

Islamic University of Gaza
Electrical Engineering Department
EELE 3360 Control Systems
Mid-term Exam (December 5, 2024)



Dr. Basil Hamed

Exam Time 10:00-12:30

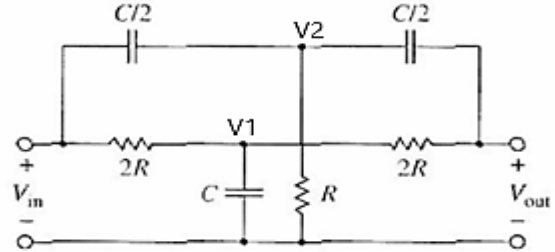
Name: SOLUTION

ID #

1. Find the transfer function $V_{out}(s) / V_{in}(s)$.

(10 Pt.)

Solution



$$\begin{cases} \frac{v_{in} - v_1}{2R} + C \frac{d}{dt} v_1 + \frac{v_{out} - v_1}{2R} = 0 \\ \frac{C}{2} \frac{d}{dt} (v_{in} - v_2) - \frac{v_2}{R} + \frac{C}{2} \frac{d}{dt} (v_{out} - v_2) = 0 \\ \frac{C}{2} \frac{d}{dt} (v_2 - v_{out}) + \frac{v_1 - v_{out}}{2R} = 0 \end{cases}$$

By using Laplace transform we have:

$$\begin{cases} \frac{V_{in}(s) - V_1(s)}{2R} + CsV_1(s) + \frac{V_{out}(s) - V_1(s)}{2R} = 0 \\ \frac{Cs}{2} (V_{in}(s) - V_2(s)) - \frac{V_2(s)}{R} + \frac{Cs}{2} (V_{out}(s) - V_2(s)) = 0 \\ \frac{Cs}{2} (V_2(s) - V_{out}(s)) + \frac{V_1(s) - V_{out}(s)}{2R} = 0 \end{cases}$$

From above equations:

$$\begin{cases} V_1(s) = \frac{1}{2(RCs + 1)} (V_{in}(s) + V_{out}(s)) \\ V_2(s) = \frac{RCS}{2(RCs + 1)} (V_{in}(s) + V_{out}(s)) \end{cases}$$

Substituting $V_1(s)$ and $V_2(s)$ into preceding equations, we obtain:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{R^2 C^2 s^2 + 1}{R^2 C^2 s^2 + 4RCs + 1}$$

2. Find the state space model for the system shown below assume the output is x .

(10 Pt.)

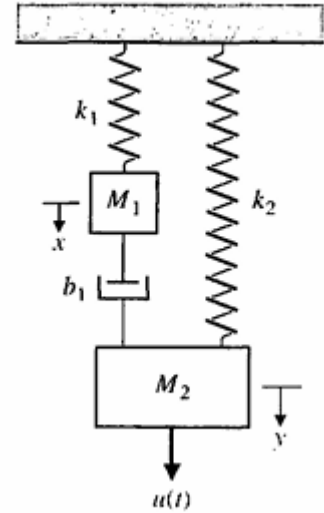
Solution

$$M_1 \ddot{x}(t) + b_1 \dot{x}(t) + k_1 x(t) = b_1 \dot{y}(t)$$

$$M_2 \ddot{y}(t) + b_1 \dot{y}(t) + k_2 y(t) = b_1 \dot{x}(t) + u(t)$$

Let

$$z_1 = x, \quad z_2 = \dot{x}, \quad z_3 = y, \quad \text{and} \quad z_4 = \dot{y}.$$

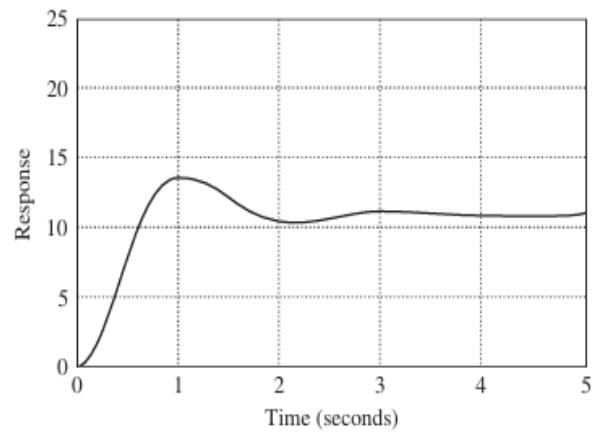


Then we write the system in state variable form as

$$\dot{\mathbf{z}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{M_1} & -\frac{b_1}{M_1} & 0 & \frac{b_1}{M_1} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{b_1}{M_2} & -\frac{k_2}{M_2} & -\frac{b_1}{M_2} \end{bmatrix} \mathbf{z} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{M_2} \end{bmatrix} u$$

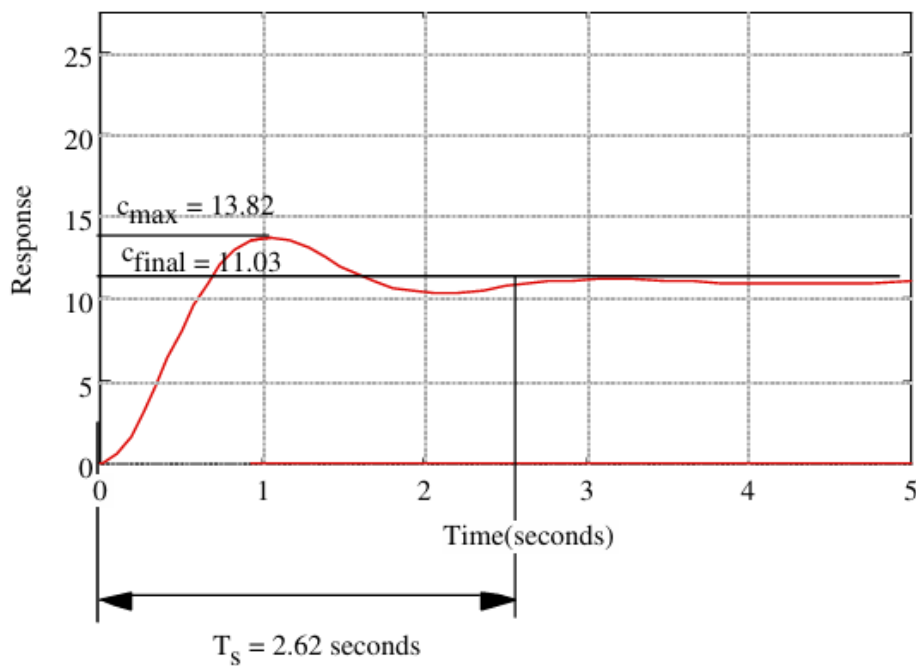
$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \mathbf{z}.$$

3. For the unit response shown in the figure below find the transfer function of the system. (10 Pt.)



b. Measuring the percent overshoot and settling time from the graph: $\%OS = (13.82-11.03)/11.03 =$

25.3%,



and $T_s = 2.62$ seconds. Estimating a second-order system, we use Eq. (4.39) to find $\zeta = 0.4$, and Eq.

(4.42) to find $\omega_n = 3.82$. Thus, $G(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$. Since $C_{final} = 11.03$, $\frac{K}{\omega_n^2} = 11.03$. Hence,

$K = 160.95$. Substituting all values,

$$G(s) = \frac{160.95}{s^2 + 3.056s + 14.59}$$

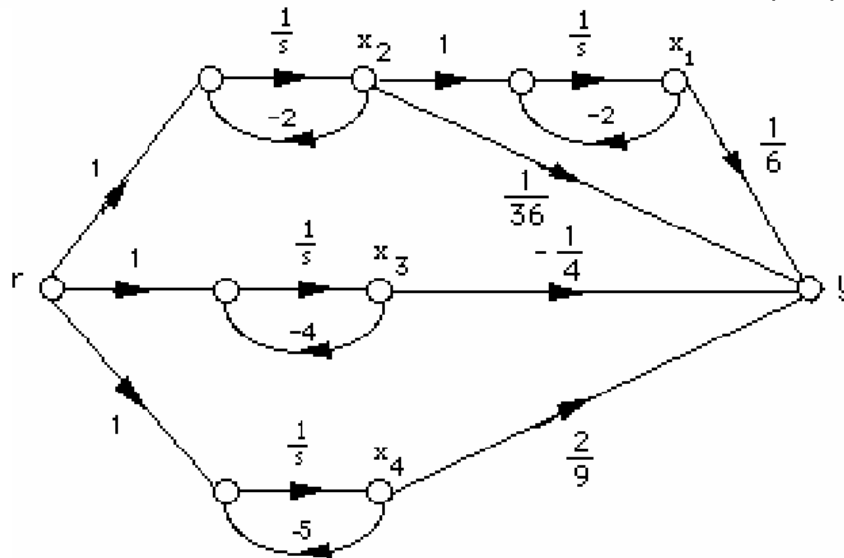
4. The system has transfer function

(10 Pt.)

$$G(s) = \frac{Y(s)}{U(s)} = \frac{(s+3)}{(s+2)^2 (s+4)(s+5)}$$

Obtain state space model for above system using parallel decomposition

$$G(s) = \frac{s+3}{(s+2)^2 (s+4)(s+5)} = \frac{2}{9} \frac{1}{s+5} - \frac{1}{4} \frac{1}{s+4} + \frac{1}{36} \frac{1}{s+2} + \frac{1}{6} \frac{1}{(s+2)^2}$$



Writing the state and output equations,

$$\dot{x}_1 = -2x_1 + x_2$$

$$\dot{x}_2 = -2x_2 + r$$

$$\dot{x}_3 = -4x_3 + r$$

$$\dot{x}_4 = -5x_4 + r$$

$$y = \frac{1}{6} x_1 + \frac{1}{36} x_2 - \frac{1}{4} x_3 + \frac{2}{9} x_4$$

In vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & -5 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} \frac{1}{6} & \frac{1}{36} & -\frac{1}{4} & \frac{2}{9} \end{bmatrix} \mathbf{x}$$

GOOD Luck



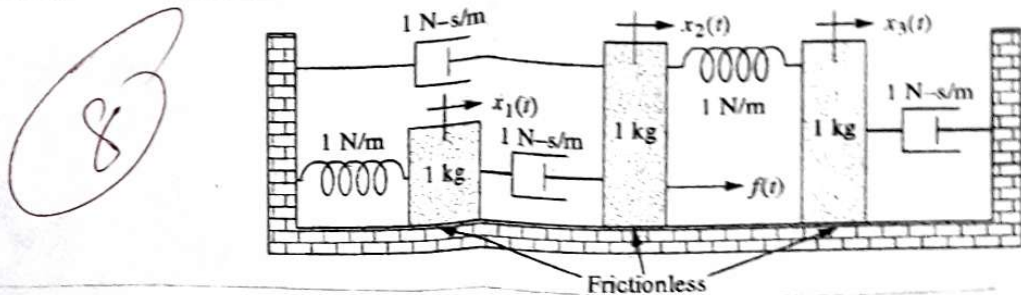
Exam Time 12:00-1:00

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1. Find the transfer function, $G(s) = X_3(s)/F(s)$, for the translational mechanical system shown in Figure (8 Pt.)



$$(\dot{x}^2 + \dot{x} + 1) \dot{x}_1(s) - \dot{x} \dot{x}_2(s) = 0 \quad \text{--- (1)}$$

$$-\dot{x} \dot{x}_1(s) + (\dot{x}^2 + 2\dot{x} + 1) \dot{x}_2(s) - \dot{x}_3(s) = F(s) \quad \text{--- (2)}$$

$$-\dot{x}_2(s) + (\dot{x}^2 + \dot{x} + 1) \dot{x}_3(s) = 0 \quad \text{--- (3)}$$

From (1) $\Rightarrow \dot{x}_2(s) = \frac{(\dot{x}^2 + \dot{x} + 1)}{\dot{x}} \dot{x}_1(s)$

بالتعويض

$$\Rightarrow -\dot{x} \dot{x}_1(s) + (\dot{x}^2 + 2\dot{x} + 1) \left(\frac{\dot{x}^2 + \dot{x} + 1}{\dot{x}} \right) \dot{x}_1(s) - \dot{x}_3(s) = F(s)$$

$$\Rightarrow (\dot{x}^3 + 3\dot{x}^2 + 3\dot{x} + \frac{1}{\dot{x}} + 3) \dot{x}_1(s) - \dot{x}_3(s) = F(s) \quad \text{--- (4)}$$

بالتعويض

$$+ \left(\frac{\dot{x}^2 + \dot{x} + 1}{\dot{x}} \right) \dot{x}_1(s) + (\dot{x}^2 + \dot{x} + 1) \dot{x}_3(s) = 0 \quad \text{--- (5)}$$

Solve to eqn 4 & 5

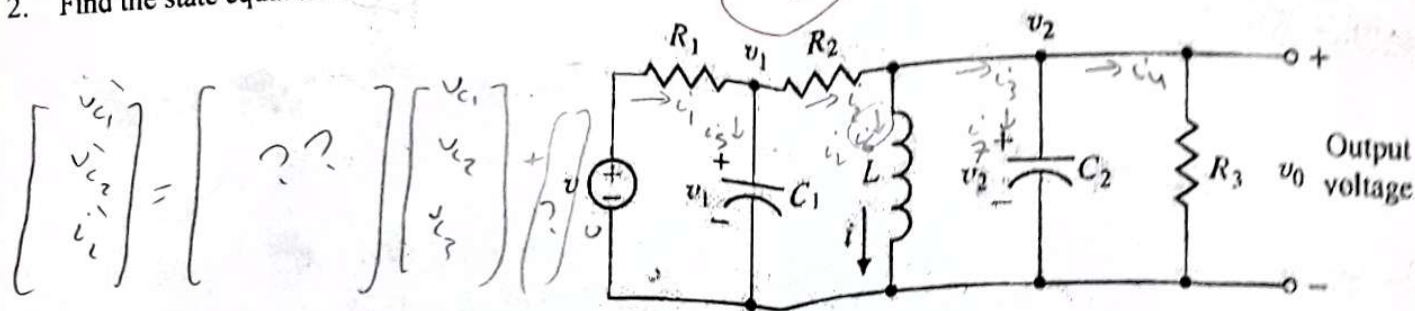
$$\dot{x}_3(s) = \frac{F(s)}{\dot{x}(\dot{x}^3 + 3\dot{x}^2 + 3\dot{x} + 3)}$$

$$\Rightarrow \frac{\dot{x}_3(s)}{F(s)} = \frac{1}{\dot{x}(\dot{x}^3 + 3\dot{x}^2 + 2\dot{x} + 3)}$$

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(8 Pt.)

