

Islamic University of Gaza
Electrical and Computer Engineering Department
EELE 3371 Control System
Final Exam
May 19, 2005

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Exam Time 10:00-12:30

Name	<u>Solution</u>
Student ID	
Grade	

GOOD LUCK

1. a) A plant is described by the following differential equations:

(5 PT)

$$\dot{x}_1 = 3x_1 + \sin x_2 + 5u$$

$$\dot{x}_2 = 4 \sin x_1 + 2u \cos x_1 + 3x_2^2$$

Find the linearized state model approximation for the above system about nominal point $x_{01} = x_{02} = u_0 = 0$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 3 & \cos x_2 \\ 4 \cos x_1 + 2u \sin x_1 & 6x_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 5 \\ 2 \cos x_1 \end{bmatrix} u$$

$\downarrow$ x_0, u_0 $\downarrow$ x_0, u_0

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 5 \\ 2 \end{bmatrix} u$$

b) Given the following connection equations for the servomotor shown in Figure 1, derive the equations of motion (governing equations) for the system. Then find the state space model that describes the system. Where:

K_τ = torque constant, K_m = back-emf constant,

$e_m(t)$ = back emf

$e_a(t)$ = system input, $\dot{\theta}_1$ = system output

$\tau(t) = K_\tau * i_a(t)$,

$e_m(t) = K_m * \frac{\partial \theta}{\partial t}$,

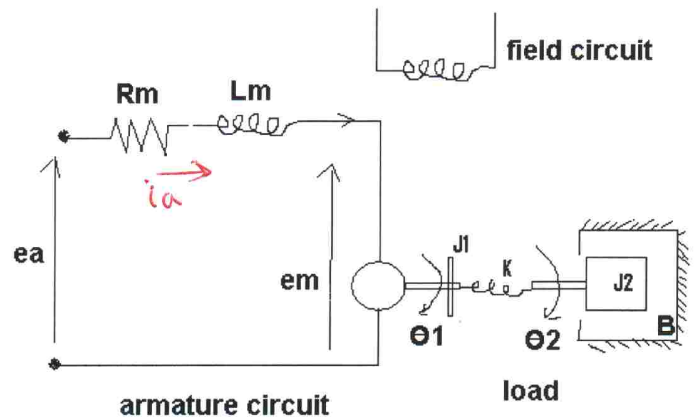


Figure 1

2.a) Determine if the system Shown in Figure 2 is controllable and observable.

(5 PT)

Where $a=1$, $b=2$,
 $c=2$, and $d=1$

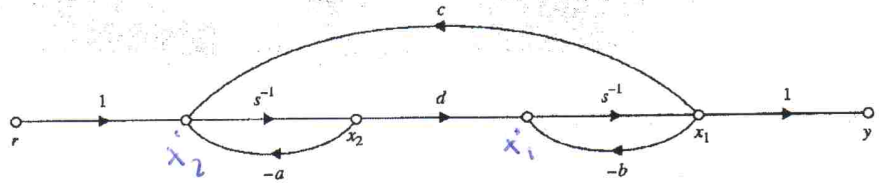


Figure 2

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$S = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}; \quad |S| \neq 0$$

controllable

$$V = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$|V| \neq 0$$

observable

b) The system is described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

Where $u(t) = t$ for $t \geq 0$. The initial conditions are $x_1(0) = 2$, $x_2(0) = 3$. Determine the state vector $x(t)$ for $t \geq 0$. (7 PT)

$$\Phi(t) = \mathcal{L}^{-1}[(sI - A)]^{-1} = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$u(s) = \frac{1}{s^2}$$

$$\therefore x(t) = \Phi(t) x(0) + \mathcal{L}^{-1} \left\{ \Phi(s) B u(s) \right\}$$

$$x(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}e^{-t} + \frac{1}{2}e^{-2t} \\ e^{-t} - e^{-2t} \end{bmatrix}$$

3. a) The block diagram of a control system is shown in Figure 3

(10 PT)

$$G_c = k, \quad G_p(s) = \frac{1}{s(s+1)}$$

$$H(s) = 1 + Ps$$

Determine the value of k and P so that the maximum overshoot in the unit step response is 0.4 and settling time is 5 sec. then obtain the rise time and the steady state error for step input

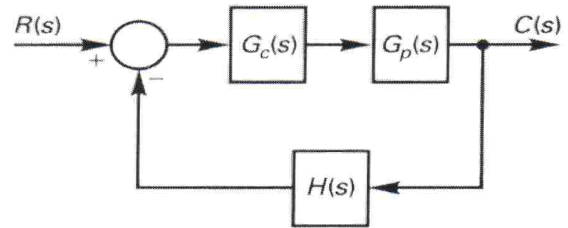


Figure 3

$$\rho = \frac{1}{\sqrt{1 + \left[\frac{\pi}{\ln(1.4)} \right]^2}} = 0.28$$

$$t_s = \frac{4.6}{\rho \omega_n} \Rightarrow \omega_n = \frac{4.6}{1.4} = 3.28$$

