

## chapter 5 : Reduction of multiple subsystems.

### Model Representation:-

- ① Transfer function Model (ch 2)
- ② State space Model (ch 3)
- ③ Block diagram and signal flow graph Model (ch 5)

Model :- النموذج هي طريقة للتعبير عن أي نظام بشكل رياضي  
Model Representation :- تمثيل النموذج، هناك أكثر من طريقة للتعبير عن أي نظام بشكل رياضي

TF  $\leftrightarrow$  Freq. domain (s) لن يكون أكثر طريقة قد

SS  $\leftrightarrow$  time domain (t)

BD  $\leftrightarrow$  Freq. domain (s)

يكون فيه نظام بلوك و قد  
تستخدم لتحويل TF

SFG  $\leftrightarrow$  Freq. domain (s)

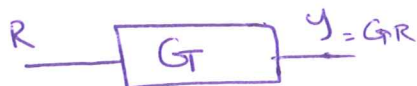
يتم استخدامه لتحويل TF

Mason's Rule

### The Interconnection types:-

- ① cascade form (product  $*$ )
- ② parallel form (algebraic summation)
- ③ feedback form

### Block diagram (BP)

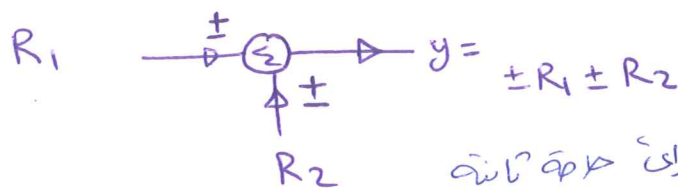


$$GT = \frac{Y}{R}$$

في هذا التتابع يكون  
عنصري BD متوحد  
مصفية متوحد  
لا يتم كل واحد  
الكل عبارة  
عن المدخل والمخرج المطلوب.

Block diagram consists of:-

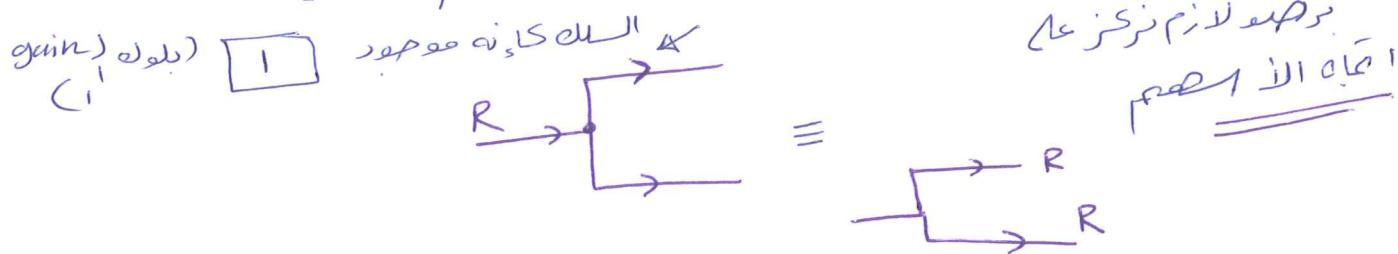
### ① Summation junction



في ال summation أدي حارة ثانية  
في هذا البند لازم شبعه لآكله لهم

كل الاشارة ( signals ) الداخلة تصح مع بعض حارة حارة كلاً بآشارة  
ومباي لآشارة بخارة مباي

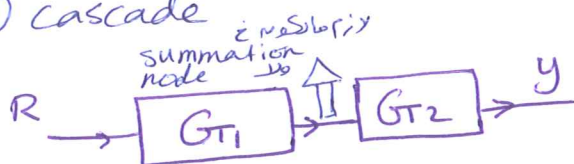
### ② pick off points ( النقطة توزيع )



برصه لازم نركز على  
اقدام ال ادهم

BD interconnection:-

### ① cascade



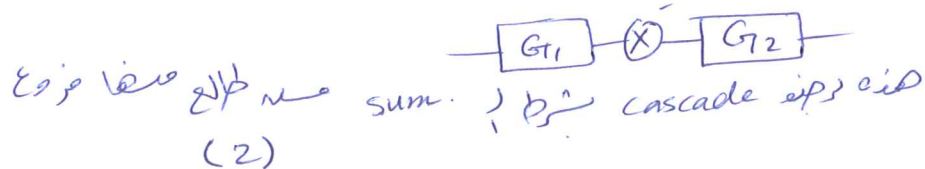
ال cascade نقل عملية افر في البوكات اجزي انه  
مخرج البوك الاول هو مدخل البوك الثاني طبقت لآكله بينهم  
لا summation ولا pick up point لو كان في بينهم ينقل  
حركه (movement)

$$TF = \frac{y}{R} = GT_1 * GT_2$$



البوك لآكله عن  
التيه

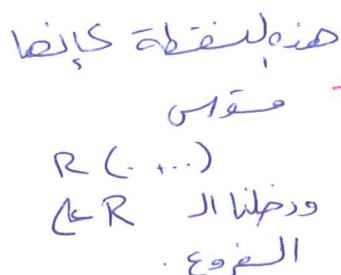
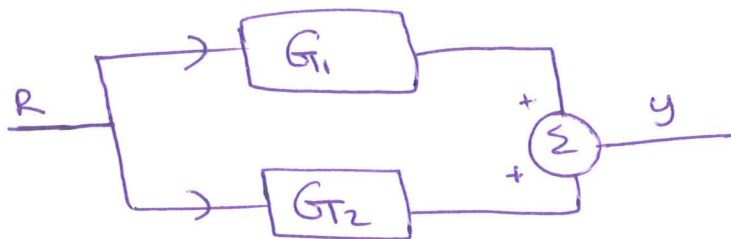
مدخله لو كان عنده



هذه رجة cascade شرط في sum. من طالع صفا فروع  
(2)

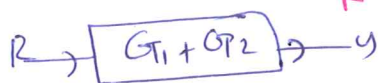
② parallel

(تَوَازِي بَعْضِ عَالَمِيَّاتٍ مَعَ رَحْمَتِ)  
بَعْضِ اَحْبَابِ (اَبِ اسْم)



$$y = y_1 + y_2 = R G_1 + R G_2$$

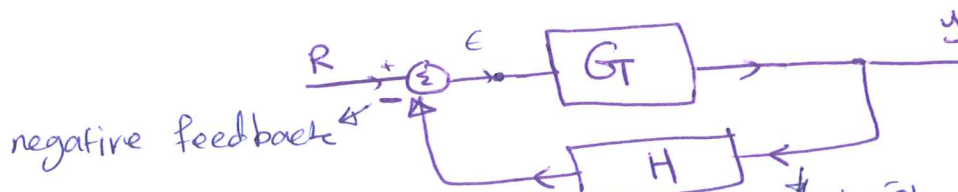
$$\frac{y}{R/y} = G_{T1} + G_{T2}$$



نوع لسته

یعنی جمع صوری

③ feedback closed loop form :-  $\frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s)H(s)}$



$$E = R - H_y$$

$$y = e \in G$$

$$E \mathfrak{g} = R - H \in G$$

$$y_e = \frac{R}{1+GTH} \rightarrow \text{Error signal}$$

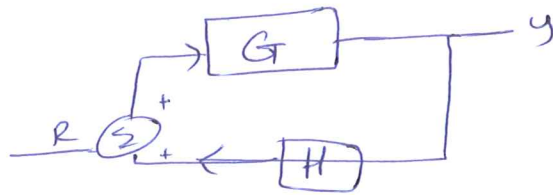
$$y = e \in G \rightarrow e = \frac{y}{G}$$

$$\frac{y}{G} = \frac{R}{1+G_H} \rightarrow \frac{y}{R} = \frac{G}{1+G_H}$$

(3)

GT ۱۰۰٪  
 البطل في  
 من اجل  
 Feedback 11  
 من اجل  
 من اجل

في حال positive feedback



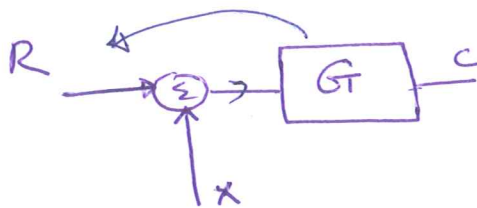
$$\frac{Y}{R} = TF = \frac{G}{1 - GH}$$

علامة إشارة ال feedback

الآن لو كان عندي BD لكن لا يمكن انه التوصل الى الأساسية  
 parallel, feedback, cascade بروح مباشرة ببساطة كل شيء قانونه  
 ولازم انتبه للاشياء.   
 لكن لو لا يمكن وجود توصلة جديدة من هنا، لتوصلة، الأساسية كوجود  
 node او summ.   
 هناك cascade ليس بينهم   
 فنعمل تحريك للبلوكات حيث يصبح (توصلة قبل التوصلة، لتقارف على  
 Block Diagram

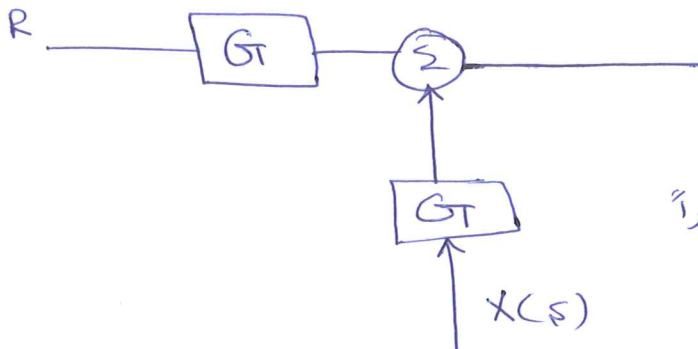
Block diagram Reduction Movement: - Movement)

① More left past summing junctions



طرحاً التحريك من اليمين  
 للتحريك اوجه التحرك لليسار  
 لليمين

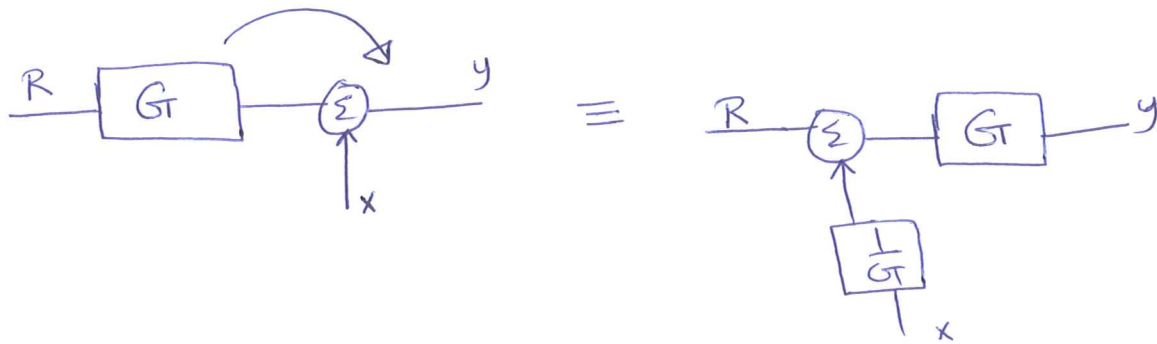
≡



تكنيصة هذه اللازمة  
 لو بي اترك بلوك للتحريك لليسار  
 عند احده  
 المرفوض بينكون بكل  
 الحيل (اللون) الموصولة على  
 هذه انما اشارته وبين انما انما المرفوض

② Move Right past summing junction.

