



Faculty of Engineering

Department of computer and Electrical Engineering

Control & Linear Systems

"Mid Exam (2025)"

Submitted By:

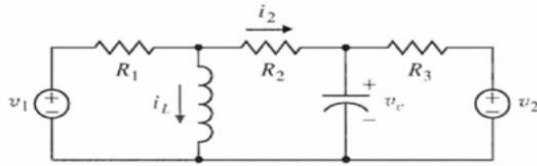
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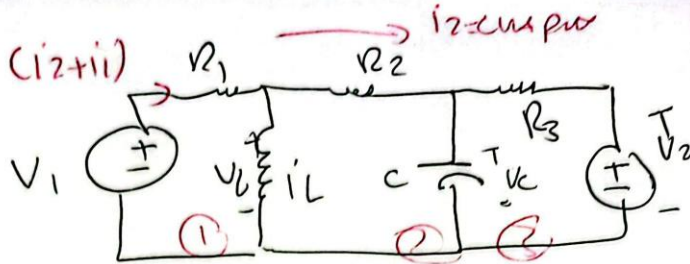
Submitted To:

Dr. Basil Hamed

1. Obtain the state model for the two-input and one-output circuit shown in Figure below, where the output is i_2 (10 Pt.)



Q1)



1) mesh (1)

$$-V_1 + (i_2 + i_L)R_1 + L \cdot \frac{di_L}{dt} = 0$$

$$L \cdot \frac{di_L}{dt} = -(i_2 + i_L)R_1 + V_1 \quad (1)$$

2) Kcl $\sim$ nod2

$$\frac{V_C - V_L}{R_2} + C \cdot \frac{dV_C}{dt} + \frac{V_C - V_2}{R_3} = 0$$

$$C \cdot \frac{dV_C}{dt} + \frac{V_C}{R_2} - \frac{V_L}{R_2} + \frac{V_C}{R_3} - \frac{V_2}{R_3} = 0$$

$$C \cdot \frac{dV_C}{dt} = -V_C \left\{ \frac{1}{R_2} + \frac{1}{R_3} \right\} + \frac{V_L}{R_2} + \frac{V_2}{R_3}$$

$$\therefore \frac{dV_C}{dt} = -V_C \left\{ \frac{R_2 + R_3}{C R_2 R_3} \right\} + \frac{V_L}{R_2 C} + \frac{V_2}{R_3 C} \quad (2)$$

دو دو

(1)

$$\frac{di_L}{dt} = \frac{V_1}{L} - \left(\frac{i_2 - i_L}{L} \right) R_1$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_1}{L} - \frac{R_1}{L} \cdot i_L - \frac{R_1}{L} \cdot i_2 \quad \text{و (3)}$$

$$i_2 = \frac{V_L - V_C}{R_2} \Rightarrow i_2 R_2 = V_L - V_C$$

$$i_2 R_2 = L \cdot \frac{di_L}{dt} - V_C$$

$$i_2 = \frac{1}{R_2} \cdot \frac{di_L}{dt} - \frac{V_C}{R_2} \quad \text{و (4)}$$

دو دو

(4) و (3)

$$\Rightarrow \frac{di_L}{dt} = \frac{V_1}{L} - i_L \left(\frac{R_1}{L} \right) - \frac{R_1}{L} \left(\frac{L}{R_2} \cdot \frac{di_L}{dt} - \frac{V_C}{R_2} \right)$$

$$\Rightarrow \frac{di_L}{dt} = \frac{V_1}{L} - i_L \left(\frac{R_1}{L} \right) - \frac{R_1}{R_2} \cdot \frac{di_L}{dt} + \frac{R_1}{L R_2} \cdot V_C$$

$$\frac{di_L}{dt} \left(1 + \frac{R_1}{R_2} \right) = -i_L \left(\frac{R_1}{L} \right) + \frac{R_1}{L R_2} V_C + \frac{V_1}{L}$$

$$\frac{di_L}{dt} \left(\frac{R_1 + R_2}{R_2} \right) = -i_L \left(\frac{R_1}{L} \right) + V_C \left(\frac{R_1}{L R_2} \right) + \frac{V_1}{L}$$

$$\frac{di_L}{dt} = -i_L \left(\frac{R_1 R_2}{L (R_1 + R_2)} \right) + V_C \left(\frac{R_1}{L (R_1 + R_2)} \right) + V_1 \left(\frac{R_2}{L (R_1 + R_2)} \right)$$

$$\frac{di_L}{dt} = i_L \left[\frac{-R_1 R_2}{L(R_1 + R_2)} \right] + V_C \left\{ \frac{R_1}{L(R_1 + R_2)} \right\} + V_1 \left\{ \frac{R_2}{L(R_1 + R_2)} \right\}$$

