

Islamic University of Gaza
Electrical and Computer Engineering Department
EELE 3371 Control System
Final Exam
May 29, 2006

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Exam Time 12:00-2:30

1. a) Figure (a) shows a first order system for which you would like to improve the response time. Assuming you place control devices on this system as shown in Figure (b), what must the values of K_A and K_T be in order to bring the settling time down to 1 second while leaving the magnitude of the steady state output unchanged? (6 PT.)

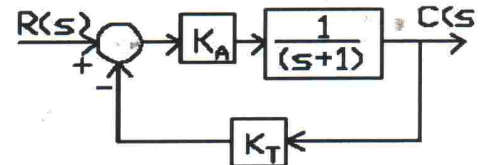
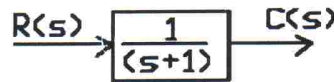


Figure (a)

Figure (b)

$$t_s = 4\tau = 1 \text{ sec} \Rightarrow \tau = 1/4$$

$$\tau = 1/4$$

$$K_{DC} = 1$$

$$\frac{C(s)}{R(s)} = \frac{K_A}{s+1+K_A K_T} = \frac{K_{DC}}{\tau s + 1} = \frac{\frac{K_A}{1+K_A K_T}}{\frac{s}{1+K_A K_T} + 1} = \frac{1}{\frac{1}{4}s + 1}$$

$$\frac{1}{1+K_A K_T} = \frac{1}{4} \Rightarrow K_A K_T = 3 ; \quad \frac{K_A}{1+K_A K_T} = 1 \Rightarrow K_A = 1+K_A K_T$$

$$\therefore K_A = 1 + 3 = 4 \quad K_T = \frac{3}{K_A} = \frac{3}{4}$$

$$\therefore K_A = 4 ; K_T = 3/4$$

- b) A second order system settles to a final position of 5cm in a time of 2 seconds in response to a unit step input. The response of this system is oscillatory with damped frequency of 3 rad/sec. Give the transfer function for this system. (8 PT.)

$$\frac{K_{DC} \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \Rightarrow t_s = 2 \text{ sec} = \frac{4.6}{\zeta \omega_n} \quad \therefore \zeta \omega_n = 2.3$$

$$K_{DC} = 5$$

$$\omega_d = \omega_n \sqrt{1-\zeta^2} = 3 \Rightarrow \omega_n \sqrt{1-\left(\frac{2.3}{\omega_n}\right)^2} = 3$$

$$\therefore \omega_n = 14.29$$

$$\therefore \frac{Y}{R} = \frac{K_{DC} \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} = \frac{5(14.29)^2}{s^2 + 4.6s + 14.29} = \frac{71.45}{s^2 + 4.6s + 14.29}$$

2. a) Write the equations of motion for the system in the time domain (4 PT.)

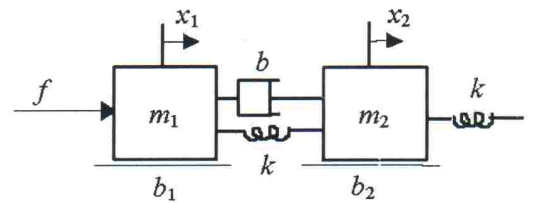


Figure 2

$$f = m_1 \ddot{x}_1 + b_1 \dot{x}_1 + b(\dot{x}_1 - \dot{x}_2) + k(x_1 - x_2)$$

$$m_2 \ddot{x}_2 + b_2 \dot{x}_2 + k x_2 = b(\dot{x}_1 - \dot{x}_2) + k(x_1 - x_2)$$

b) The system is described by

(8 PT.)

$$\frac{dx_1}{dt} = x_1 + u(t)$$

$$\frac{dx_2}{dt} = x_1 + x_2 + u(t)$$

Where $u(t) = 1$ for $t \geq 0$. The initial conditions are $x_1(0) = 1$, $x_2(0) = 1$. Determine state vector $x(t)$ for $t \geq 0$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$\Phi(t) = \mathcal{L}^{-1} \{ (sI - A)^{-1} \}$$

$$\Phi(t) = \mathcal{L}^{-1} \left[\begin{bmatrix} \frac{s-1}{(s-1)^2} & 0 \\ \frac{1}{(s-1)^2} & \frac{s-1}{(s-1)^2} \end{bmatrix} \right] = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$$

$$u(s) = \frac{1}{s}$$

$$\therefore x(t) = \Phi(t) x(0) + \mathcal{L}^{-1} \{ \Phi(s) B u(s) \}$$

$$\Phi(s) B u(s) = \begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{s-1} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{1}{s} = \begin{bmatrix} \frac{1}{s(s-1)} \\ \frac{1}{s(s-1)^2} + \frac{1}{s(s-1)} \end{bmatrix}$$

$$\therefore x(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 + e^t \\ 1 + te^t - e^t - 1 + e^t \end{bmatrix}$$

$$\therefore x(t) = \begin{bmatrix} e^t \\ te^t + e^t \end{bmatrix} + \begin{bmatrix} -1 + e^t \\ te^t \end{bmatrix}$$

3. a) The block diagram of a control system is shown in Figure 3

$$G_c = k, \quad G_p(s) = \frac{1}{s^3 + 6s^2 + 10s} = \frac{1}{s(s+3 \pm j)}$$

$$H(s) = 1$$

Sketch the root locus with respect to K for the system showing in

Figure 3. Also find the value of K with locations of poles when the R-L crosses the imaginary- axis, if any. (10. PT.)

