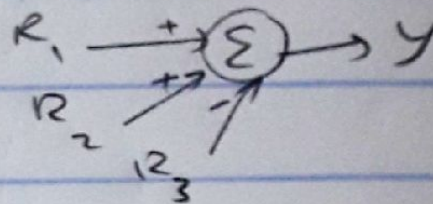


* Chapter 5 ::

* Summing Junction ::

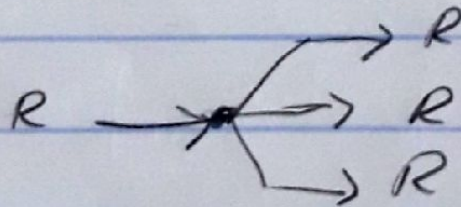
The output of summing junction is equal of All input signals.

$$Y = R_1 + R_2 + (-R_3)$$

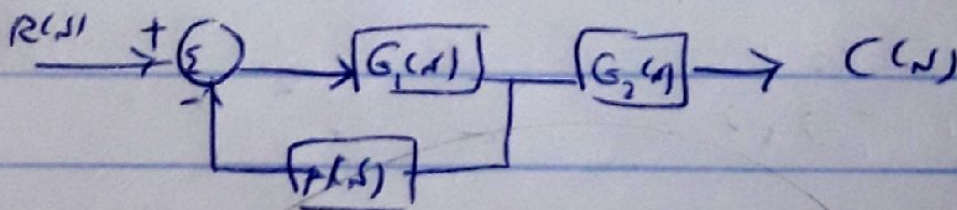


* Pick off Points ::

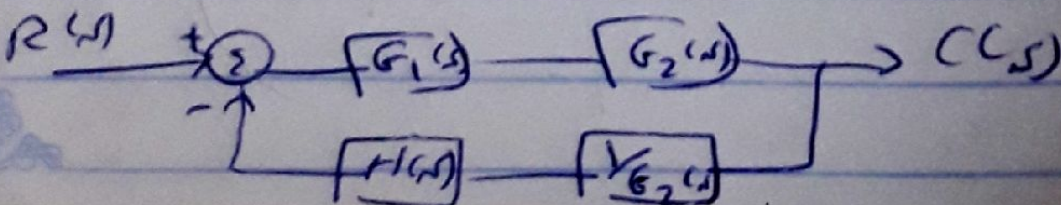
This points is used to distributes any input signal to different several output points.

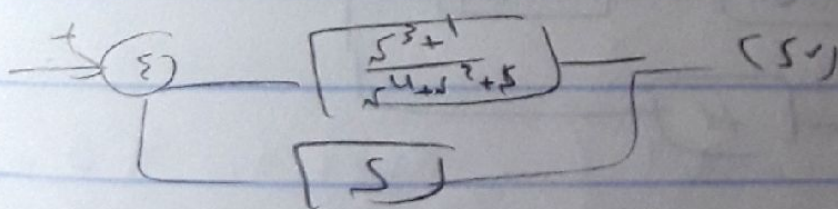
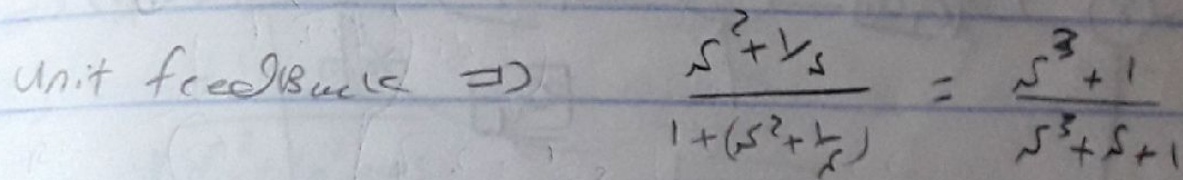


(*)



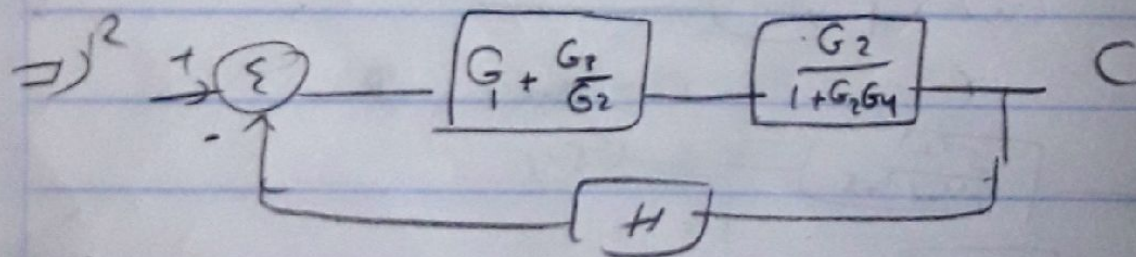
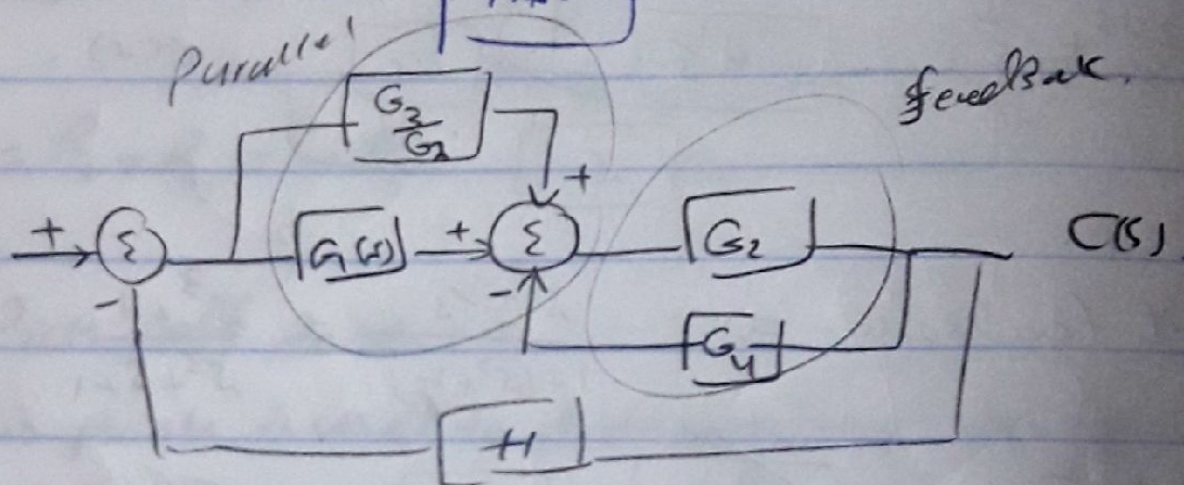
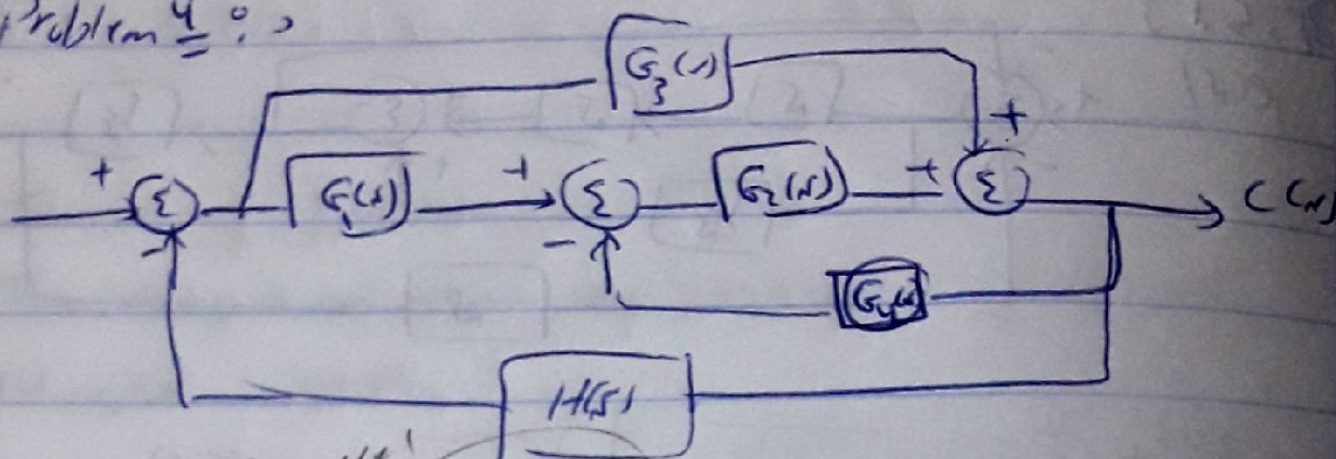
≡



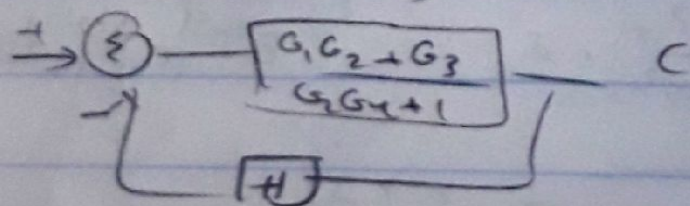


$$\begin{array}{c|c} P(s) & C(s) \\ \hline s^3 + 1 & \\ \hline 2s^4 + s^2 + 2s & \end{array}$$

Problem 4: 0.3



$$\left(\frac{G_1 + \frac{G_3}{G_2}}{G_2} \right) \left(\frac{G_2}{1 + G_2 G_4} \right) = \frac{G_1 G_2 + G_3}{G_2 G_4 + 1}$$



$$\frac{C(s)}{R(s)} = \frac{\frac{G_1 G_2 + G_3}{G_2 G_4 + 1}}{1 + \frac{G_1 G_2 + G_3}{G_2 G_4 + 1} H}$$

* Ex 5.2: >

If you have $G(s) = \frac{16}{s(s+a)}$ and Askied to

Design a closed loop system with $OS\% = 5\%$.

$$T(s) = \frac{16}{s^2 + as + 16} = \frac{K_{dc} \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\Rightarrow \omega_n^2 = 16 \Rightarrow \omega_n = \boxed{4}$$

$$\zeta = \frac{-\ln 0.05}{\sqrt{\pi^2 + \ln^2 0.05}} = \boxed{0.69}$$

$$a = 2\zeta\omega_n \Rightarrow a = 5.52$$

$$K_1 > \frac{a^2}{4} \Rightarrow \text{underdamped.}$$

Complex Poles

#

$$s_{1,2} = -\frac{a}{2} \pm \sqrt{\frac{a^2}{4} - K_1}$$

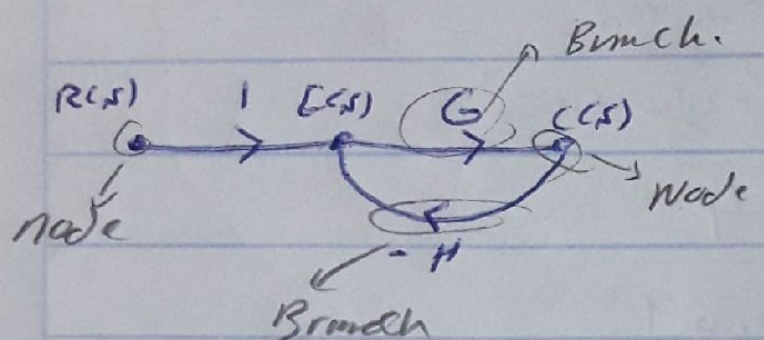
5

5.4 signal Flow graphs (SFG).

* SFG are an alternative to block diagram Represent.

* SFG consist only of:-

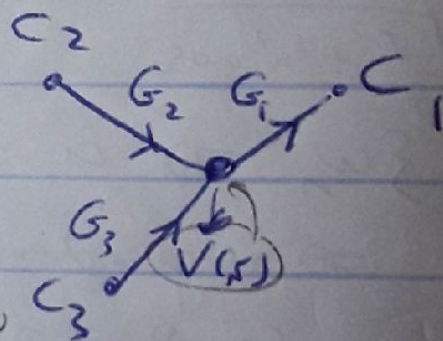
- 1) Branches $\rightarrow$ which used to represent system.
- 2) Nodes $\rightarrow$ which used to represent signal.



Example:-

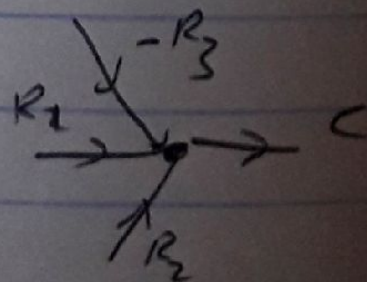
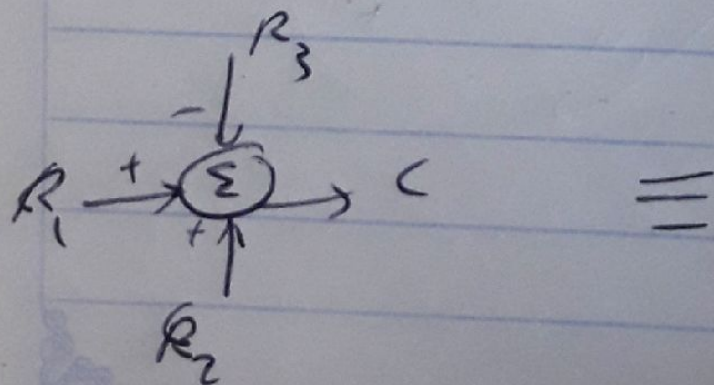
$$U(s) = C_2 G_2 + C_3 G_3$$

$$C_1 = G_1 U(s) \quad \text{not}$$



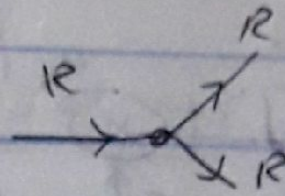
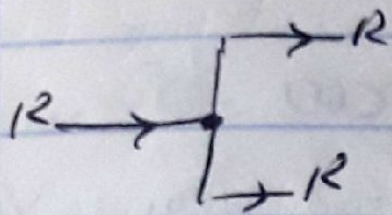
* How we can convert from Block to SFG.

* Summing Junction.