②) At $t = 0$

$$\frac{dv_C}{dt} = -V_C \left\{ \frac{R_2 + R_3}{C R_2 R_3} \right\} + \frac{V_L}{R_2 C} + \frac{V_2}{R_3 C}$$

$$\frac{dv_C}{dt} = -V_C \left\{ \frac{R_2 + R_3}{C R_2 R_3} \right\} + \frac{1}{R_2 C} \left\{ L \cdot \frac{di_L}{dt} \right\} + \frac{V_2}{R_3 C}$$

$$\frac{dv_C}{dt} = -V_C \left\{ \frac{R_2 + R_3}{C R_2 R_3} \right\} + \frac{1}{R_2 C} \left\{ \frac{di_L}{dt} \right\} + \frac{V_2}{R_3 C}$$

$$\frac{dv_C}{dt} = -V_C \left\{ \frac{R_2 + R_3}{C R_2 R_3} \right\} + \frac{1}{R_2 C}$$

$$\left\{ i_L \left(\frac{-R_1 R_2}{L(R_1 + R_2)} \right) + V_C \left(\frac{R_1}{L(R_1 + R_2)} \right) + V_1 \left(\frac{R_2}{L(R_1 + R_2)} \right) \right\}$$

(2) $\vec{P}$

$$\frac{dv_C}{dt} = V_C \left\{ \frac{R_1}{R_2 C (R_1 + R_2)} - \frac{R_2 + R_3}{C R_2 R_3} \right\} - i_L \left\{ \frac{R_1}{C (R_1 + R_2)} \right\} + V_1 \left(\frac{1}{(R_1 + R_2) C} \right) + \frac{V_2}{R_3 C}$$

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Also we

$$i_2 = \frac{L}{R_2} \left\{ \frac{di_L}{dt} \right\} - \frac{V_c}{R_2}$$

$$i_2 = \frac{L}{R_2} \left\{ i_L \left(\frac{-R_1 R_2}{L(R_1 + R_2)} \right) + V_c \left(\frac{R_1}{L(R_1 + R_2)} \right) + V_1 \left(\frac{R_2}{L(R_1 + R_2)} \right) - \frac{V_c}{R_2} \right\}$$

$$i_2 = i_L \left(\frac{-R_1}{R_1 + R_2} \right) - \frac{V_c}{R_1 + R_2} + \frac{V_1}{R_1 + R_2}$$

نفس الطريقة 5.2

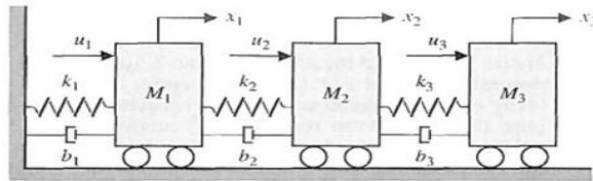
$$\begin{Bmatrix} \frac{di_L}{dt} \\ \frac{dV_c}{dt} \end{Bmatrix} = \begin{Bmatrix} \frac{-R_1 R_2}{L(R_1 + R_2)}; \frac{R_1}{L(R_1 + R_2)} \\ \frac{-R_1}{C(R_1 + R_2)}; \frac{R_1}{R_2 C(R_1 + R_2)}; \frac{R_2 + R_3}{C R_2 R_3} \end{Bmatrix}$$

$$\begin{Bmatrix} i_L \\ V_c \end{Bmatrix} + \begin{Bmatrix} \frac{R_2}{L(R_1 + R_2)}; \frac{1}{C(R_1 + R_2)} \\ \frac{1}{R_3 C} \end{Bmatrix} \begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix}$$

