatrix} \dot{i}_1 \\ \dot{i}_{L1} \\ \dot{i}_{L2} \\ \dot{v}_C \end{bmatrix} = \begin{bmatrix} -i_{L1} \\ i_{L1} - i_{L2} \\ i_{L2} - i_3 \\ i_3 \end{bmatrix} + \begin{bmatrix} v(t) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\dot{v}_{L1} = -i_2 + i_3 + v_C$$

$$\begin{aligned} i_2 &= i_{L1} - i_{L1} \\ i_3 &= v(t) - v_{L1} \end{aligned}$$

$$\Rightarrow i_2 = i_{L1} + v_{L1} - v(t)$$

$$i_3 = i_2 - i_{L2}$$

$$v_{L1} = 2i_2 - i_{L2} + v_C$$

$$= 2i_1 + 2v_{L1} - 2v(t) - i_{L2} + v_C$$

$$\Rightarrow v_{L1} = -2i_1 + i_{L2} - v_C + 2v(t)$$

$$\therefore \frac{di_1}{dt} = \frac{v_{L1}}{L_1} = v_{L1} = -2i_1 + i_{L2} - v_C + 2v(t) \quad \text{--- (1)}$$

$$\begin{aligned}
 v_{L_2} &= \dot{i}_3 + v_c \\
 &= \dot{i}_2 - \dot{i}_{L_2} + v_c \\
 &= \dot{i}_{L_1} + v_{L_1} - v(t) - \dot{i}_{L_2} + v_c.
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow v_{L_2} &= \dot{i}_{L_1} + (2\dot{i}_{L_1} + \dot{i}_{L_2} - v_c + 2v(t)) - v(t) - \dot{i}_{L_2} + v_c \\
 &= -\dot{i}_{L_2} + v(t).
 \end{aligned}$$

$$\therefore \frac{dv_{L_2}}{dt} = \frac{v_{L_2}}{L_2} = -\dot{i}_{L_2} + v(t). \quad \text{--- (2)}$$

$$\begin{aligned}
 \dot{i}_c(t) &= \dot{i}_3 = \dot{i}_2 - \dot{i}_{L_2} \\
 &= \dot{i}_{L_1} + v_{L_1} - v(t) - \dot{i}_{L_2} \\
 &= \dot{i}_{L_1} + (-2\dot{i}_{L_1} + \dot{i}_{L_2} - v_c + 2v(t)) - v(t) - \dot{i}_{L_2} \\
 &= -\dot{i}_{L_1} - v_c + v(t).
 \end{aligned}$$

$$\therefore \frac{dv_c}{dt} = \frac{\dot{i}_c(t)}{C} = \dot{i}_c(t) = -\dot{i}_{L_1} - v_c + v(t) \quad \text{--- (3)}$$

$$\begin{bmatrix} \frac{di_{L_1}}{dt} \\ \frac{di_{L_2}}{dt} \\ \frac{dv_c}{dt} \end{bmatrix} = \begin{bmatrix} -2 & 1 & -1 \\ -1 & 0 & 0 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} i_{L_1} \\ i_{L_2} \\ v_c \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} v(t).$$

#

$$\dot{L}_1 = \dot{L}_2 + \dot{L}_3 + v_c$$

$$\dot{L}_2 = \dot{L}_1 - \dot{L}_1, \quad \dot{L}_1 = v_c - \dot{L}_1$$

$$\Rightarrow \dot{L}_2 = v_c - \dot{L}_1 - \dot{L}_1$$

$$\begin{aligned} \dot{L}_3 &= \dot{L}_2 - \dot{L}_2 \\ &= v_c - \dot{L}_1 - \dot{L}_1 - \dot{L}_2 \end{aligned}$$

$$\dot{L}_1 = (v_c - \dot{L}_1 - \dot{L}_1) + (v_c - \dot{L}_1 - \dot{L}_1 - \dot{L}_2) + v_c$$