2. Find the state equation for the circuit shown below.



$$i_5 = C_1 \frac{dv_{c1}}{dt} \quad , \quad i_5 = i_1 - i_2 \quad , \quad i_1 = \frac{v_i - v_{c1}}{R_1} \quad , \quad i_2 = \frac{v_{c1} - v_{c2}}{R_2}$$

$$\Rightarrow i_5 = \frac{1}{R_1} v_i - \frac{1}{R_1} v_{c1} - \frac{1}{R_2} v_{c1} + \frac{1}{R_2} v_{c2}$$

$$\therefore \dot{v}_{c1} = \frac{i_5}{C_1} = \frac{1}{R_1 C_1} v_i - \frac{1}{R_1 C_1} v_{c1} - \frac{1}{R_2 C_1} v_{c1} + \frac{1}{R_2 C_1} v_{c2}$$

$$= \frac{1}{R_1 C_1} v_i - \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} \right) v_{c1} + \frac{1}{R_2 C_1} v_{c2} \quad \text{--- (1)}$$

$$L \frac{di_L}{dt} = v_{c2} \Rightarrow \dot{i}_L = \frac{v_{c2}}{L} \quad \text{--- (2)}$$

$$i_7 = C_2 \frac{dv_{c2}}{dt} \quad , \quad i_7 = i_3 - i_4$$

$$= i_2 - i_L - i_4$$

$$= \frac{v_{c1}}{R_2} - \frac{v_{c2}}{R_2} - i_L - \frac{v_{c2}}{R_3} = \frac{v_{c1}}{R_2} - \left(\frac{1}{R_2} + \frac{1}{R_3} \right) v_{c2} - i_L$$

$$\therefore \frac{dv_{c2}}{dt} = \frac{i_7}{C_2}$$

$$= \frac{v_{c1}}{C_2 R_2} - \frac{1}{C_2} \left(\frac{1}{R_2} + \frac{1}{R_3} \right) v_{c2} - \frac{1}{C_2} i_L \quad \text{--- (3)}$$

$$v_0 = v_{c2} \quad \text{--- (4)}$$

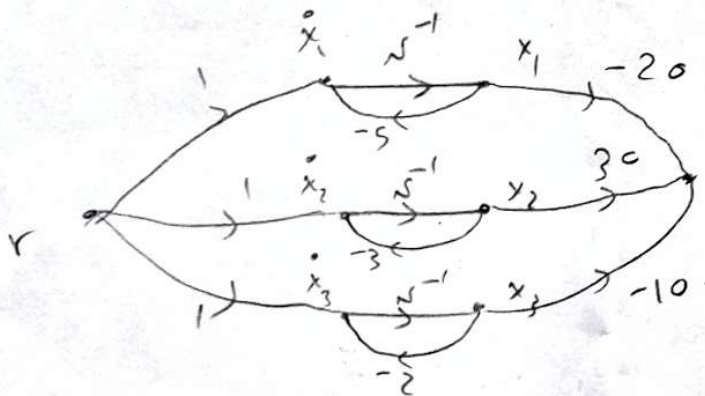
$$\therefore \begin{bmatrix} \dot{v}_{c1} \\ \dot{v}_{c2} \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_1} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) & \frac{1}{R_2 C_1} & 0 \\ \frac{1}{C_2 R_2} & -\frac{1}{C_2} \left(\frac{1}{R_2} + \frac{1}{R_3} \right) & -\frac{1}{C_2} \\ 0 & \frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} v_{c1} \\ v_{c2} \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1 C_1} \\ 0 \\ 0 \end{bmatrix} v_i$$

$$v_0 = [0 \ 1 \ 0] \begin{bmatrix} v_{c1} \\ v_{c2} \\ i_L \end{bmatrix}$$

with complex.

3. Given the transfer function, $G(s) = \frac{Y(s)}{R(s)} = \frac{30(s+1)}{(s^3+10s^2+31s+30)}$
using; Parallel method. (7 Pt.)

$$\frac{Y(s)}{R(s)} = \frac{-20}{s+5} + \frac{30}{s+3} - \frac{10}{s+2}$$



$$\dot{x}_1 = r - 5x_1$$

$$\dot{x}_2 = r - 3x_2$$

$$\dot{x}_3 = r - 2x_3$$

$$y = -20x_1 + 30x_2 - 10x_3$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} r$$

$$y = [-20 \quad 30 \quad -10] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

#

Without complex

Determine state space model of the system

without complex

$$\begin{array}{r} s^2 + 7s + 10 \\ s+3 \overline{) s^3 + 10s^2 + 31s + 30} \\ \underline{s^3 + 3s^2} \\ 7s^2 + 21s + 30 \\ \underline{7s^2 + 21s} \\ 30 \end{array}$$

$$(s^3 + 10s^2 + 31s + 30) = (s+3)(s^2 + 7s + 10) = (s+3)(s+5)(s+2)$$

$$\frac{Y(s)}{R(s)} = \frac{30(s+1)}{(s+5)(s+3)(s+2)}$$

$$= \frac{A}{s+5} + \frac{B}{s+3} + \frac{C}{s+2}$$

$$A = \frac{30(s+1)}{(s+3)(s+2)} \Big|_{s=-5} = -20$$

$$B = \frac{30(s+1)}{(s+5)(s+2)} \Big|_{s=-3} = 30$$

$$C = \frac{30(s+1)}{(s+5)(s+3)} \Big|_{s=-2} = -10$$

$$\frac{Y(s)}{R(s)} = \frac{-20}{s+5} + \frac{30}{s+3} - \frac{10}{s+2}$$

By general formula.

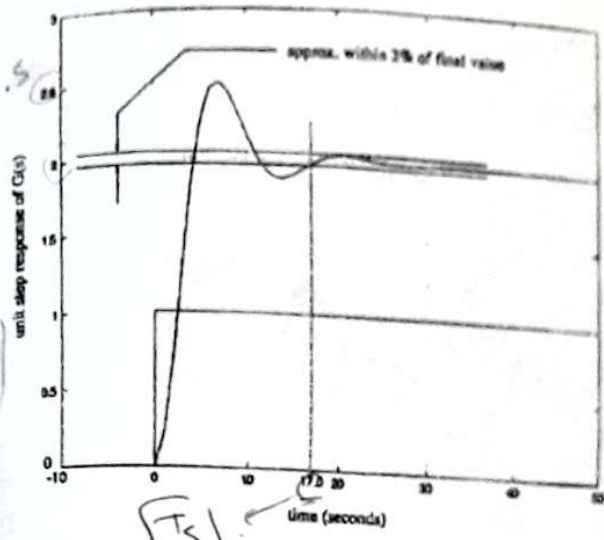
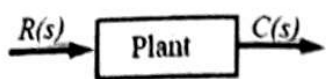
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} r$$

$$y = [-20 \quad 30 \quad -10] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

#

(7)

4. A plant has the response, $c(t)$, to a unit step, $r(t) = u(t)$ as shown below. From the graph, estimate the settling time, % overshoot and the transfer function. (7 Pt.)



(1) $OS\% = \frac{2.5 - 2}{2} \times 100 = 25$

(2) $T_s = 17 \text{ sec.}$

$T_s = \frac{4}{\xi \omega_n}$, $OS\% = 100 e^{-\frac{\pi \xi}{\sqrt{1-\xi^2}}}$

To Find ξ, ω_n ??

$T_s = \frac{4}{\xi \omega_n} \Rightarrow 17 = \frac{4}{\xi \omega_n} \Rightarrow \omega_n = \frac{4}{17 \xi} = \frac{0.2353}{\xi}$

$OS\% = 100 e^{-\frac{\pi \xi}{\sqrt{1-\xi^2}}}$
 $25 = 100 e^{-\frac{\pi \xi}{\sqrt{1-\xi^2}}} \Rightarrow 0.25 = e^{-\frac{\pi \xi}{\sqrt{1-\xi^2}}} \rightarrow \text{solve eqn}$

$\Rightarrow \xi = 0.4037$

$\Rightarrow \omega_n = \frac{4}{17 \xi} = \frac{4}{17(0.4037)} = 0.5828$

$\Rightarrow \omega_n = 0.5828$

$K_{dc} = 2$

$\therefore G(s) = \frac{K_{dc} \omega_n^2}{s^2 + 2 \xi \omega_n s + \omega_n^2}$

$= \frac{(2)(0.5828)^2}{s^2 + 2(0.4037)(0.5828)s + (0.5828)^2}$

$= \frac{0.67937}{s^2 + 0.47057s + 0.33968}$

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under damped

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EELE 3360 Control Systems
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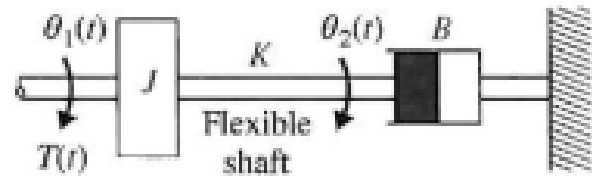


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1. Find the transfer function $\Theta_1(s) / T(s)$.

(8 Pt.)



Solution

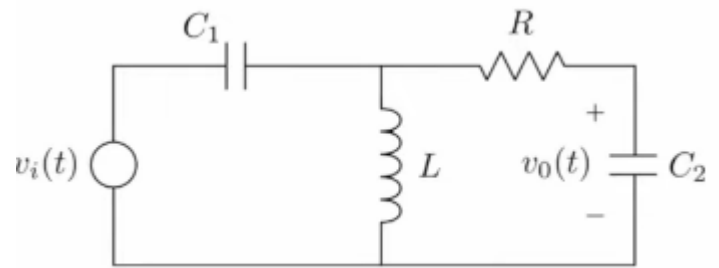
$$\frac{d^2\theta_1}{dt^2} = -\frac{K}{J}(\theta_1 - \theta_2) + \frac{1}{J}T \quad K(\theta_1 - \theta_2) = B \frac{d\theta_2}{dt}$$

$$\begin{aligned} s^2 \Theta_1(s) &= -\frac{k}{J}(\Theta_1(s) - \Theta_2(s)) + \frac{1}{J}T(s) & k(\Theta_1(s) - \Theta_2(s)) &= Bs\Theta_2(s) \\ \therefore \Theta_2(s) &= \frac{k}{Bs+k} \Theta_1(s) \\ \rightarrow (s^2 + \frac{k}{J}) \Theta_1(s) &= \frac{k}{J} \Theta_2(s) + \frac{1}{J}T(s) \end{aligned}$$

$$\frac{\Theta_1(s)}{T(s)} = \frac{Bs + K}{s(BJs^2 + JKs + BK)}$$

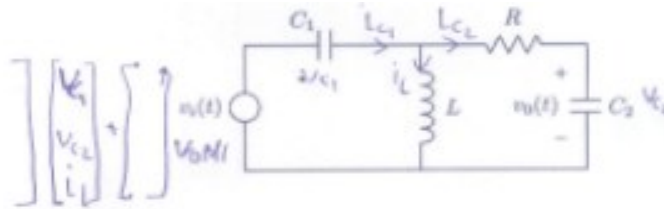
2. Find the state space model for the circuit shown below.

(10 Pt.)



Solution

$$\begin{bmatrix} \frac{dv_{c1}}{dt} \\ \frac{dv_{c2}}{dt} \\ \frac{di_L}{dt} \end{bmatrix} =$$



$$v_i = v_{c1} + R i_{c2} + v_{c2} \Rightarrow i_{c2} = -\frac{v_{c1}}{R} - \frac{v_{c2}}{R} + \frac{v_i}{R}$$

$$i_{c1} = i_L + i_{c2} \Rightarrow C_1 \frac{dv_{c1}}{dt} = i_L - \frac{v_{c1}}{R} - \frac{v_{c2}}{R} + \frac{v_i}{R}$$

$$\therefore \frac{dv_{c1}}{dt} = -\frac{v_{c1}}{RC_1} - \frac{v_{c2}}{RC_1} + \frac{1}{C_1} i_L + \frac{v_i}{RC_1}$$

$$C_2 \frac{dv_{c2}}{dt} = i_{c2} = -\frac{v_{c1}}{R} - \frac{v_{c2}}{R} + \frac{v_i}{R}$$

$$\therefore \frac{dv_{c2}}{dt} = -\frac{v_{c1}}{RC_2} - \frac{v_{c2}}{RC_2} + \frac{v_i}{RC_2}$$

$$L \frac{di_L}{dt} = v_i - v_{c1} \Rightarrow \frac{di_L}{dt} = \frac{v_i}{L} - \frac{v_{c1}}{L}$$

$$\begin{bmatrix} \frac{dv_{c1}}{dt} \\ \frac{dv_{c2}}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC_1} & -\frac{1}{RC_1} & \frac{1}{C_1} \\ -\frac{1}{RC_2} & -\frac{1}{RC_2} & 0 \\ -\frac{1}{L} & 0 & 0 \end{bmatrix} \begin{bmatrix} v_{c1} \\ v_{c2} \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{RC_1} \\ \frac{1}{RC_2} \\ \frac{1}{L} \end{bmatrix} v_i(t)$$

$$[v_o(t)] = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_{c1} \\ v_{c2} \\ i_L \end{bmatrix}$$

3. The dynamics of a system are governed by the equation

(6 Pt.)

$$\begin{aligned}\dot{x}_1 &= x_2^2 + x_2^3 + 5x_1 - 90 + u \\ \dot{x}_2 &= \sin(x_1) + x_2 - 1.243 + 3u\end{aligned}$$

Find a linearized model at the equilibrium point $x_1 = x_2 = 2$

Solution

$$\dot{x}_1 = x_2^2 + x_2^3 + 5x_1 - 90 + u = f_1$$

$$\dot{x}_2 = \sin(x_1) + x_2 - 1.243 + 3u = f_2$$

$$A^* = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}_{x_0, u_0} = \begin{bmatrix} 5 & 2x_2 + 3x_2^2 \\ \cos x_1 & 1 \end{bmatrix}_{x_0, u_0}$$

$$A^* = \begin{bmatrix} 5 & 16 \\ -0.416 & 1 \end{bmatrix}$$

$$B^* = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

114.598

$$\Delta \dot{\underline{x}} = \begin{bmatrix} 5 & 16 \\ -0.416 & 1 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} \Delta u$$

4. The system has transfer function

(6 Pt.)

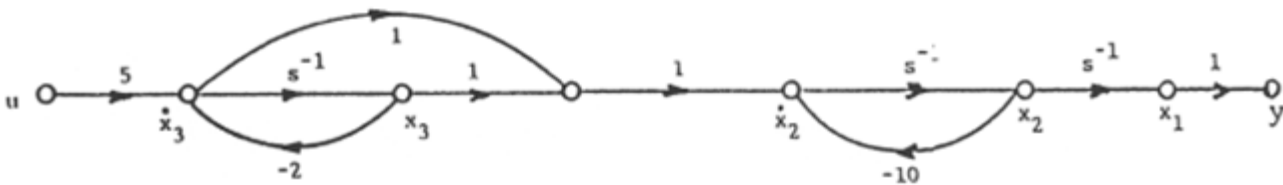
$$G(s) = \frac{Y(s)}{U(s)} = \frac{5(s+1)}{s(s+2)(s+10)}$$

Obtain state space model for above system using cascade (series) decomposition

Solution

$$G(s) = \frac{Y(s)}{U(s)} = \frac{5s+5}{s(s+2)(s+10)} = \left(\frac{5}{s}\right) \left(\frac{s+1}{s+2}\right) \left(\frac{1}{s+10}\right)$$

State diagram:



State equations: $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -10 & -1 \\ 0 & 0 & -2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}$$

$$y(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}u(t)$$

$$\mathbf{C} = [1 \ 0 \ 0]$$

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EELE 3360 Control Systems
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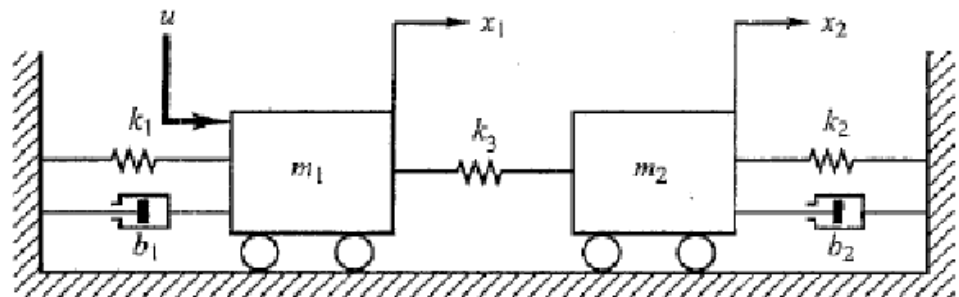


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1. Find the transfer function $X_2(s) / U(s)$.