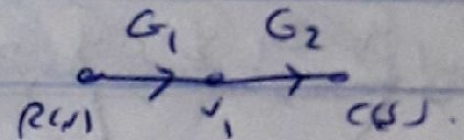
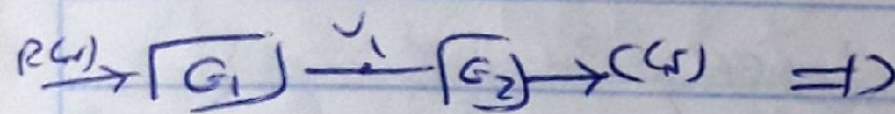


$$C = R_1 + R_2 - R_3$$

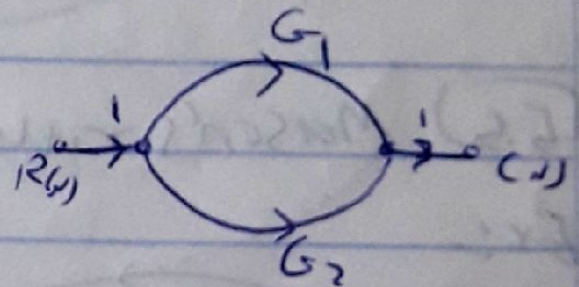
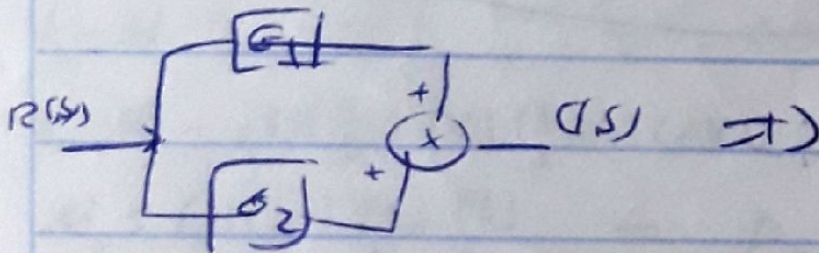
* Pickoff point.



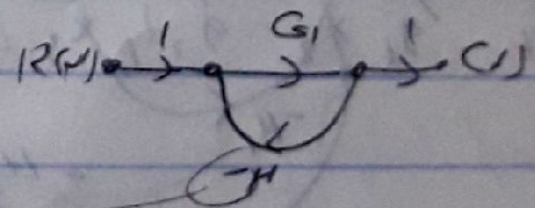
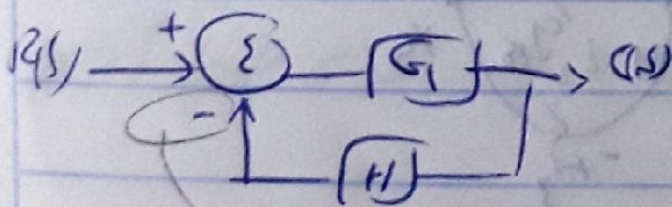
① Cascade:



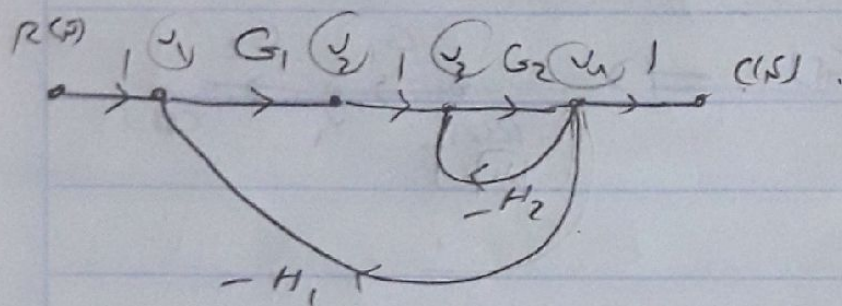
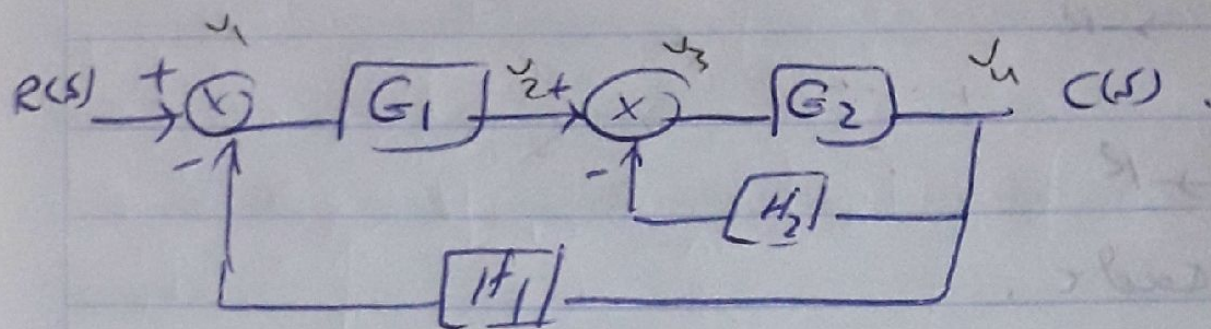
② Parallel:-



③ Feedback:-

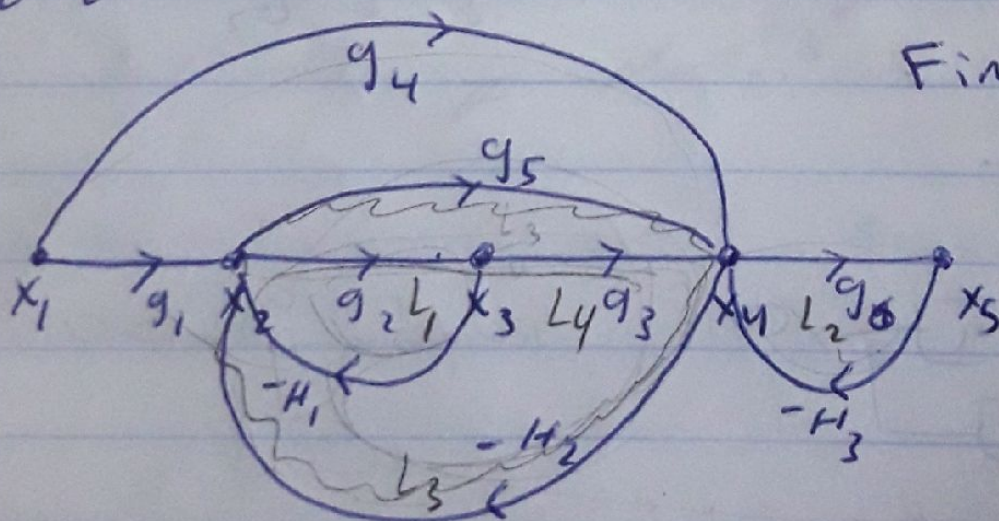


* Example :- Convert Block to SFG.



(E.S) Mason's rule:-

Ex:-



$$P = 3, \quad M_1 = g_1 g_2 g_3 g_6$$

$$M_2 = g_4 g_6$$

$$M_3 = g_1 g_5 g_6$$

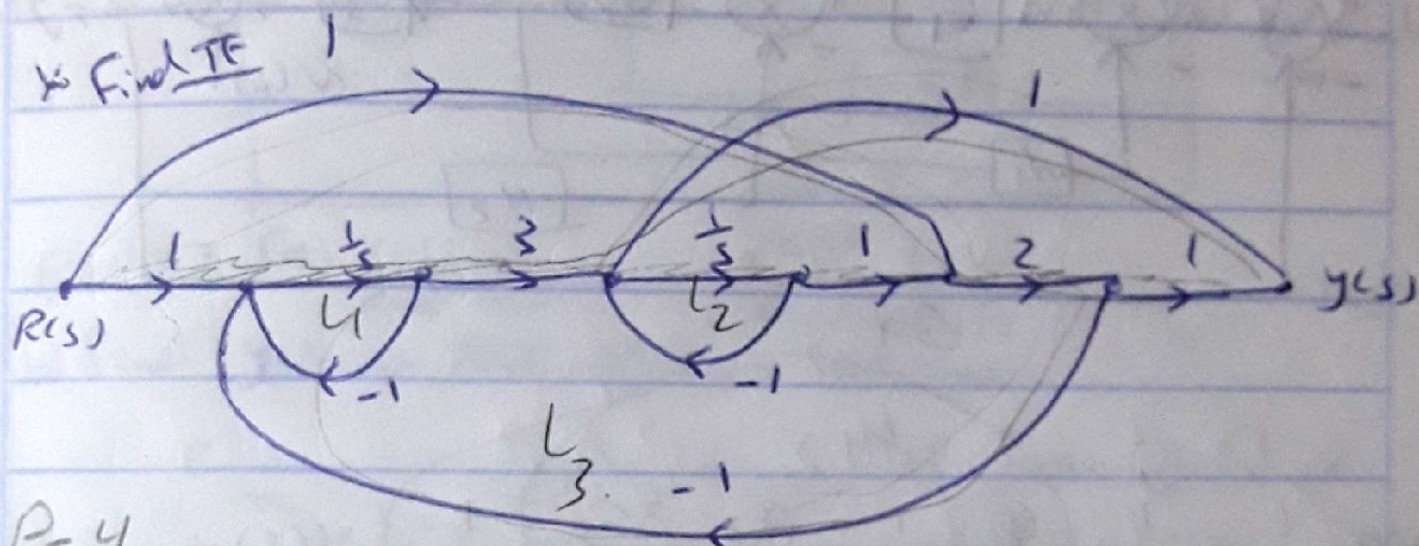
$$\Delta_1 = 1$$

$$\Delta_2 = 1 - L_1$$

$$\Delta_3 = 1$$

$$4 \text{ loops} \Rightarrow \Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_2)$$

$$\therefore G(s) = \frac{M_1 \Delta_1 + M_2 \Delta_2 + M_3 \Delta_3}{\Delta}$$



$$P = 4$$

$$\Rightarrow M_1 = (1) \left(\frac{1}{5}\right) (3) \left(\frac{1}{5}\right) (1) (2) (1) \Rightarrow \Delta_1 = 1$$

$$M_2 = (1) \left(\frac{1}{5}\right) (3) (1) \Rightarrow \Delta_2 = 1$$

$$M_3 = (1) (2) (1) \Rightarrow \Delta_3 = 1 - (L_1 + L_2) + (L_1 L_2)$$

$$M_4 = (1) (2) (-1) \left(\frac{1}{5}\right) (3) (1) \Rightarrow \Delta_{11} = 1$$

of loop = 3

$$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_2)$$

$$L_1 = \left(\frac{1}{5}\right) (-1) = -\frac{1}{5}$$

$$L_2 = \left(\frac{1}{5}\right) (-1) = -\frac{1}{5}$$

$$L_3 = \left(\frac{1}{5}\right) (3) \left(\frac{1}{5}\right) (1) (2) (-1) = -\frac{6}{5}$$

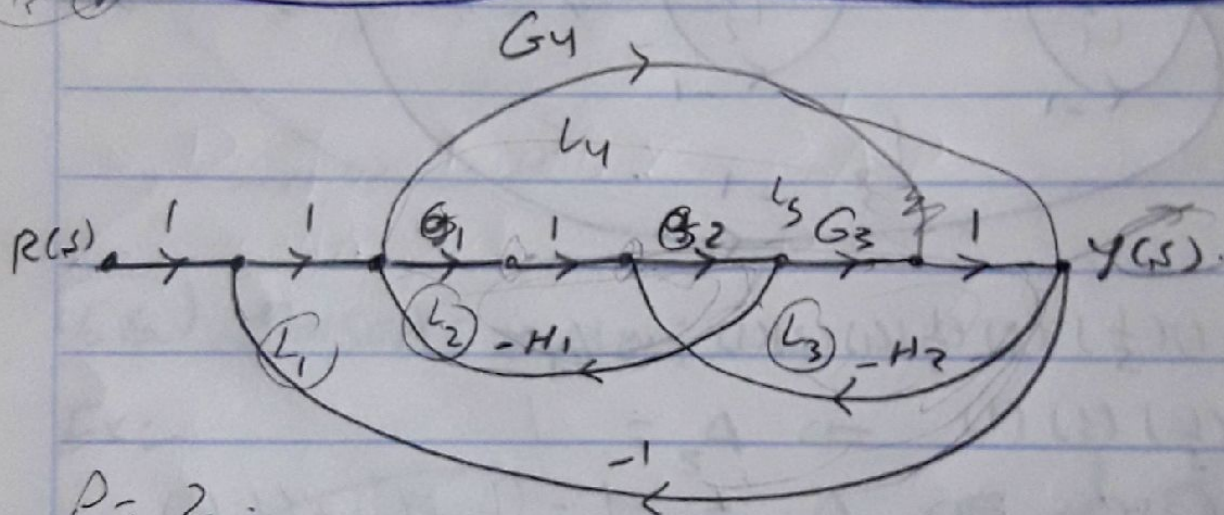
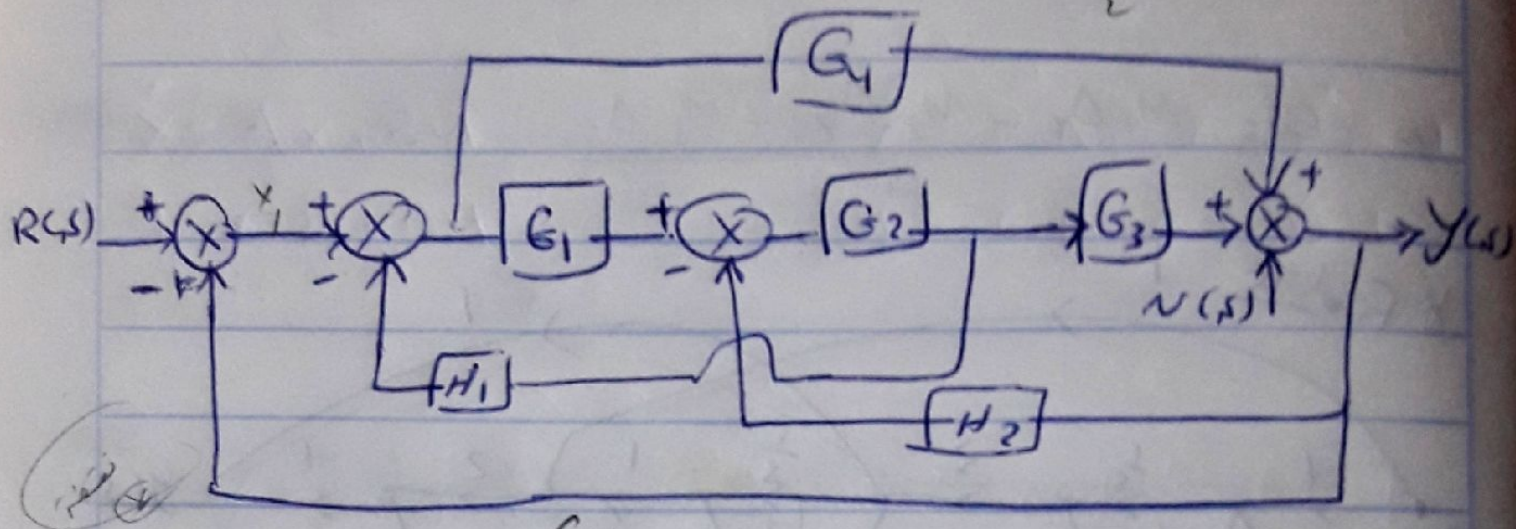
$$\Rightarrow G(s) = \frac{M_1 \Delta_1 + M_2 \Delta_2 + M_3 \Delta_3 + M_4 \Delta_{11}}{\Delta}$$

$$= \frac{-216 (s^2 + 2s + 1)}{s^4 (s^2 + 2s + 7)}$$

$$s^4 (s^2 + 2s + 7)$$

Ex: 2. Find $X(s)/R(s)$??