$$3\dot{L}_1 = -2\dot{L}_1 - \dot{L}_2 + v_c + 2v_c$$

$$\Rightarrow \dot{L}_1 = -\frac{2}{3}\dot{L}_1 - \frac{1}{3}\dot{L}_2 + \frac{1}{3}v_c + \frac{2}{3}v_c$$

$$\dot{L}_2 = \dot{L}_3 + v_c$$

$$= v_c - \dot{L}_1 - \dot{L}_1 - \dot{L}_2 + v_c$$

$$= -\frac{1}{3}\dot{L}_1 - \frac{2}{3}\dot{L}_2 + \frac{2}{3}v_c + \frac{1}{3}v_c$$

$$\dot{L}_3 = c \frac{dv_c}{dt} \Rightarrow v_c = \frac{\dot{L}_3}{c}$$

#

1129 Convert T.F to S.S :>

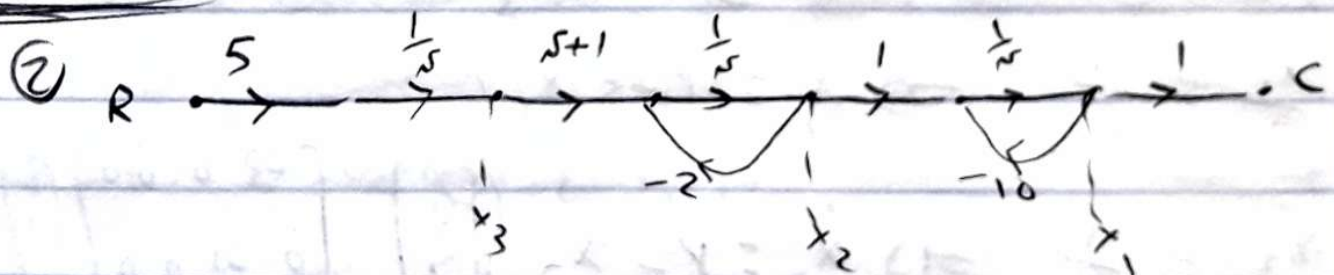
(I) By Parallel (II) By Cascade.

$$G(s) = \frac{5(s+1)}{s(s+2)(s+10)}$$

$$\textcircled{1} G(s) = \frac{-9/16}{s+10} + \frac{5/16}{s+2} + \frac{1/4}{s}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -10 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} -9/16 & 5/16 & 1/4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \#$$



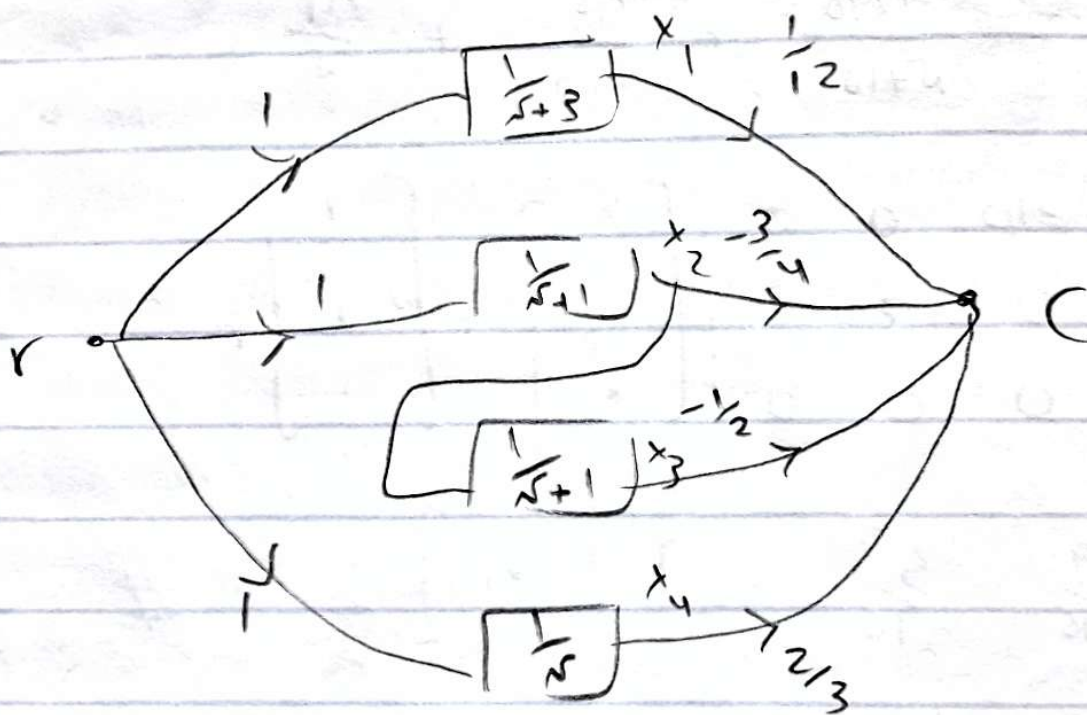
$$\begin{aligned} \dot{x}_1 &= x_2 - 10x_1 \\ \dot{x}_2 &= x_3 + x_2 - 2x_2 \\ \dot{x}_3 &= 5v \\ \Rightarrow \dot{x}_2 &= -2x_2 + x_3 + 5v \\ y &= x_1 \end{aligned}$$

