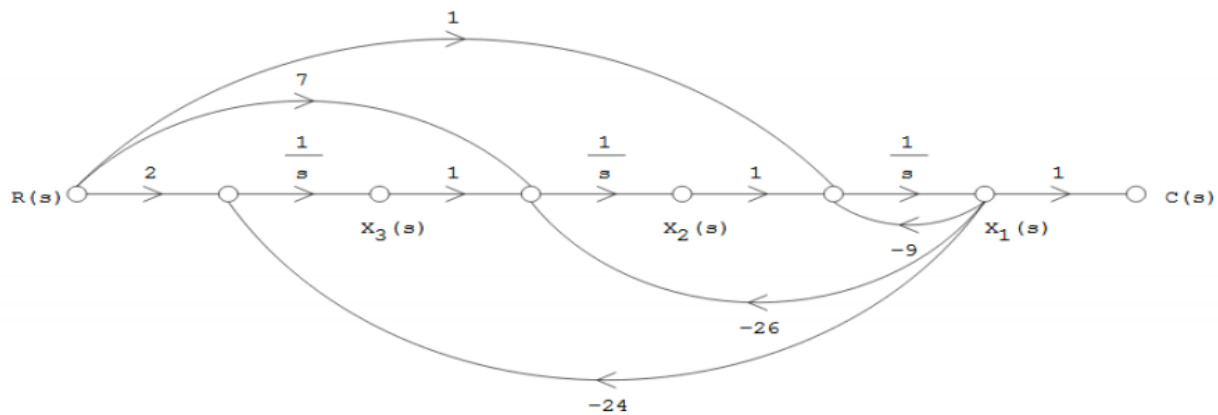


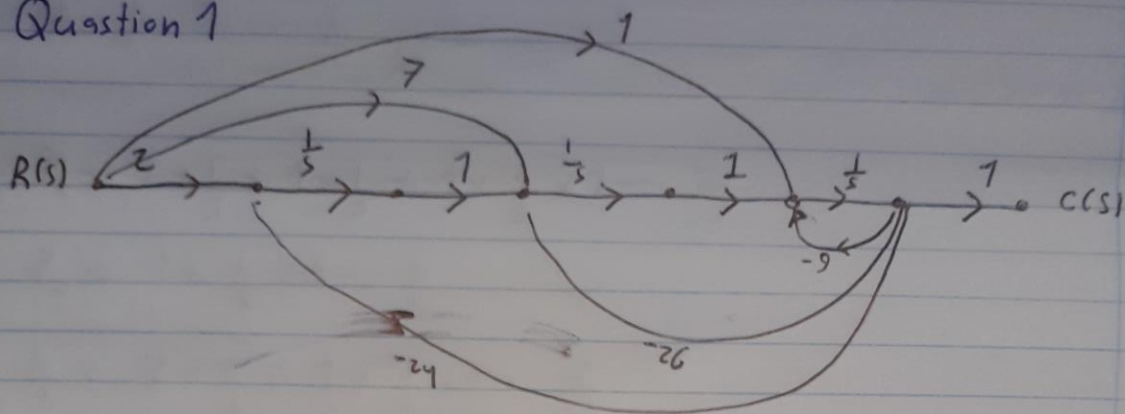
Name/Jaafar Wael Jaafar

ID/120180534

1. For system given in the Figure, find
 - i. Write the state-space equations of the above signal-flow graph in the form (5 PT)
 - ii. Transfer Function using Mason Gain Formula. $T(s) = C(s)/R(s)$ (5 PT)



Question 1



$$\dot{X}_1 = -9X_1 + X_2 + R(s) \quad \text{--- (2)}$$

$$\dot{X}_2 = -26X_1 + X_3 + 7R(s) \quad \text{--- (1)}$$

$$\dot{X}_3 = -24X_1 + 2R(s) \quad \text{--- (3)}$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} -9 & 1 & 0 \\ -26 & 0 & 1 \\ -24 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix} R$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

of forward paths = 3

$$P_1 = 2 \times \frac{1}{s} \times 1 \times \frac{1}{s} \times 1 \times \frac{1}{s} \times 1 = \frac{2}{s^3}$$

$$P_2 = 1 \times \frac{1}{s} \times 1 = \frac{1}{s}$$

$$P_3 = 7 \times \frac{1}{s} \times 1 \times \frac{1}{s} \times 1 = \frac{7}{s^2}$$

of loop

$$L_1 = \frac{-9}{s}$$

$$L_2 = \frac{-26}{s^2}$$

$$L_3 = \frac{-242}{s^3}$$

$$\Delta = 1 - (L_1 + L_2 + L_3)$$

$$1 - \left(\frac{-9}{s} + \frac{-26}{s^2} + \frac{-242}{s^3} \right) = \boxed{}$$