نقلنا صمد الطريقة من على السابقة  
تماماً



تأثيره الطريقة السابقة :-  
1- في حال كنت تريد ازالة بلوك للبارصه ابعينه بروج لفرع البلوك في كل  
الفرع وبتليه صفره

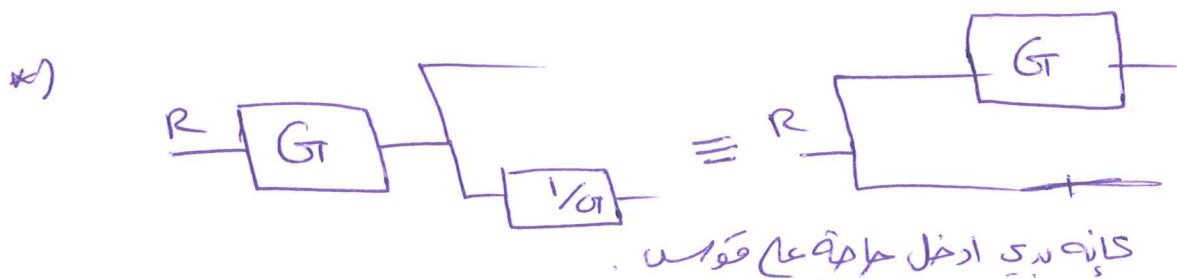
2- في حال كنت تريد ازالة بلوك للبارصه ابعينه بروج لفرع البلوك في  
الفرع نفس اتجاه (direct from input to output) ولباغ بفتح على الـ G وخرجاً بيتان  
صفره

③ Move Right or left past pick point junction

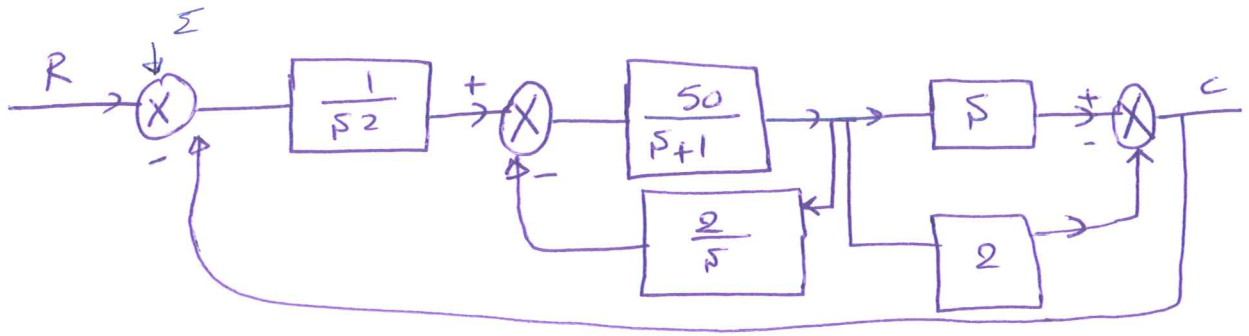
النقل كذا (نقل)



$$R(1+G) \equiv RG(1+\frac{1}{G})$$



5.1 Reduce the block diagram shown below to a single TF = C/R use the Block diagram Reduction.

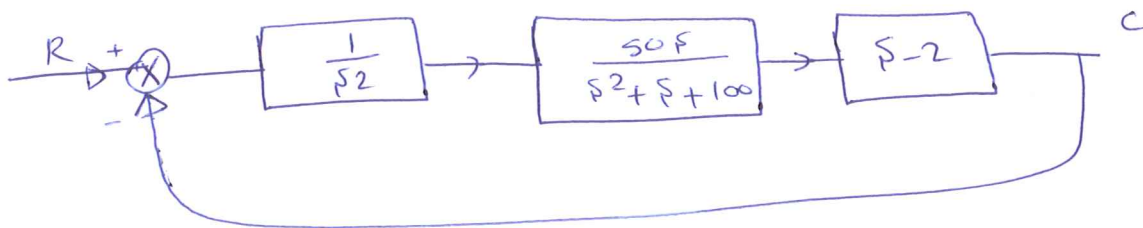


2 and s parallel

$s-2$

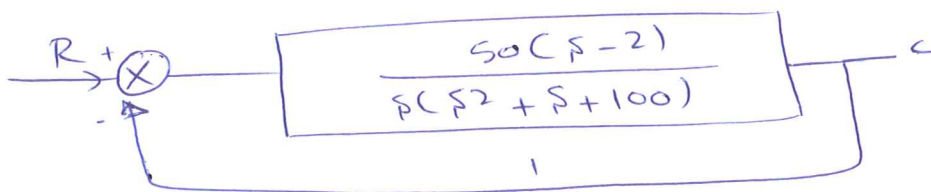
$\frac{50}{s+1}$  and  $\frac{2}{s}$  feed back

$$\frac{\frac{50}{s+1}}{1 + \frac{50}{s+1} \cdot \frac{2}{s}} = \frac{50s}{s^2 + s + 100}$$



$\frac{1}{s^2}$  and  $\frac{50s}{s^2 + s + 100}$  and  $s-2$  all cascade

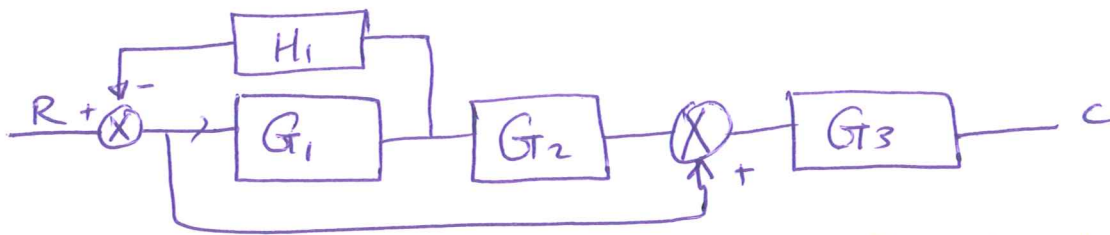
$$\frac{50s}{s^2 + s + 100} \cdot \frac{1}{s^2} \cdot (s-2) = \frac{50(s-2)}{s(s^2 + s + 100)}$$



unity negative feedback

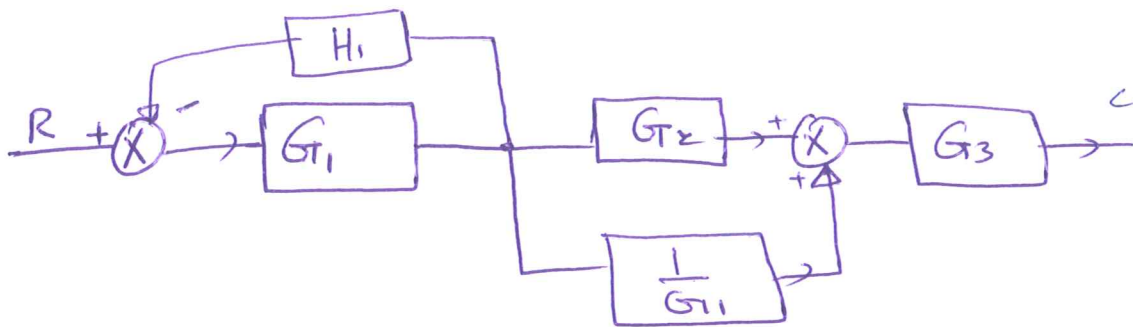
$$T = \frac{C}{R} = \frac{50(s-2)}{s(s^2 + s + 100)} \cdot \frac{1}{1 + \frac{50(s-2)}{s(s^2 + s + 100)}} = \frac{50(s-2)}{s^2 + s^2 + 150s + 100}$$

5.2 find the closed-loop TF function:  $T = C/R$  for the system shown below, using block diagram reduction.



نلاحظ ان النظام يحتوي على حلقة مغلقة مع feedback cascade, parallel, لذلك لا بد من فصل خزانة.

لنقوم بفتح حلقة  $G_1$  التي ليس لها مخرج.



$G_2$  and  $\frac{1}{G_1}$  parallel

$$\left( \frac{1}{G_1} + G_2 \right)$$

$H_1$  and  $G_1$  feedback

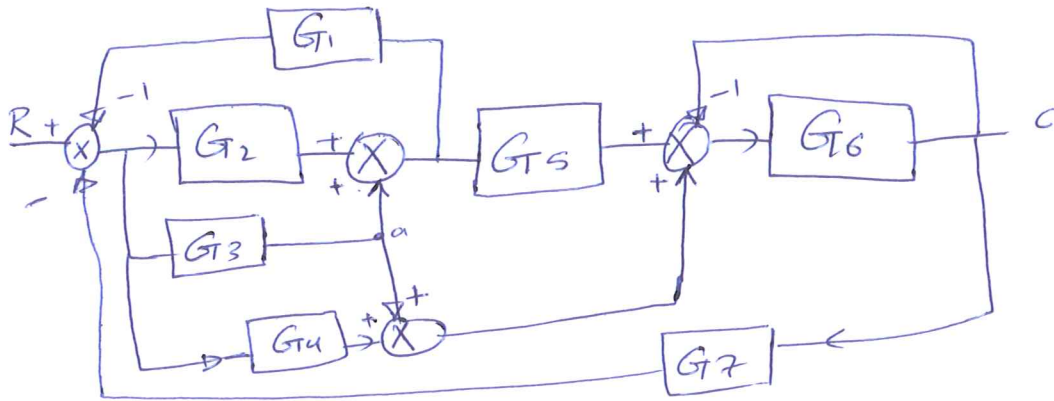
$$\frac{G_1}{1 + G_1 H_1}$$



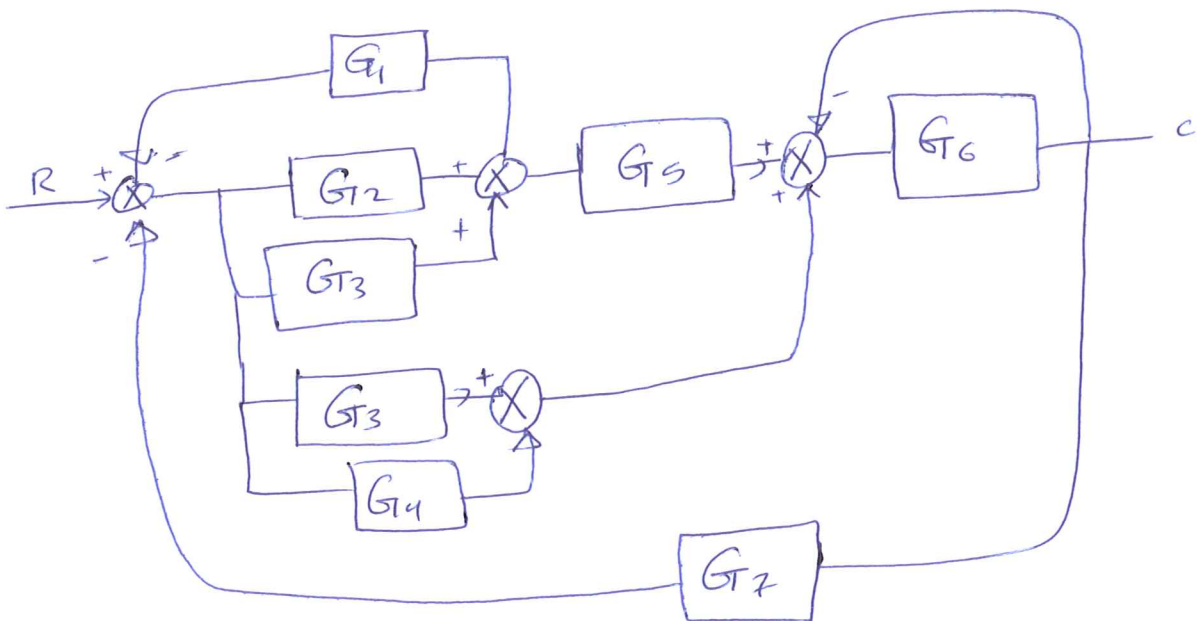
Now all cascade

$$\left( \frac{G_1}{1 + G_1 H_1} \right) \left( \frac{1}{G_1} + G_2 \right) (G_3) = \frac{(G_1 G_2 + 1) G_3}{(1 + G_1 H_1)}$$

5.3 Find the equivalent tf,  $T=C/R$ , for the system shown below



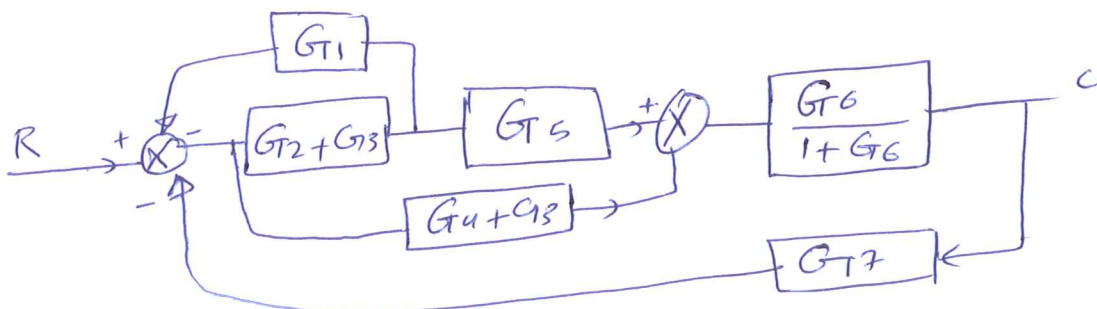
Move  $G_3$  from the node a then split it