(10 Pt.)



$$m_1 \ddot{x}_1 = -k_1 x_1 - b_1 \dot{x}_1 - k_3 (x_1 - x_2) + u$$

$$m_2 \ddot{x}_2 = -k_2 x_2 - b_2 \dot{x}_2 - k_3 (x_2 - x_1)$$

Rewriting, we have

$$m_1 \ddot{x}_1 + b_1 \dot{x}_1 + k_1 x_1 + k_3 x_1 = k_3 x_2 + u$$

$$m_2 \ddot{x}_2 + b_2 \dot{x}_2 + k_2 x_2 + k_3 x_2 = k_3 x_1$$

Assuming the zero initial condition and taking the Laplace transforms of these two equations, we obtain

$$(m_1 s^2 + b_1 s + k_1 + k_3) X_1(s) = k_3 X_2(s) + U(s) \quad (1)$$

$$(m_2 s^2 + b_2 s + k_2 + k_3) X_2(s) = k_3 X_1(s) \quad (2)$$

By eliminating $X_2(s)$ from Equations (1) and (2), we get

$$(m_1 s^2 + b_1 s + k_1 + k_3) X_1(s) = \frac{k_3^2}{m_2 s^2 + b_2 s + k_2 + k_3} X_1(s) + U(s)$$

Hence

$$\frac{X_1(s)}{U(s)} = \frac{m_2 s^2 + b_2 s + k_2 + k_3}{(m_1 s^2 + b_1 s + k_1 + k_3)(m_2 s^2 + b_2 s + k_2 + k_3) - k_3^2}$$

From Equation (2), we obtain

$$\frac{X_2(s)}{X_1(s)} = \frac{k_3}{m_2 s^2 + b_2 s + k_2 + k_3}$$

Hence

$$\frac{X_2(s)}{U(s)} = \frac{X_2(s)}{X_1(s)} \cdot \frac{X_1(s)}{U(s)} = \frac{k_3}{(m_1 s^2 + b_1 s + k_1 + k_3)(m_2 s^2 + b_2 s + k_2 + k_3) - k_3^2}$$

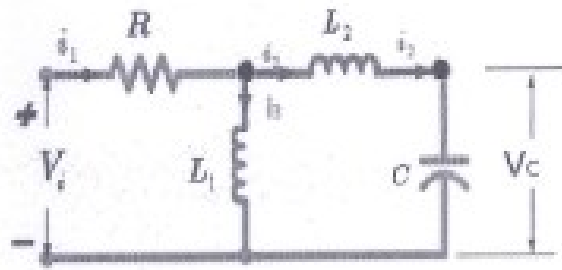
2. Find the state equation for the circuit shown below.

(10 Pt.)

$$V_i = R i_1 + L_1 \frac{di_1}{dt}$$

$$\frac{di_1}{dt} = -\frac{R}{L_1} i_1 + \frac{1}{L_1} V_i$$

$$\frac{di_3}{dt} = -\frac{R}{L_1} (i_2 + i_3) + \frac{1}{L_1} V_i$$



$$\therefore \frac{di_3}{dt} = -\frac{R}{L_1} L_2 - \frac{R}{L_1} i_3 + \frac{1}{L_1} V_i$$

$$V_i = R i_1 + L_2 \frac{di_2}{dt} + V_C$$

$$\frac{di_2}{dt} = -\frac{R}{L_2} i_1 - \frac{1}{L_2} V_C + \frac{1}{L_2} V_i$$

$$\frac{di_2}{dt} = -\frac{R}{L_2} (i_2 + i_3) - \frac{1}{L_2} V_C + \frac{1}{L_2} V_i$$

$$\therefore \frac{di_2}{dt} = -\frac{R}{L_2} L_2 - \frac{R}{L_2} i_3 - \frac{1}{L_2} V_C + \frac{1}{L_2} V_i$$

$$C \frac{dV_C}{dt} = i_2 \Rightarrow \frac{dV_C}{dt} = \frac{1}{C} i_2$$

$$\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \\ \frac{dV_C}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L_1} & -\frac{R}{L_1} & -\frac{1}{L_1} \\ -\frac{R}{L_2} & -\frac{R}{L_2} & -\frac{1}{L_2} \\ \frac{1}{C} & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ V_C \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ \frac{1}{L_2} \\ 0 \end{bmatrix} V_i$$

3. The dynamics of a system are governed by the ordinary differential equation

(4 Pt.)

$$\ddot{y} + 5\dot{y} + 10y + 4y^5 + \frac{2}{\pi} \sin\left(\frac{\pi}{2}y\right) = u$$

Linearize the above nonlinear system at $y_0 = 1, \ddot{y}_0 = \dot{y}_0 = 0, u_0 = \frac{1}{2}$

Let $x_1 = y$
 $x_2 = \dot{x}_1 = \dot{y}$

$$\dot{x}_2 + 5x_2 + 10x_1 + 4x_1^5 + \frac{2}{\pi} \sin\left(\frac{\pi}{2}x_1\right) = u$$

$$\dot{x}_1 = x_2 = f_1$$

$$\dot{x}_2 = -10x_1 - 4x_1^5 - \frac{2}{\pi} \sin\left(\frac{\pi}{2}x_1\right) - 5x_2 + u = f_2$$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \bigg|_{x_0, u_0} = \begin{bmatrix} 0 & 1 \\ -30 & -5 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ -30 & -5 \end{bmatrix}; \quad B = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\therefore \dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -30 & -5 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

4. The system has transfer function

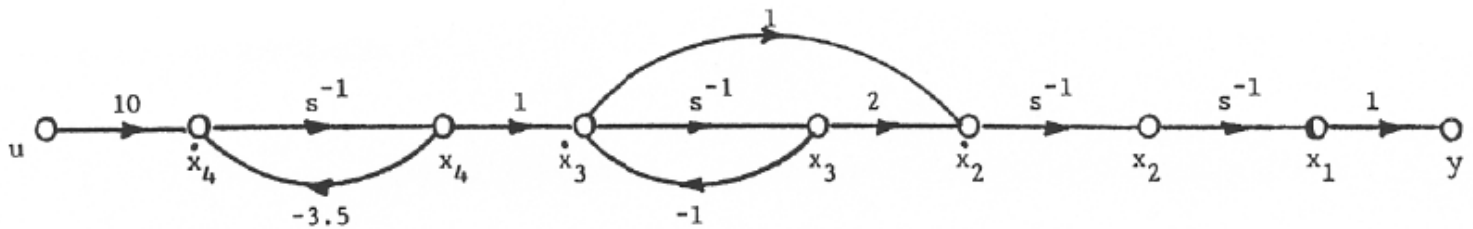
(6 Pt.)

$$G(s) = \frac{Y(s)}{U(s)} = \frac{10(s+2)}{s^2(s+1)(s+3.5)}$$

Obtain state space model for above system using cascade (series) decomposition

$$G(s) = \frac{Y(s)}{U(s)} = \frac{10(s+2)}{s^2(s+1)(s+3.5)} = \left(\frac{10}{s^2} \right) \left(\frac{s+2}{s+1} \right) \left(\frac{1}{s+3.5} \right)$$

State diagram:



State equations: $\dot{x}(t) = Ax(t) + Bu(t)$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -3.5 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \end{bmatrix}$$

Output equation $y = [1 \ 0 \ 0 \ 0]x$

GOOD Luck

Islamic University of Gaza
Electrical Engineering Department
EELE 3360 Control Systems
Mid-term Exam (April 24, 2021)

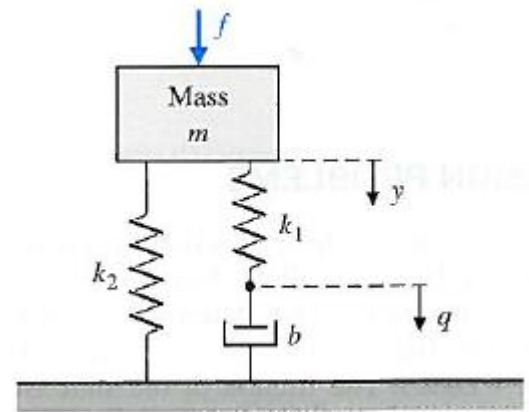


Dr. Basil Hamed
Name:

Exam Time 12:00-1:00
ID #

1. Determine the differential equations describes the motion of the system

(5 Pt.)

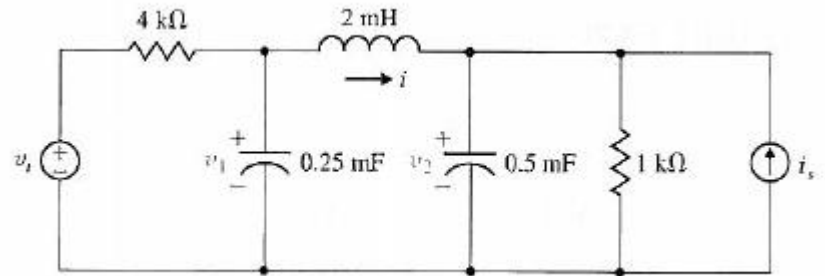


Solution

$$\begin{aligned} m\ddot{y} + k_2\dot{y} + k_1(y - q) &= f \\ -b\dot{q} + k_1(y - q) &= 0 \end{aligned}$$

2. Find the state equation for the circuit shown below.

(8 Pt.)



Solution

The node equations are

$$\begin{aligned} 0.00025 \frac{dv_1}{dt} + i_L - \frac{v_i - v_1}{4000} &= 0 \\ 0.0005 \frac{dv_2}{dt} - i_L + \frac{v_2}{1000} - i_3 &= 0 \\ 0.002 \frac{di_L}{dt} + v_2 - v_1 &= 0 . \end{aligned}$$

Define the state variables

$$x_1 = v_1 \quad x_2 = v_2 \quad x_3 = i_L .$$

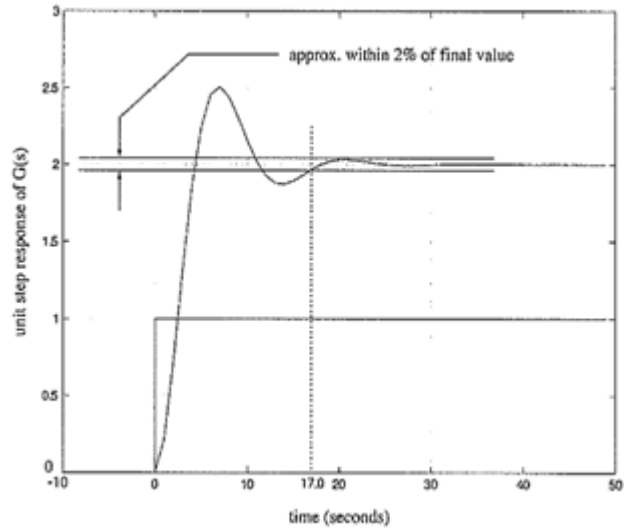
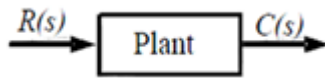
Then,

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

where

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & -4000 \\ 0 & -2 & 2000 \\ 500 & -500 & 0 \end{bmatrix} , \quad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 2000 \\ 0 & 0 \end{bmatrix}$$

3. A plant has the response, $c(t)$, to a unit step, $r(t) = u(t)$ as shown below, From the graph, estimate the settling time, % overshoot and the transfer function (7 Pt.)



From the plot, DC gain of system = 2 — ①

% overshoot = 25%

$$\rightarrow e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.25$$

$$\rightarrow \pi\xi/\sqrt{1-\xi^2} = 1.386 \rightarrow \frac{\xi}{\sqrt{1-\xi^2}} = 0.441$$

$$\text{or } \xi^2 = 0.194(1-\xi^2) \rightarrow \xi^2 = 0.1629$$

$$\text{or } \xi = 0.403 \text{ — ②}$$

$$T_s = 4\tau = \frac{4}{\xi\omega_n} = 17$$

$$\rightarrow \xi\omega_n = \frac{4}{17} = 0.235 \text{ — ③}$$

From ② & ③, $\omega_n = \frac{0.235}{0.403} = 0.583$

$$G(s) = K \left(\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \right)$$

$$= \frac{2 \cdot 0.583^2}{s^2 + 2(0.403)(0.583)s + 0.583^2}$$

$$= \frac{0.6817}{s^2 + 0.469s + 0.339}$$

4. The system has transfer function

(5 Pt.)

$$\frac{Y(s)}{R(s)} = T(s) = \frac{1}{s^3 + 10s^2 + 31s + 30}$$

Obtain state space model for above system using Parallel Method

Solution

$$\dot{\mathbf{x}} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -5 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} r$$
$$y = \begin{bmatrix} \frac{1}{6} & \frac{1}{3} & -\frac{1}{2} \end{bmatrix} \mathbf{x} .$$

GOOD Luck

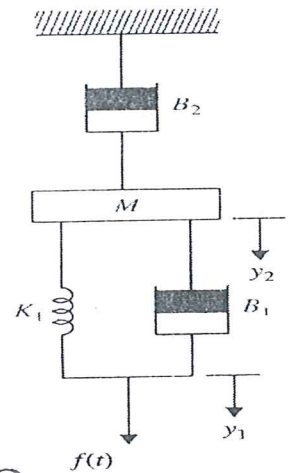
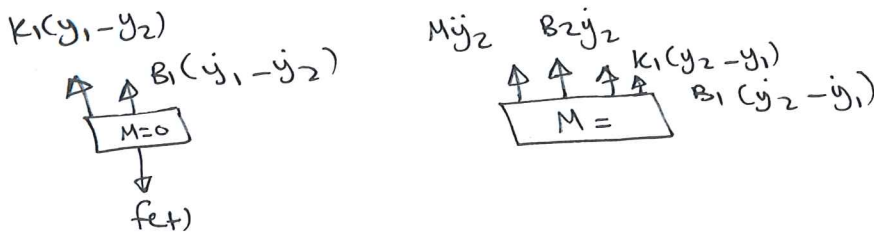
Islamic University of Gaza
Electrical Engineering Department
EELE 3360 Control Systems
Med-term Exam (April 18, 2010)



Dr. Basil Hamed
Name:

Exam Time 8:00-9:30
ID #

1. For the Mechanical system shown, Write the differential equations, and find the state equation. (7 Pt.)



$$f(t) = B_1 \dot{y}_1 - B_1 \dot{y}_2 + k_1 y_1 - k_1 y_2 \rightarrow ①$$

$$M \ddot{y}_2 + B_2 \dot{y}_2 + B_1 \dot{y}_2 - B_1 \dot{y}_1 + k_1 y_2 - k_1 y_1 = 0$$

$$M \ddot{y}_2 + (B_2 + B_1) \dot{y}_2 - B_1 \dot{y}_1 + k_1 y_2 - k_1 y_1 = 0 \rightarrow ②$$

$$z_1 = y_2, \quad z_2 = \dot{y}_2, \quad z_3 = y_1$$

$$f(t) = B_1 \dot{z}_3 - B_1 z_2 + k_1 z_3 - k_1 z_1$$

$$\dot{z}_3 = \frac{1}{B_1} f(t) + z_2 - \frac{k_1}{B_1} z_3 + \frac{k_1}{B_1} z_1 \rightarrow ①^*$$

$$M \dot{z}_2 + (B_2 + B_1) z_2 - B_1 \dot{z}_3 + k_1 z_1 - k_1 z_3 = 0$$

$$\dot{z}_2 = \frac{B_1}{M} \dot{z}_3 - \frac{(B_1 + B_2)}{M} z_2 - \frac{k_1}{M} z_1 + \frac{k_1}{M} z_3$$

$$\dot{z}_2 = \frac{B_1}{M B_1} f + \frac{B_1}{M} z_2 - \frac{k_1}{M} z_3 + \frac{k_1}{M} z_1 - \frac{(B_1 + B_2)}{M} z_2 -$$

$$\frac{k_1}{M} z_1 + \frac{k_1}{M} z_3$$

$$\dot{z}_2 = \frac{f}{M} - \frac{B_2}{M} z_2 \rightarrow (2)^*$$

$$\dot{z}_1 = z_2 \rightarrow (3)^*$$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{B_2}{M} & 0 \\ \frac{K_1}{B_1} & 1 & -\frac{K_1}{B_1} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \\ \frac{1}{B_1} \end{bmatrix} f$$