$$\sigma = 3 - 6 = 3 \text{ asympt} \quad \sigma_m = \frac{0 - 3 + j - 3 - j}{3} = -2$$

$$\theta_m = \pm 60^\circ, +180^\circ$$

$$B.P.: N(s)D'(s) - N'(s)D(s) = 0$$

$$3s^2 + 12s + 10 = 0$$

$$\text{Roots} = -2 \pm \sqrt{2/3}$$

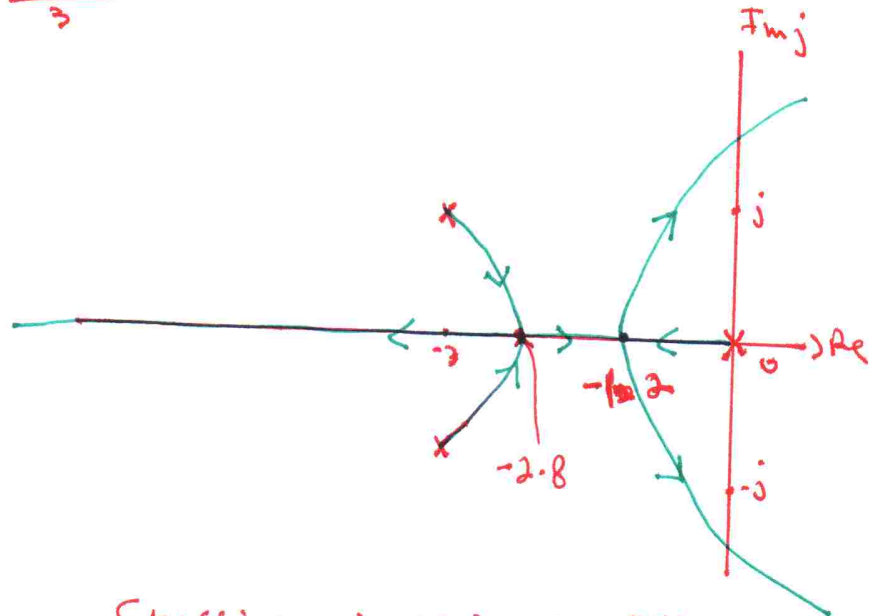
$$\approx -2.8, -1.2$$

To Find K:

$$D(s) = s^3 + 6s^2 + 10s + k$$

s^3	1	10
s^2	6	k
s	$\frac{60-k}{6}$	0
s	k	

$$0 < K < 60$$



Crossing imaginary axis

when $k=60$

$$\therefore 6s^2 + 60 = 6(s^2 + 10) = 0$$

$$\therefore s_{1,2} \approx \pm j\sqrt{10}$$

b) Write state-space equations for the transfer function (using canonical form)

(4 PT.)

$$G(s) = \frac{3s^3 + 5s + 1}{s^4 + 7s + 4s + 3}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & -4 & -7 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & 5 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

4. Consider the plant transfer function shown in Figure. 3

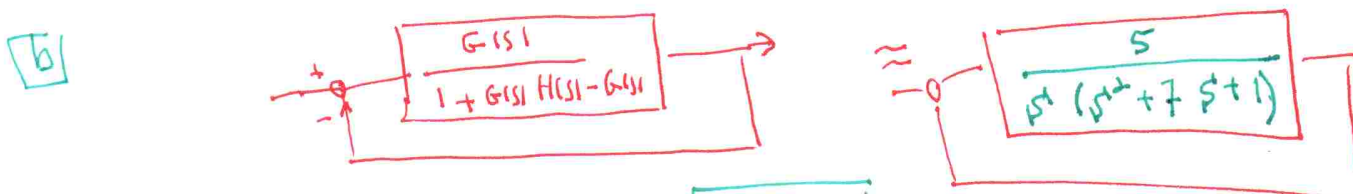
(5 PT.)

$$G_p(s) = \frac{1}{s^2(s+7)}$$

(a) Find steady-state error due to a unit step, a unit ramp, and a unit parabolic. For $G_C(s) = K$, and unity feedback.

(b) For the gain $G_C(s) = 5$ and feedback $H(s) = 0.2(s+5)$, find the steady-state error due to a unit step, a unit ramp, and a unit parabolic.

[a] System is unstable because charact. eq = $s^3 + 7s^2 + k$
there is no ESS for unstable system.



system is stable and is type 1

$e_{ss}|_{\text{step}} = 0$, $e_{ss}|_{\text{ramp}} = 1/k_v$; $k_v = \lim_{s \rightarrow 0} s K G(s) H(s) = 5$

$\therefore e_{ss}|_{\text{ramp}} = 1/5 = 0.2$; $e_{ss}|_{\text{parabla}} = \infty$

C) Choose the right answer;

(5 Points)

- i. A glass bulb thermometer is immersed in a hot water bath or 1 minute reads 98.2% of final temperature. The thermal time constant of the thermometer is?
 - a) 2.3456
 - b) 0.2489
 - c) 0.4978
 - d) 4.017
- ii. The transfer function of the plant is $G(s) = \frac{1}{s-1}$ and the feedback path $H(s) = (s-b)$. the value of b for Which the system will be stable is
 - a) $b \geq 0$.
 - b) $b \leq 0$.
 - c) $b > 1$
 - d) $b < -1$.
- iii. The effect of addition of a pole to an open-loop transfer function will
 - a) Pull the root locus towards the imaginary axis.
 - b) Pull the root locus to the right of imaginary axis.
 - c) Have no effect on the root locus.
 - d) Pull the root locus to the left of imaginary axis.
- iv. When the gain k of a system is increased, the steady state error of the system
 - a) Increases.
 - b) Remains unchanged.
 - c) May increase or decrease.
 - d) Decreases.
- v. Open loop control
 - a) Takes account of actual output and compares this to desired output
 - b) has feedback
 - c) Is blind to actual output.
 - d) Results more accurate than closed loop system