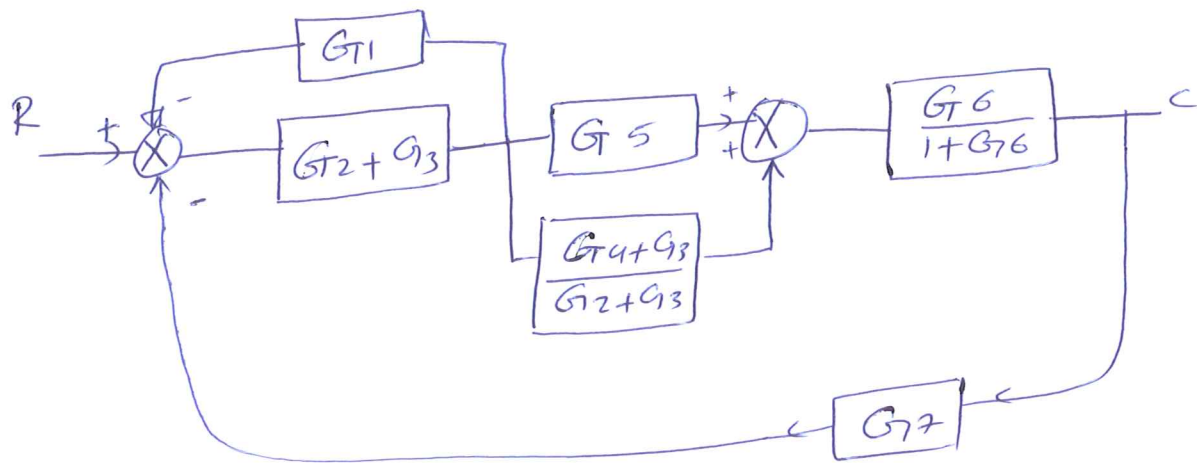
$G_2$  and  $G_3$  parallel  $G_2 + G_3$

$G_3$  and  $G_4$  parallel  $G_3 + G_4$

$G_6$  and 1 feedback  $\frac{G_6}{1 + G_6}$

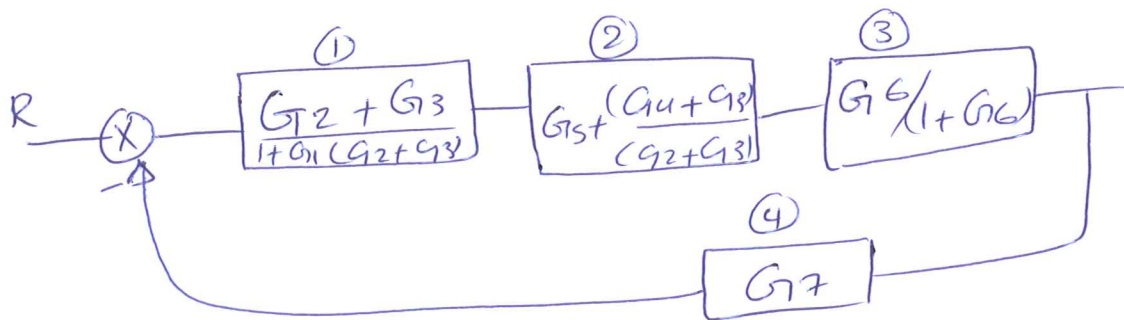


push  $(G_2 + G_3)$  to the left past the pick point



$G_1$  and  $G_2 + G_3$  feedback  $\frac{G_2 + G_3}{1 + G_1(G_2 + G_3)}$

$G_5$  and  $\frac{G_4 + G_3}{G_2 + G_3}$  parallel  $G_5 + \left( \frac{G_4 + G_3}{G_2 + G_3} \right)$



1 and 2 and 3 cascade  $① * ② * ③ =$

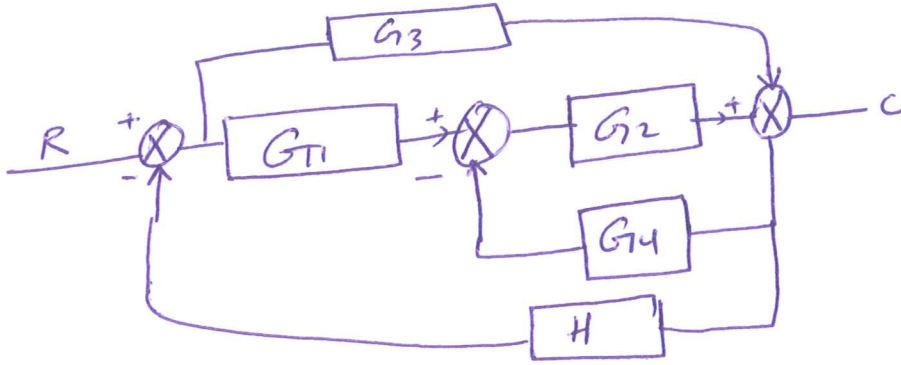
$$= \frac{G_2 + G_3}{1 + G_1(G_2 + G_3)} \cdot \left[ G_5 + \left( \frac{G_4 + G_3}{G_2 + G_3} \right) \right] \cdot \left( \frac{G_6}{1 + G_6} \right)$$

$= M$

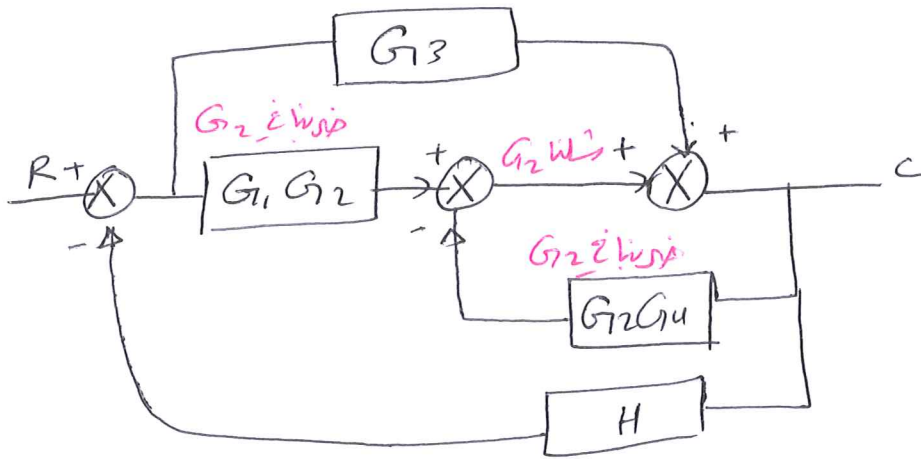
then  $M$  feedback with  $G_7 \rightarrow \frac{M}{1 + G_7 M}$

$$\frac{C}{R} = \frac{G_6 G_4 + G_6 G_3 + G_6 G_5 G_3 + G_6 G_5 G_2}{1 + G_6 + G_3 G_1 + G_2 G_1 + G_7 G_6 G_4 + G_7 G_6 G_3 + G_7 G_6 G_5 G_3 + G_7 G_6 G_5 G_2 + G_6(G_4 + G_3)}$$

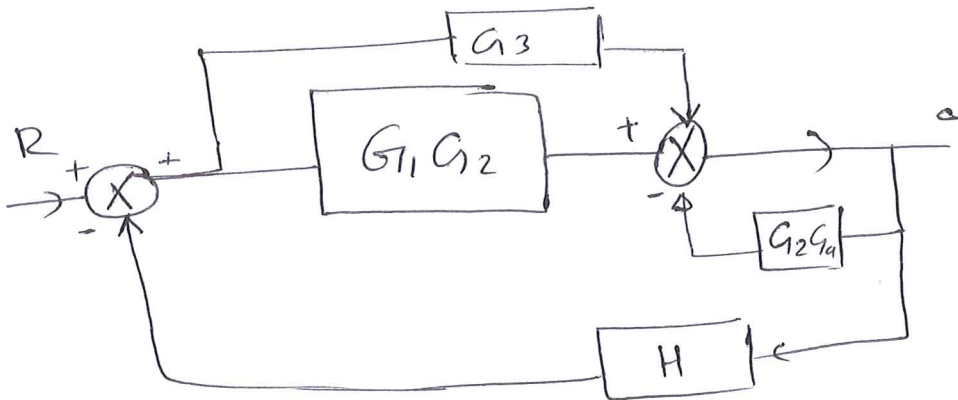
5.4 Reduce the system shown below to a single TF =  $C/R$ .



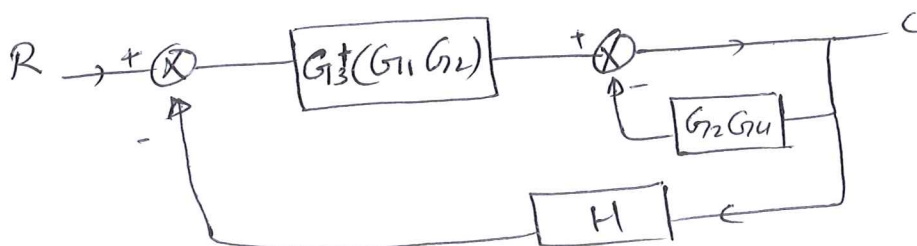
push  $G_2$  to the left past the summing junction



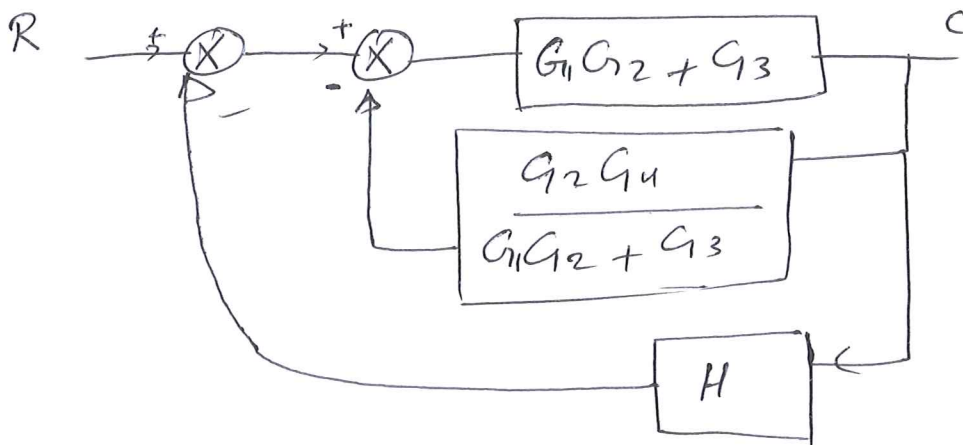
Summation junction Gain = 1  
Summation is no longer needed