2. a) Linearise the following differential equation around the point $x_1(t)=x_2(t)=u(t)=0$ (4 Pt.)

$$\dot{x}_1(t) - 2x_1(t) - u(t) * x_2(t) = 0$$

$$\dot{x}_2(t) - 3x_2(t) - \sin(x_1(t)) - u(t) = 0, \text{ and obtain the state equation for the system.}$$

$$\dot{x}_1 = 2x_1 + u x_2$$

$$\dot{x}_2 = 3x_2 + \sin(x_1) + u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & u \\ \cos x_1 & 3 \end{bmatrix}_{x_1=x_2=u=0} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} x_2 \\ 1 \end{bmatrix}_{x_1=x_2=u=0} u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

- b) Solve the following state equation and output equation for $y(t)$, where $u(t)$ is the unit step. (4 Pt.)

$$\dot{x} = \begin{bmatrix} -2 & 0 \\ -1 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x \quad x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$x(t) = \mathcal{L}^{-1}[(sI - A)^{-1}] x(0) + \mathcal{L}^{-1}[(sI - A)^{-1} B u(s)]$$

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} s+2 & 0 \\ 1 & s+1 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{\text{adj}(sI - A)}{\det(sI - A)} = \begin{bmatrix} s+1 & 0 \\ -1 & s+2 \end{bmatrix} / (s^2 + 3s + 2)$$

$$(sI - A)^{-1} = \begin{bmatrix} \frac{s+1}{(s+1)(s+2)} & 0 \\ \frac{-1}{(s+1)(s+2)} & \frac{(s+2)}{(s+1)(s+2)} \end{bmatrix}$$

$$\frac{(\bar{s}+1)}{(\bar{s}+1)(\bar{s}+2)} = \frac{1}{(\bar{s}+2)} \xrightarrow{\mathcal{L}^{-}} e^{-2t}$$

$$\frac{(\bar{s}+2)}{(\bar{s}+1)(\bar{s}+2)} = \frac{1}{(\bar{s}+1)} \xrightarrow{\mathcal{L}^{-}} e^{-t}$$

$$\frac{-1}{(\bar{s}+1)(\bar{s}+2)} = \frac{A_1}{\bar{s}+1} + \frac{B_1}{\bar{s}+2}$$

$$A_1 = \left. \frac{-1}{\bar{s}+2} \right|_{\bar{s}=-1} = \frac{-1}{1} = -1$$

$$B_1 = \left. \frac{-1}{\bar{s}+1} \right|_{\bar{s}=-2} = \frac{-1}{-1} = 1$$

$$\frac{-1}{(\bar{s}+1)(\bar{s}+2)} = \frac{-1}{\bar{s}+1} + \frac{1}{\bar{s}+2} \xrightarrow{\mathcal{L}^{-}} -e^{-t} + e^{-2t}$$

$$\mathcal{L}^{-}(\bar{s}I - A)^{-} = \phi(t) = \begin{bmatrix} e^{-2t} & 0 \\ -e^{-t} + e^{-2t} & e^{-t} \end{bmatrix}$$

$$\phi(t) x(0) = \begin{bmatrix} e^{-2t} & 0 \\ -e^{-t} + e^{-2t} & e^{-t} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} e^{-2t} \\ -e^{-t} + e^{-2t} \end{bmatrix}$$

$$(\bar{s}I - A)^{-} B u(\bar{s}) =$$

$$\frac{1}{\bar{s}^2 + 3\bar{s} + 2} \begin{bmatrix} \bar{s}+1 & 0 \\ -1 & \bar{s}+2 \end{bmatrix} * \begin{bmatrix} 1 \\ 1 \end{bmatrix} * \frac{1}{\bar{s}} = \begin{bmatrix} \frac{1}{\bar{s}(\bar{s}+2)} \\ \frac{1}{\bar{s}(\bar{s}+2)} \end{bmatrix}$$

$$\mathcal{L}^{-1}((sI-A)^{-1}Bu(s)) = \mathcal{L}^{-1}\begin{bmatrix} \frac{1}{s(s+2)} \\ \frac{1}{s(s+2)} \end{bmatrix}$$

$$\mathcal{L}^{-1} \frac{1}{s(s+2)} \rightarrow ??$$

$$\frac{1}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

$$A = \frac{1}{s+2} \Big|_{s=0} = \frac{1}{2}$$

$$B = \frac{1}{s} \Big|_{s=-2} = -\frac{1}{2}$$

$$\frac{1}{s(s+2)} \xrightarrow{\text{partial}} \frac{1/2}{s} + \frac{-1/2}{s+2} \xrightarrow{\mathcal{L}^{-1}} \frac{1}{2}u(t) + \frac{-1}{2}e^{-2t}$$

$$x(t) = \begin{bmatrix} e^{-2t} \\ -e^{-t} + e^{-2t} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}u(t) + \frac{-1}{2}e^{-2t} \\ \frac{1}{2}u(t) + \frac{-1}{2}e^{-2t} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}e^{-2t} + \frac{1}{2}u(t) \\ -e^{-t} + \frac{1}{2}e^{-2t} + \frac{1}{2}u(t) \end{bmatrix}$$

$$y(t) = cx(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2}e^{-2t} + \frac{1}{2}u(t) \\ -e^{-t} + \frac{1}{2}e^{-2t} + \frac{1}{2}u(t) \end{bmatrix}$$

$$= -e^{-t} + \frac{1}{2}e^{-2t} + \frac{1}{2}u(t)$$



Islamic University of Gaza
Electrical and Computer Engineering Department
EELE 3360 Control Systems
Med-term Exam (April 28, 2009)

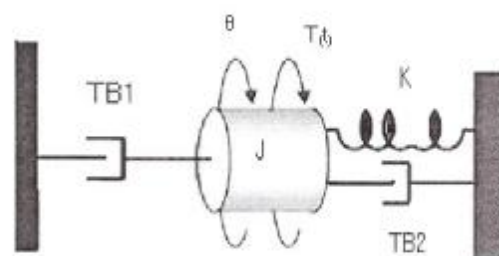


Dr. Basil Hamed

Exam Time 11:00-12:30

1. Consider the following mechanical system where $T(t)$ is applied torque; $TB1 = 10\dot{\theta} + \dot{\theta}^3$ and $TB2 = 5\dot{\theta}$ are damping force; $K=20$ and $J=2$ are spring stiffness and inertia
 - a. Find the mechanical system differential equation. (3 Points)
 - b. Linearize the above nonlinear system at $\theta_0 = 0$, $\dot{\theta}_0 = 1$, $\ddot{\theta}_0 = 0$ (4 Points)
 - c. Find the Linearize transfer function. (3 Points)

a) $T(t) = K\theta + TB1 + TB2 + J\ddot{\theta}$
 $= J\ddot{\theta} + 10\dot{\theta} + \dot{\theta}^3 + 5\dot{\theta} + K\theta$
 $T(t) = 2\ddot{\theta} + 15\dot{\theta} + \dot{\theta}^3 + 20\theta$



b) $\ddot{\theta} = -15/2 \dot{\theta} - 1/2 \dot{\theta}^3 - 10\theta + 1/2 T(t)$
 Let $x_1 = \theta$; $x_2 = \dot{x}_1 = \dot{\theta} \approx f_1$
 $\therefore \dot{x}_2 = -15/2 x_2 - 1/2 x_2^3 - 10x_1 + 1/2 T(t) \approx f_2$
 $A^* = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}_{x_{0,t_0}} = \begin{bmatrix} 0 & 1 \\ -10 & -15/2 - 3/2 x_2^2 \end{bmatrix}_{x_{0,t_0}} = \begin{bmatrix} 0 & 1 \\ -10 & -18/2 \end{bmatrix}$
 $B^* = \begin{bmatrix} \frac{\partial f_1}{\partial T} \\ \frac{\partial f_2}{\partial T} \end{bmatrix}_{x_{0,t_0}} = \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}$

$\therefore \dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -10 & -18/2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/2 \end{bmatrix} T(t)$; $y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

c) $\frac{\Theta(s)}{T(s)} = \frac{1/2}{s^2 + 18/2 s + 10} \approx \frac{1}{2s^2 + 18s + 20}$

$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -20 & -18 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} T(t)$; $y = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{x}$

2. The system is described by

$$A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = [2 \quad 1]$$

$$d = 0$$

a) Evaluate state transition matrix.

(3 PT.)

$$\Phi(s) = (sI - A)^{-1} = \begin{bmatrix} s & -1 \\ 3 & s+4 \end{bmatrix}^{-1} = \frac{1}{(s+1)(s+3)} \begin{bmatrix} s+4 & 1 \\ -3 & s \end{bmatrix}$$

$$\Phi(t) = \mathcal{L}^{-1} \{ \Phi(s) \} = \frac{1}{2} \begin{bmatrix} 2e^{-t} - e^{-3t} & e^{-t} - e^{-3t} \\ -3e^{-t} + 3e^{-3t} & -e^{-t} + 3e^{-3t} \end{bmatrix}$$

b) What is the response of the system due to input of e^{-2t} and initial condition $x_1(0) = -1, x_2(0) = 2$.

(5 PT.)

$$x(t) = \Phi(t) x(0) + \mathcal{L}^{-1} \{ \Phi(s) B U(s) \}$$

$$U(s) = \frac{1}{s+2}$$

$$x(t) = \frac{1}{2} \begin{bmatrix} 2e^{-t} - e^{-3t} & e^{-t} - e^{-3t} \\ -3e^{-t} + 3e^{-3t} & -e^{-t} + 3e^{-3t} \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} + \mathcal{L}^{-1} \left\{ \frac{1}{(s+1)(s+3)} \begin{bmatrix} s+4 & 1 \\ -3 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{1}{s+2} \right\}$$

$$x(t) = \frac{1}{2} \begin{bmatrix} -e^{-t} - e^{-3t} \\ e^{-t} + 3e^{-3t} \end{bmatrix} + \mathcal{L}^{-1} \left[\frac{\frac{1}{(s+1)(s+2)(s+3)}}{\frac{s}{(s+1)(s+2)(s+3)}} \right]$$

$$x(t) = \frac{1}{2} \begin{bmatrix} -e^{-t} - e^{-3t} \\ e^{-t} + 3e^{-3t} \end{bmatrix} + \frac{1}{2} \begin{bmatrix} e^{-t} - 2e^{-2t} + e^{-3t} \\ -e^{-t} + 4e^{-2t} - 3e^{-3t} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2e^{-2t} \\ 4e^{-2t} \end{bmatrix}$$

$$\therefore \underline{y(t)} = C x(t) = [2 \quad 1] \begin{bmatrix} -e^{-2t} \\ 2e^{-2t} \end{bmatrix} = -2e^{-2t} + 2e^{-2t} = \underline{0}$$

c) What is the system transfer function?

(2 PT.)

$$G(s) = \frac{s+2}{s^2 + 4s + 3}$$

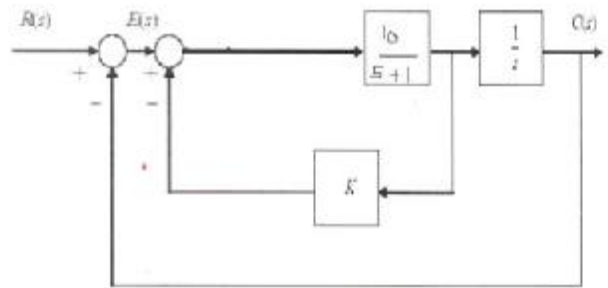
3. a) Find the closed loop transfer function.

(2 PT.)

$$G_1 = \frac{10/s+1}{1 + \frac{10k}{s+1}} = \frac{10}{s^2 + 1 + 10k}$$

$$\therefore T(s) = \frac{10}{(s^2 + 1 + 10k)s}$$

$$\therefore T(s) = \frac{10}{s^2 + (1+10k)s + 10}$$



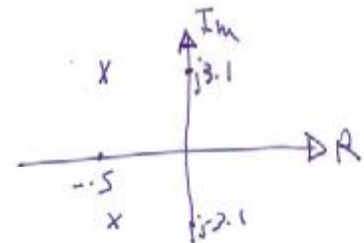
b) For $K=0$ will the system oscillate, given step input?

(2 PT.)

$$\text{when } K=0 \quad \therefore T(s) = \frac{10}{s^2 + s + 10} = \frac{10}{(s + 0.5 + j3.1)(s + 0.5 - j3.1)}$$

Complex Poles

Yes the system will oscillate



c) What value of K will makes the system critically damped?

(2 PT.)

when the system is critically damped $\zeta = 1$

charact. eqn

$$s^2 + (1+10k)s + 10 = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\omega_n^2 = 10 \Rightarrow \omega_n = 3.2 \quad \zeta = 1$$

$$\therefore 1+10k = 2\zeta\omega_n = 2(3.2) = 6.4$$

$$\therefore k = \frac{5.4}{10} = 0.54 \quad \text{for system to be critically damped}$$

d) Assume now that $\zeta = 0.5$. What is the steady state value of $c(t)$ for step input of amplitude of 2.5, what is % overshoot and what is settling time? (4 PT.)

$$\frac{C(s)}{R(s)} = \frac{10}{s^2 + 2\zeta\omega_n s + 10} = \frac{10}{s^2 + 2(0.5)(3.2)s + 10}$$

$$\boxed{R(s) = 2.5/s}$$

$$\therefore C(s) = \frac{10}{s^2 + 3.2s + 10} \cdot \frac{2.5}{s}$$

To find $c(t)$ steady state value apply Final value

Theorem:

$$c(t) = \lim_{t \rightarrow \infty} s C(s) = \lim_{s \rightarrow 0} \frac{2.5}{s^2 + 3.2s + 10} = \frac{2.5}{10} = \boxed{2.5}$$

$$\% \text{ overshoot} = 100 e^{-\pi\zeta/\sqrt{1-\zeta^2}} = \boxed{16.3\%}$$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.5(3.2)} = \underline{2.5 \text{ sec}}$$

GOOD Luck

Dr. Basil Hamed

Exam Time 11:00-12:30

1. In the system shown in Fig. 1 :

- Write the differential equation of the system.
- Write the state equations

(4 Points)

(3 Points)

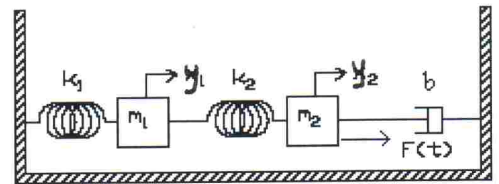
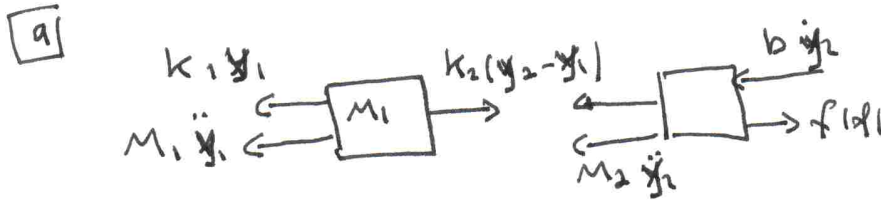