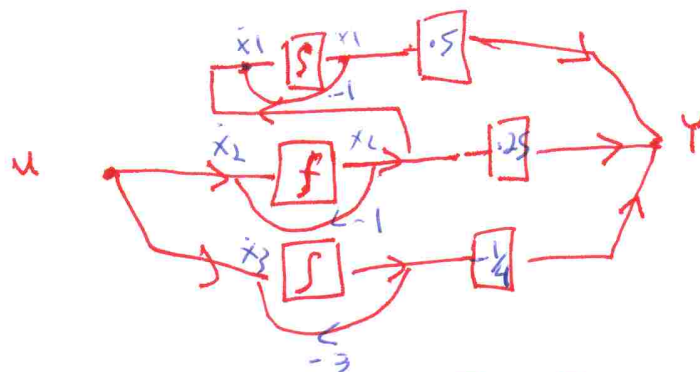
$$\frac{C}{R} = \frac{k G_p}{1 + k G_p H} = \frac{k}{s^2 + (kP + 1)s + k} = \frac{\omega_n^2}{s^2 + 2 \rho \omega_n s + \omega_n^2}$$

b) Using parallel decomposition technique, obtain the state-space model of the system

$$\frac{Y(s)}{U(s)} = \frac{s+2}{(s+1)^2(s+3)}$$

(5 PT)

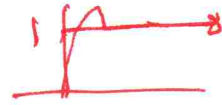
$$\frac{s+2}{(s+1)^2(s+3)} = \frac{0.5}{(s+1)^2} + \frac{0.25}{s+1} + \frac{-0.25}{s+3}$$



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u$$

$$v = \begin{bmatrix} 0.5 & 0.25 & -0.25 \end{bmatrix} x$$

$$k_{DC} \lim_{s \rightarrow 0} \frac{k}{s^2 + (kp+1)s + k} = \underline{\underline{1}}$$



$$k = \omega_n^2 = (3.28)^2 = \underline{\underline{10.79}}$$

$$\rho = \frac{2\zeta\omega_n - 1}{k} = 0.077$$

$$t_r = \frac{1.8}{\omega_n} = \frac{1.8}{3.28} = 0.548$$

$$\boxed{e_{ss}|_{\text{step}} = 0}$$

4.a) Find the range of values for k so that the system represented by the Figure 3 will be stable.

$$G_p(s) = \frac{1}{s(s^3 + s^2 + s + 1)}, \quad H(s) = 1, \quad G_c(s) = k \quad (8 \text{ PT})$$

char. eq $1 + kGH = 0 \Rightarrow s^4 + s^3 + s^2 + s + k = 0$

s^4	1	1	k
s^3	1	1	0
s^2	ϕ	k	
s^1	$k - \frac{k}{\phi} \approx -\frac{k}{\phi}$		
s^0	k		

we conclude that the system is unstable for all values of k

b) Choose the right answer;

(5 PT)

i. When the pole of a system is moved towards the imaginary axis, then

- a) Settling time of the system decreases. b) Steady state error is reduced to zero
☒ c) Settling time of the system increases d) Settling time increases by 20% of initial value

ii. The case in Routh Hurwitz in which a particular row elements are zero, show that

- a) There is a pole at the origin b) The system is unstable
c) The system is stable ☒ d) Some roots are distributed symmetrically about the origin

iii. A system is required to have a peak overshoot of 4.32 % and natural frequency of 5 rad/sec. the required location of dominant pole is:

- a) $-3.535 + j 5.535$ b) $-2.535 + j 2.525$
c) $-2.535 + j 3.535$ ☒ d) $-3.535 + j 3.535$

iv. The number of integrators in the system transfer function $G(s)$ indicates

- a) Number of finite poles b) Number of zeros at origin
c) Number of finite zeros ☒ d) Number of poles at origin

v. Type number of a system gets decreased if

- a) First an integrator and then a differentiator is included in the system.
b) An integrator is included in the forward path.
☒ c) A differentiator is included in the forward path
d) A differentiator is included in a parallel path.

EXTRA CREDIT: (3 PT)

Sketch the root locus with respect to K for the system showing in Figure 3. Also find the value of K when the R-L crosses the imaginary- axis, if any.

$$G_c = K, \quad G_p(s) = \frac{(s+3)}{s(s+1)(s^2 + 4s + 5)}, \quad H(s) = 1$$