Ex 2



$$P = 2$$

$$M_1 = G_1 G_2 G_3 \Rightarrow \Delta_1 = 1$$

$$M_2 = G_4 \Rightarrow \Delta_2 = 1$$

of loops = 5

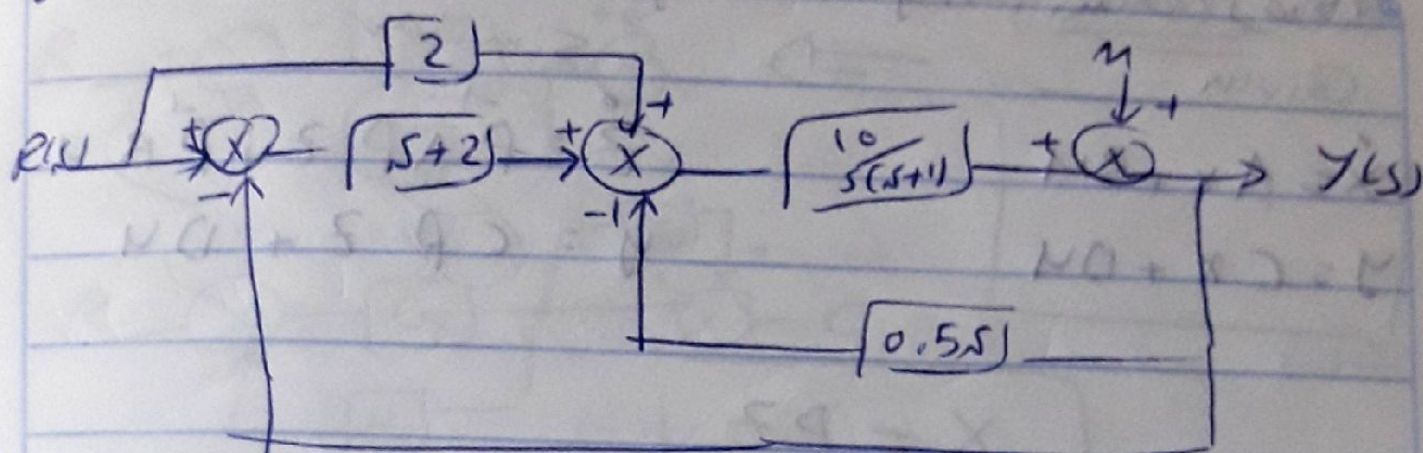
$$L_1 = G_1 G_2 G_3 (-1), \quad L_2 = G_1 G_2 (-H_1)$$

$$L_3 = G_2 G_3 (-H_2), \quad L_4 = G_4 (-1) = -G_4$$

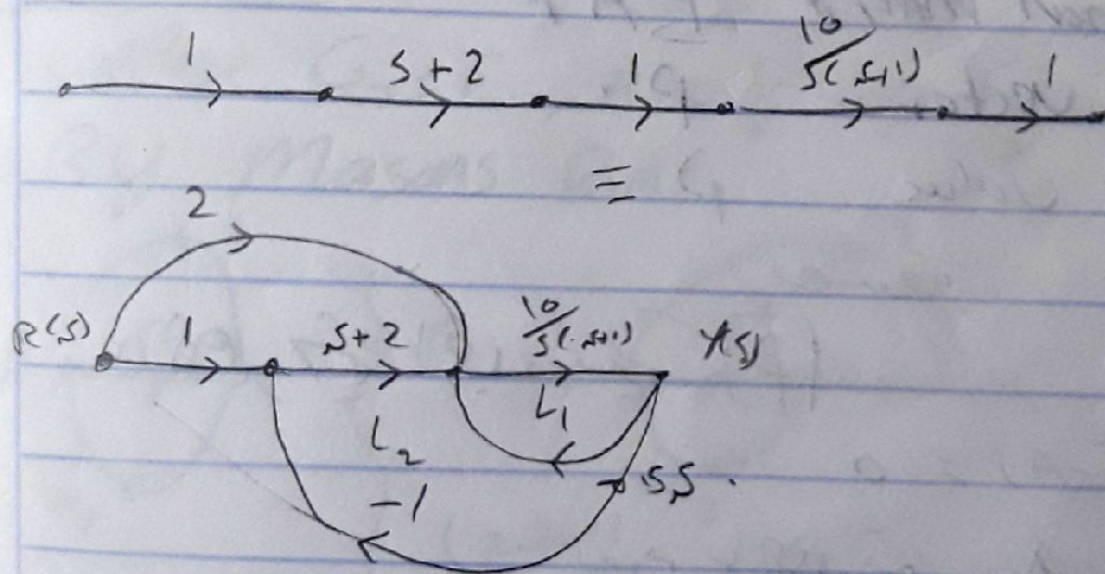
$$L_5 = G_4 (-H_2) (G_2) (-H_1)$$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + (0)$$

Ex: 8



Find $\Rightarrow$ (a) $\frac{Y(s)}{R(s)} \Big|_{N(s)=0}$ (b) $\frac{Y(s)}{N(s)} \Big|_{R(s)=0}$
 (c) Find $y(t)$ at $R(s), N(s)$ are applied.



(a) $\frac{Y(s)}{R(s)} \Big|_{N(s)=0}$; $P=2$, $M_1 \frac{10}{s(s+1)}$ $\Rightarrow \Delta_1 = 1$
 $M_2 = (2) \left(\frac{10}{s(s+1)} \right) \Rightarrow \Delta_2 = 1$

of loops $= \Delta = 1 - (L_1 + L_2)$

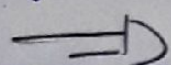
(b) $\frac{Y(s)}{N(s)} \Big|_{R(s)=0}$

$P_1 = 1$; $M_1 = 1$; $\Delta_1 = 1$

(c) $y(t) = \mathcal{L}^{-1} \left\{ \frac{Y(s)}{R(s)} \Big|_{N(s)=0} R(s) + \frac{Y(s)}{N(s)} \Big|_{R(s)=0} N(s) \right\}$ Superposition

(5.2) Similarity Transformation

Given



Similar.

$$\begin{cases} \dot{X} = AX + BU \\ Y = CX + DU \end{cases}$$

$$\begin{cases} \dot{Z} = \bar{P}^{-1} A \bar{P} Z + \bar{P}^{-1} B U \\ Y = C \bar{P} Z + D U \end{cases}$$

$$\begin{cases} X = P Z \\ Z = P^{-1} X \end{cases}$$

- * Diagonal matrix $\bar{P}^{-1} A \bar{P}$
- * eigen vector P
- * eigen value P

مصفوفة

* $\bar{P}$ هم $\bar{P}^{-1}$ اي داي رسي

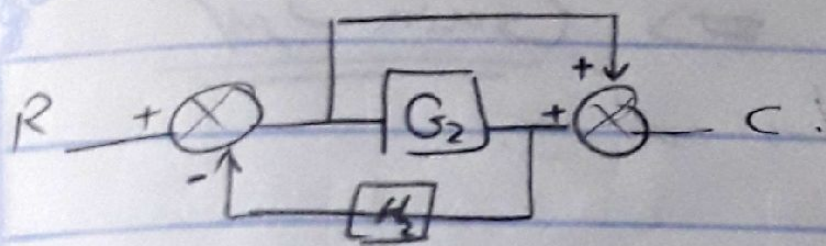
$$1) \det(\lambda I - A) = 0$$

نقوم بايجاد قيم λ

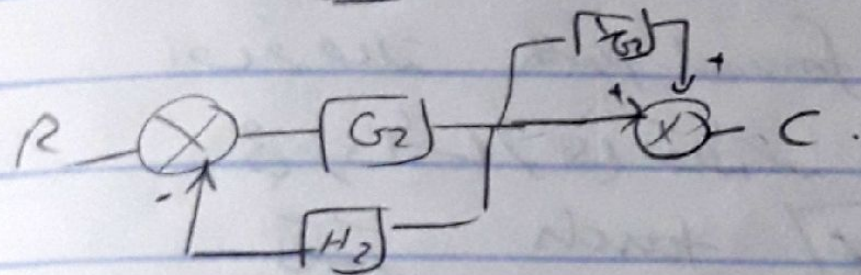
ثم نلاحظ ان $\lambda I - A$ كذا هو مصفوفة
من قيم λ ونحن نلاحظ ان لا يوجد الا عمود من

مصفوفة P

#

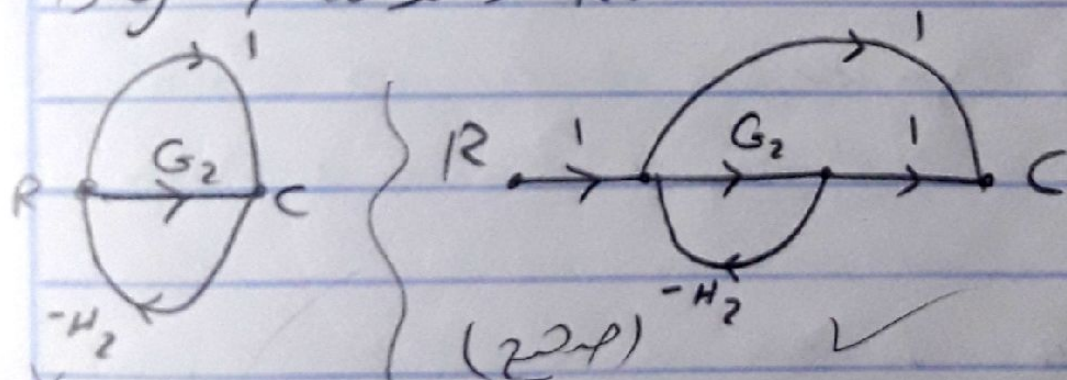


$$C = \frac{1}{1 + G_2 H_2} R$$



$$R \rightarrow \left[\frac{G_2}{1 + G_2 H_2} \right] \rightarrow \left[\frac{1}{G_2 + 1} \right] \rightarrow C$$

suppose $G_2 = s$, $H_2 = \frac{1}{s} \Rightarrow \frac{C}{R} = \frac{s+1}{2}$.
By Mason's Rule.



X sub 1001

$$M_1 = 1 \quad \Delta_1 = 1$$

$$M_2 = G_2 \quad \Delta_2 = 1$$

$$L = -G_2 H_2$$

$$\Delta = 1 - L$$

$$= 1 + G_2 H_2$$

suppose $G_2 = s$, $H_2 = \frac{1}{s}$.

$$\Rightarrow \frac{1+s}{1+1}$$

$$\frac{1+s}{2}$$

Chapter 6 :

* How can test the stability of system ??

- T.F $\Rightarrow$ - If all Poles LHP $\rightarrow$ Stable
- If one Pole RHP $\rightarrow$ Unstable
- if one Pole imaginary not repeated $\rightarrow$ marginal stable
- S.S $\Rightarrow$ eigen value ≤ 0 Stable
- eigen value > 0 Unstable.

Notes : if there one pole at zero $\rightarrow$ stable
if repeated poles at zero $\rightarrow$ unstable

* Routh - Hurwitz Criterion.

By this method we can test any system without find its poles.

- The first column of Routh table has been formed R-H criterion.

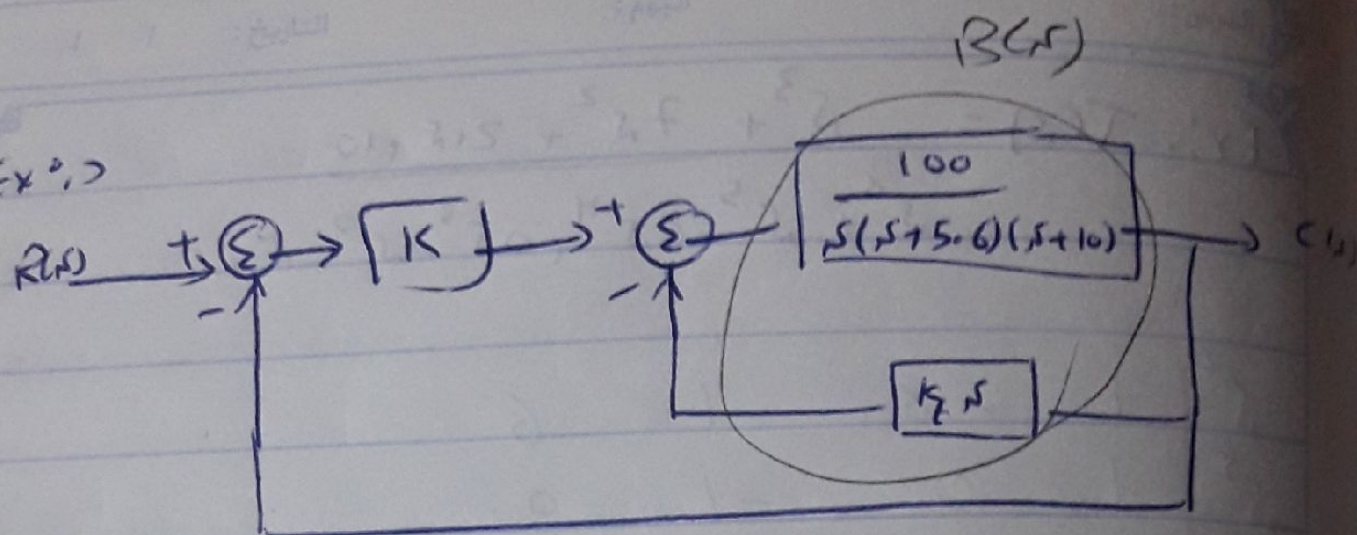
② - The number of Poles in RHP is equal the number of sign changes in the first column.