Fig. 1

$$f(t) = M_2 \ddot{y}_2 + b \dot{y}_2 + k_2 (y_2 - y_1)$$

$$k_2 (y_2 - y_1) = k_1 y_1 + M_1 \ddot{y}_1$$

b

Let $x_1 = y_1$
 $x_2 = \dot{x}_1 = \dot{y}_1$

$x_3 = y_2$
 $x_4 = \dot{x}_3 = \dot{y}_2$

$$\therefore f(t) = M_2 \dot{x}_4 + b x_4 + k_2 (x_3 - x_1)$$

$$\therefore \dot{x}_4 = + \frac{k_2}{M_2} x_1 - \frac{k_2}{M_2} x_3 - \frac{b}{M_2} x_4 + \frac{1}{M_2} f(t)$$

$$k_2 (x_3 - x_1) = k_1 x_1 + M_1 \dot{x}_2$$

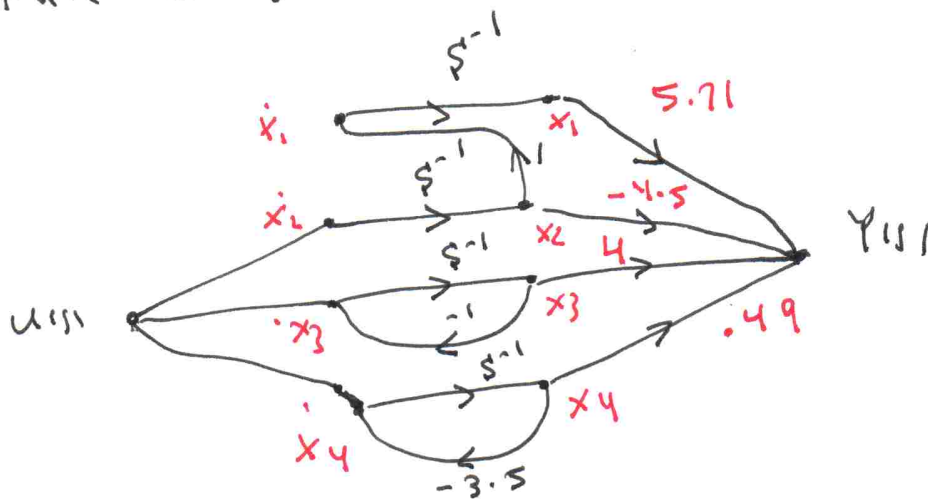
$$\dot{x}_2 = -\frac{(k_1 + k_2)}{M_1} x_1 + \frac{k_2}{M_1} x_3$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{(k_1 + k_2)}{M_1} & 0 & \frac{k_2}{M_1} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} f(t)$$

2. Draw state diagram for the system $G(s) = \frac{10(s+2)}{s^2(s+1)(s+3.5)}$ by means of Parallel decomposition. Make certain that the state diagram contain a minimum number of integrators. Write the state equation from state diagram. (7 Points)

$$G(s) = \frac{Y(s)}{U(s)} = \frac{10(s+2)}{s^2(s+1)(s+3.5)} = \frac{-4.5}{s^1} + \frac{.49}{s+3.5} + \frac{4}{s^1+1} + \frac{5.71}{s^2}$$

State diagram



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 5.71 & -4.5 & 4 & .49 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

3. The system is described by

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = [1 \quad 1]$$

$$d = 0$$

a) Evaluate state transition matrix. (3 PT.)

b) What is the response of the system due to input of unit impulse and initial condition $x_1(0) = 1, x_2(0) = 0$. (5 PT.)

$$\Phi(s) = \mathcal{L}^{-1}\{(sI - A)^{-1}\}$$

$$a) \quad \Phi(s) = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}}{s^2 + 3s + 2}$$

$$\Phi(s) = \mathcal{L}^{-1}\{(sI - A)^{-1}\} = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$\underline{u(s) = 1}$$

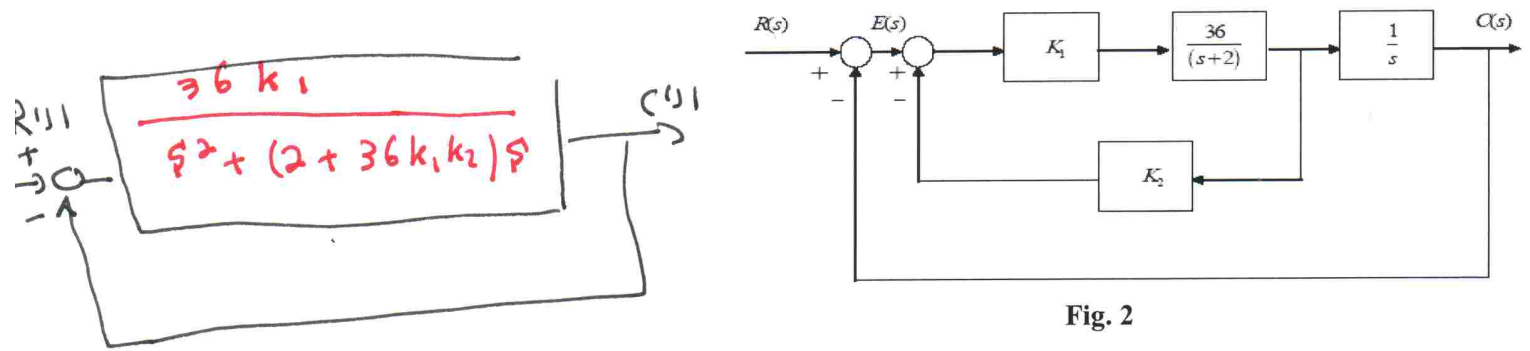
$$b) \quad \underline{x(s)} = \Phi(s) x(0) + \mathcal{L}^{-1}\{\Phi(s) B u(s)\}$$

$$x(s) = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \mathcal{L}^{-1}\left\{ \frac{\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{s^2 + 3s + 2} \right\}$$

$$x(s) = \begin{bmatrix} 2e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} \end{bmatrix} + \begin{bmatrix} e^{-t} - e^{-2t} \\ -e^{-t} + 2e^{-2t} \end{bmatrix} = \begin{bmatrix} 3e^{-t} - 2e^{-2t} \\ -3e^{-t} + 4e^{-2t} \end{bmatrix}$$

$$\boxed{y(s) = C x(s) = [1 \quad 1] \begin{bmatrix} 3e^{-t} - 2e^{-2t} \\ -3e^{-t} + 4e^{-2t} \end{bmatrix} = \boxed{2e^{-2t}}}$$

4. Consider the block diagram shown in Fig. 2. Determine appropriate value for K_1 , and K_2 to achieve overshoot of 20 %, and settling time of 2 second for a unit step input. (8PT.)



$$\therefore \frac{C(s)}{R(s)} = \frac{36 K_1}{s^2 + (2 + 36 K_1 K_2)s + 36 K_1} \quad (1)$$

$$\zeta = 20\% \Rightarrow \zeta = \frac{-\ln(0.21)}{\sqrt{\pi^2 + \ln^2(0.21)}} = 0.45$$

$$T_s = 2 \text{ sec} \Rightarrow \omega_n = \frac{4}{T_s} = 4.44$$

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{19.75}{s^2 + 3.996s + 19.75} \quad (2)$$

$$\text{eq (1)} = \text{eq (2)} \quad 36 K_1 = 19.75 \quad \therefore K_1 = 0.548$$

$$2 + 36 K_1 K_2 = 3.996 \Rightarrow K_2 = 0.1$$

EXTRA CREDIT:

Name the textbook used in this course and name the author of the textbook

(2 Points)

* Control Systems Engineering

GOOD Luck

* Norman Nise

Islamic University of Gaza
Electrical and Computer Engineering Department
EELE 3360 Control Systems
Med-term Exam (April 12, 2007)

Dr. Basil Hamed

Exam Time 11:00-12:30

1. A vehicle towing a trailer through a spring-damper coupling hitch is shown in Fig. 1, $y_1(t)$, the displacement of the towing vehicle; $y_2(t)$, the displacement of the trailer; and $f(t)$, the force of the towing vehicle.

- Write the differential equation of the system.
- Write the state equations

(4 Points)
(3 Points)

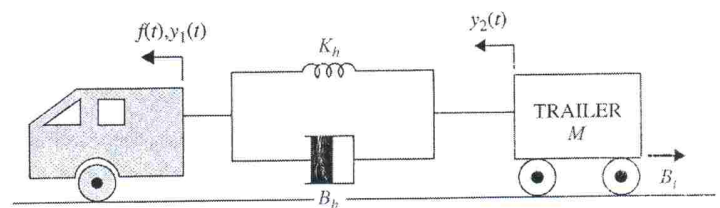
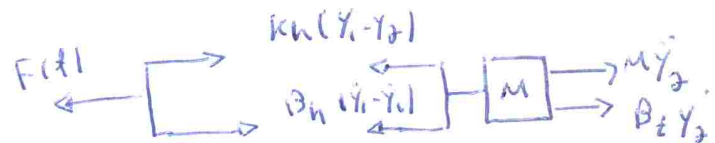


Fig. 1

a

$$f(t) = k_h(y_1 - y_2) + B_h(\dot{y}_1 - \dot{y}_2)$$

$$k_h(y_1 - y_2) + B_h(\dot{y}_1 - \dot{y}_2) = M\ddot{y}_2 + B_t\dot{y}_2$$



b

state variables:

$$\text{Let } x_1 = y_1 - y_2$$

$$x_2 = \dot{y}_2$$

$$\therefore f(t) = k_h x_1 + B_h \dot{x}_1$$

$$k_h x_1 + B_h \dot{x}_1 = M \dot{x}_2 + B_t x_2$$

$$\therefore \dot{x}_1 = -\frac{k_h}{B_h} x_1 + \frac{1}{B_h} f(t)$$

$$\dot{x}_2 = -\frac{B_t}{M} x_2 + \frac{1}{M} f(t)$$

2. Represent the electrical network shown in Fig. 2 in state space, where $v_o(t)$ is the output. (8 Points)

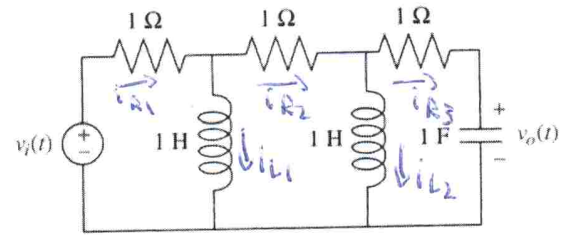


Fig. 2

$$V_i = i_{R1} + i_{R2} + i_{R3} + V_o$$

$$i_{R1} = i_{L1} + i_{L2}$$

$$i_{R2} = i_{L2} + i_{R3}$$

$$i_{R3} = C \frac{dv_o}{dt} = \frac{dv_o}{dt}$$

$$\therefore V_i = i_{L1} + 2i_{L2} + 3 \frac{dv_o}{dt} + V_o$$

$$\therefore \frac{dv_o}{dt} = \frac{1}{3} V_i - \frac{1}{3} i_{L1} - \frac{2}{3} i_{L2} - \frac{1}{3} V_o$$

$$L \frac{di_{L1}}{dt} = V_i - i_{R1} = V_i - i_{L1} - i_{L2} - C \frac{dv_o}{dt}$$

$$\therefore \frac{di_{L1}}{dt} = \frac{2}{3} V_i - \frac{2}{3} i_{L1} - \frac{1}{3} i_{L2} + \frac{1}{3} V_o$$

$$L \frac{di_{L2}}{dt} = i_{R3} + V_o = C \frac{dv_o}{dt} + V_o$$

$$\frac{di_{L2}}{dt} = \frac{1}{3} V_i - \frac{1}{3} i_{L1} - \frac{2}{3} i_{L2} + \frac{2}{3} V_o$$

$$\begin{bmatrix} \frac{di_{L1}}{dt} \\ \frac{di_{L2}}{dt} \\ \frac{dv_o}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ -\frac{1}{3} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ v_o \end{bmatrix} + \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} V_i$$

$$\begin{bmatrix} v_o \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ v_o \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} V_i$$

3. The system is described by

(8 PT.)

$$\frac{dx_1}{dt} = x_1 + u(t)$$

$$\frac{dx_2}{dt} = x_1 + x_2 + u(t)$$

Where $u(t) = 1$ for $t \geq 0$. The initial conditions are $x_1(0) = 1$, $x_2(0) = 1$. Determine state vector $x(t)$ for $t \geq 0$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$\Phi(t) = \mathcal{L}^{-1}(\mathcal{L}(I - A)^{-1})$$

$$\Phi(t) = \mathcal{L}^{-1} \begin{bmatrix} \frac{s-1}{(s-1)^2} & 0 \\ \frac{1}{(s-1)^2} & \frac{s-1}{(s-1)^2} \end{bmatrix} = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$$

$$u(s) = 1/s \quad \therefore x(t) = \Phi(t) x(0) + \mathcal{L}^{-1} \{ \Phi(s) B u(s) \}$$

$$\Phi(s) B u(s) = \begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{s-1} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{1}{s} = \begin{bmatrix} \frac{1}{s(s-1)} \\ \frac{1}{s(s-1)^2} + \frac{1}{s(s-1)} \end{bmatrix}$$

$$\therefore x(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 + e^t \\ 1 + te^t - e^t - 1 + e^t \end{bmatrix}$$

$$\therefore x(t) = \begin{bmatrix} e^t \\ te^t + e^t \end{bmatrix} + \begin{bmatrix} -1 + e^t \\ te^t \end{bmatrix}$$

$$x(t) = \begin{bmatrix} -1 + 2e^t \\ 2te^t + e^t \end{bmatrix}$$

4. A second order system settles to a final position of 5cm in a time of 2 seconds in response to a unit step input. The response of this system is oscillatory with damped frequency of 3 rad/sec. Give the transfer function for this system. (7 PT.)

$$t_s = 2 \text{ sec}$$

$$k_{DC} = 5$$

$$\omega_d = 3$$

$$\boxed{\frac{k_{DC} \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 3 \quad ; \quad t_s = \frac{4}{\zeta\omega_n} \Rightarrow \zeta\omega_n = \frac{4}{2} = 2$$

$$\therefore \omega_d = \omega_n \sqrt{1 - \left(\frac{2}{\omega_n}\right)^2} = 3$$

$$\therefore \omega_n^2 = 13$$

$$\therefore \omega_n = \underline{\underline{3.6}}$$

$$\therefore \frac{Y}{R} = \frac{k_{DC} \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{5(13)}{s^2 + 4s + 13}$$

$$\boxed{\therefore \frac{Y}{R} = \frac{65}{s^2 + 4s + 13}}$$

GOOD Luck

Islamic University of Gaza
Electrical and Computer Engineering Department
EELE 3371 Control System
Med-term Exam
March 26, 2006

Dr. Basil Hamed

Exam Time 11:00-12:30

Name	<u>Solution</u>
Student ID	
Grade	

GOOD LUCK

1. a) The Dynamics of certain plant can be represented by the following differential equation

$$\ddot{y}(t) + 0.1 \dot{y}(t) + \sin(y(t)) = u(t)$$

This may also be written in the form

$$f(u, y, \dot{y}, \ddot{y}) = \ddot{y}(t) + 0.1 \dot{y}(t) + \sin(y(t)) - u(t)$$

It is desired to regulate the plant around the equilibrium, $u_0 = \frac{1}{\sqrt{2}}$, $y_0 = \frac{\pi}{4}$, $\dot{y}_0 = 0$ Find the Linear State Space model of the plant (7 PT.)

Let $\begin{cases} x_1 = y \\ x_2 = \dot{x}_1 = \dot{y} \end{cases}$

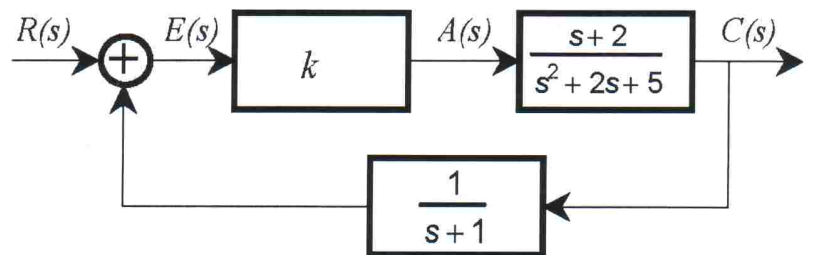
$$\dot{x}_1 = x_2 = f_1$$

$$\dot{x}_2 = -\sin x_1 - 0.1 x_2 + u \quad f_2$$

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -\cos x_1 & -0.1 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u = \begin{bmatrix} 0 & 1 \\ -0.707 & -0.1 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$x_{01} u_0$

b) Find $E(s)/R(s)$ and the characteristic equation of the system (5 PT.)