$G_3$  and  $G_1G_2$  parallel  $G_3 + G_1G_2$

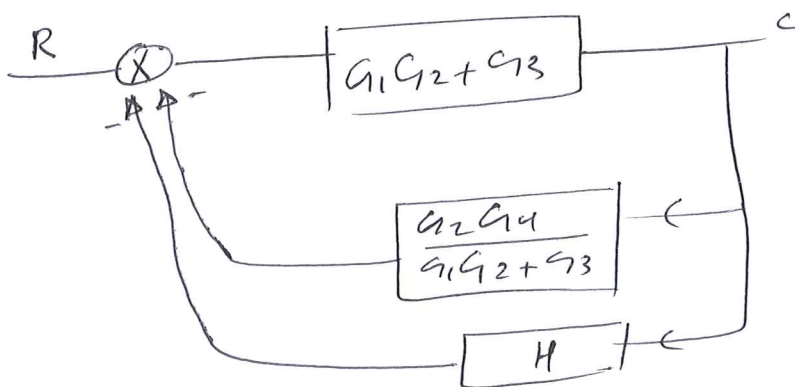


push  $G_3 + G_2G_1$  to the right past the summing junction

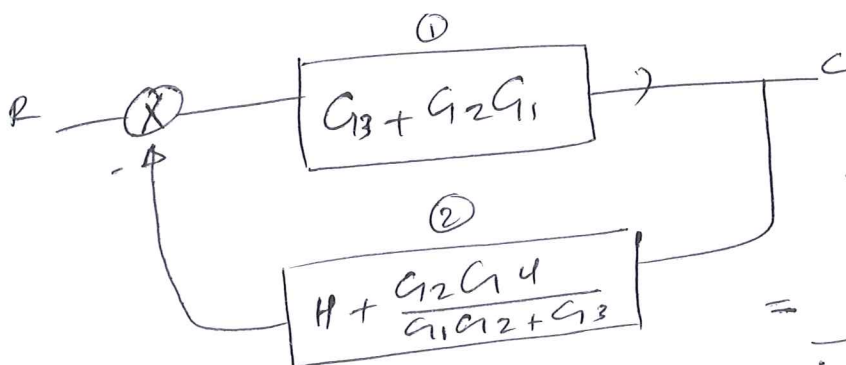


2 summation between it Gain = 1 then  
collapse to 1 summation

Now  $H$  and  $\frac{G_2G_4}{G_1G_2 + G_3}$  parallel



$$H + \frac{G_2 G_4}{G_1 G_2 + G_3}$$

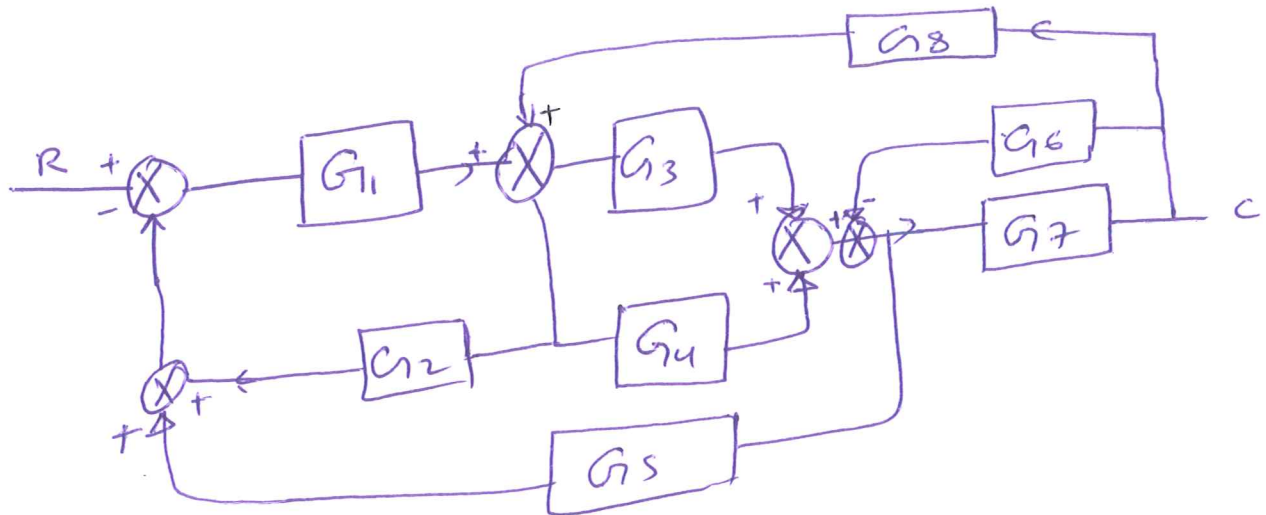


Now feed back

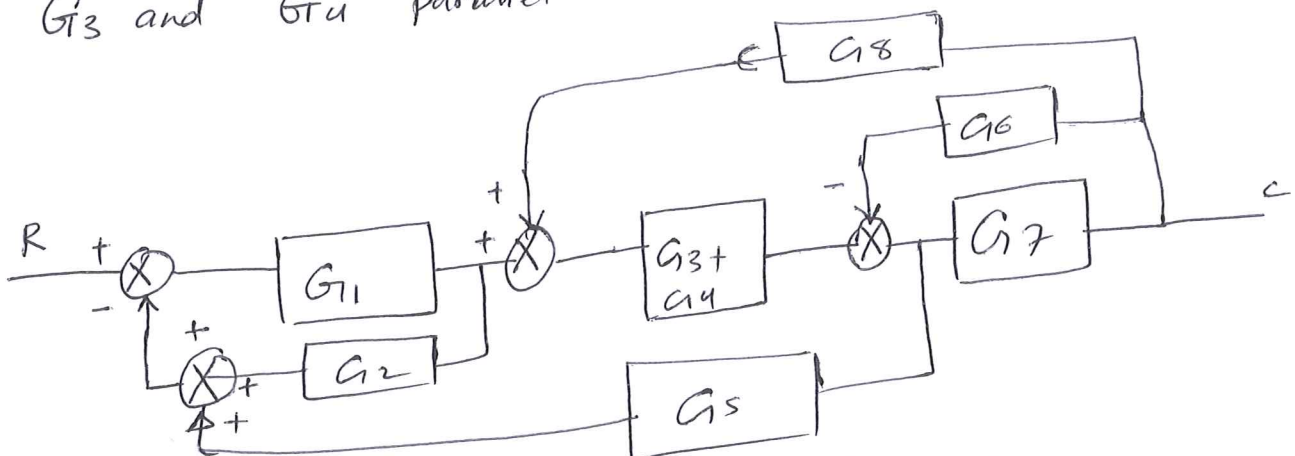
$$\frac{\textcircled{1}}{1 + \textcircled{1}\textcircled{2}} = \frac{G_3 + G_1G_2}{1 + H(G_3 + G_1G_2) + G_2G_4}$$

5.5

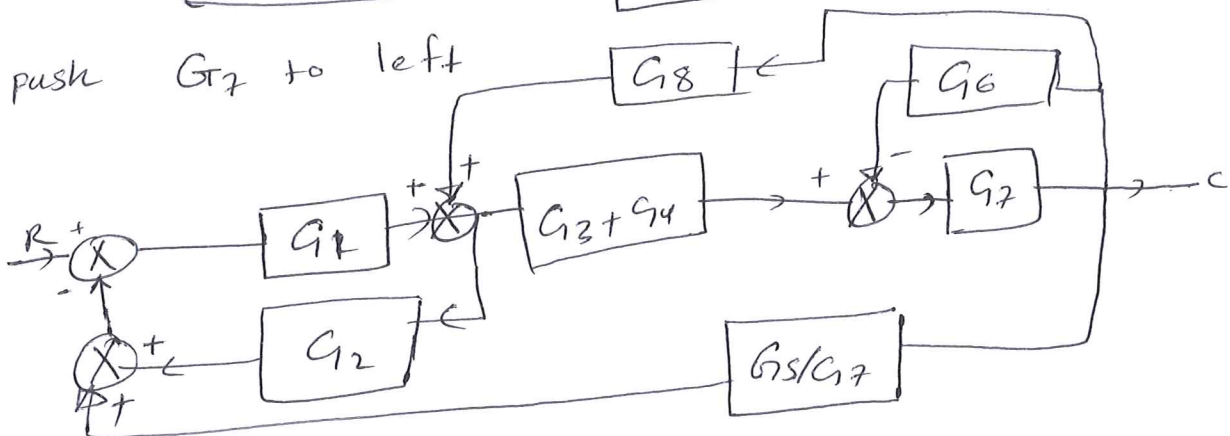
Find  $T = C/R$  by using BD Reduction



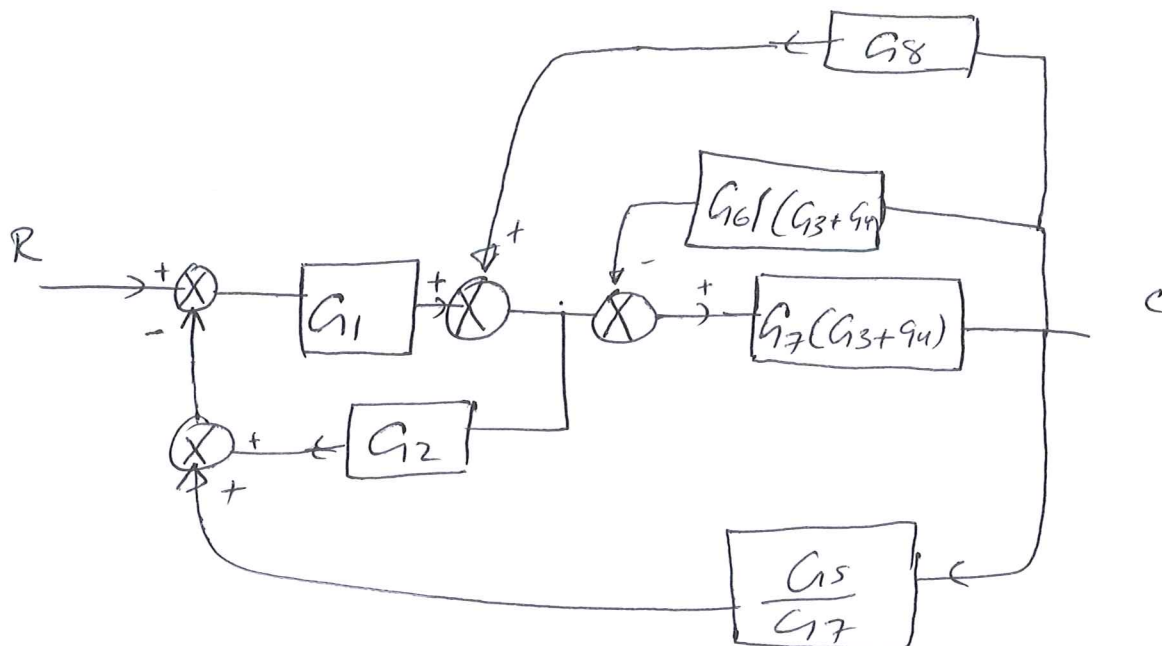
$G_3$  and  $G_4$  parallel



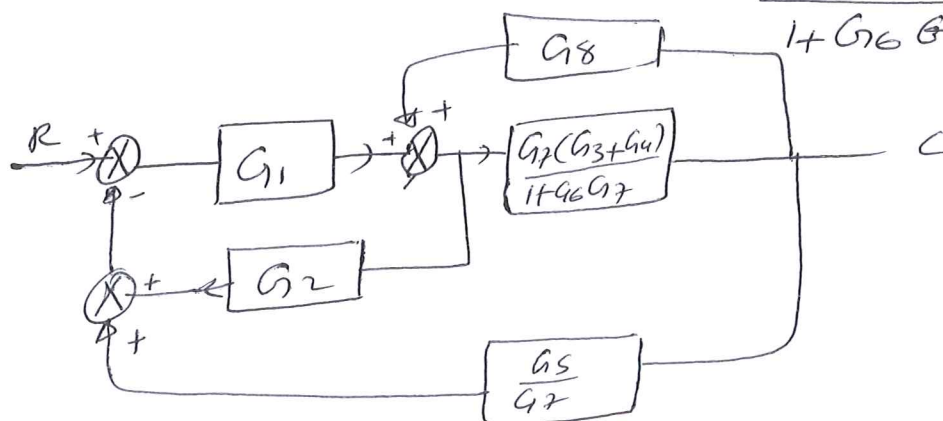
push  $G_7$  to left



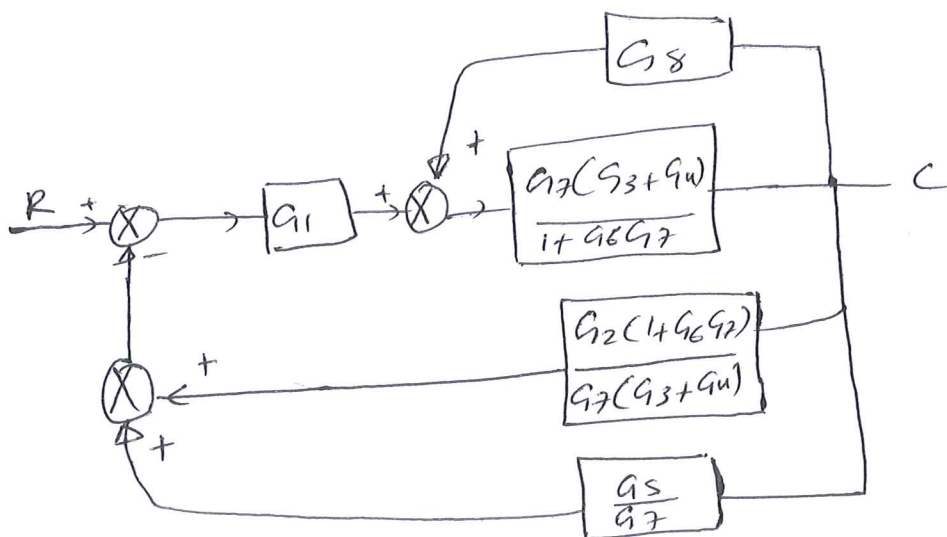
push  $G_3 + G_4$  to the right past the summing junction.