Ex: $T(s) = \frac{s^3 + 7s^2 + 21s + 10}{s^6 + s^5 - 6s^4 + 0s^3 - s^2 - s + 6}$

s^6	1	-6	-1	6	
s^5	1	0	-1	0	
s^4	-6	0	6	0	
s^3	0	(-24)	0	0	$\rightarrow -6s^4 + 6$
s^2	0	(-24)	6		$-24s^3$
s^1	$\frac{144}{2}$				
s^0	6				

+ , - , + $\Rightarrow$ 2 times 2 Poles RHP.
4 Poles RHP.

Ex 2.2



Find range of K_f so the system will be stable if $K=10$.

$$\frac{100}{s(s+5.6)(s+10)}$$

$$\left(-1 + \frac{100}{s(s+5.6)(s+10)} \right)$$

$$B(s) = \frac{100}{s(s+5.6)(s+10) + 100K_f s}$$

$$T(s) = \frac{K B(s)}{1 + K B(s)} = \frac{1000}{s(s+5.6)(s+10) + 100K_f s + 1000}$$

$$\Rightarrow s^3 + 15.6s^2 + (56 + 100K_f)s + 1000 = 0$$

s^3	1	$56 + 100K_f$
s^2	15.6	1000
s^1	$-1000 + 873.6 + 1560K_f$	
s^0	1000	0

$$-1000 + 873.6 + 1560K_f > 0$$

$$K_f > 0.081$$

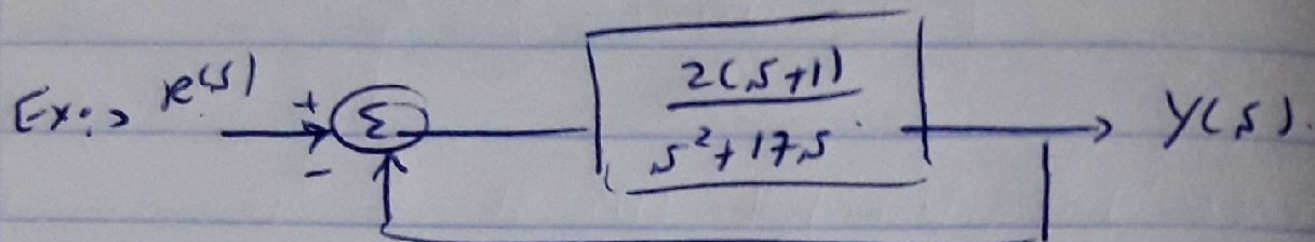
$\Rightarrow$ stable if $K_f > 0.081$

unstable if $K_f < 0.081$

marginal if $K_f = 0.081$

[Signature]

Chapter 7.2



① Test stability.

$$H(s) = \frac{2(s+1)}{s^2+19s+2}$$

Stable Poles in LHP.

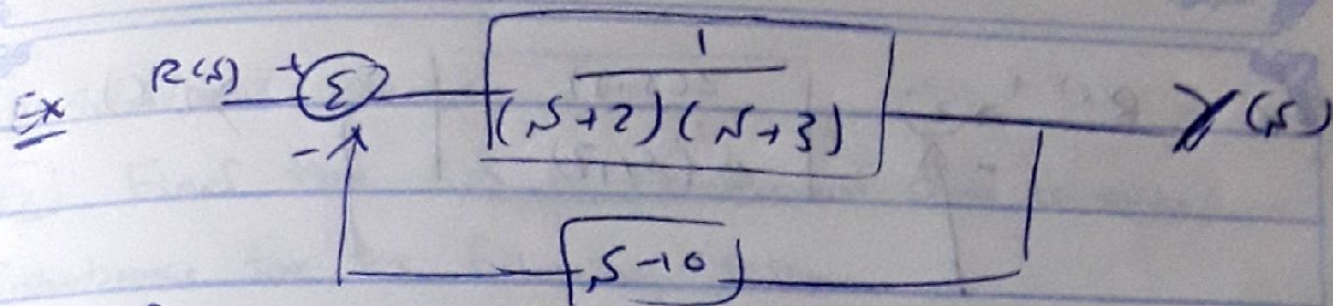
② $G(s) = \frac{2(s+1)}{s(s+17)}$ type 1

$\therefore$ ① $15u(t) \rightarrow \text{S.S.E} = 0.$

② $15tu(t) \rightarrow K_v = \lim_{s \rightarrow 0} sG(s) = \frac{2}{17}$

$\Rightarrow \text{S.S.E} = \frac{1}{K_v} = \frac{17}{2} \times 15 = 127.5$

③ $15t^2u(t) \rightarrow \text{S.S.E} = \infty.$



Find S.S.E.

① Check the stability.

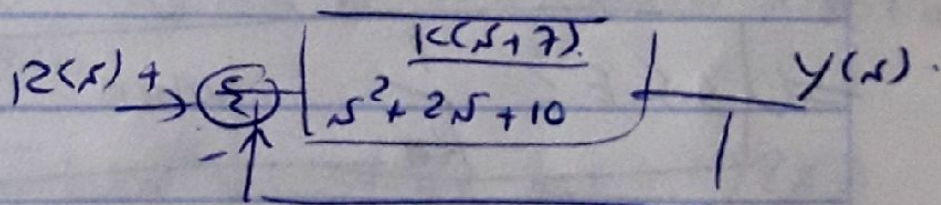
$$T(s) = \frac{\frac{1}{(s+2)(s+3)}}{1 + \frac{s-10}{(s+2)(s+3)}} = \frac{1}{(s+2)(s+3) + (s-10)}$$

$$= \frac{1}{s^2 + 5s - 4} \Rightarrow \text{System is unstable.}$$

No S.S.E.

~~$e_{ss} = \infty$~~

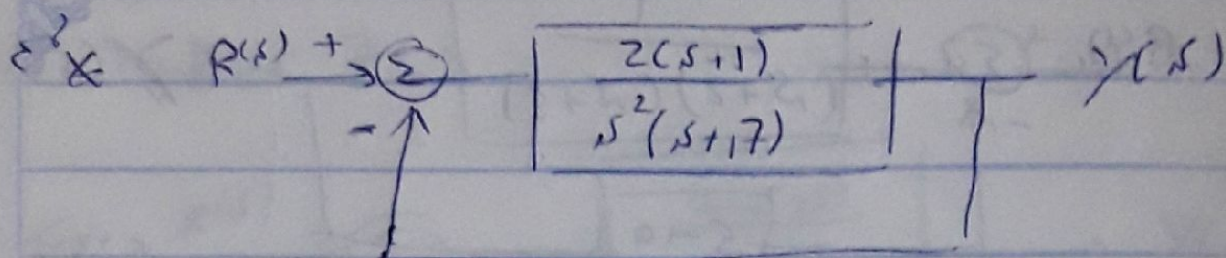
Ex. Find the sensitivity of S.S.E. to changes in k .



$$T(s) = \frac{\frac{k(s+7)}{s^2+2s+10}}{1 + \frac{k(s+7)}{s^2+2s+10}} = \frac{k(s+7)}{s^2+2s+10+k(s+7)}$$

$$K_p = \lim_{s \rightarrow 0} G(s) = \frac{7k}{10} \Rightarrow e_{ss} = \frac{1}{1 + \frac{7k}{10}} = \frac{10}{10+7k}$$

$$S_{e_{ss}} = \frac{\frac{10}{10+7k}}{\left(\frac{10}{10+7k}\right)^2} \cdot \frac{-7(10)}{(10+7k)^2} = \frac{-710}{10+7k}$$



Find S.S.E for $15u(t)$, $15tu(t)$, $15t^2u(t)$.

the system type $\underline{2} \Rightarrow 1$.

① $15u(t) \Rightarrow \text{S.S.E} = 0$.

② $15tu(t) \Rightarrow \text{S.S.E} = 0$.

③ $15t^2u(t) \Rightarrow$

$$K_a = \lim_{s \rightarrow 0} s^2 G(s)$$

$$= \lim_{s \rightarrow 0} s^2 \left(\frac{2(s+1)}{s^2(s+17)} \right) = \left[\frac{2}{17} \right]$$

$$\Rightarrow \boxed{\text{S.S.E} = \frac{1}{K_a} \text{ when } \frac{1}{2}t^2u(t)}$$

$$\Rightarrow \text{S.S.E} = \left(\frac{1}{K_a} \right) (2 \times 15)$$

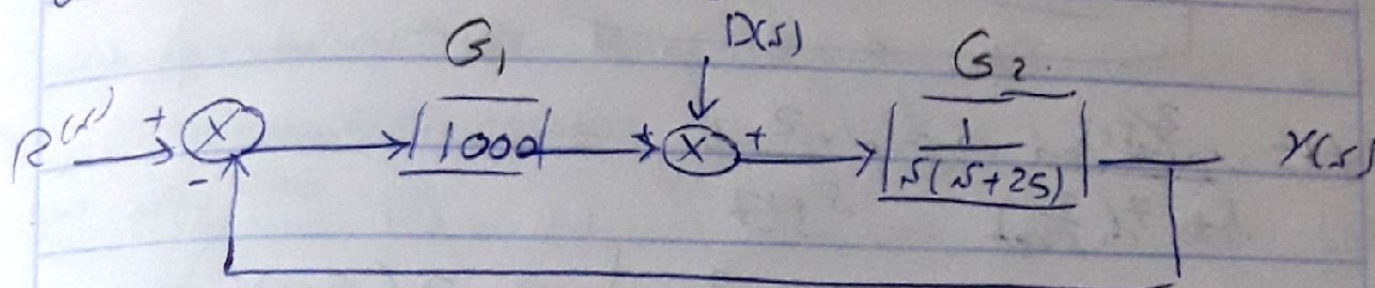
$$= \left(\frac{17}{2} \right) (30) = 255$$

50

#

* Additional Exercise 2

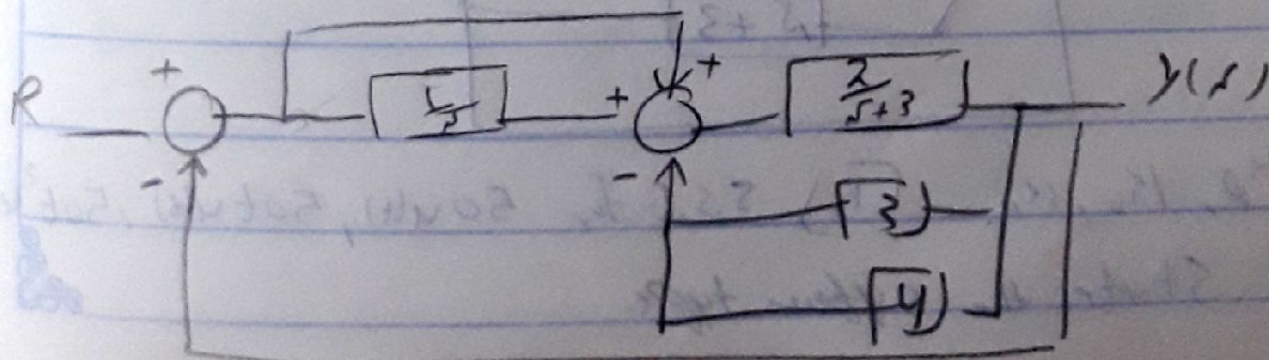
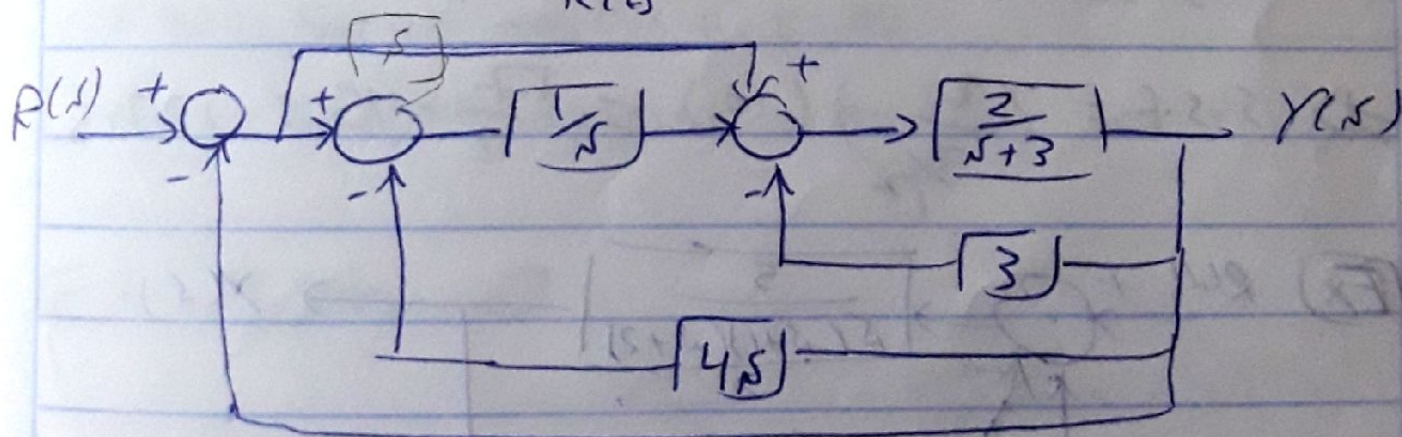
[Ex] Find the S.S.E Component due to a step disturbance for the following system.