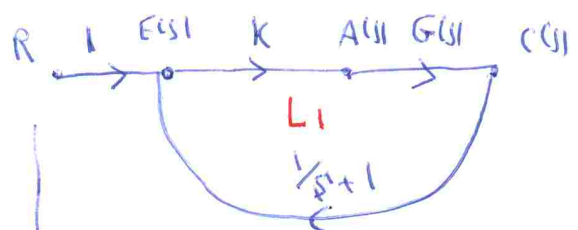


$$\frac{E}{R} = \frac{1}{\Delta} \sum_{k=1}^P \Delta_k M_k$$

$$P=1 \quad M_1=1 \quad \Delta_1=1$$

$$\Delta = 1 - L_1 = 1 - \left(k \frac{s+2}{s^2+2s+5} \frac{1}{s+1} \right)$$

$$\Delta = \frac{s^3 + 3s^2 + 7s + 5 - k(s+2)}{s^3 + 3s^2 + 7s + 5}$$



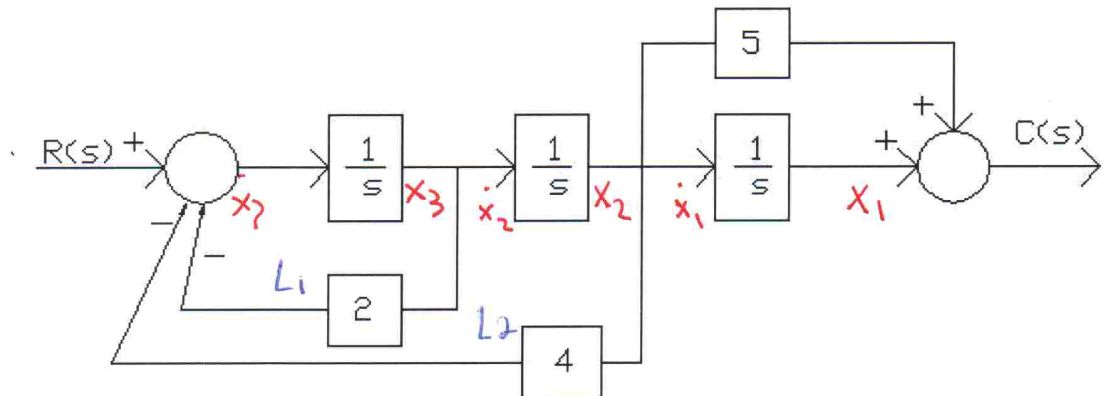
character. eq: $\Delta = 0$

$$\frac{E}{R} = \frac{s^3 + 3s^2 + 7s + 5}{s^3 + 3s^2 + 7s + 5 - k(s+2)}$$

$$s^3 + 3s^2 + 7s + 5 - k(s+2) = 0$$

2. a) Write state equations and output equation for the system shown in Fig 1.

(4 PT.)



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(t) \quad \text{Fig. 1}$$

$$y = \begin{bmatrix} 1 & 5 & 0 \end{bmatrix} \underline{x}$$

b) Write a transfer function for the block diagram shown in Fig. 1

(4 PT.)

$$\frac{C}{R} = \frac{1}{\Delta} \sum_{k=1}^P M_k \Delta_k$$

$$P = 2$$

$$M_1 = \frac{1}{s^3}, \Delta_1 = 1$$

$$M_2 = \frac{s}{s^2}, \Delta_2 = 1$$

$$\frac{C}{R} = \frac{\frac{s}{s^2} + \frac{1}{s^3}}{\frac{s^2 + 2s + 4}{s^2}}$$

$$\frac{C}{R} = \frac{s^2 + 1}{s(s^2 + 2s + 4)}$$

$$\Delta = 1 - (L_1 + L_2)$$

$$\Delta = 1 - \left(-\frac{2}{s} - \frac{4}{s^2} \right)$$

$$\Delta = \frac{s^2 + 2s + 4}{s^2}$$

$$\frac{C}{R} = \frac{s^2 + 1}{s^3 + 2s^2 + 4s}$$

3. A journal bearing, mounted on shafts that are assumed to deform, is modeled by the diagram shown in Fig 2.

a) Write the differential equations of motion for the system, with the applied torque on the outer bearing. (5 PT.)

b) Obtain the state equations for the system (5 PT)

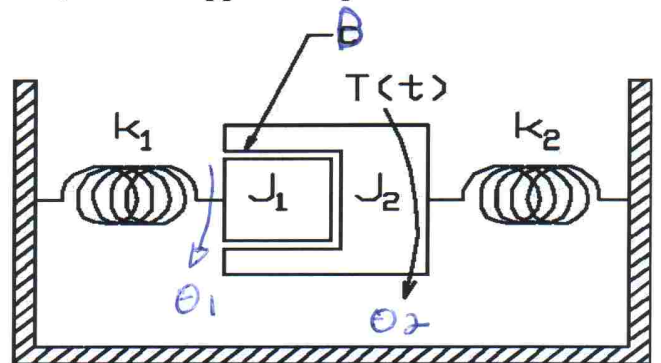


Fig. 2

a

$$k_1 \theta_1 + J_1 \ddot{\theta}_1 = B(\dot{\theta}_2 - \dot{\theta}_1)$$

$$T = J_2 \ddot{\theta}_2 + k_2 \theta_2 + B(\dot{\theta}_2 - \dot{\theta}_1)$$

b

$$\begin{aligned} x_1 &= \theta_1 \\ x_2 &= \dot{\theta}_1 = \dot{x}_1 \end{aligned}$$

$$\begin{aligned} x_3 &= \theta_2 \\ x_4 &= \dot{\theta}_2 = \dot{x}_3 \end{aligned}$$

$$k_1 x_1 + J_1 \dot{x}_2 = B(x_4 - x_2)$$

$$\dot{x}_2 = -\frac{k_1}{J_1} x_1 - \frac{B}{J_1} x_2 + \frac{B}{J_1} x_4$$

$$T = J_2 \dot{x}_4 + k_2 x_3 + B(x_4 - x_2)$$

$$\dot{x}_4 = \frac{B}{J_2} x_2 - \frac{k_2}{J_2} x_3 - \frac{B}{J_2} x_4 - \frac{1}{J_2} T$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{J_1} & -\frac{B}{J_1} & 0 & \frac{B}{J_1} \\ 0 & 0 & 0 & 1 \\ \frac{B}{J_2} & -\frac{k_2}{J_2} & -\frac{B}{J_2} & -\frac{1}{J_2} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{1}{J_2} \end{bmatrix} T$$

Islamic University of Gaza
Electrical and Computer Engineering Department
EELE 3371 Control System
Med-term Exam
March 31, 2005

Dr. Basil Hamed

Exam Time 10:00-12:00

Name	<u>SOLUTION</u>
Student ID	
Grade	

GOOD LUCK

1. a) For the circuit shown in Figure 1, find the state model with the state variables as defined in the circuit diagrams (4 Points)

b) Find state diagram (4 Points)

c) Find the transfer function. (4 Points)

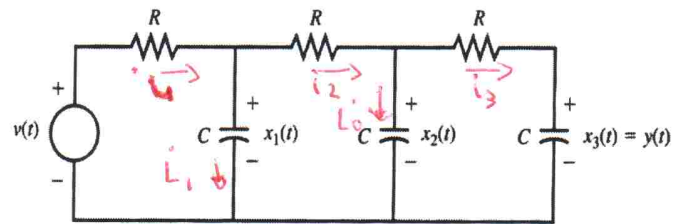


Figure 1

a) KVL around right loop:

$$x_2(t) = R i_3(t) + x_3(t)$$

$$\dot{x}_2(t) = R C \dot{x}_3 + x_3(t)$$

$$\dot{x}_3 = \frac{1}{RC} x_2 - \frac{1}{RC} x_3$$

KVL around center loop:

$$x_1(t) = R i_2(t) + x_2(t)$$

$$\dot{x}_1(t) = R C \dot{x}_3 + R C \dot{x}_2 + x_2$$

$$R C \dot{x}_2 = x_1 - R C \left[\frac{1}{RC} x_2 - \frac{1}{RC} x_3 \right] + x_2$$

$$\dot{x}_2 = \frac{1}{RC} x_1 - \frac{2}{RC} x_2 + \frac{1}{RC} x_3$$

KVL around left loop:

$$v(t) = R i_1(t) + x_1(t)$$

$$v(t) = R C [\dot{x}_1 + \dot{x}_2 + \dot{x}_3] + x_1$$

$$R C \dot{x}_1 = -R C \left[\frac{1}{RC} x_1 - \frac{2}{RC} x_2 + \frac{1}{RC} x_3 \right] + \frac{1}{RC} x_2 - \frac{1}{RC} x_3 + x_1 + v(t)$$

$$x_3(t) = \frac{1}{C} \int i_3(t) dt$$

$$C \dot{x}_3 = i_3(t)$$

$$\dot{i}_2 = \dot{i}_3 + \dot{i}_0$$

$$i_0 = C \dot{x}_2$$

$$\dot{i}_2 = C \dot{x}_3 + C \dot{x}_2$$

$$i(t) = i_1(t) + i_2(t)$$

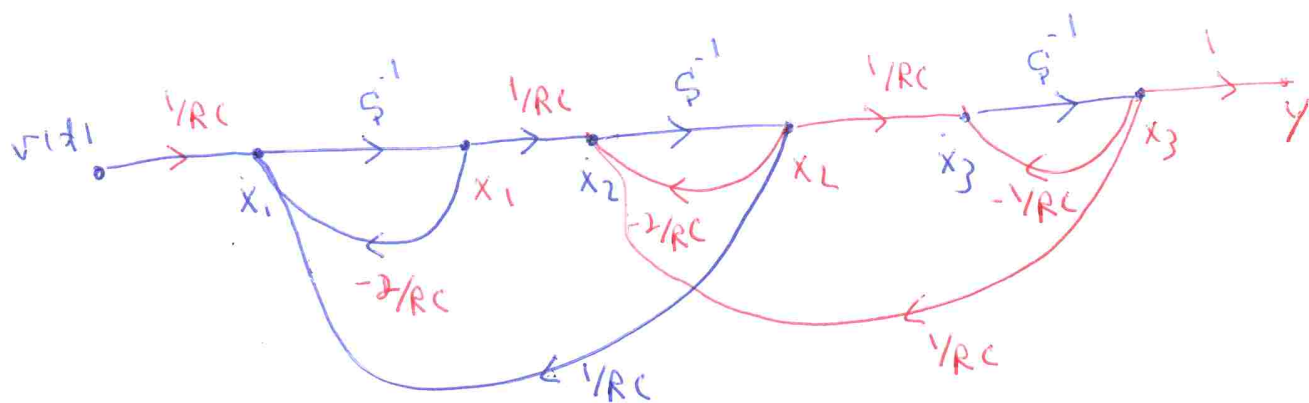
$$i(t) = C \dot{x}_1 + C \dot{x}_2 + C \dot{x}_3$$

$$\dot{x}_1 = -2/RC x_1(t) + 1/RC x_2(t) + 1/RC v(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2/RC & 1/RC & 0 \\ 1/RC & -2/RC & 1/RC \\ 0 & 1/RC & -1/RC \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1/RC \\ 0 \\ 0 \end{bmatrix} v(t)$$

$$[y(t)] = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(b) state diagram:



2. Consider the mechanical system shown in Figure 2, where $m = 0.1$, $k_1 = 6$, $k_2 = 4$, $b = 0.4$. Assume that u is the input to the system and is a force, y and z are displacement. Considering displacement y is the output, obtain the transfer function. Also, obtain a state model representation for the system. (10 Points)

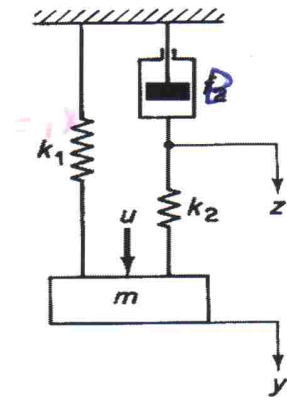
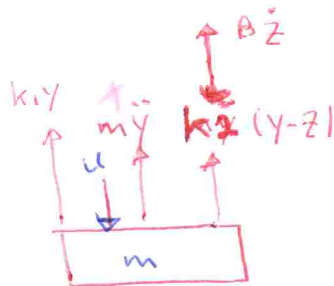


Figure 2

$$u = m\ddot{y} + k_1 y + k_2 (y - z)$$

$$k_2 (y - z) = B \dot{z}$$

By taking Laplace transform of above eqs.

$$[m s^2 + k_1 + k_2] Y(s) = U(s) + k_2 Z(s)$$

$$k_2 Y(s) = (k_2 + B s) Z(s)$$

Eliminating $Z(s)$ from these 2 eqs:

$$[m s^2 + k_1 + k_2] Y(s) = U(s) + \frac{k_2}{k_2 + B s} Y(s)$$

$$\therefore [m s^2 + k_1 + k_2 - \frac{k_2}{k_2 + B s}] Y(s) = U(s)$$

$$\therefore \frac{Y(s)}{U(s)} = \frac{k_2 + B s}{(m k_2 + m B s) s^2 + (k_1 + k_2)(k_2 + B s) - k_2^2}$$

$$\frac{Y(s)}{U(s)} = \frac{4 + 0.4 s}{(0.1 \times 4 \times 10) s^2 + 10 (4 + 0.4 s) - 16}$$

$$\frac{Y}{U} = \frac{10s + 100}{s^3 + 10s^2 + 100s + 600}$$

Find state model :

$$\text{Let } x_1 = Y, \quad x_2 = \dot{x}_1 = \dot{Y}, \quad x_3 = Z$$

$$m \ddot{x}_2 + k_1 x_1 + k_2 (x_1 - x_3) + u$$

$$\boxed{\ddot{x}_2 = -\frac{(k_1 + k_2)}{m} x_1 + \frac{k_2}{m} x_3 + \frac{1}{m} u}$$

$$k_2 (x_1 - x_3) = B x_3$$

$$\boxed{\ddot{x}_3 = \frac{k_2}{B} x_1 - \frac{k_2}{B} x_3}$$

$$\therefore \dot{x}_1 = x_2$$

$$\dot{x}_2 = -100 x_1 + 40 x_3 + 10 u$$

$$\dot{x}_3 = 10 x_1 - 10 x_3$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -100 & 40 & 0 \\ 10 & 0 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix} u$$

$$[Y] = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

3. Consider the closed-loop control system in Figure 3(a) and its simulation diagram in part (b) of the Figure.

- Write the state equations for the control system from the simulation diagram. (4 Points)
- Find $G_c(s)$, the compensator transfer function, and $G_p(s)$, the plant transfer (6 Points)

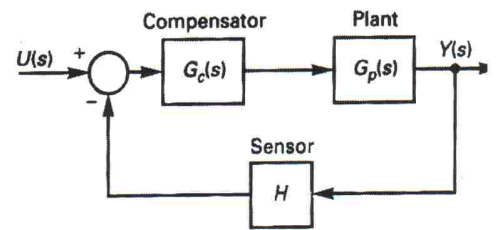


Figure 3(a)