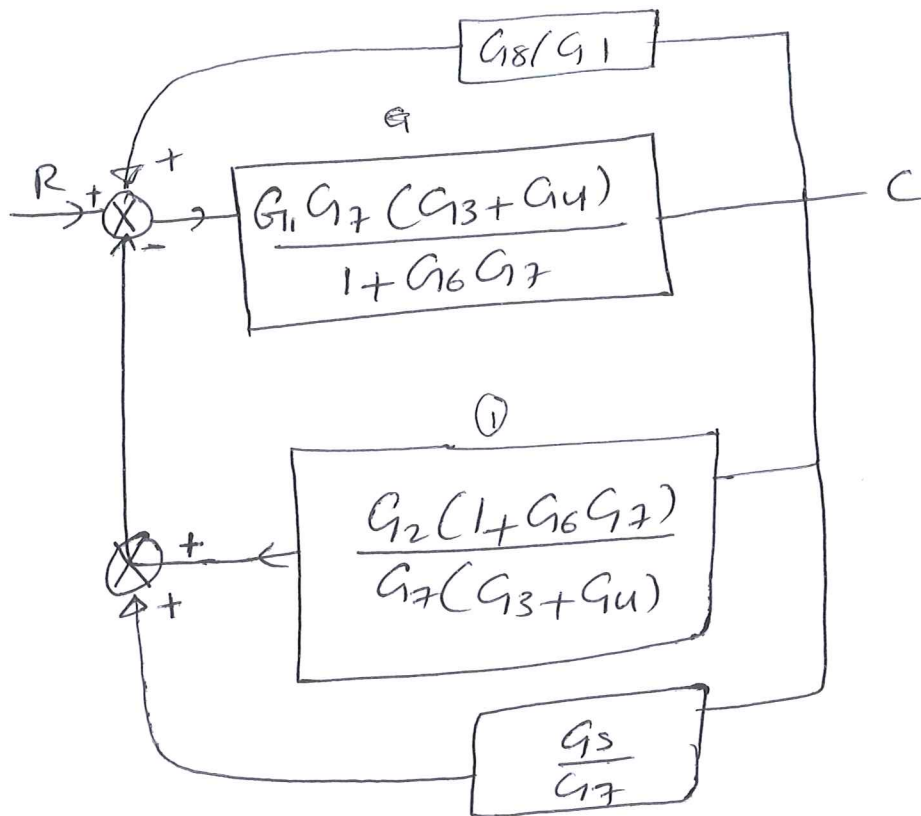
$G_7(G_3+G_4)$  and  $\frac{G_6}{G_3+G_4}$  feedback  $\frac{G_7(G_3+G_4)}{1 + G_7 G_6 (G_3+G_4) (G_3+G_4)}$   
 $= \frac{G_7(G_3+G_4)}{1 + G_6 G_7}$



push  $\frac{G_7(G_3+G_4)}{1+G_6G_7}$  to the left past the pickoff point



push  $G_1$  to the right past the summing junction



Now  $\frac{G_5}{G_7}$  and  $G_8/G_1$  and Block ① are parallel

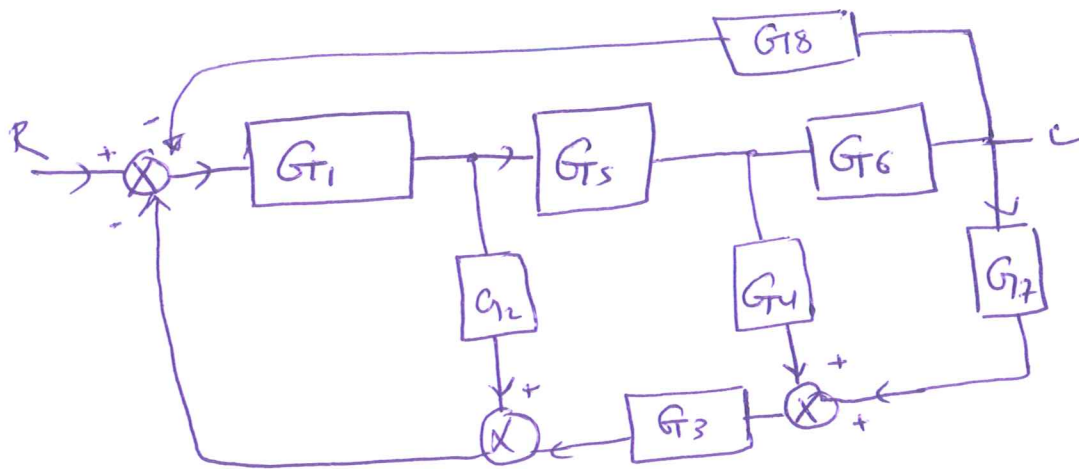
$$H = \frac{G_5}{G_7} + \frac{G_2(1+G_6G_7)}{G_7(G_3+G_4)} - \frac{G_8}{G_7}$$

ملاحظات  
بالإضافة  
Negative feedback  
هو إشارة موجبة  
في  
positive  
feedback

$$T = \frac{G}{1+GH} =$$

$$\frac{G_7 G_1 (G_4 + G_3)}{( [G_7 G_6 + 1] G_2 G_1 + [G_4 + G_3] [G_5 G_1 - G_8 G_7] ) + (G_7 G_6 + 1)}$$

5.6 use BD Reduction to find  $T = C/R$



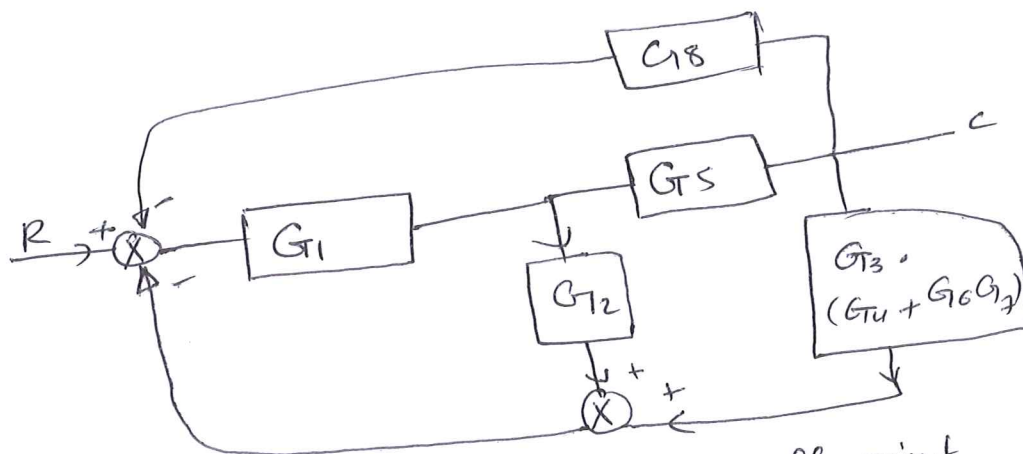
$G_6$  and  $G_7$  series (cascade)  $G_6 \cdot G_7$

then parallel with  $G_4$

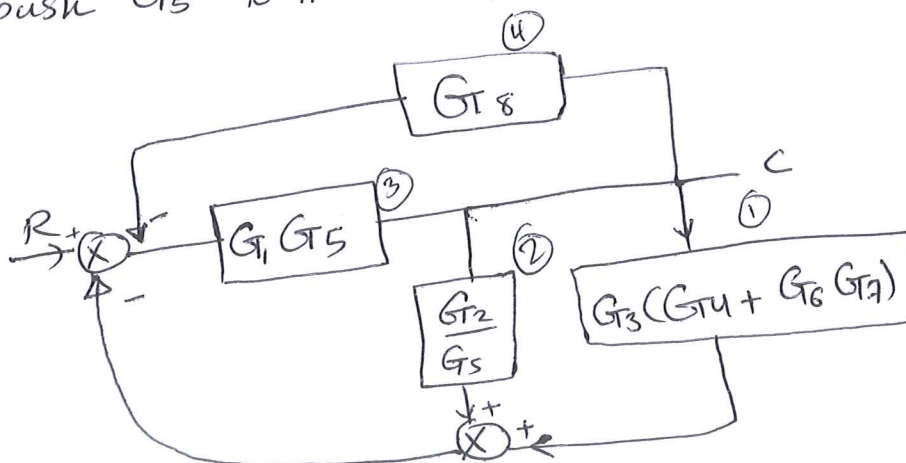
$$G_4 + G_6 G_7$$

then cascade with  $G_3$

$$G_3 (G_4 + G_6 G_7)$$



push  $G_5$  to the left past the pickoff point



(15)

Now ① and ② parallel and ③

$$G_3(G_4 + G_6 G_7) + G_8$$

على الإشارة لا بد من  
إشارة الإشارة لا بد من  
الأصل هو

$$G_3(G_4 + G_6 G_7) - G_8$$

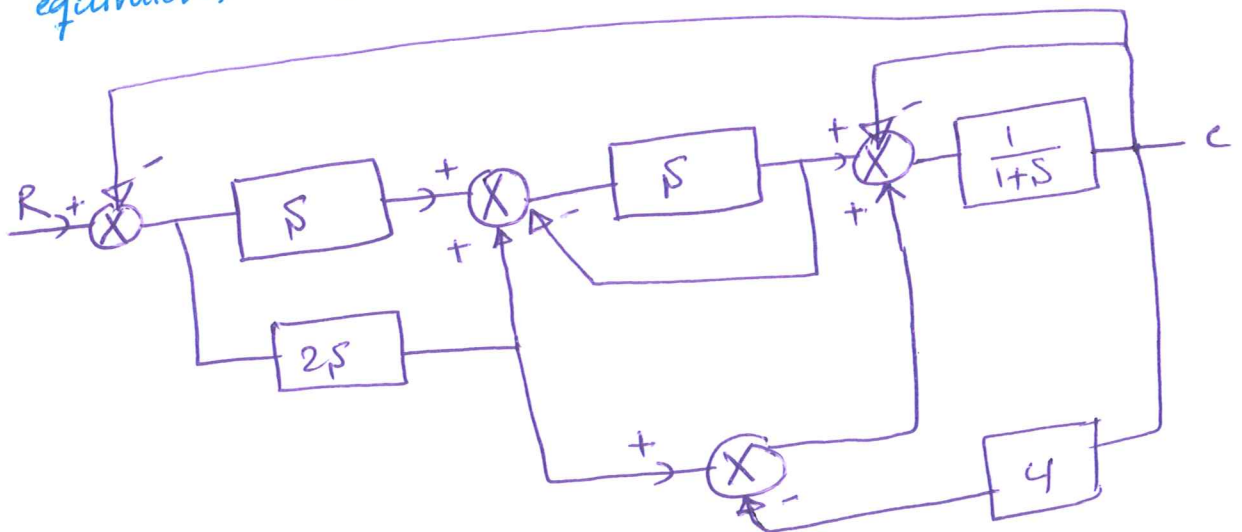
في إشارة - على إشارة  
إشارة الإشارة لا بد من