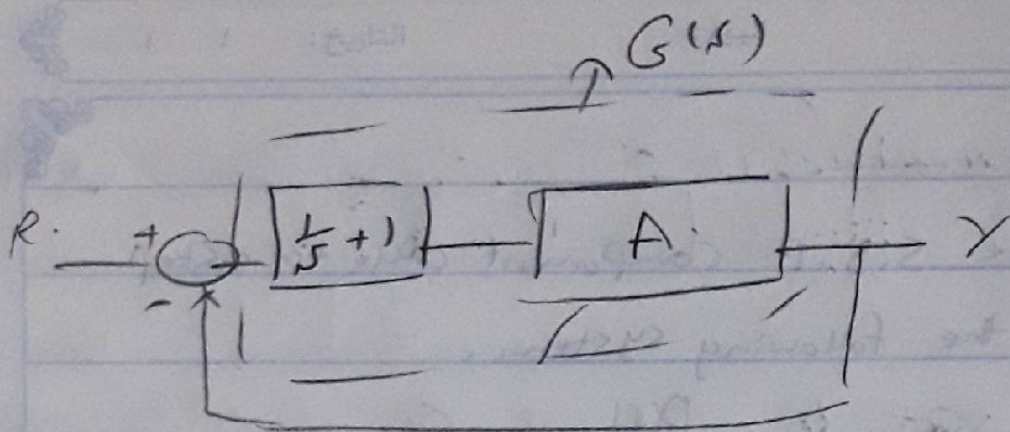


$$S.S.E = \lim_{s \rightarrow 0} \frac{s G_2 D(s)}{1 + G_1 G_2}$$

$$= - \lim_{s \rightarrow 0} \frac{s \cdot \left(\frac{1}{s(s+25)} \right) \frac{1}{s}}{1 + \frac{1000}{s(s+25)}} = - \frac{1}{1000} \quad \neq$$

[Ex] Find S.S.E For (a) $15u(t)$ (b) $15t u(t)$ (c) $15t^2 u(t)$
 $R(t)$





$$A = \frac{2/s+3}{1 + 7(\frac{2}{s+3})} = \frac{2}{s+17}$$

$$\therefore G(s) = \left(\frac{1}{s} + 1\right) \left(\frac{2}{s+17}\right) = \frac{2(s+1)}{s(s+17)}$$

open loop.

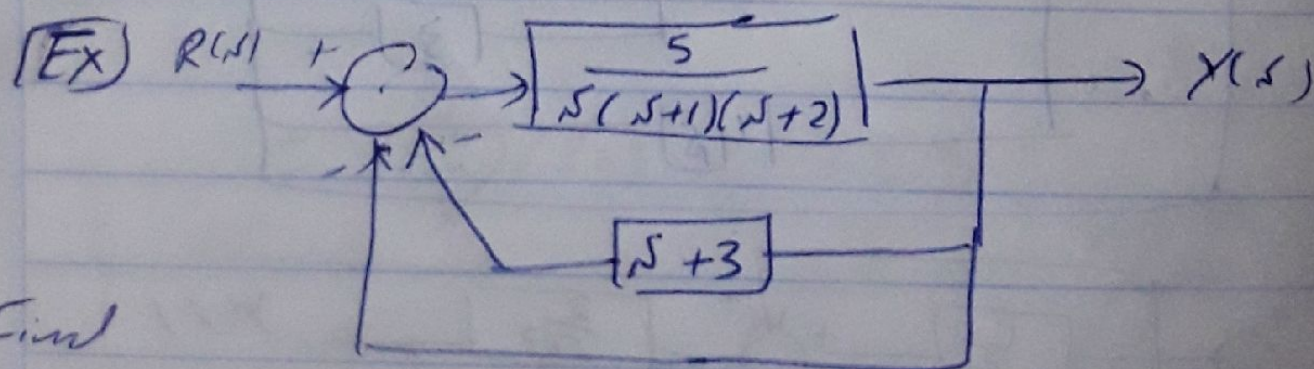
type 1 $\Rightarrow$ s.s.f (1) $15 u(t) = 0$.

(3) $15 t^2 u(t) = \infty$.

(2) s.s.f for $15 t u(t) = ??$

$$K_v = \lim_{s \rightarrow 0} s G(s) = \frac{2}{17}$$

$$\Rightarrow \text{s.s.f} = \left(\frac{2}{2/17}\right)(15) = \frac{17}{2} \times 15 = 127.5$$



Find

- (a) K_p, K_v, K_a (b) s.s.f for $50 u(t), 50 t u(t), 50 t^2 u(t)$
 (c) State the system type.

open loop TF

$$\hat{G}(s) = \frac{5/s(s+1)(s+2)}{1 + \frac{5(s+2)}{s(s+1)(s+2)}} = \frac{5}{s^3 + 3s^2 + 7s + 15}$$

the system type 0.

$$(a) \Rightarrow K_p = \lim_{s \rightarrow 0} G(s) = \frac{5}{15} = \frac{1}{3}$$

$$\Rightarrow K_v = 0, K_a = 0.$$

$$(b) R(t) = 50u(t) \Rightarrow \text{s.s.f} = \left(\frac{1}{1+K_p} \right) 50$$

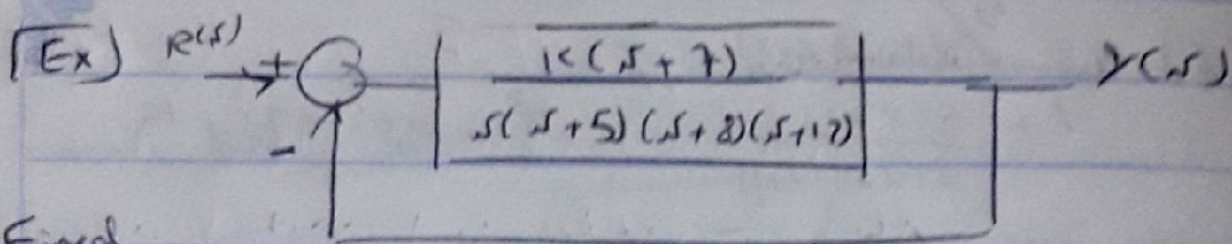
$$= \frac{50}{1 + \left(\frac{1}{3} \right)} = 37.5$$

$$R(t) = 50tu(t) \Rightarrow \text{s.s.f} = \infty$$

$$R(t) = 50t^2u(t) \Rightarrow \text{s.s.f} = \infty$$

(c) type 0.

#



Find

- Value of K s.s.e = 0.01 for input $\frac{1}{10}t$.
- Value of K_v for the value of K found in a.
- What is the minimum possible s.s.e for the input

type 1 $\Rightarrow$ s.s.e = $\left(\frac{1}{K_v}\right) \left(\frac{1}{10}\right) = 0.01 \Rightarrow K_v = 10$.

$K_v = \lim_{s \rightarrow 0} s G(s) = 10$.

$\Rightarrow \lim_{s \rightarrow 0} \frac{K(s+7)}{(s+5)(s+8)(s+12)} = 10 \Rightarrow \frac{7K}{(5)(8)(12)} = 10 \Rightarrow K = 685.7$

s.s.e = $\lim_{s \rightarrow 0} \frac{s R(s)}{1 + G(s)}$ هناك علاقة عكسية بين s.s.e و K

لذلك نقوم بإيجاد القيمة الدنيا لـ K لكي يكون النظام مستقر

Characteristic equation: ثم نكتب المعادلة السوية

$s^4 + 25s^3 + 196s^2 + (480 + K)s + 7K = 0$.

s^4	1	196	7K
s^3	25	480 + K	0
s^2	$\frac{4470 - K}{25}$	7K	0
s^1	$-\frac{1}{25} \frac{4470 - K}{25}$	$\frac{4470 - K}{25}$	0
s^0	$\frac{4470 - K}{25}$		

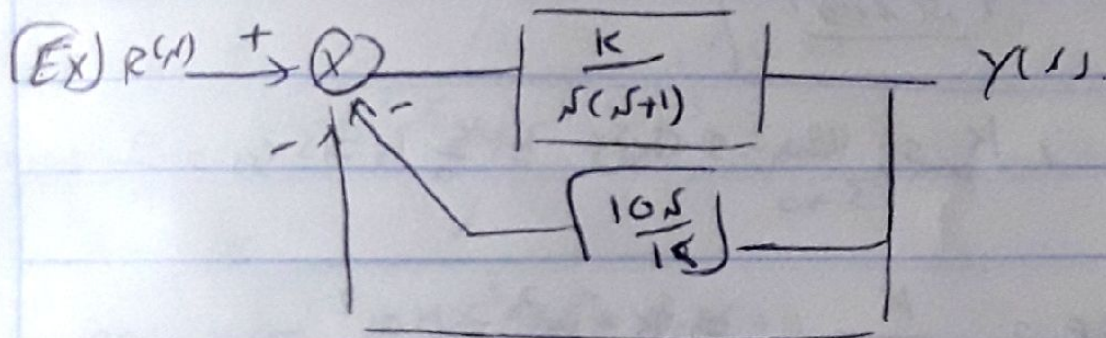
$$\frac{4420 - K}{25} > 0 \Rightarrow K < 4420.$$

$$\frac{-K^2 - 435K + 2121600}{4420 - K} > 0 \Rightarrow (K - 1255.22)(K + 1690) < 0$$

range of stability

$$0 < K < 1255.22$$

$$PSS_{\min} = \frac{0.1}{K_U} = \frac{0.1}{\lim_{s \rightarrow 0} s G(s)} = 5.46 \times 10^{-3}$$



Design the value of K , so that the input $100t$ u(t), there will be 0.01 error in the steady state.

$$G(s) = \frac{K/s(s+1)}{1 + \left(\frac{K}{s(s+1)}\right)\left(\frac{10s}{K}\right)} = \frac{K}{s(s+1)} \quad \text{type 1}$$

$$\Rightarrow K_p = \lim_{s \rightarrow 0} s G(s) = \frac{K}{1} \Rightarrow s.s.e = \left(\frac{1}{K_p}\right) 100$$

$$\Rightarrow \frac{1}{\frac{K}{1}} > 100 \Rightarrow \frac{100}{K} = 0.01 \Rightarrow K < 110000.$$

* Ex) 2nd order, unity Feedback system is to follow a ramp input
 $- s.s.e = 0.01$ of the input velocity.

- natural freq of closed loop = 10 rad/s

find (a) system type (b) exact expression of forward Path T.F.

(c) closed loop system damping ratio.

(a) type 1.

(b) $P \rightarrow \otimes \rightarrow \frac{K}{s(s+a)} \rightarrow Y(s)$ general form 2nd

$$e_{ss} = \frac{1}{K_v} \quad | \quad K_v = \lim_{s \rightarrow 0} s G(s) = \frac{K}{a} \Rightarrow e_{ss} = \frac{a}{K} = 0.01$$

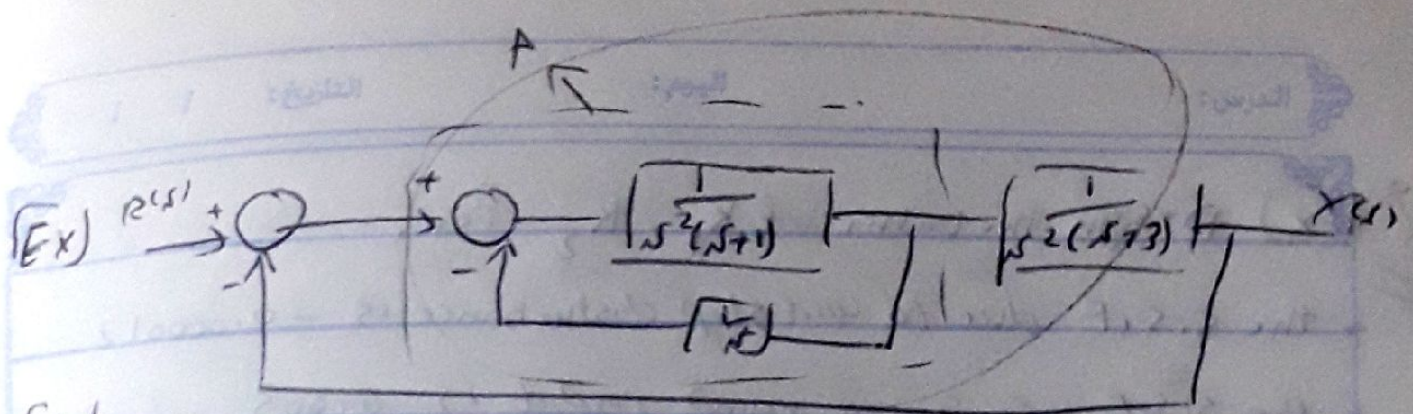
$$\text{closed loop T.F} = \frac{K}{s^2 + as + K} \Rightarrow K = \omega_n^2 = (10)^2 \Rightarrow K = 100.$$

$$\Rightarrow \frac{a}{K} = 0.01 \Rightarrow a = 1 \Rightarrow G(s) = \frac{100}{s(s+1)}$$

$$(c) T(s) = \frac{K}{s^2 + as + K} = \frac{100}{s^2 + s + 100}$$

$$a = ? \quad \omega_n = 10 \Rightarrow \zeta = \frac{1}{2 \times 10} = \frac{1}{20} = 0.05.$$

#



Find

(a) closed loop T.F (b) system type (c) s.s.e for $5u(t)$

(d) s.s.e for $5tu(t)$

$$A = \frac{Y_{s^2(s+1)}}{1 + \frac{1}{s^3(s+1)}} = \frac{s}{s^4 + s^3 + 1}$$

$$\text{open loop } G(s) = \frac{1}{s(s^5 + 4s^4 + 3s^3 + s + 3)}$$

$$\text{closed } T(s) = \frac{1}{s^6 + 4s^5 + 3s^4 + s^3 + 3s + 1}$$

$$(b) \text{ Type } 1 \Rightarrow K_v = \lim_{s \rightarrow 0} s G(s) = \frac{1}{3}$$

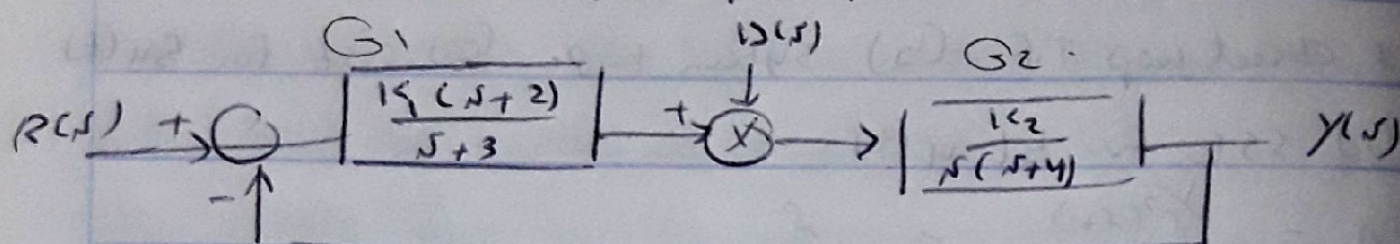
$$(c) \text{ s.s.e } 5u(t) = 0$$

$$(d) \text{ s.s.e } 5tu(t) = \frac{1}{(\frac{1}{3})} \times 5 = 15 \quad \#$$

Ex) Design the value of K_1, K_2 for.