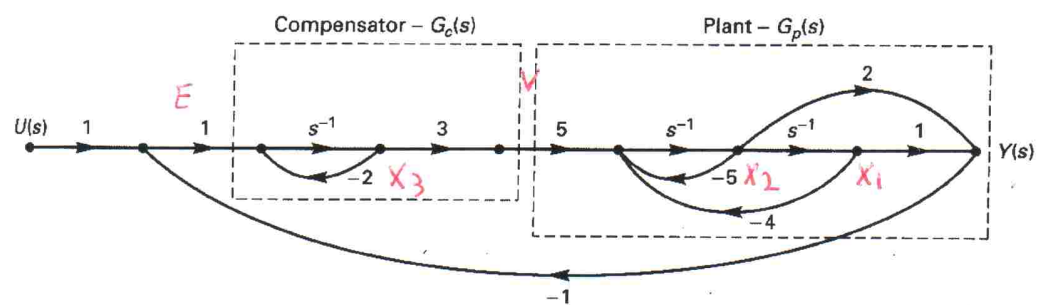


Figure 3(b)

a) state eqs

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -5x_2 - 4x_1 + 15x_3$$

$$\dot{x}_3 = -2x_3 + [u - y] = -2x_3 + u - [x_1 + 2x_2]$$

$$\dot{x}_3 = -x_1 - 2x_2 - 2x_3 + u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -4 & -5 & 15 \\ -1 & -2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$[y] = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\text{ii) } G_c(s) = \frac{V(s)}{E(s)} = \frac{M_1 \Delta_1 + M_2 \Delta_2}{\Delta}$$

$$M_1 = -2/s$$

$$M_2 = 2$$

$$\Delta = 1 - (-3/s)$$

$$\Delta_1 = 1, \Delta_2 = 1$$

$$\therefore G_c(s) = \frac{-2/s + 2}{1 + 3/s} = \frac{2s - 2}{s + 3}$$

$$\text{i) } G_c(s) = \frac{V(s)}{E(s)} = \frac{M_1 \Delta_1}{\Delta} = \frac{3/s \cdot 1}{1 + 2/s}$$

$$\boxed{G_c(s) = \frac{3}{s + 2}}$$

$$\text{ii) } G_p(s) = \frac{Y(s)}{V(s)} = \frac{M_1 \Delta_1 + M_2 \Delta_2}{\Delta}$$

$$M_1 = 5/s^2$$

$$\Delta_1 = 1$$

$$M_2 = 10/s$$

$$\Delta_2 = 1$$

$$\Delta = 1 + \frac{5}{s} + \frac{4}{s^2} = \frac{s^2 + 5s + 4}{s^2}$$

$$\therefore \boxed{G_p(s) = \frac{5/s^2 + 10/s}{s^2 + 5s + 4} = \frac{5(2s + 1)}{s^2 + 5s + 4}}$$

Islamic University of Gaza
Electrical and Computer Engineering Department
EELE 3371 Control System
Med-Term Exam
April 4, 2004

Name:

Solution

ID#

Exam Time: 11:00- 12:30

Instructor: Dr. Basil Hamed

Grade:

GOOD LUCK

1. Consider the closed-loop control system in Figure 1(a) and its simulation diagram in part (b) of the figure.

(a) Write the state equations for the control system from the simulation diagram. (6 Points)

(b) Find $G_c(s)$, the compensator transfer function, and $G_p(s)$, the plant transfer function, directly from the simulation diagram. (6 Points)

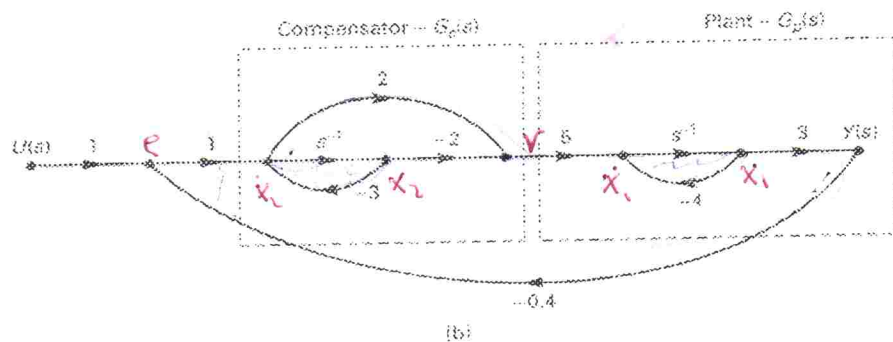
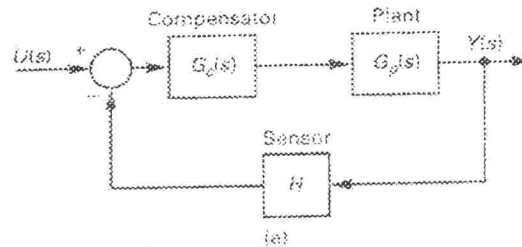


Figure 1

$$5U - 4X_1$$

$$2\dot{X}_2 - 2X_2$$

$$= e - 3X_2$$

$$= u - 0.4Y \Rightarrow Y = 3X_1$$

$$\dot{X}_2 = u - 1.2X_1 - 3X_2$$

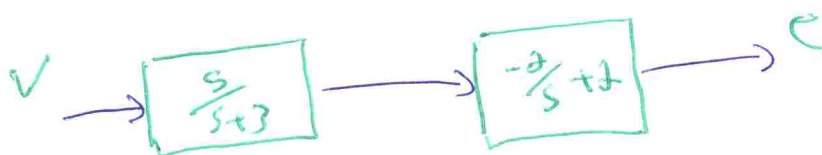
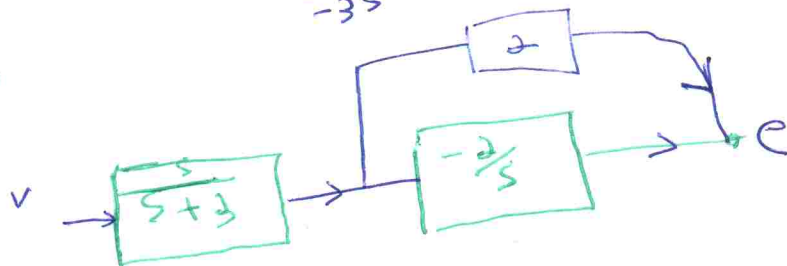
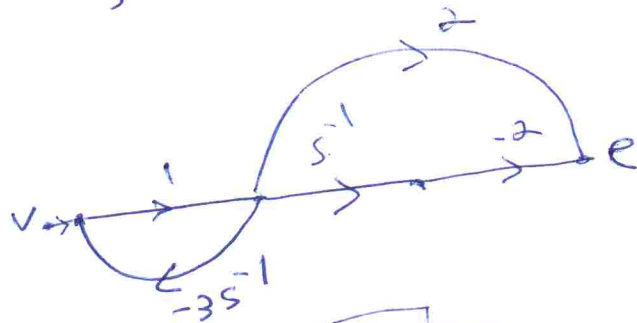
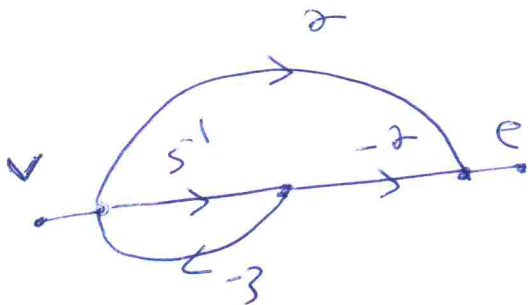
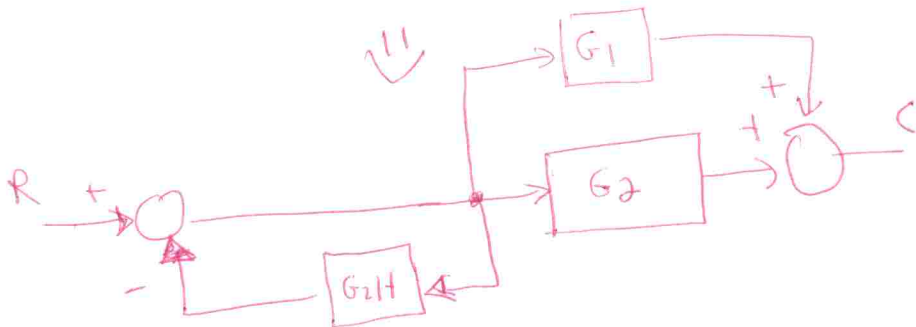
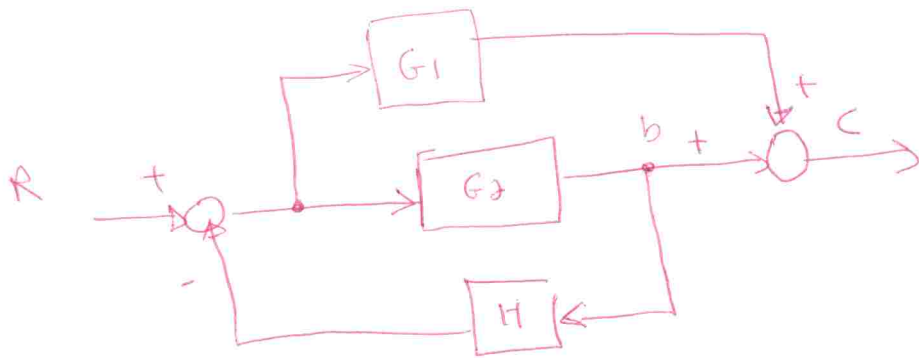
$$= 5(2\dot{X}_2 - 2X_2) - 4X_1$$

$$\dot{X}_1 = 10u - 12X_1 - 30X_2 - 10X_2 - 4X_1$$

$$\dot{X}_1 = -16X_1 - 40X_2 + 10u$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -16 & -40 \\ -1.2 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 10 \\ 1 \end{bmatrix} u$$

$$[Y] = \begin{bmatrix} 3 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$



$$\frac{e}{V} = \left(\frac{s}{s+3} \right) \left(-\frac{2}{s} + 2 \right) = \left(\frac{s}{s+3} \right) \left(\frac{-2 + 2s}{s} \right) = \boxed{\frac{2s-2}{s+3}}$$

$$G_c(s) = \frac{V(s)}{E(s)}$$

$$\frac{V(s)}{E(s)} = \frac{V(s)/u(s)}{E(s)/u(s)} = \frac{M_1 \Delta_1 + M_2 \Delta_2}{M_0 \Delta_0}$$

$$M_1 = -2/s$$

$$\Delta_1 = 1 - (-4/s) = 1 + 4/s$$

$$M_2 = 2$$

$$\Delta_2 = 1 - (-4/s) = 1 + 4/s$$

$$M_0 = 1$$

$$\Delta_0 = 1 - (-3/s - 4/s) + [(-3/s)(-4/s)]$$

$$\frac{V(s)}{E(s)} = \frac{-2/s (s+4) + 2 (s+4/s)}{1 + 3/s + 4/s + 12/s^2}$$

$$\frac{V(s)}{E(s)} = \frac{\frac{s+4}{s} [-2/s + 2]}{\frac{s^2 + 7s + 12}{s^2}} = \frac{(s+4)(-2+2s)}{(s+4)(s+3)}$$

$$G_c(s) = \frac{V(s)}{E(s)} = \frac{2s-2}{s+3}$$

$$\frac{w}{u} \quad G_p(s) = \frac{Y(s)}{V(s)} = \frac{Y(s)/u(s)}{V(s)/u(s)} = \frac{(M_1 \Delta_1 + M_2 \Delta_2) \Big|_{\text{from } u \rightarrow Y}}{(M_1 \Delta_1 + M_2 \Delta_2) \Big|_{\text{from } u \rightarrow V}}$$

from $u \rightarrow Y$

$$M_1 = -30/s^2$$

$$\Delta_1 = 1$$

$$M_2 = 30/s$$

$$\Delta_2 = 1$$

$$\frac{Y_{III}}{V_{III}} = \frac{\left(-\frac{30}{s}\right)(1) + \left(\frac{30}{s}\right)(1)}{\frac{s+4}{s} \left(-\frac{2}{s} + 2\right)} = \frac{\frac{-30 + 30s}{s}}{\frac{(s+4)(2s-2)}{s}}$$

$$\boxed{\frac{Y_{III}}{V_{III}} = \frac{30(s-1)}{2(s+4)(s-1)} = \frac{15}{s+4}}$$

$$\boxed{G_{p1}(s) = \frac{Y_{III}}{V_{III}} = \frac{15}{s+4}}$$

$\Rightarrow R$
=

$$\frac{V}{F} = \frac{M_1 \Delta_1 + M_2 \Delta_2}{\Delta_c} = \frac{-\frac{2}{s}(1) + 2(1)}{1 - (-\frac{3}{s})} = \frac{\frac{-2+2s}{s}}{\frac{s+3}{s}}$$

$$\boxed{\therefore G_{c1}(s) = \frac{V}{F} = \frac{2s-2}{s+3}}$$

$$G_{p2}(s) = \frac{Y}{V} = \frac{M_1 \Delta_1}{\Delta_p} = \frac{\left[s\left(\frac{1}{s} + 3\right)\right](1)}{1 - (-\frac{4}{s})} = \frac{15/s}{\frac{s+4}{s}}$$

$$\boxed{\therefore G_{p2}(s) = \frac{15}{s+4}}$$

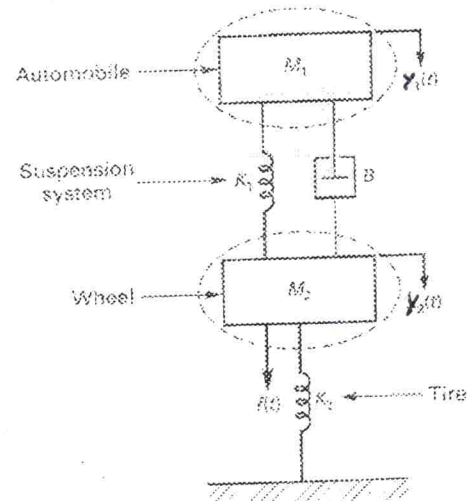
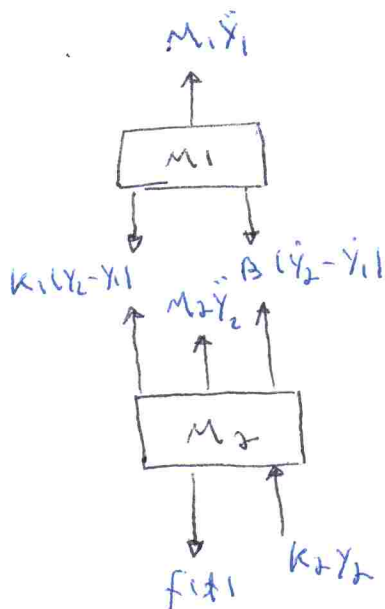
2. Consider the mechanical system shown in Figure 2. This system is a simplified model of an automobile suspension system of one wheel, with M_1 the mass of the automobile, B the shock absorber, K_1 the springs, M_2 the mass of the wheel, and K_2 the elastance of the tire.

a) Find the state model of the system

(8 Points)

b) Find state diagram of the system shown in Figure 2.