Negative feedback  
TF لا بد من  
الأصل هو

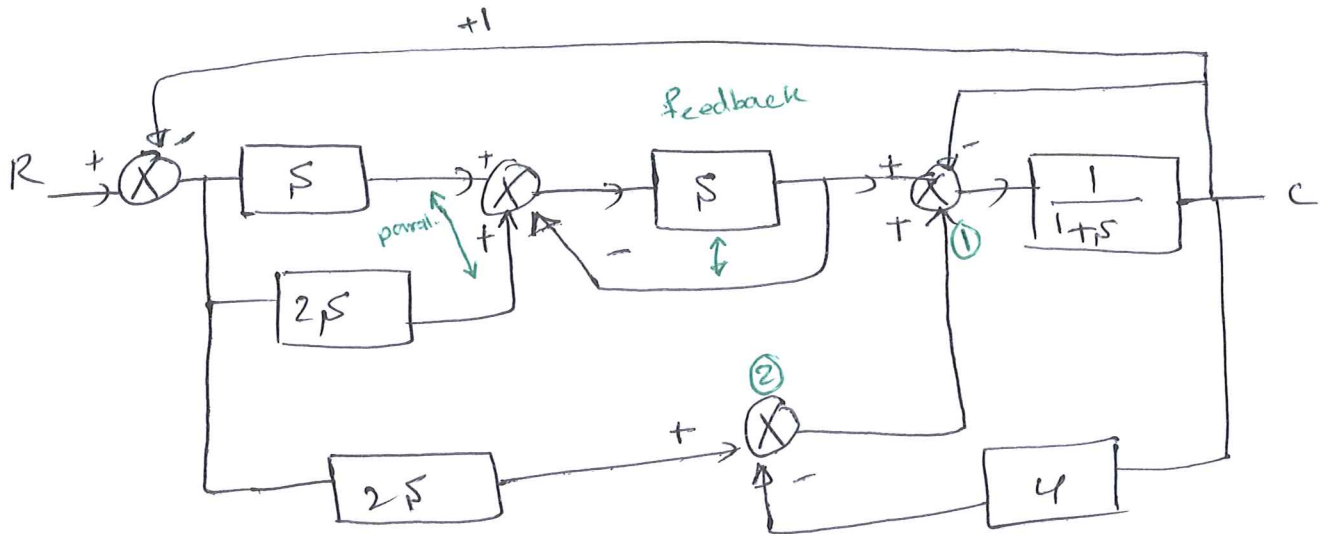
Now  $G_3(G_4 + G_6 G_7) + G_8$  Feedback with  $G_1 G_5$

$$T = \frac{G_2 G_1}{1 + G_1 (G_8 G_5 + G_7 G_6 G_5 G_3 + G_5 G_4 G_3 + G_2)}$$

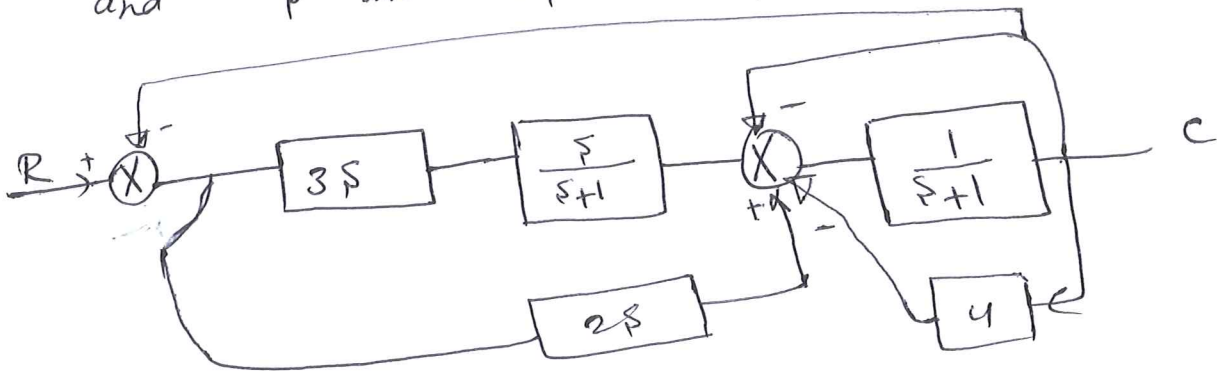
5.7 Find the unity feedback system that is equivalent to the system shown below



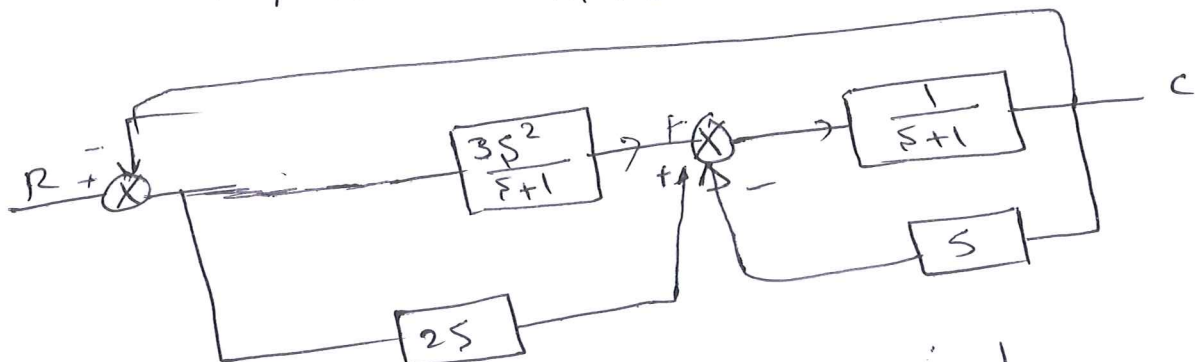
push 25 to the right past the pick off point



collapse and summation ① and ②  
 $S$  and  $2S$  are parallel  $S+2S=3S$

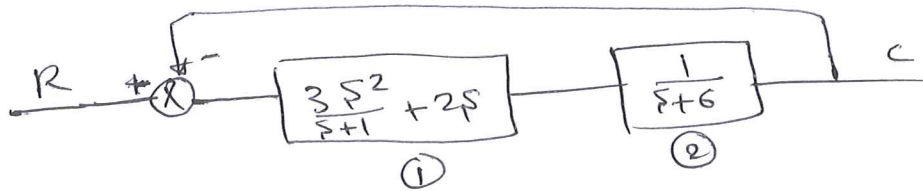

$$3f \text{ and } \frac{f}{f+1} \text{ cascade} \rightarrow \frac{3f^2}{f+1}$$

4 and 1 parallel  $1+4 = 5$



$\frac{1}{s+1}$  and  $s$  feed back  $\rightarrow \frac{1}{s+6}$

$$\frac{3s^2}{s+1} \text{ and } 2s \xrightarrow{\text{Parallel}} \frac{3s^2}{s+1} + 2s$$



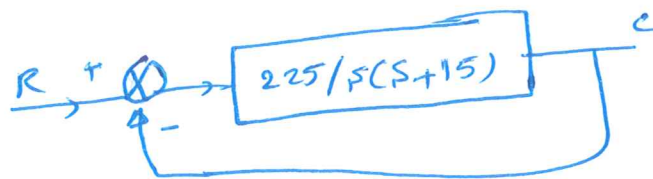
① and ② cascade then unity Negative feedback

$$T = \frac{5s^2 + 2s}{6s^2 + 9s + 6}$$

Analysis and Design of feedback system.

(controller)  $k$  give us  
 $\uparrow$   $\downarrow$   $\uparrow$   $\downarrow$   $\uparrow$   $\downarrow$   
 OS,  $T_s$  ---  $\downarrow$

S.11 For the system shown below  
 find OS,  $T_s$ ,  $T_p$  for a step input.



$$\frac{225}{s(s+15)} \text{ and } \text{feedback } T = \frac{225}{s^2 + 15s + 225}$$

second order TF formula  $\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$$\omega_n^2 = 225$$

$$\omega_n = 15$$

$$2\zeta\omega_n = 15$$

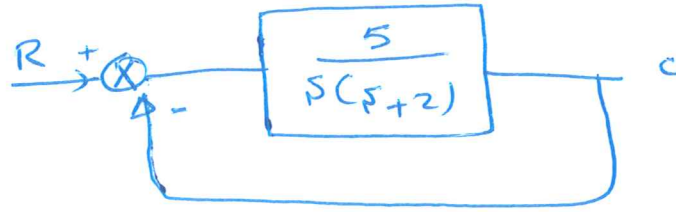
$$\zeta = \frac{1}{2}$$

$$OS = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100\% = 16.3\%$$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{7.5} = 0.533$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.242$$

S.12 for the system shown below, find the output  $c(t)$ , if the input  $r(t)$  is a step unit.



$$\frac{C}{R} = \frac{\frac{5}{s(s+2)}}{1 + \frac{5}{s(s+2)}} = \frac{5}{s(s+2) + 5} = \frac{5}{s^2 + 2s + 5}$$

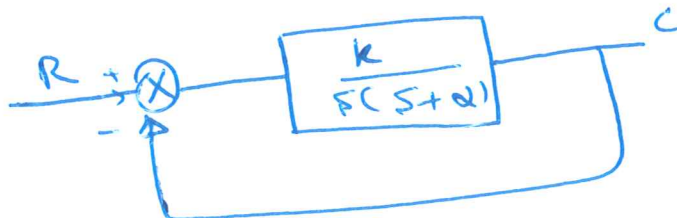
$r(t)$  a step  $\longrightarrow R(s) = \frac{1}{s}$

$$C(s) = \frac{5}{s^2 + 2s + 5} \cdot R = \frac{5}{s^2 + 2s + 5} \cdot \frac{1}{s} = \frac{5}{s(s^2 + 2s + 5)}$$

$\mathcal{L}^{-1}$  المبرورون نقل

$$c(t) = 1 - e^{-t} \left( \cos 2t + \frac{1}{2} \sin 2t \right)$$

S.15 for the system shown below, find  $k$  and  $\alpha$  to yield a settling time of 0.15s and a 30% overshoot



$$\frac{C}{R} = \frac{\frac{k}{s(s+\alpha)}}{1 + \frac{k}{s(s+\alpha)}} = \frac{k}{s^2 + \alpha s + k}$$

$$0.5 = 0.3 = e^{-\pi \beta / \sqrt{1-\beta^2}}$$

$$\boxed{\beta = 0.358}$$

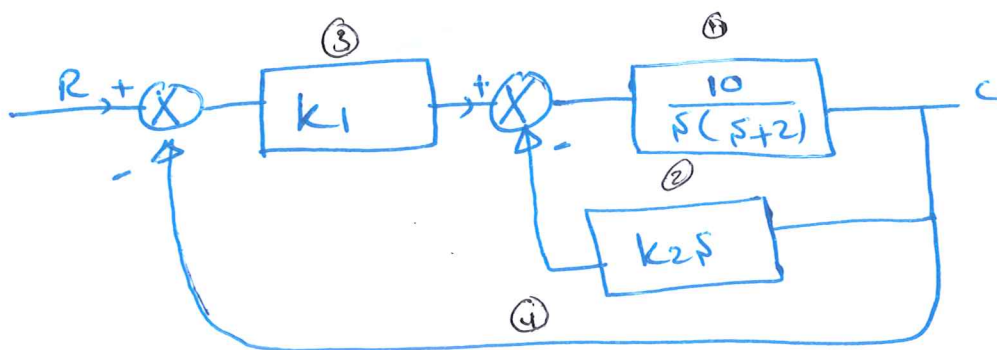
$$\frac{4}{\beta \omega_n} = 0.15 = \frac{4}{0.358 \omega_n} \rightarrow \omega_n = 74.49$$

$$\frac{k}{s^2 + \alpha s + k} = \frac{k d e \omega_n^2}{s^2 + 2 \beta \omega_n s + \omega_n^2}$$

$$k = \omega_n^2 = 5549$$

$$\alpha = 2 \beta \omega_n = 53.33$$

S.16 Find the values of  $k_1$  and  $k_2$  to yield a peak time of 1.5 s and a settling time of 3.2 s for the closed loop system's step response