- the S.S.E due to unit step disturbance is -0.000012 .

- the S.S.E $\propto s$ ramp input is 0.003



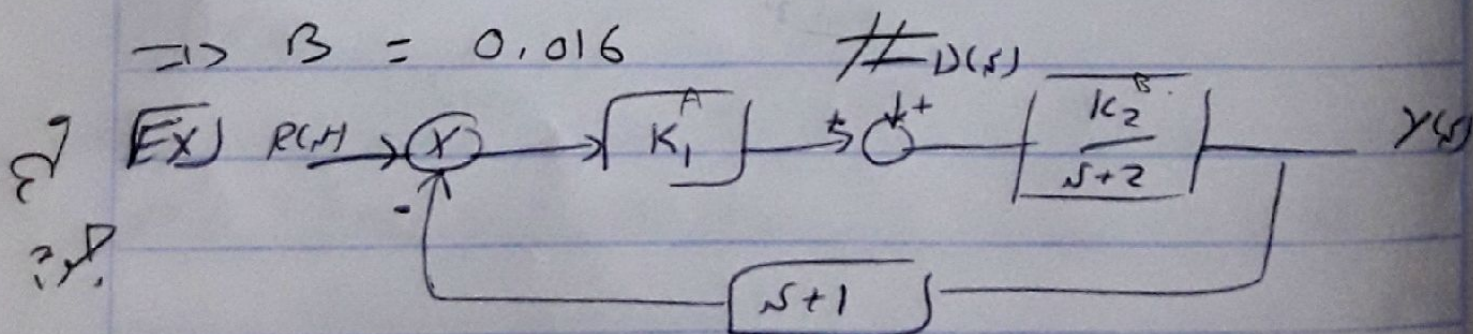
S.S.E for input $= \lim_{s \rightarrow 0} \frac{s R(s)}{1 + G_1 G_2} \cdot \frac{1}{s^2}$ - by complex.

$$\Rightarrow \left| \frac{6}{AB} \right| = 0.003$$

S.S.E for disturbance $\Rightarrow \lim_{s \rightarrow 0} \frac{s G_2 D(s)}{1 + G_1 G_2} = -0.000012$
by complex

$$\Rightarrow \frac{3}{2A} = 0.000012 \Rightarrow A = 125000$$

$$\Rightarrow B = 0.016$$



Find the sensitivity of S.S.E for change in K_1 & K_2
where $K_1 = 100$ & $K_2 = 0.1$, Assume step inputs for
Both input & disturbance.

$$\text{S.S.E for input} = \lim_{s \rightarrow 0} \frac{s R(s)}{1 + G_1 G_2 G_3} = \frac{2}{K_1 K_2 + 2}$$

$$\text{S.S.E for disturbance} = - \lim_{s \rightarrow 0} \frac{s G_2 G_3 D(s)}{1 + G_1 G_2 G_3} = \frac{-K_2}{K_1 K_2 + 2}$$

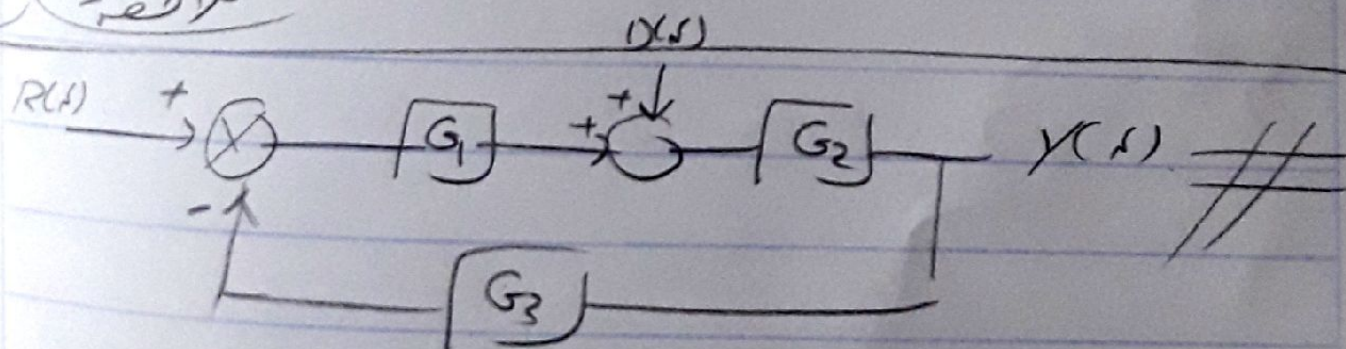
$$\therefore \text{S.S.E} = \frac{2}{K_1 K_2 + 2} - \frac{K_2}{K_1 K_2 + 2} = \frac{2 - K_2}{K_1 K_2 + 2}$$

$$\frac{\partial}{\partial K_1} \text{S.S.E} = \frac{K_1}{\text{S.S.E}} \frac{d \text{S.S.E}}{d K_1}$$

$$= \left(\frac{K_1}{2 - K_2 / (K_1 K_2 + 2)} \right) \left(\frac{K_2 (K_2 - 2)}{(K_1 K_2 + 2)^2} \right) = - \frac{K_1 K_2}{K_1 K_2 + 2} = \frac{-10}{10 + 2} = \frac{-10}{12} = -0.833$$

$$\frac{\partial}{\partial K_2} \text{S.S.E} = \frac{K_2}{\text{S.S.E}} \frac{d \text{S.S.E}}{d K_2} = \frac{2 K_2 (K_1 + 1)}{(K_2 - 2)(K_1 K_2 + 2)} = \frac{-101}{114} = -0.8859$$

or



$$\text{S.S.E} = \lim_{s \rightarrow 0} \frac{s R(s)}{1 + G_1 G_2 G_3} - \lim_{s \rightarrow 0} \frac{s G_2 G_3 D(s)}{1 + G_1 G_2 G_3}$$

* Chapter 8: R-L techniques.

Ex: sketch R-L for $G(s)$ with unity feedback.

$$G(s) = \frac{s+3}{s(s+1)(s+2)(s+4)}$$

$$1/K \cdot G(s) \cdot H(s) = \frac{K(s+3)}{s(s+1)(s+2)(s+4)}$$

$$\text{Poles} = 0, -1, -2, -4 \Rightarrow \# P = 4$$

$$\text{Zeros} = -3 \Rightarrow \# Z = 1$$

$$\Rightarrow \sigma = 4 - 1 = 3$$

$$\sigma_m = \frac{(0 - 1 - 2 - 4) - (-3)}{3} = \frac{-4}{3} = -1.33$$

$$\theta_m = r \frac{180}{3} = r(60), \text{ but } r = \pm 1, +3.$$

$$\Rightarrow \theta_m = +60^\circ, -60^\circ, +180^\circ$$

$$\text{Break point: } \frac{dG(s)}{ds} \cdot H(s) = 0$$

$$\Rightarrow -3s^4 + 26s^3 + 77s^2 + 84s + 24 = 0$$

$$s_1 = +11.129 \times$$

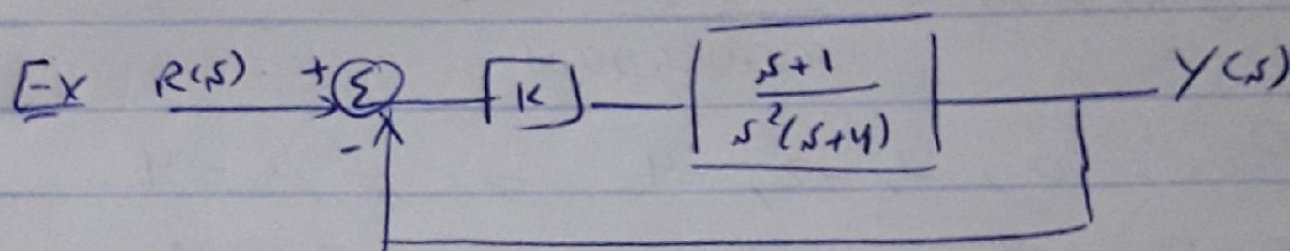
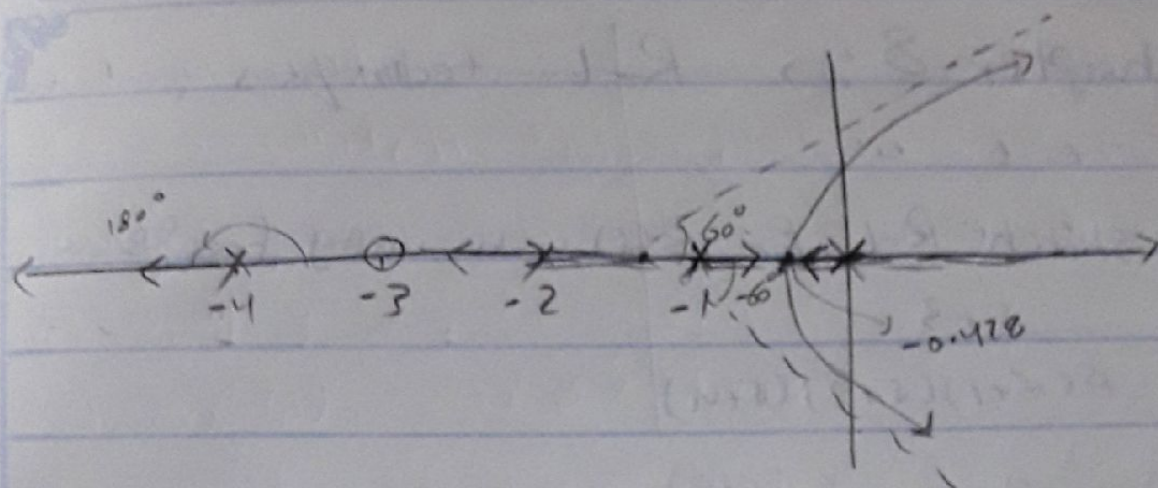
$$s_2 = -0.429 \times$$

$$s_3 = -1.04 + j0.75 \times$$

$$s_4 = -1.04 - j0.75 \times$$

خطوط انتقال تتقاطع في نقطة

B.P. نقطة تقاطع الخطوط



Sketch R-L & check if it crosses the Imag axis or not.

$$K G(s) H(s) = \frac{K(s+1)}{s^2(s+4)}$$

$$P: 0, 0, -4 \Rightarrow \# P = 3 \quad Z: -1 \Rightarrow \# Z = 2 \quad \alpha = 2$$

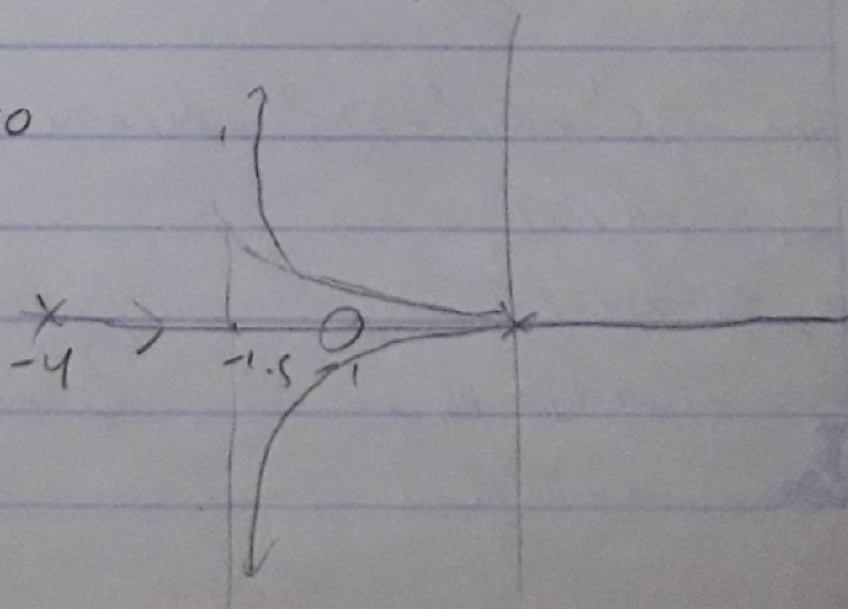
$$\sigma_m = \frac{-4+1}{2} = -1.5, \quad \theta_m = \pm \frac{180}{2} = +90^\circ, -90^\circ$$

$$B.P.: \frac{d}{ds} \left(\frac{s+1}{s^2(s+4)} \right) = 0$$

$$s_1 = 0$$

$$s_2 = -1.75 + j0.9$$

$$s_3 = -1.75 - j0.9$$



You can check if cross or not By using Routh Hurwitz

$$1 + K G H = 0$$

$$\Rightarrow s^3 + 4s^2 + Ks + K = 0$$

$$s^3 \quad 1 \quad K \quad 0$$

$$s^2 \quad 4 \quad K \quad 0$$

$$s^1 \quad \frac{3K}{4} \quad 0$$

$$s^0 \quad K$$

$$b_1 = -\frac{1(K - 4K)}{4} = \frac{3K}{4}$$

$$c_1 = -\frac{(4 \times 0 - 3\frac{K^2}{4})}{3\frac{K}{4}} = K$$

to be stable the first column must be have same sign

$$\text{So } K > 0$$

$$\frac{3K}{4} > 0 \Rightarrow K > 0$$

The system stable for all $K > 0$, so the system always stable that means the RL does not cross the j-axis & there is no poles in RHP.