$$\Delta_1 = 1$$

$$\Delta_2 = 1$$

$$\Delta_3 = 1$$

$$T(s) = \frac{1}{\Delta} \sum_{k=1}^3 P_k A_k$$

$$T(s) = \frac{P_1 A_1 + P_2 A_2 + P_3 A_3}{\Delta}$$

$$T(s) = \frac{s^2 + 7s + 2}{s^3 + 9s^2 + 26s + 24}$$

2. Check whether the following system given by its characteristic equation is stable or not and shows the location of roots on the s-plane (10 PT)

$$Q(s) = s^5 + 10s^4 + 45s^3 + 90s^2 + 164s + 200 = 0$$

Question 2/

$$Q(s) = s^5 + 10s^4 + 45s^3 + 90s^2 + 164s + 200 = 0$$

the two condition are satisfied

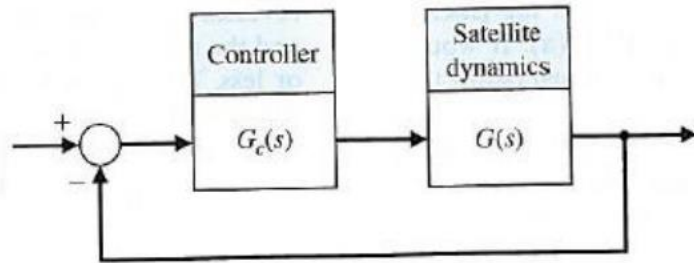
s^5	1	45	
s^4	161	269	200
s^3	369	6416	
s^2	65	9	
s^1	-116	13	
s^0	20		

the system is a sign change two time
 $\therefore$ the system is unstable

there are two roots in the right Half plane

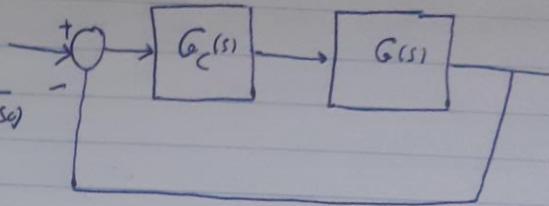
3. A system shown in the figure below, find root locus and the range of **K** that makes the system stable and the crossover points of the imaginary axis. (10 PT)

$$G_c(s)G(s) = \frac{K(s+2)(s+3)}{s^2(s+1)(s+10)(s+50)}.$$



Question 3:-

$$G_c(s) G(s) = \frac{K(s+2)(s+3)}{s^2(s+1)(s+10)(s+50)}$$



$$G_c(s) G(s) = \frac{K(s+2)(s+3)}{s^2(s+1)(s+10)(s+50)}$$

Poles: 5 0, 0, -1, -10, -50
Zeros: 2 -2, -3

$$\Delta = \text{Poles} - \text{zeros}$$

$$5 - 2 = 3$$

$$\sigma_m = \frac{\sum P - \sum Z}{\Delta} = \frac{(0+0-1-10-50) - (-2-3)}{3} = \frac{-61+5}{3}$$

$$= -18.66$$

$$\theta = (\pm 1, \pm 3) * \frac{180^\circ}{\Delta} \implies \boxed{\pm 90^\circ} / \boxed{180^\circ}$$

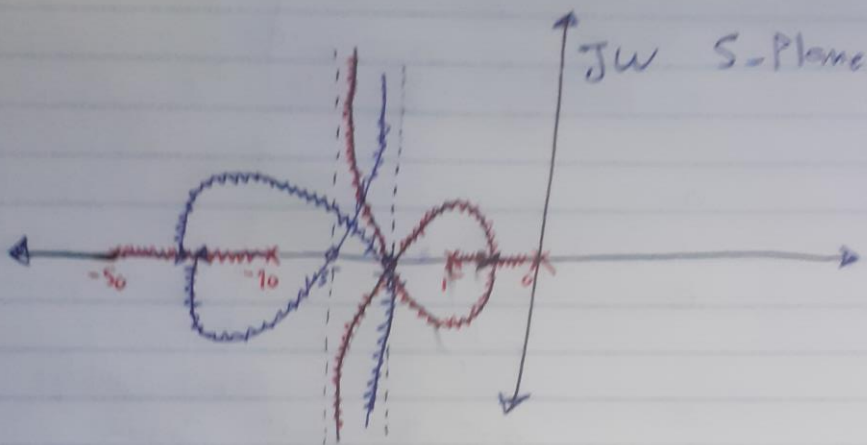
The Break Point

$$\frac{d}{ds} \left(\frac{(s+2)(s+3)}{s^2(s+1)(s+10)(s+50)} \right)$$

$$= - \frac{3s^5 + 142s^4 + 150s^3 + 7064s^2 + 12580s + 6000}{s^3(s+1)^2(s+10)^2(s+50)^2}$$