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -10 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix} v \\ y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \# \end{cases}$$

$C(A-B)^{-1}B$  T.F is correct

Convert to S.S :  $\frac{s+2}{s(s+1)^2(s+3)}$

① Partial,  $G(s) = \frac{y_2}{s+3} - \frac{3y_4}{s+1} - \frac{y_2}{(s+1)^2} + \frac{y_3}{s}$



$$\frac{x_1}{R} = \frac{1}{s+3} \Rightarrow \dot{x}_1 = r - 3x_1$$

$$\frac{x_2}{R} = \frac{1}{s+1} \Rightarrow \dot{x}_2 = r - x_2$$

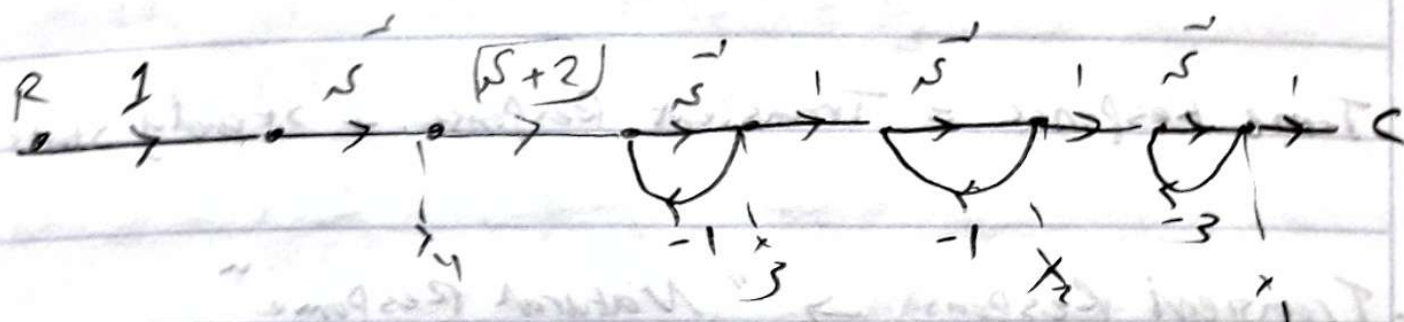
$$\frac{x_3}{R} = \frac{1}{s+1} \Rightarrow \dot{x}_3 = x_2 - x_3$$

$$\frac{x_4}{R} = \frac{1}{s} \Rightarrow \dot{x}_4 = r$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -3 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} \frac{1}{12} & -\frac{3}{4} & \frac{1}{2} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

2) Cascad.



$$\dot{x}_1 = x_2 - 3x_1$$

$$\dot{x}_2 = x_3 - x_2$$

$$\dot{x}_3 = \dot{x}_4 + 2x_4 - x_3 \Rightarrow \dot{x}_3 = r + 2x_4 - x_3$$

$$\dot{x}_4 = r$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} r$$