Also we

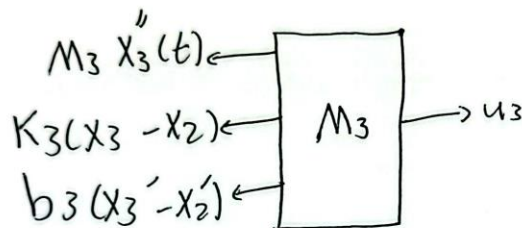
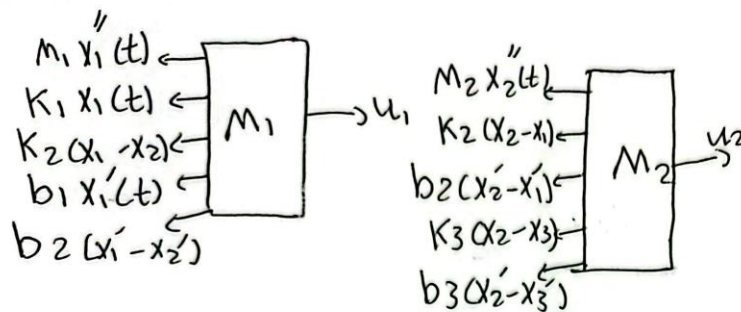
$$i_2 = \begin{Bmatrix} \frac{-R_1}{R_1 + R_2} & \frac{-1}{R_1 + R_2} \end{Bmatrix} \begin{Bmatrix} i_L \\ V_c \end{Bmatrix} + \begin{Bmatrix} \frac{1}{R_1 + R_2} & 0 \end{Bmatrix} \begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix}$$

2. For the three-cart system illustrated in Figure below, obtain the equations of motion. The system has three inputs u_1 , u_2 , and u_3 and three outputs x_1 , x_2 , and x_3 . (7 Pt.)



Q2))

* state diagram :

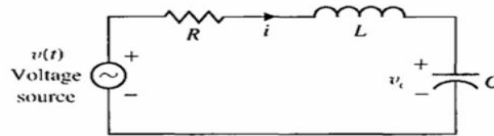


$$\begin{aligned}
 M_1 \ddot{x}_1 + b_1 \dot{x}_1 + b_2 \dot{x}_1 - b_2 \dot{x}_2 + K_1 x_1 + K_2 x_1 - K_2 x_2 &= u_1 \quad (1) \\
 M_2 \ddot{x}_2 + b_2 \dot{x}_2 - b_2 \dot{x}_1 + b_3 \dot{x}_2 - b_3 \dot{x}_3 + K_2 x_2 - K_2 x_1 & \\
 + K_3 x_2 - K_3 x_3 &= u_2 \quad (2) \\
 M_3 \ddot{x}_3 + b_3 \dot{x}_3 - b_3 \dot{x}_2 + K_3 x_3 - K_3 x_2 &= u_3 \quad (3)
 \end{aligned}$$

in $C \& \text{ domain}$

$$\begin{aligned}
 (M_1 s^2 + b_1 s + b_2 s + K_1 + K_2) x_1 - (b_2 s + K_2) x_2 &= u_1 \quad (1) \\
 -(b_2 s + K_2) x_1 - (b_3 s + K_3) x_3 + (M_2 s^2 + b_2 s + b_3 s + K_2 + K_3) x_2 &= u_2 \quad (2) \\
 -(b_3 s + K_3) x_2 + (M_3 s^2 + b_3 s + K_3) x_3 &= u_3 \quad (3)
 \end{aligned}$$

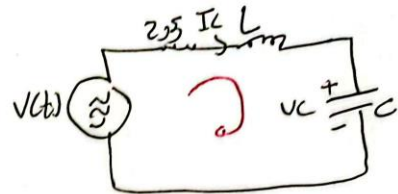
3. Consider the RLC circuit shown below when $R = 2.5$, $L = 1/4$, and $C = 1/6$;
- Find the state equations (4 Pt.)
 - Determine the transition matrix ($\Phi(t)$) of the network, (5 Pt.)
 - When the initial inductor current is 0.1 amp, $v_c(0) = 0$, and $v(t) = 0$, determine the response of the system, (4 Pt.)