$$-(3x^8 + 142x^7 + 150x^6 + 7064x^5 + 72520x + 6000) = 0$$

x is imaginary point



the close loop system

$$\begin{aligned} & \frac{K(s+2)(s+3)}{s^2(s+1)(s+10)(s+50)} \\ & 1 + \frac{K(s+2)(s+3)}{s^2(s+1)(s+10)(s+50)} \\ & = \frac{K(s+2)(s+3)}{s^2(s+1)(s+10)(s+50) + K(s+2)(s+3)} \\ & = \frac{s^6 + 67s^4 + 500s^3 + 500s^2 + Ks^2 + 5Ks + 6K}{s^6 + 67s^4 + 500s^3 + s^2(500+K) + 5Ks + 6K} \end{aligned}$$

$$s^5 + 61s^4 + 500s^3 + s^2(500 + K) + 5Ks + 6K$$

s^5	1	500	~~$(500 + K)$~~	$6K$
s^4	61	$(500 + K)$	$6K$	
s^3	$\frac{30000 + K}{61}$	$-\frac{5K(500 + K) - 500(6K)}{500 + K}$		
s^2	$\frac{5K^2 - 2K(500 + K)}{500 + K}$			
s^1	K			
s^0				

system stable

When $\frac{30000 + K}{61} > 0$

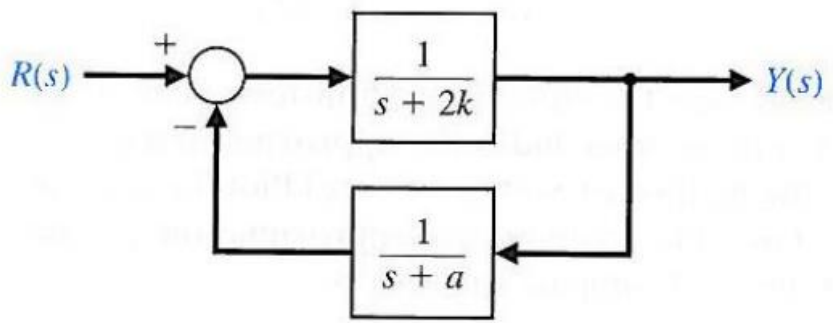
$$30000 + K > 0 \quad K > -30000$$

and $K < 0$

$\therefore$ system stable When $0 > K > -30000$

the system is unstable When $K \leq -30,000$

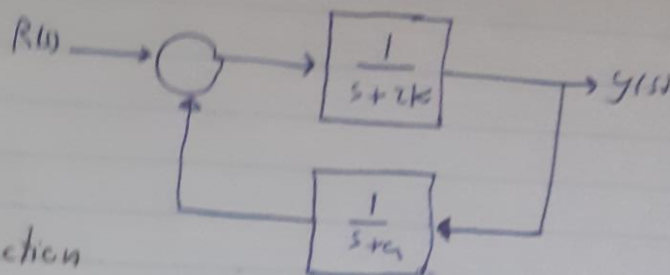
4. Consider the closed loop system shown in the figure below. Determine values of the parameters **k** and **a** so that the following specifications are satisfied:
- a) The steady state error to unit step is zero (5 PT)
 - b) The closed loop system has a percent overshoot of less than 5 % (5 PT)



Question 4

1

a)



Find the close loop function

$$T(s) = \frac{Y(s)}{R(s)} = \frac{s+a}{s^2 + (2K+a)s + 2aK+1}$$

since $R(s) = \frac{1}{s}$ unit step

$$E(s) = R(s) [1 - T(s)]$$

$$E(s) = \frac{s^2 + (2K+a-1)s + 2aK+1-a}{s^2 + (2K+a)s + 2aK+1} * \frac{1}{s}$$

$$e(\infty) = \lim_{s \rightarrow 0} s E(s)$$

$$e(\infty) = \lim_{s \rightarrow 0} \cancel{s} \frac{s^2 + (2K+a-1)s + 2aK+1-a}{s^2 + (2K+a)s + 2aK+1} * \frac{1}{\cancel{s}}$$

$$e(\infty) = \frac{2aK+1-a}{2aK+1}$$

For the zero state error $e(\infty) = 0$

$$\frac{2aK+1-a}{2aK+1} = 0$$

$$2aK+1-a = 0$$

$$\boxed{K = (a-1)/2a} \quad (1)$$

the state error due to unit step $K = (a-1)/2a$

2

b) according the close loop system $\boxed{0.5 = 5\%}$

$$\xi = \frac{-\ln\left(\frac{5\%}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{5\%}{100}\right)}} = \boxed{0,92}$$

$$\frac{y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$\frac{y(s)}{R(s)} = \frac{s+a}{s^2 + (2K+a)s + 2aK}$$