① and ② feedback then cascade with 3 then feedback with ④

$$\frac{C}{R} = \frac{10 k_1}{s^2 + (10 k_2 + 2) s + 10 k_1}$$

$$\frac{4}{\beta \omega_n} = 3.2 \rightarrow \beta \omega_n = 1.25$$

$$(10 k_2 + 2) = 2 \beta \omega_n = 2 \times (1.25) = 2.5$$

$$\boxed{k_2 = 0.05}$$

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\beta^2}} = 1.5, \quad \omega_n \sqrt{1-\beta^2} = \frac{\pi}{1.5} = 2.09$$

$$T_s = \frac{4}{8\omega_n} \rightarrow 8\omega_n = 1.25 \rightarrow \beta = \frac{1.25}{\omega_n}$$

$$\omega_n \sqrt{1 - \frac{(1.25)^2}{\omega_n^2}} = 2.09$$

$$\omega_n^2 - (1.25)^2 = (2.09)^2$$

$$\omega_n^2 = (1.25)^2 + (2.09)^2 =$$

$$\omega_n^2 = 10k_1$$

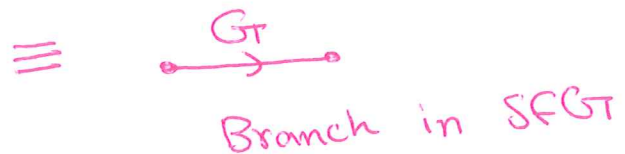
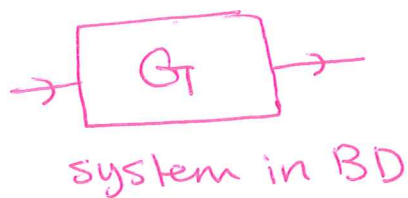
$$k_1 = \frac{\omega_n^2}{10} = \frac{(1.25)^2 + (2.09)^2}{10} = 0.593$$

Signal flow graphs (SFGT)

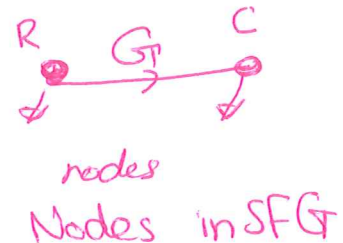
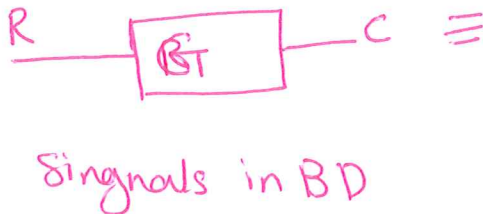
طريقة اخرى للتحليل  
عن  
system

\* SFGT consists of:-

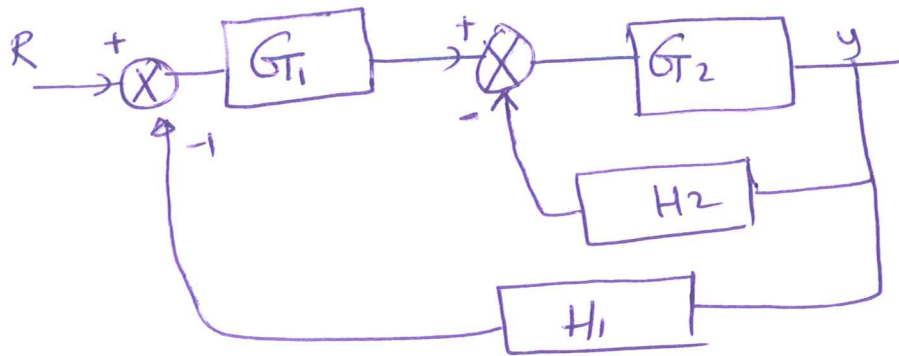
① Branches



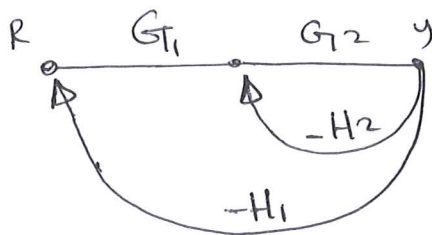
② Nodes



Q:- Convert the BD to SFGT



كل بلوك يقبل لورانس غير ذلك لنور



Mason's Rule: طريقة لتحويل SFGT وإيجاد TF

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{\Delta} \sum_{k=1}^P M_k \Delta_k$$

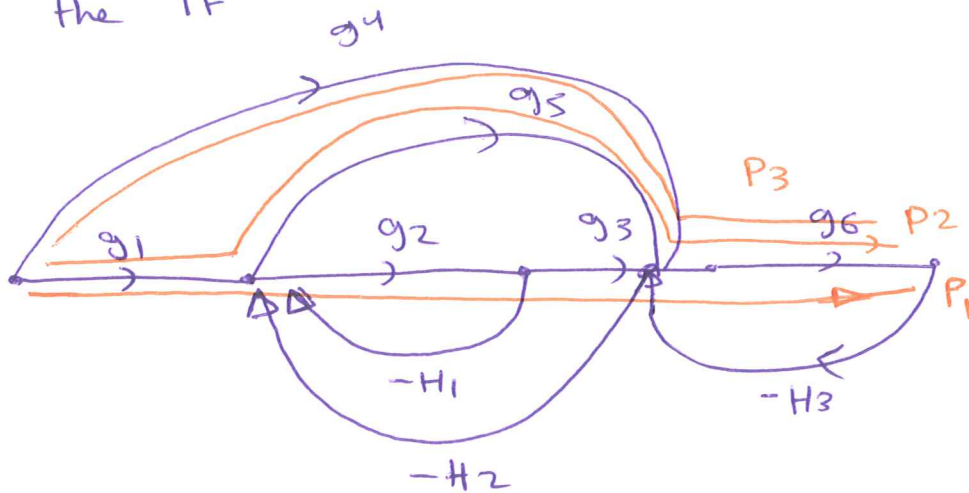
$P \rightarrow$  \* forward path جميع المسارات غير المتكررة  
التي توصل بين مدخل المخرج (من الإدخال للإدخال)

$M_k \rightarrow$  the gain of  $k^{th}$  forward path خلال أي مسار  
جميع ضربات ال gains

$\Delta = 1 - \sum \text{single loop gain} + \sum \text{non-touching 2 loops} - \dots$   
 $\sum \text{non touching 3 loops} + \dots$

$\Delta_k = 1 - \sum \text{loop gain terms in } \Delta \text{ that touch the } k^{th} \text{ forward path.}$

Find the TF



$$G_T = \frac{1}{\Delta} \sum_{k=1}^P M_k \Delta_k$$

$P \rightarrow$  عدد المسارات التي تصل بين المدخل والمخرج  
مع الاستبعاد لبقية المسارات  
مع الاستبعاد لبقية المسارات = 3

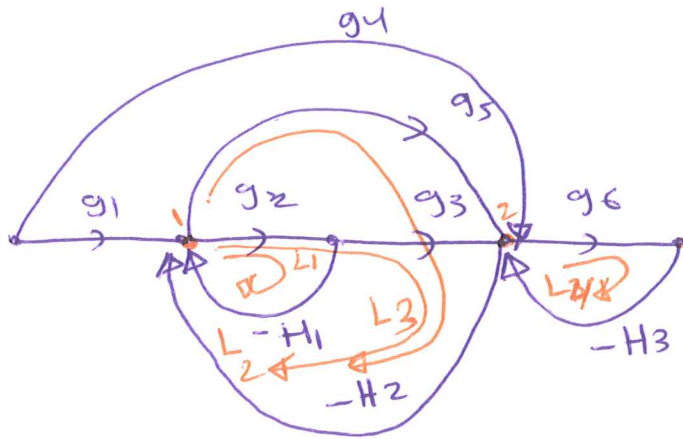
$M_k \rightarrow$  gain في المسار رقم  $k$   
بعض

$$P_1 \rightarrow M_1 = g_1 g_2 g_3 g_6$$

$$P_2 \rightarrow M_2 = g_1 g_5 g_6$$

$$P_3 \rightarrow M_3 = g_4 g_6$$

$L \rightarrow$  عدد الـ loops  
loop في حلقة يعني بين المدخل والمخرج  
وارجو ان لا تخطئوا ان الـ loop يكون  
loop في الحلقة اي في loop



عند اختيار اللول ننظر الى بنود التي عندها 1 في المسار الاتجاهي

$$L_1 = -g_2 H_1, \quad L_2 = -g_2 g_3 H_2$$

$$L_3 = -g_5 H_2, \quad L_4 = -g_6 H_3$$

$$\Delta = 1 - \sum \text{single loop} + \sum \text{non touching 2 loop} - \dots$$

$$= 1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_4)$$

نلاحظ انه  $L_1, L_2, L_3$  مشتركة في البنود 1 لذلك مضاعفتهم  
عنايتهم في  $\Delta$

نلاحظ انه  $L_4, L_2, L_3$  مشتركة في البنود 2 لذلك مضاعفتهم في بنود عنايتهم  
فذلك  $L_1, L_4$  هما متقاطعتا غير مشتركة

$$\Delta_k = 1 - \sum \text{loop gain in terms of } \Delta \text{ that touch the } k\text{th forward path}$$

$\Delta_k$  + يتبقى من  $\Delta$  كل اللول التي تتقاطع مع  $M_k$

$$\Delta_1 \Rightarrow M_1 \text{ بدون تقاطع مع}$$

في اللول التي من متقاطعت معها  
كلهم متقاطعتهم اذا كان لهم متقاطعوا مع  $\Delta$

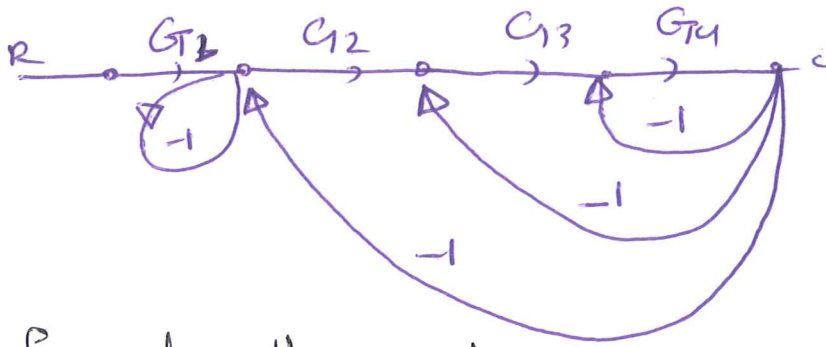
$$\Delta_1 = 1$$

$$\Delta_2 = 1$$

$\Delta_3 = 1 - L_1 \rightarrow$  من منزل مع  $M_3$  و  $Op$  بنور

$$G_T = \frac{M_1 \Delta_1 + M_2 \Delta_2 + M_3 \Delta_3}{\Delta}$$

s.26 Find  $T = C/R$



$P \rightarrow \times$  forward path = 1  
عكس المسار

$$M_1 = G_1 G_2 G_3 G_4$$

$L \rightarrow$  loops

$$L_1 = -GTC$$

$$L_2 = -G_3 G_4$$

$$L_3 = -G_2 G_3 G_4$$

$$L_9 = -1$$

$L_1$  is a  $\mathbb{Z}$ -module,  $L_2$  is a  $\mathbb{Z}$ -module,  $L_1 \oplus L_2$  is a  $\mathbb{Z}$ -module

حلقة حركية انه انحراف  
 $G_1$  بعد ا - بعد  $G_2$  بعد  
 $G_3$  بعد  $G_4$   
وهذا حل خالص لانه  
في الحرف ما ينفع اخر  
على النور اكثر من  
عيرة .  
الحرف ما ينفع فيه  
Loop

$$D = 1 - \sum \text{single loop} + \sum \text{non touching 2 loop} - \dots$$

$$1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_4 + L_2 L_4)$$

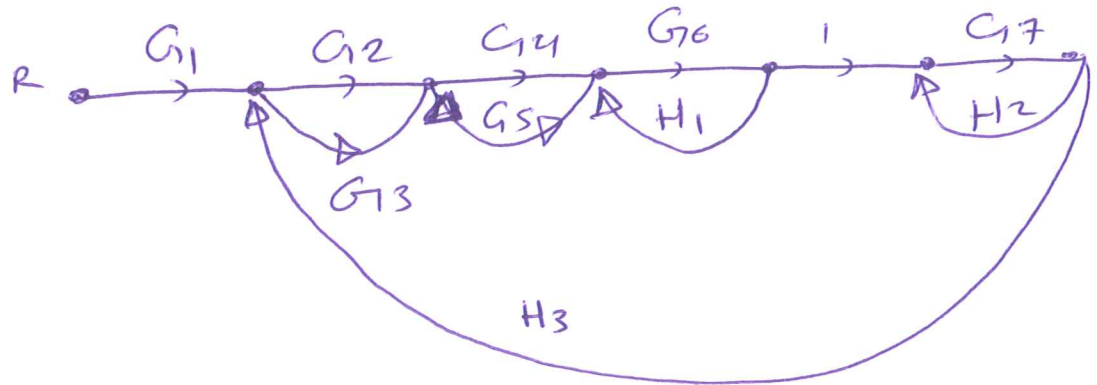
$$1 + (G_4 + G_3 G_4 + G_2 G_3 G_4 + 1) + (G_4 + G_3 G_4)$$

$$\Delta_1 = 1$$

$$T = \frac{C}{R} = \frac{M_1 \Delta_1}{\Delta} = \frac{G_1 G_2 G_3 G_4}{2 + G_2 G_3 G_4 + 2G_4 + 2G_3 G_4}$$