➤ **3.A**

3A)) $R = 2.5$ $L = \frac{1}{4}$ $C = \frac{1}{6}$

State variable i_L , v_c



$$-v(t) + i_L R + v_L + v_c = 0$$

$$L \dot{i}_L = v(t) - i_L R - v_c \quad (1)$$

$$\dot{i}_L = \frac{1}{L} v(t) - \frac{R}{L} i_L - v_c$$

$$\dot{i}_L = 4v(t) - 10i_L - 4v_c \quad (2)$$

$$i_c = i_L = C \dot{v}_c \Rightarrow \dot{v}_c = 6i_L \quad (3)$$

$\therefore$ State equation

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_c \end{bmatrix} = \begin{bmatrix} -10 & -4 \\ 6 & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \end{bmatrix} v(t)$$

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➤ 3.B

$$(sI - A) = \begin{Bmatrix} s & 0 \\ 0 & s \end{Bmatrix} - \begin{Bmatrix} -10 & -4 \\ 6 & 0 \end{Bmatrix} = \begin{Bmatrix} s+10 & 4 \\ -6 & s \end{Bmatrix}$$

$$(sI - A)^{-1} = \begin{bmatrix} \frac{s}{(s+10)(s+6)} & \frac{-4}{s^2+10s+24} \\ \frac{6}{s^2+10s+24} & \frac{3}{s+4} - \frac{2}{s+6} \end{bmatrix}$$

$$\phi = s^{-1} \{ (sI - A)^{-1} \}$$

$$= \begin{bmatrix} \frac{3 - 2e^{2b}}{e^b} & \frac{-2(-1 + e^{2b})}{e^{6b}} \\ \frac{3(-1 + e^{2b})}{e^{6b}} & \frac{-2 + 3e^{2b}}{e^{6b}} \end{bmatrix}$$

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➤ 3.C

$$3.C)) \quad I(0) = 0,1, \quad V_C(0) = 0, \quad V(t) = 0$$

$$\therefore X(s) = (sI - A)^{-1} \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}.$$

$$C_1 = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} + (sI - A)^{-1} \begin{Bmatrix} 4 \\ 0 \\ 0 \end{Bmatrix} \times 0$$

$$X_S = \begin{bmatrix} \frac{s}{10(s+4)(s+6)} \\ \frac{3}{5(s^2+10s+24)} \end{bmatrix}$$

$$X(t) = \int^{-1} (X(s)) = \begin{bmatrix} \frac{3-2e^{2t}}{10e^{6t}} \\ \frac{3(-1+e^{2t})}{10e^{6t}} \end{bmatrix}$$

$$y(t) = \begin{bmatrix} \frac{3-2e^{2t}}{10e^{6t}} \\ \frac{3(-1+e^{2t})}{10e^{6t}} \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} = \frac{3(-1+e^{2t})}{10e^{6t}}$$

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4. The system has transfer function

(10 Pt.)

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{Js^2 + (a + K k_f)s + K}$$

Assume $J = 1 \text{ kg-m}^2$ and $a = 1 \text{ N-m/rad/sec}$. If the maximum overshoot of the unit-step input is 0.2 and the peak time is 0.1 sec,

(a) Find its damping ratio and natural frequency.

(b) Find the gain K and k_f . Also, calculate the settling time.

➤ A

Q4)) $G(s) = \frac{Y(s)}{U(s)} = \frac{K}{Js^2 + (a + K k_f)s + K}$ $K k_f = \frac{Ks}{s}$

$J = 1 \text{ kg-m}^2$, $a = 1 \text{ N-m/rad/sec}$

max OS = 0.2

Peak time = 0.1 s (TP)

[1] damping ratio and natural freq

$\frac{K k_f}{s^2 + 2\zeta \omega_n s + \omega_n^2} = \text{تنظيم الجهد}$

$\omega_n^2 = K$, $2\zeta \omega_n = a + K k_f$

$\therefore \%OS = 0.2$

$\zeta = \frac{-\ln OS}{\sqrt{\pi^2 + \ln(OS)^2}} = \frac{-\ln \frac{0.2}{1.0}}{\sqrt{\pi^2 + \ln(\frac{0.2}{1.0})^2}} = \boxed{0.456}$

$TP \cdot \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$

$\omega_n = \frac{\pi}{TP \sqrt{1-\zeta^2}} = \boxed{35.3 \text{ rad/s}}$

$\therefore$ damping ratio = 0.456

natural freq = 35.3 rad/s

B

$$\omega_n^2 = \frac{K}{J}$$

$$(3523)^2 = \frac{K}{1} \quad \therefore K = 1,246 \times 10^3$$

$$T_s = \frac{4}{\xi \omega_n} = \frac{4}{0,456 \times 3523} = 0,2495$$

$$K_s = \frac{2\xi\omega_n - a}{K} = \frac{2 \times 0,456 \times 3523 - 1}{1,246 \times 10^3}$$

$$K_s = 2,50 \times 10^{-2}$$

$$K_s = 0,025$$

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