$$(w_n)^2 = 2aK \quad w_n = \sqrt{2aK}$$

$$2\xi w_n = 2K+a \quad \xi = \frac{2K+a}{2w_n} \Rightarrow \xi = \frac{2K+a}{2\sqrt{2aK}}$$

$$0,9 = \frac{2K+a}{2\sqrt{2aK}}$$

$$w_n^2 = 2aK + 1$$

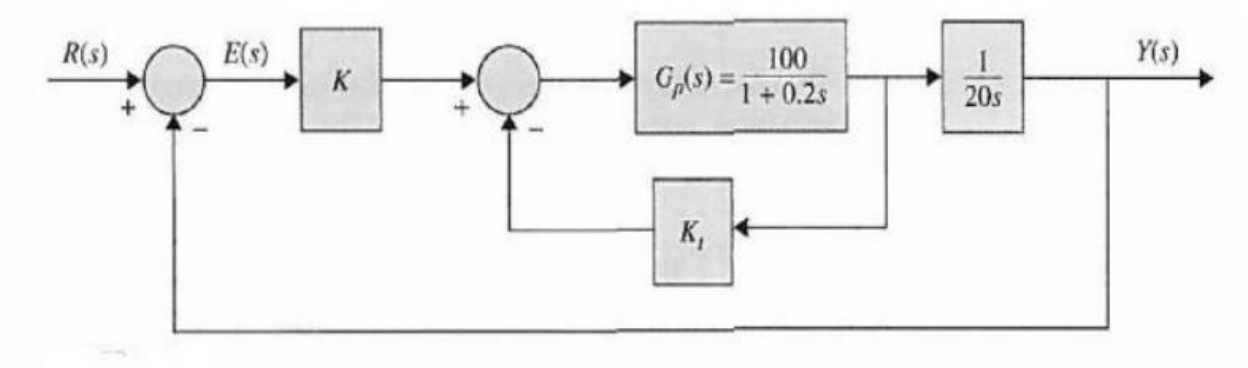
$$\left(\frac{2K+a}{1,38}\right)^2 = 2aK + 1$$

substitute $K = (a-1)/2a$ from equation (1)

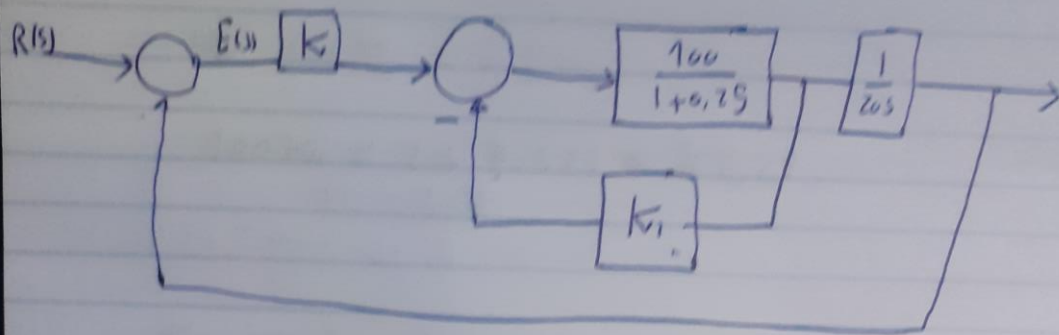
$a = 0,4287$ and $\boxed{1,343}$
 substitute the value of a in equation (1)

$$\boxed{K = 0,2138}$$

5. For the control system shown in Fig. below, find the values of K and K_t so that the maximum overshoot of the output is approximately 10 % and a settling time of 0.0625 sec (10 Pt)



Question 5//



$$C(s) = \frac{\frac{100}{1+0,25s}}{1 + \frac{100}{1+0,25s} \cdot K_1} = \boxed{\frac{100}{1+0,25s+100K}}$$

$$\frac{C(s)}{R(s)} = \frac{100}{1+0,25s+100K} \cdot \frac{K}{20s}$$

$$\frac{C(s)}{R(s)} = \boxed{\frac{5K}{0,25s^2+100Ks+5}}$$

$$\xi = \frac{-\ln\left(\frac{10\%}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{10\%}{100}\right)}} = \boxed{\xi = 0,591}$$

$$T_s = \frac{4}{\xi \omega_n}$$

$$\omega_n = \frac{\xi T_s}{4} \Rightarrow \omega_n = 108,29$$

$$K_1 \cdot 100 K_1 = 2 \{ W_n$$

$$100 K_1 = 2 * 0,591 * 108,29$$

$$K_1 = 1,27$$

$$K_{dc} \cdot W_n^2 = 5 K_1$$

$$1 * W_n^2 = 5 K_1$$

$$K_1 = 23$$