Part 2

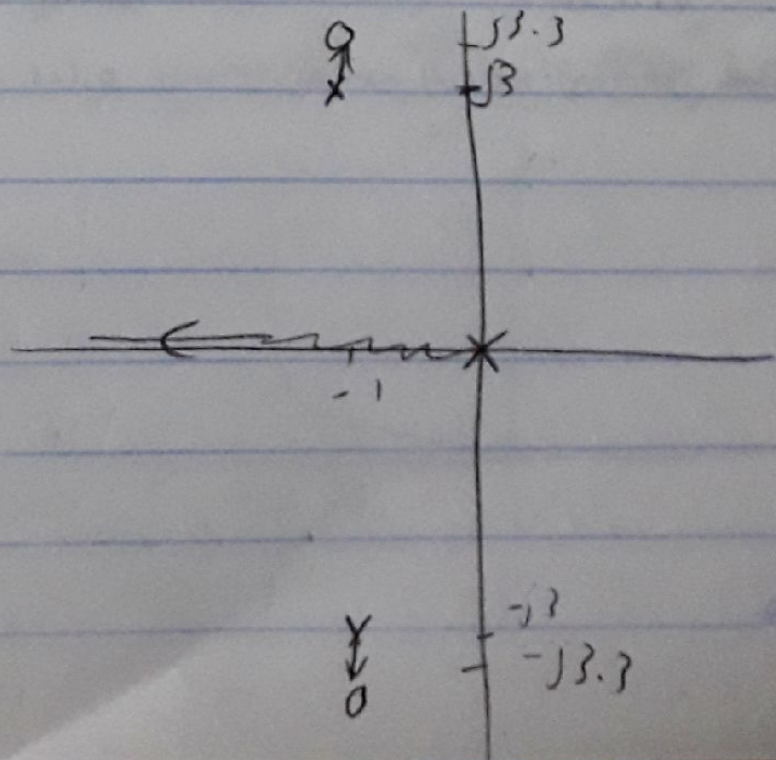
Ex: Sketch the RL with respect R for the following system and given the imaginary axis crossing if any

$$\begin{aligned} \text{Ex) } KG(s) &= \frac{K(s^2 + 2s + 12)}{s(s^2 + 2s + 10)} \\ &= \frac{(s + 1 - j3.3)(s + 1 + j3.3)}{s(s + 1 - j3)(s + 1 + j3)} \end{aligned}$$

$$\begin{aligned} P: & -1 + j3, -1 - j3, 0 \\ Z: & -1 + j3.3, -1 - j3.3 \end{aligned}$$

$$\sigma = 3 - 2 = 1 \Rightarrow \theta = +180^\circ$$

$$\sigma = -$$



$$b) G(s) = \frac{1}{s(s+1 \pm j3)}$$

$$K_{GH} = \frac{K}{s(s+1-j3)(s+1+j3)}$$

$$P: 0, -1+j3, -1-j3.$$

$$Z: \emptyset.$$

$$q = 3 - 0 = 3 \Rightarrow \theta_m = \pm 60^\circ + 180^\circ$$

$$\sigma_m = -\frac{2-0}{3} = -0.66$$

$$B.P. = \frac{d}{ds} K_{GH} = 0 \Rightarrow s_1 = -0.6 - j1.69$$

$$s_2 = -0.6 + j1.69$$

Complex number, So No B.P. intersection real axis

$$1 + K_{GH} = 0.$$

$$s^3 + 2s^2 + 10s + K = 0$$

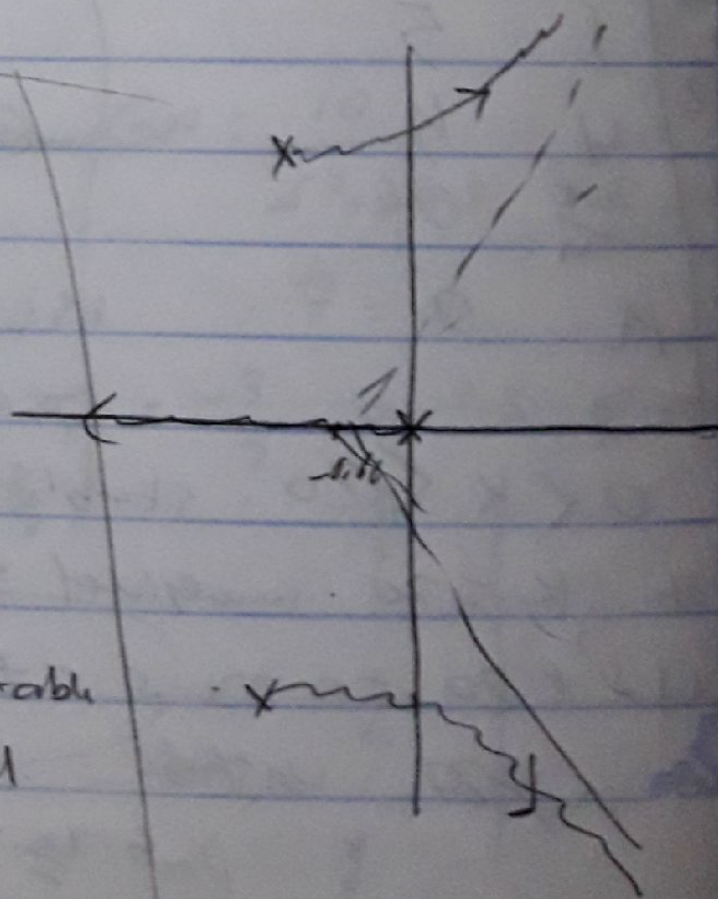
s^3	1	10
s^2	2	K
s^1	$\frac{10-K}{2}$	0
s^0	K	

$$0 < K < 20 \text{ Stable}$$

$$\text{at } K = 20 \text{ marginal}$$

$$2s^2 + 20 = 0 \Rightarrow s = \pm j\sqrt{10}$$

$$K > 20 \text{ unstable}$$



$$[C] \quad K G(s) = \frac{K}{s(s^2 + 4s + 5)} = \frac{K}{s(s+2-j)(s+2+j)}$$

$$P: 0, -2+j, -2-j$$

$$Z: \emptyset$$

$$n = 3 - 0 = 3 \Rightarrow \Theta = \pm 60, +180.$$

$$\sigma_m = \frac{-4 - 0}{3} = -\frac{4}{3} = -1.33$$

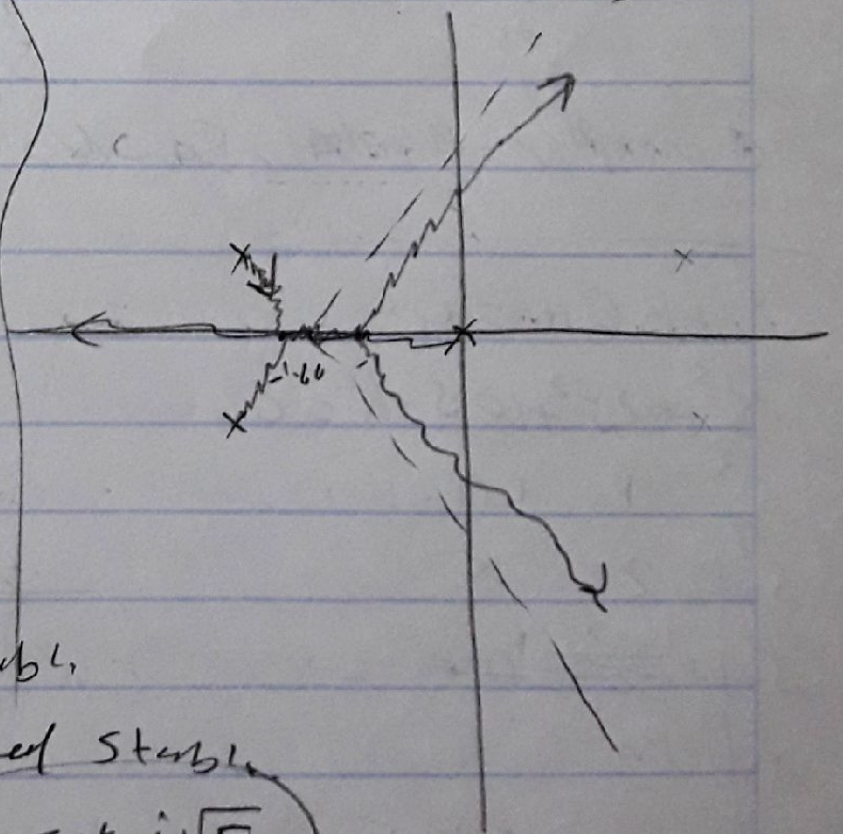
$$(BP) \quad \frac{d}{ds}(GH) = 0 \Rightarrow \begin{matrix} s_1 = -1 \\ s_2 = -1.66 \end{matrix}$$

یک
آن نقطه
القطبیه

$$1 + KGH = 0$$

$$s^3 + 4s^2 + 5s + K = 0$$

s^3	1	5
s^2	4	K
s^1	$\frac{20-K}{4}$	0
s^0	K	0



$$0 < K < 20 \text{ - stable}$$

$$\text{at } K = 20 \text{ marginal stable}$$

$$4s^2 + 20 = 0 \Rightarrow s = \pm j\sqrt{5}$$

$$K > 20 \text{ unstable}$$

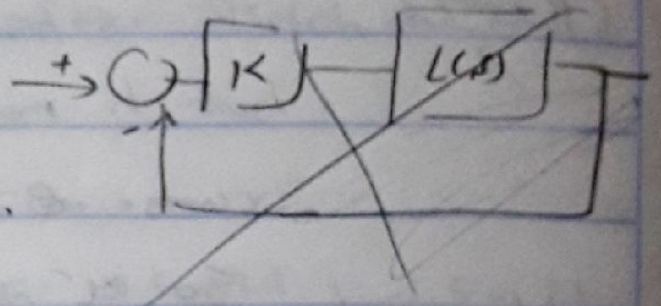
نقطه مرزی پایداری

* Additional exercise 30.

EX] For the following closed loop transfer function:

- Sketch the Root locus for the system.
- Check if the Root locus cross the j-axis or not.

$$T(s) = \frac{10}{s^2 + 3s + 7 + 10K}$$



Characteristic eqn $1 + KGH = 0$.

$$\Rightarrow s^2 + 3s + 7 + 10K = 0$$

$$\Rightarrow s^2 + 3s + 7 + 10K = 1 + KGH$$

$$\Rightarrow KGH = s^2 + 3s + 7 + 10K$$

$$1 + KGH = 0$$

نقطة 4: هذه المعادلة

$$\frac{s^2 + 3s + 7}{s^2 + 3s + 7} + \frac{10K}{s^2 + 3s + 7} = 0$$

$$\Rightarrow KGH = \frac{10K}{s^2 + 3s + 7} \Rightarrow GH = \frac{10}{s^2 + 3s + 7}$$

$$P \Rightarrow -1.5 + j2.1, -1.5 - j2.1, \quad \tau = \emptyset$$

$$\Rightarrow \alpha = 2 - 0 = 2 \Rightarrow \sigma_m = -\frac{3}{2} = -1.5, \quad \theta_m = \pm 90^\circ$$

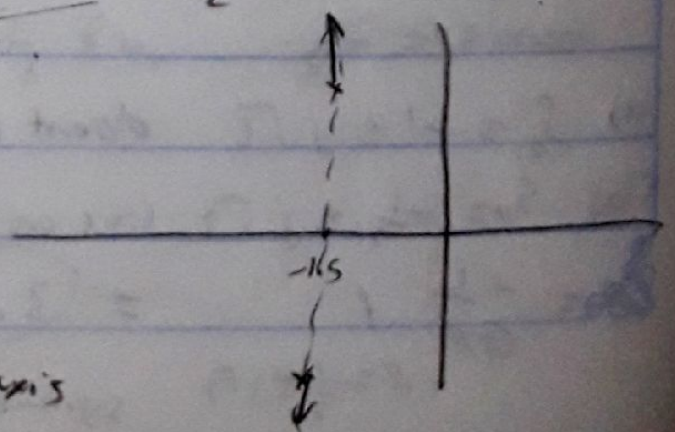
$$\Rightarrow \text{B.P} = \frac{d}{ds} GH = 0 \Rightarrow s = -\frac{3}{2}$$

NO B.P because

no value on real

axis.

R-L does not cross j-axis



الزخم RL في الجذور

$$1 + KGH = 0 \Rightarrow s^2 + 3s + 7 + 10K = 0.$$

$$\begin{array}{r|l} s^2 & 1 \quad 7 + 10K \\ s^1 & 3 \quad 0 \\ s^0 & (7 + 10K) \end{array}$$

$$7 + 10K > 0.$$

$$\Rightarrow K > -0.7.$$

but K must be positive. So the range of stability $K > 0$ stable

So RL doesn't cross the j-axis.