27

Find  $T = C/R$



$P \rightarrow 4$

$$M_1 = G_1 G_2 G_4 G_6 G_7$$

$$M_2 = G_1 G_2 G_5 G_6 G_7$$

$$M_3 = G_1 G_3 G_4 G_6 G_7$$

$$M_4 = G_1 G_3 G_5 G_6 G_7$$

Loops  $\rightarrow L_1 = G_2 G_4 G_6 G_7 H_3$

$$L_2 = G_2 G_5 G_6 G_7 H_3$$

$$L_3 = G_3 G_4 G_6 G_7 H_3$$

$$L_4 = G_3 G_5 G_6 G_7 H_3$$

$$L_5 = G_6 H_1$$

$$L_6 = G_7 H_2$$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6) - (L_5 L_6)$$

$$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1$$

$$T = \frac{M_1 \Delta_1 + M_2 \Delta_2 + M_3 \Delta_3 + M_4 \Delta_4}{\Delta}$$

(26)

## Similarity Transformations :-

ممكن انه SS. من و هو بيني بكنه نلاقي ان SS. متشابه لنظام

وعنا نتاكد انهم متشابهين لحوصلهم (TF) لازم يكون هو

الذي ليه نوعي SS. وبدي احبي واحد آخر متا به ؟!

$$\dot{Z} = P^{-1}APZ + P^{-1}Bu$$

$$y = CPZ + Du$$

$$Z = P^{-1}x$$

**5.43** Given the system represented in S.S. as follows:

$$\dot{x} = \begin{bmatrix} -1 & -7 & 6 \\ -8 & 4 & 8 \\ 4 & 7 & -8 \end{bmatrix} x + \begin{bmatrix} -5 \\ -7 \\ 5 \end{bmatrix} r$$

$$y = [-9 \quad -9 \quad -8] x$$

Convert the system to one where the new state vector  $Z$ , is

$$Z = \begin{bmatrix} -4 & 9 & -3 \\ 0 & -4 & 7 \\ -1 & -4 & -9 \end{bmatrix} x$$

Solution:-

$$Z = P^{-1}x$$

$$P^{-1} = \begin{bmatrix} -4 & 9 & -3 \\ 0 & -4 & 7 \\ -1 & -4 & -9 \end{bmatrix}$$

\* inverse of matrix  $3 \times 3$

$$C^{-1} = \frac{\text{cofactor matrix } (C)}{\det(C)}$$

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

step ① :-

$$\begin{bmatrix} \begin{vmatrix} C_{22} & C_{23} \\ C_{32} & C_{33} \end{vmatrix} & \begin{vmatrix} C_{21} & C_{23} \\ C_{31} & C_{33} \end{vmatrix} & \begin{vmatrix} C_{21} & C_{22} \\ C_{31} & C_{32} \end{vmatrix} \\ \begin{vmatrix} C_{12} & C_{13} \\ C_{32} & C_{33} \end{vmatrix} & \begin{vmatrix} C_{11} & C_{13} \\ C_{31} & C_{33} \end{vmatrix} & \begin{vmatrix} C_{11} & C_{12} \\ C_{31} & C_{32} \end{vmatrix} \\ \begin{vmatrix} C_{12} & C_{13} \\ C_{22} & C_{23} \end{vmatrix} & \begin{vmatrix} C_{11} & C_{13} \\ C_{21} & C_{23} \end{vmatrix} & \begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix} \end{bmatrix}$$

قانون حذف الصف  
والعمود الذي فيه  
التي نريد  
التي نريد  
التي نريد

Step ② :-

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

نقل الإشارة  
من الصف والعمود  
التي نريد  
التي نريد  
التي نريد

Step 3:- Mirror the matrix

cofactor matrix

التي نريد

$$\det(C) =$$

$$\begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$$

$$(C_{11}C_{22}C_{33} + C_{12}C_{23}C_{31} + C_{13}C_{21}C_{32}) - (C_{13}C_{22}C_{31} + C_{11}C_{23}C_{32} + C_{12}C_{21}C_{33})$$

\* inverse of matrix  $2 \times 2$

$$A^{-1} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{ad - bc} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{\det(A)}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

الآن بالعودة للـ سؤال

$$P^{-1} = \begin{bmatrix} -4 & 9 & -3 \\ 0 & -4 & 7 \\ -1 & -4 & -9 \end{bmatrix}$$

$$\textcircled{1} \left[ \begin{array}{c|c|c} \begin{vmatrix} -4 & 7 \\ -4 & -9 \end{vmatrix} & \begin{vmatrix} 0 & 7 \\ -1 & -9 \end{vmatrix} & \begin{vmatrix} 0 & -4 \\ -1 & -4 \end{vmatrix} \\ \hline \begin{vmatrix} 9 & -3 \\ -4 & -9 \end{vmatrix} & \begin{vmatrix} -4 & -3 \\ -1 & -9 \end{vmatrix} & \begin{vmatrix} -4 & 9 \\ -1 & -4 \end{vmatrix} \\ \hline \begin{vmatrix} 9 & -3 \\ -4 & 7 \end{vmatrix} & \begin{vmatrix} -4 & -3 \\ 0 & 7 \end{vmatrix} & \begin{vmatrix} -4 & 9 \\ 0 & -4 \end{vmatrix} \end{array} \right]$$

$$= \begin{bmatrix} 64 & 7 & -4 \\ -93 & 33 & 25 \\ 59 & -28 & -16 \end{bmatrix}$$

$$\textcircled{2} \text{ multiply by } \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$= \begin{bmatrix} 64 & -7 & -4 \\ -93 & 33 & 25 \\ 59 & -28 & -16 \end{bmatrix}$$

③ Mirror

$$\begin{bmatrix} 64 & -7 & -4 \\ -93 & 33 & 25 \\ 59 & -28 & -16 \end{bmatrix}$$

cofactor matrix

$$\begin{bmatrix} 64 & -93 & -4 \\ -7 & 33 & -28 \\ -4 & 25 & -16 \end{bmatrix}$$

④  $\det(P^-)$

$$\begin{bmatrix} -4 & 9 & -3 \\ 0 & -4 & 7 \\ -1 & -4 & -9 \end{bmatrix}$$

$$= [(-4 \times -4 \times -9) + (9 \times 7 \times -1) + (-3 \times 0 \times -4)] - [ (9 \times 0 \times -9) + (-4 \times 7 \times -4) + (-3 \times -4 \times -1) ]$$

$$= -307$$

$$P = \begin{bmatrix} 64 & -93 & -4 \\ -7 & 33 & -28 \\ -4 & 25 & -16 \end{bmatrix} = \frac{-307}{\begin{bmatrix} -0.208 & +0.302 & +0.013 \\ 0.0228 & -0.1075 & 0.0912 \\ 0.130 & 0.081 & 0.0521 \end{bmatrix}}$$

$$P^{-1}AP = \begin{bmatrix} 18.59 & 25.47 & 5.615 \\ -12.9 & -28.88 & -8.39 \\ -0.57 & 11.41 & 5.29 \end{bmatrix}$$

$$P^{-1}B = \begin{bmatrix} -58 \\ 63 \\ -12 \end{bmatrix}$$

$$CP = [1.56 \quad 3.04 \quad 2.73]$$

$$\dot{Z} = P^{-1}APZ + P^{-1}Bu, \quad y = CPZ$$

diagonalizing a system matrix

diagonal  $\Rightarrow$  diagonalizing a system matrix

$$\dot{Z} = P^{-1}APZ + P^{-1}Bu$$

$$y = CPZ + Du$$

eigen value vector

$\Rightarrow P$

is a

\* how you can find the P-matrix

① find eigen value

$$\det(\lambda I - A) = 0$$

② Now substitute to find eigen vector

$$(\lambda I - A)P_i = 0$$

5.45 Diagonalize the following system.

$$\dot{x} = \begin{bmatrix} -5 & -5 & 4 \\ 2 & 0 & -2 \\ 0 & -2 & -1 \end{bmatrix} x + \begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix} r$$

$$y = \begin{bmatrix} -1 & 1 & 2 \end{bmatrix} x$$

$$(\lambda I - A) = \begin{bmatrix} \lambda + 5 & 5 & -4 \\ -2 & \lambda & 2 \\ 0 & 2 & \lambda + 1 \end{bmatrix}$$

$$\det(\lambda I - A) = 0$$

$$\begin{aligned} & (\lambda + 5) \lambda (\lambda + 1) + 16 - (-10(\lambda + 1) + 4(\lambda + 5)) = 0 \\ & \lambda (\lambda + 5) (\lambda + 1) + 16 + 6\lambda + (-10) = 0 \\ & \lambda^3 + 6\lambda^2 + 5\lambda + 6\lambda + 6 = 0 \\ & \lambda^3 + 6\lambda^2 + 11\lambda + 6 = 0 \end{aligned}$$

$$\lambda^3 + 6\lambda^2 + 11\lambda + 6 = 0$$



بخطين حيز لاجل صفة  
طولة كيب باع  
الجزء

$$(\lambda + 3)(\lambda + 2)(\lambda + 1) = 0$$

$$\lambda = -3 \quad \lambda = -2 \quad \lambda = -1$$

$$\text{Now } (\lambda I - A)(X) = 0$$

$$\begin{bmatrix} \lambda + 5 & 5 & -4 \\ -2 & \lambda & 2 \\ 0 & 2 & \lambda + 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

مربعاتهم في صفر

$$(\lambda + 5)x_1 + 5x_2 - 4x_3 = 0$$

$$-2x_1 + \lambda x_2 + 2x_3 = 0$$

$$2x_2 + (\lambda + 1)x_3 = 0$$

$$\text{if } \lambda = -1 \quad \text{مؤثر في صفر}$$

$$4x_1 + 5x_2 - 4x_3 = 0$$

$$-x_2 - 2x_1 + 2x_3 = 0$$

$$2x_2 = 0 \rightarrow x_2 = 0$$

$$x_1 = x_3$$

$$x_1 = x_3 = 1 \quad x_2 = 0$$

$$\text{if } \lambda = -2$$

$$3x_1 + 5x_2 - 4x_3 = 0$$

$$-2x_1 - 2x_2 + 2x_3 = 0$$

$$2x_2 - x_3 = 0 \rightarrow x_2 = \frac{x_3}{2}$$

$$-2x_1 - 2 \frac{x_3}{2} + 2x_3 = 0$$

$$x_1 = \frac{x_3}{2}$$

$$x_2 = 1$$

$$x_3 = \frac{1}{2} \quad x_1 = \frac{1}{4}$$

$$\text{if } \lambda = -3$$

$$x_1 = x_2 = \frac{x_3}{2}$$

$$4x_3 - 5x_2 + 2x_1 = 0$$

$$-2x_3 - 3x_2 + 2x_1 = 0$$

$$2x_2 - 2x_3 = 0$$

$$\Rightarrow x_2 = x_3$$

$$-2x_1 - 3x_2 + 2x_2 = 0$$

$$x_1 = \frac{1}{2}x_2$$

$$x_1 = \frac{1}{2}x_2 = \frac{1}{2}x_3$$

$$x_1 = 1$$

$$x_2 = \frac{1}{2}$$

$$x_3 = \frac{1}{2}$$

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & \frac{1}{2} \\ 1 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & \frac{1}{2} \\ 1 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}^{-1} \begin{bmatrix} -5 & -5 & 4 \\ 2 & 0 & -2 \\ 0 & -2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & \frac{1}{2} \\ 1 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

$$P^{-1}B = \begin{bmatrix} -12 \\ 8 \\ -3 \end{bmatrix}$$

$$CP = [1 \ 4 \ 7]$$