(4 Points)



$$f(t) = M_2 \ddot{y}_2 + B(\dot{y}_2 - \dot{y}_1) + k_1(y_2 - y_1) + k_2 y_2$$

$$B(\dot{y}_2 - \dot{y}_1) + k_1(y_2 - y_1) = M_1 \ddot{y}_1$$

Figure 2

Let $x_1 = y_1$ $x_3 = y_2$
 $x_2 = \dot{x}_1 = \dot{y}_1$ $x_4 = \dot{x}_3 = \dot{y}_2$
 sub. above in eq II

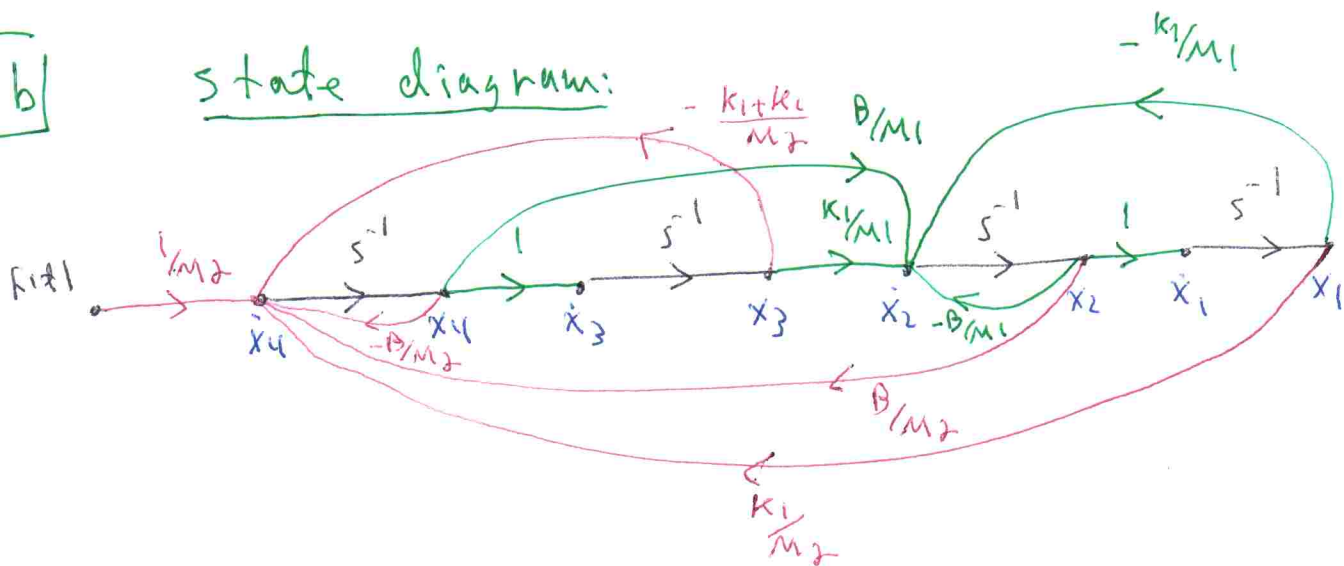
$$\dot{x}_4 = -\frac{B}{M_2} (x_4 - x_2) - \frac{k_1}{M_2} (x_3 - x_1) - \frac{k_2}{M_2} x_3 + \frac{1}{M_2} f(t)$$

$$\dot{x}_2 = \frac{k_1}{M_1} (x_3 - x_1) + \frac{B}{M_1} (x_4 - x_2)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{M_1} & -\frac{B}{M_1} & \frac{k_1}{M_1} & \frac{B}{M_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{M_2} & \frac{B}{M_2} & -\frac{k_1+k_2}{M_2} & -\frac{B}{M_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{M_2} \end{bmatrix} \quad \text{firl}$$

$$Y_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$Y_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$



3. Consider the system

$$\begin{aligned}\frac{dx_1}{dt} &= \sin x_2 + x_3 + 4u = f_1 \\ \frac{dx_2}{dt} &= x_2 + 2 \frac{dx_1}{dt} x_3 = f_2 = x_2 + 2x_3 \sin x_2 + 2x_3^2 + 8x_3 u \\ \frac{dx_3}{dt} &= x_2 u + x_1 \cos x_3 = f_3\end{aligned}$$

Find the linearized state model approximation for the above system about nominal point $x_{01} = x_{02} = x_{03} = u_0 = 0$. (6 Points)

$$A^* = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix}_{x_0, u_0} = \begin{bmatrix} 0 & \cos x_2 & 1 \\ 0 & 1 + 2x_3 \cos x_2 & 2 \sin x_2 + 4x_3 + 8u \\ \cos x_3 & u & -x_1 \sin x_3 \end{bmatrix}_{x_0, u_0}$$

$$= \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad B^* = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \\ \frac{\partial f_3}{\partial u} \end{bmatrix}_{x_0, u_0} = \begin{bmatrix} 4 \\ 8x_3 \\ x_2 \end{bmatrix}_{x_0, u_0} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

EXTRA CREDIT:

Name the textbook used in this course and name of author of the textbook (2 Points)

Text book: Automatic Control Systems 7th ed.

Author: Benjamin Kuo

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} u$$

Islamic University of Gaza
Electrical and Computer Engineering Department
ENGE 3371 Control Systems
Midterm Exam
April, 13th 2003

Name: _____
ID# _____

11:00- 12:30
Dr. Basil Hamed

1. consider the system

$$\dot{x}_1 = x_2^2 + x_1 \cos x_2 = f_1$$

$$\dot{x}_2 = x_2 + (x_1 + 1)x_1 + x_1 \sin x_2 = f_2$$

Find the linearized approximation about nominal point $x_0=0$. (6 Pt.)

$$\frac{\partial f_1}{\partial x_1} = \cos x_2$$

$$\frac{\partial f_1}{\partial x_2} = 2x_2 - x_1 \sin x_2$$

$$\frac{\partial f_2}{\partial x_1} = 2x_1 + 1 + \sin x_2$$

$$\frac{\partial f_2}{\partial x_2} = 1 + x_1 \cos x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \cos x_2 & 2x_2 - x_1 \sin x_2 \\ 2x_1 + 1 + \sin x_2 & 1 + x_1 \cos x_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

2. For the electrical system shown in Fig. 1, Find

a) State Equations

(6 Pt.)

b) Transfer function $V_o(s) / V_{in}(s)$.

(6 Pt.)

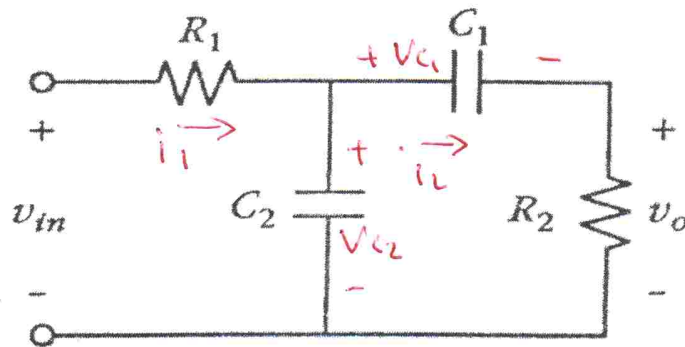


Figure 1

$$V_{in} = R_1 i_1 + V_{C2}$$

$$V_{C2} = R_2 i_2 + V_{C1}$$

$$\frac{dV_{C1}}{dt} = \frac{1}{C_1} i_2$$

$$\frac{dV_{C2}}{dt} = \frac{1}{C_2} (i_1 - i_2)$$

$$i_2 = \frac{V_{C2} - V_{C1}}{R_2}$$

$$i_1 = \frac{V_{in} - V_{C2}}{R_1}$$

$$\therefore \frac{dV_{C2}}{dt} = \frac{1}{C_2} \left[\frac{V_{in} - V_{C2}}{R_1} - \frac{V_{C2} - V_{C1}}{R_2} \right]$$

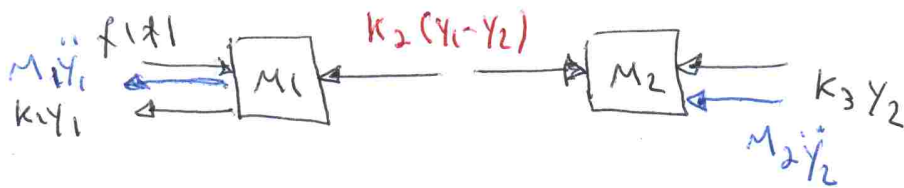
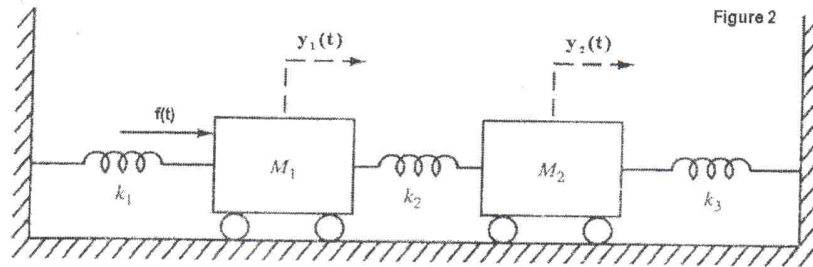
$$\therefore \frac{dV_{C2}}{dt} = \frac{1}{C_2} \left[\frac{V_{in}}{R_1} - \frac{(R_1 + R_2)}{R_1 R_2} V_{C2} + \frac{V_{C1}}{R_2} \right]$$

$$\frac{dV_{C1}}{dt} = \frac{1}{C_1} \left[\frac{V_{in} - V_{C2}}{R_1} - \frac{V_{C2} - V_{C1}}{R_2} \right]$$

$$\therefore \begin{bmatrix} \frac{dV_{C1}}{dt} \\ \frac{dV_{C2}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_1 R_2} & \frac{1}{C_1 R_2} \\ \frac{1}{R_2 C_2} & -\frac{(R_1 + R_2)}{C_2 R_1 R_2} \end{bmatrix} \begin{bmatrix} V_{C1} \\ V_{C2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{R_1 C_2} \end{bmatrix} V_{in}$$

$$V_o = V_{C2} - V_{C1} = \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} V_{C1} \\ V_{C2} \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} V_{in}$$

3. Consider the system consisting of two masses and three springs shown in Fig. 2 the masses are on wheels which are assumed to be frictionless. The input $f(t)$ to the system is the force $f(t)$ applied to the first mass. Find
- state equations, (7 pt.)
 - state diagram. (5 pt.)



Diff Eq:

$$f(t) = k_1 y_1 + k_2 (y_1 - y_2) + M_1 \ddot{y}_1$$

$$k_2 (y_1 - y_2) = k_3 y_2 + M_2 \ddot{y}_2$$

state eq:

$$\ddot{y}_1 = -\frac{(k_1 + k_2)}{M_1} y_1 + \frac{k_2}{M_1} y_2 + \frac{1}{M_1} f(t)$$

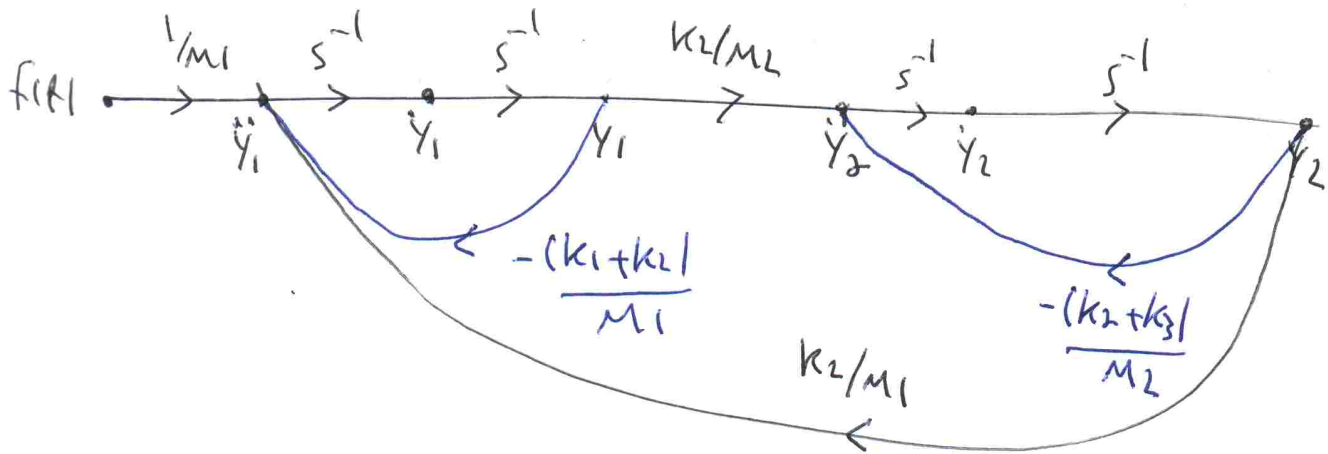
$$\ddot{y}_2 = \frac{k_2}{M_2} y_1 - \frac{(k_2 + k_3)}{M_2} y_2$$

Let $x_1 = y_1$, $x_3 = y_2$

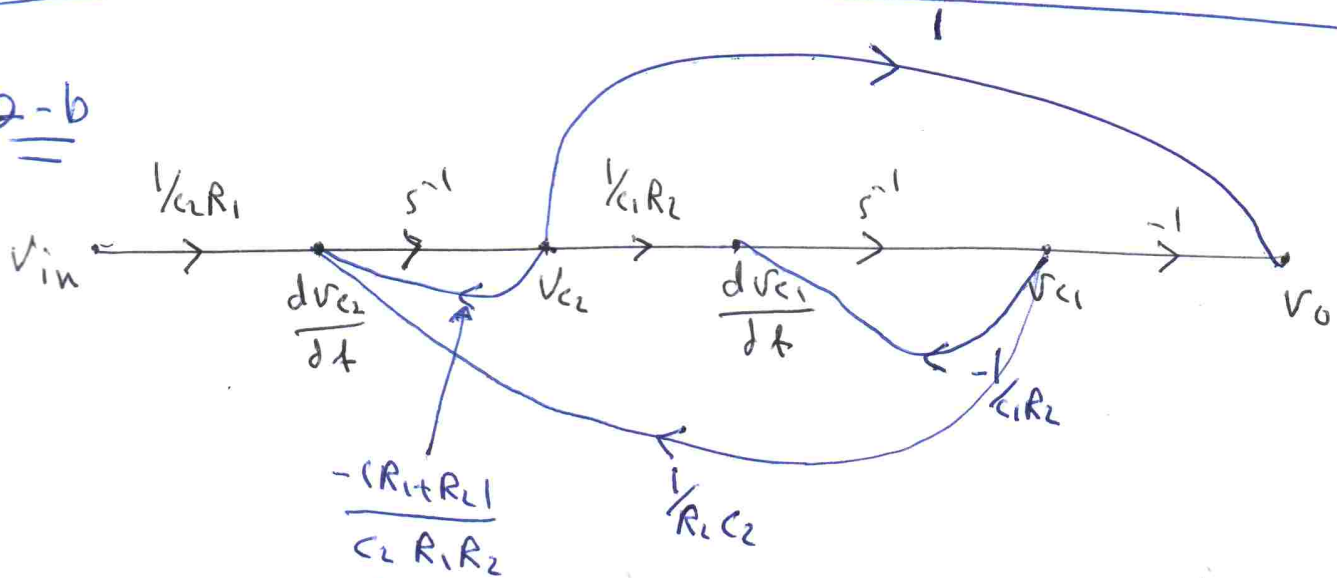
$x_2 = \dot{y}_1 = \dot{x}_1$, $x_4 = \dot{y}_2 = \dot{x}_3$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{(k_1 + k_2)}{M_1} & 0 & \frac{k_2}{M_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{M_2} & 0 & -\frac{(k_2 + k_3)}{M_2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M_1} \\ 0 \\ 0 \end{bmatrix} f(t)$$

3b



2-b



$$\frac{V_0}{V_{in}} = \frac{1}{\Delta} \sum_{k=1}^P M_k \Delta_k$$

$$P=2$$

$$L_1 = \frac{-(R_1+R_2)}{C_2 R_1 R_2 S}$$

$$L_2 = -\frac{1}{C_1 R_2 S}$$

$$L_3 = \frac{1}{C_1 C_2 R_2^2 S^2}$$

$$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_2)$$

$$M_1 = -\frac{1}{C_1 C_2 R_1 R_2 S^2}$$

$$M_2 = \frac{1}{C_2 R_1 S}$$

$$\Delta_1 = 1$$

$$\Delta_2 = 1 - L_2$$

$$\therefore V_0 = M_1 \Delta_1 + M_2 \Delta_2$$