[Ex] For the following Transfer function Plant

$$G(s) = \frac{1}{s(s+1)}$$

for the following closed loop poles which are located.

$$a) s_0 = -1 \pm j\sqrt{3} \quad b) s_0 = -\frac{1}{2} \pm j\sqrt{3}.$$

Show that it can be achieved by the controller K or not.

$$KGH = \frac{K}{s(s+1)}$$

$$P: 0, -1, Z: \emptyset.$$

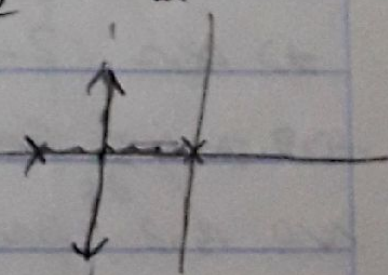
$$d = 2, \sigma_m = -\frac{1}{2}, \theta_m = \pm 90^\circ.$$

$$B.P = \frac{d}{ds} GH = 0$$

$$\Rightarrow s = -\frac{1}{2}$$

$$a) s_0 = -1 \pm j\sqrt{3} \text{ doesn't lie on RL.}$$

$$b) s_0 = -\frac{1}{2} \pm j\sqrt{3} \text{ lies on RL.}$$



$$1 + KGH = 0$$

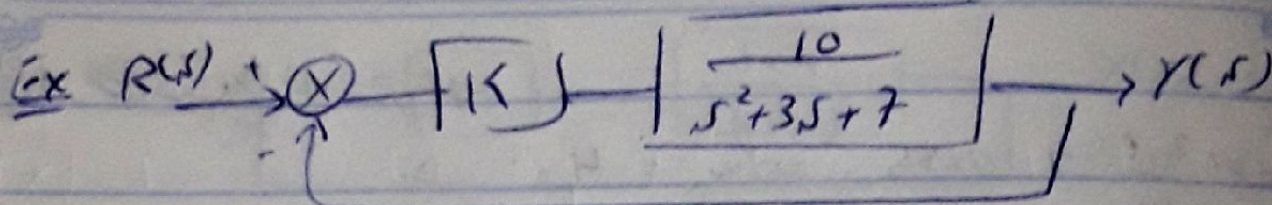
$$K = -\frac{1}{G}$$

$$K = \frac{-1}{G} = 3.25$$

$$s = -\frac{1}{2} \pm j\sqrt{3}$$

المركبة ذات ادين متساويين لكن متعاكسين

#



Design specification.

$$OS\% \leq 20\%$$

$$T_r \leq 0.55 \text{ sec}$$

$$T_s \leq 3.1 \text{ sec}$$

Find the value of K

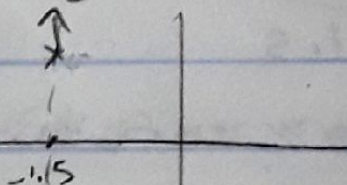
After sketch the R.L.

$$KGH = \frac{10K}{s^2 + 3s + 7}, \quad P = -1.5 + j2.1, -1.5 - j2.1$$

$$Z = \emptyset$$

$$\Rightarrow \alpha = 2, \quad \sigma_n = -\frac{3}{2} = -1.5, \quad \theta_n = \pm 90^\circ$$

No BP:



$$OS\% = 100 e^{-\frac{\pi \zeta}{\sqrt{1-\zeta^2}}} = 20 \Rightarrow \zeta = 0.456$$

$$T_s = \frac{4}{\zeta \omega_n} = 3.1 \Rightarrow \omega_n = \frac{4}{(3.1)(0.456)} = 2.89 \text{ rad/sec}$$

$$T_r = \frac{2.2}{\omega_n} = 0.76 \text{ not satisfied } T_r \leq 0.55 \text{ sec}$$

هناك علاقة كسرية بين t_s و ω_n ، ولأن ω_n تزيد، t_s تنقص.

$$\Rightarrow s_{1,2} = -\zeta \omega_n \pm j \omega_n \sqrt{1-\zeta^2}$$

$$\sigma = -2.28$$

R.L. is $\sigma = -2.28$

لذلك نكتب ان دة 0.55

$$T_s = \frac{2.2}{\omega_n} = 0.55 \Rightarrow \omega_n = 4$$

$$\xi \omega_n = 1.5 \Rightarrow \xi = \frac{1.5}{4} = 0.375 \quad \times$$

كانت قيمة 0.456 وبارساي 20% > 0.5

لوجود خطأ في السؤال ، الغي $T_r < 0.55$ بل يجب ان يكون $T_r > 0.55$

$$\text{at } OS = 20\% \Rightarrow \xi = 0.456$$

$$\text{at } T_s = 3.1 \text{ sec} \Rightarrow \omega_n = 2.89$$

$$s_{1,2} = -\xi \omega_n \pm j \omega_n \sqrt{1-\xi^2}$$

$$\xi \omega_n = 1.5$$

نبي حيتي
العدد الاقرب

$$0.456 \omega_n = 1.5 \Rightarrow \omega_n = 3.289$$

مع زيادة ω_n بعد ξ

$$\therefore \xi = 0.456, \omega_n = 3.289$$

$$\Rightarrow s_{1,2} = -1.5 \pm j 2.927$$

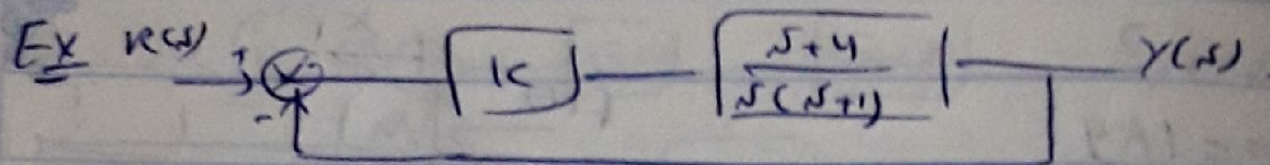
$$\Rightarrow K = -\frac{1}{G_H} \Big|_{s=-1.5+j2.927} = 2.619$$

$$\text{OR another sol} \Rightarrow \omega_n = 2.89, \xi = 1.5 \Rightarrow \xi = 0.519$$

$$\Rightarrow s_{1,2} = -1.5 \pm j 2.47$$

$$K = -\frac{1}{G_H} \Big|_{s=-1.5+j2.47} = 7.4$$

~~///~~



Find R-L, and range of K Controller.

$$KGH = \frac{K(s+4)}{s(s+1)}, \quad p = 0, -1, \quad z = -4$$

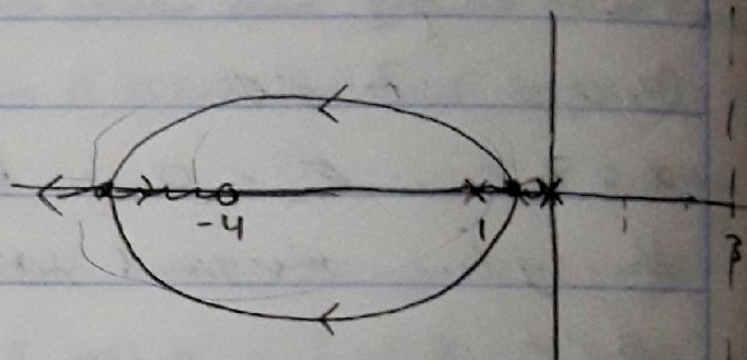
$$\Rightarrow \alpha = 2 - 1 = 1, \quad \sigma_m = \frac{-1 + 4}{1} = 3.$$

$$\phi_m = +180^\circ,$$

$$B.P = \frac{d}{ds} GH = 0$$

$$\Rightarrow s = -7.46 \quad \checkmark$$

$$= -0.535 \quad \checkmark$$



$$1 + KGH = s^2 + s + Ks + 4K = 0.$$

s^2	1	$4K$
s^1	$1+K$	0
s^0	$4K$	

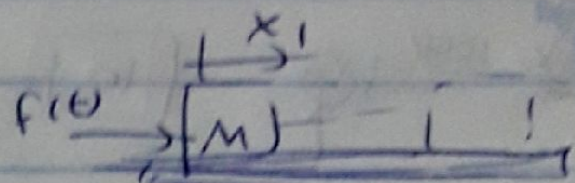
$$1 + K > 0.$$

$$\Rightarrow K > -1, \quad K \text{ must be } > 0$$

So $K > 0$ and system always stable.

Ex 8.7

$$m = 1 \text{ kg}$$



show that if K-controller

can be control this system

$$F(s) = m s^2 x(s) \Rightarrow \frac{x(s)}{F(s)} = \frac{1}{s^2}$$

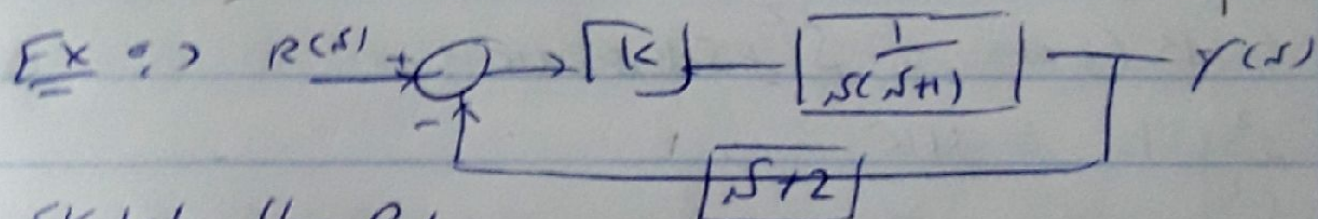
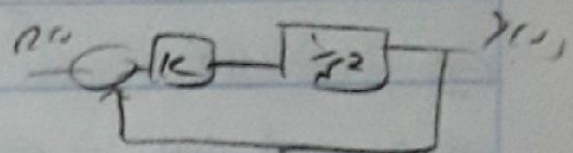
$$K_{GH} = \frac{K}{s^2}$$

$$P = 0, 0, \quad Z = \emptyset$$

$$\alpha = 2, \quad \sigma_m = 0, \quad \theta_m = \pm 90^\circ$$

the system marginally stable.

$$B.P. = 0$$



Sketch the R-L.

$$K_{GH} = \frac{K(s+2)}{s(s+1)}, \quad P = 0, -1, \quad Z = -2$$

$$\Rightarrow \alpha = 2 - 1 = 1$$

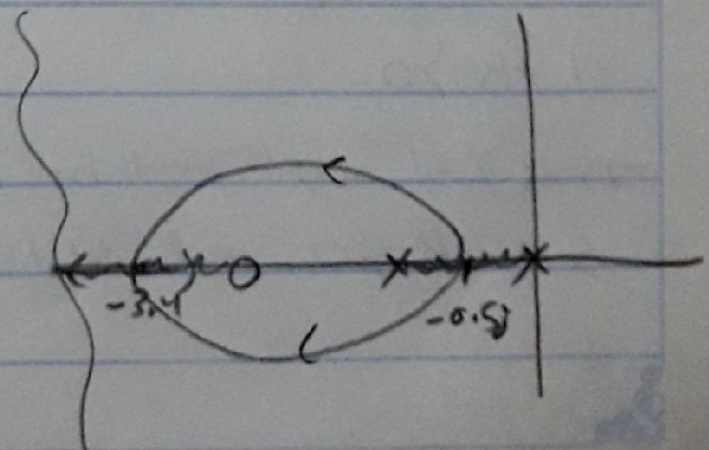
$$\sigma_m = \frac{-1 + 2}{2} = 0.5$$

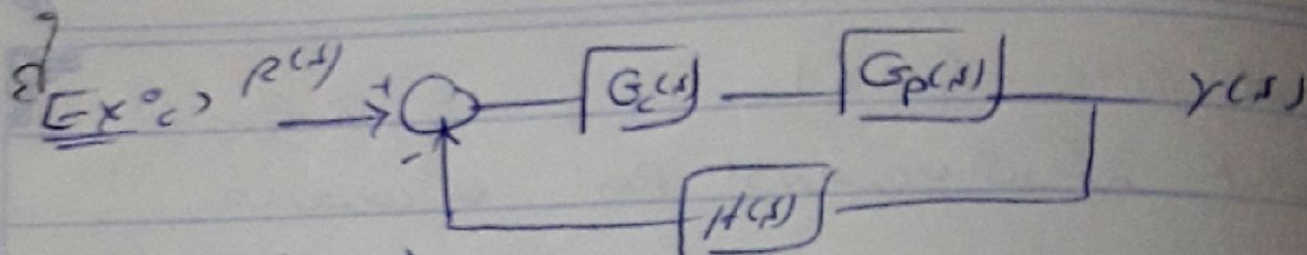
$$\theta_m = \pm 180^\circ$$

$$B.P. = \frac{2}{2} K_{GH} = 0$$

$$\Rightarrow s = -3.41$$

$$= -0.585$$





$$G_c(s) = K(s+3)$$

$$G_p(s) = \frac{0.5}{s}$$

$$H(s) = \frac{1}{(s-1)}$$

(a) Sketch R-L

(b) Value of K with location pole at $s = -1$

(c) Find the range of K for which the roots of the system chara. equation are all real.

(d) Show if K can satisfy $0.5 \leq 4.32$

$$(a) G_c G_p H = \frac{0.5K(s+3)}{s(s-1)}$$

$$T_s \leq 2.3 \text{ sec}$$

$$P: 0, +1, Z: -3 \Rightarrow d = 1 \Rightarrow \sigma_m = 1 - \frac{(-3)}{1} = 4 \Rightarrow \sigma_m = -1.5$$

$$B.P. = \frac{\sigma_m}{2} = -0.75 \Rightarrow s = -0.75$$

$$B.P. = \frac{\sigma_m}{2} = \frac{0.5(s+3)}{s(s-1)} \Rightarrow s = 0.464$$

$$s = -6.464$$

$$(b) 1 + G_c G_p H = 0$$

$$s^2 - s + 0.5Ks + 1.5K = 0$$

$$s^2 \quad | \quad 1 \quad 0.5K$$

$$s \quad | \quad 0.5K - 1 \quad 0$$

$$s^0 \quad | \quad 1.5K$$

$$0.5K - 1 > 0 \Rightarrow K > 2$$

$$\therefore K > 2$$

At $K = 2$ system marginal stable.

$$\Rightarrow s^2 + 3 = 0 \Rightarrow s = \pm j\sqrt{3}$$

~~✗~~

(C) at Poles -6.464

س.پ

فانجھو کہ Real ان تقویٰ ایڈریجہ کا ہے

$$K = \frac{1}{G(s)} \Big|_{s=-6.464} = 27.856.$$

So at $K \gg 27.856$ roots is real all.

(D) $\sigma \leq -4.3$, $\tau_s \leq 2.3$ sec.

$$OS = 100 e^{\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}} \Rightarrow \zeta = 0.707.$$

$$\tau_s = \frac{4}{\omega_n \zeta} \Rightarrow \omega_n = \frac{4}{(2.3)(0.707)} = 2.459 \text{ rad/sec}.$$

$$\Rightarrow s_{1,2} = -\zeta \omega_n \pm j \omega_n \sqrt{1-\zeta^2} \\ = -1.73 \pm j 1.739 \rightarrow \sqrt{3}.$$

back to the value of marginal poles $s_{1,2} = \pm j\sqrt{3}$.

at this point real equal zero. $= \pm j1.73$.

So there no K controller can be satisfy these condition