

Islamic University of Gaza
Electrical and Computer Engineering Department
EELE 3371 Control System
Final Exam
June 16, 2004

EXAM TIME: 9:00 – 11:00 (2 hours exam)

Name	Solution
Student ID	
Instructor	Dr. Basil Hamed
Grade	

GOOD LUCK

1. A system is characterized by the following state equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

- Find the transfer function of the system. (5 Points)
- Compute the state transition matrix (4 Points)
- Show that if the above system is state controllable and state observable. (2 Points)
- Obtain the solution to the state equation for a unit step input under zero initial condition. (4 Points)

$$T.F = C [sI - A]^{-1} B + D$$

$$[sI - A]^{-1} = \begin{bmatrix} s+3 & -1 \\ 2 & s \end{bmatrix}^{-1} = \frac{1}{(s+2)(s+1)} \begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix}$$

$$C [sI - A]^{-1} B = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{s}{(s+2)(s+1)} & \frac{1}{(s+2)(s+1)} \\ \frac{-2}{(s+2)(s+1)} & \frac{s+3}{(s+2)(s+1)} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T.F = \frac{1}{(s+2)(s+1)} = \frac{1}{s^2 + 3s + 2}$$

$$\Phi(t) = \mathcal{L}^{-1} [(sI - A)^{-1}] = \mathcal{L}^{-1} \begin{bmatrix} \frac{s}{(s+2)(s+1)} & \frac{1}{(s+2)(s+1)} \\ \frac{-2}{(s+2)(s+1)} & \frac{s+3}{(s+2)(s+1)} \end{bmatrix}$$

$$\therefore \Phi(t) = \begin{bmatrix} -e^{-t} + 2e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & 2e^{-t} - e^{-2t} \end{bmatrix}$$

No pole zero cancellation

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system is controllable & observable.

$$x(t) = \Phi(t) x(0) + \int_0^t \phi(t-\tau) B u(\tau) d\tau$$

$$x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore x(t) = \int_0^t \begin{bmatrix} -e^{-(t-\tau)} + 2e^{-2(t-\tau)} & e^{-(t-\tau)} - e^{-2(t-\tau)} \\ -2e^{-(t-\tau)} + 2e^{-2(t-\tau)} & 2e^{-(t-\tau)} - 1e^{-2(t-\tau)} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} d\tau$$

$$x(t) = \int_0^t \begin{bmatrix} e^{-(t-\tau)} - e^{-2(t-\tau)} \\ 2e^{-(t-\tau)} - 1e^{-2(t-\tau)} \end{bmatrix} d\tau$$

$$x(t) = \begin{bmatrix} \frac{e^{-(t-\tau)}}{+1} - \frac{e^{-2(t-\tau)}}{+2} \\ 2e^{-(t-\tau)} - \frac{1e^{-2(t-\tau)}}{2} \end{bmatrix}_0^t$$

$$x(t) = \begin{bmatrix} \frac{1}{2} - e^{-t} + \frac{1}{2} e^{-2t} \\ \frac{3}{2} - 2e^{-t} + \frac{1}{2} e^{-2t} \end{bmatrix}$$

2. a) Consider the system shown in Figure 1; determine the value of k such that the damping ratio ζ is 0.5. Then obtain the rise time, maximum overshoot percent, and settling time. (10 Points)

$$G_c(s) = 1, \quad G_p(s) = \frac{16}{s(s+0.8)}$$

$$H(s) = 1 + Ks$$

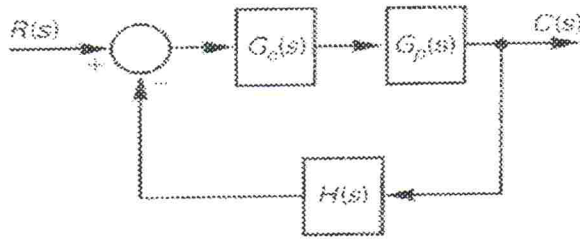


Figure 1

$$\frac{C}{R} = \frac{16}{s^2 + (0.8 + 16k)s + 16}$$

$$\therefore \omega_n = 4 \quad 2\zeta\omega_n = 0.8 + 16k \quad \Rightarrow \zeta = 0.5$$

$$\therefore \underline{k = 0.2}$$

$$\times \quad t_r = \frac{1.8}{\omega_n} = \frac{1.8}{4} = \underline{0.45 \text{ sec}}$$

$$\times \quad \% \text{ MP} = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100 = \underline{16.3 \%}$$

$$\times \quad t_s = \frac{4.6}{\zeta\omega_n} = \frac{4.6}{0.5(4)} = \underline{2.3 \text{ sec}}$$

b) Find the range of values for k so that the system represented by Figure 1 will be stable and find the frequency of sustained Oscillation (10 Points)

$$G_c = K, \quad G_p(s) = \frac{1}{s(s^2 + 1.4s + 1)}, \quad H(s) = 1$$

$$\frac{C}{R} = \frac{k G_p}{1 + k G_p}$$

charact. eq: $1 + k G_p = s(s^2 + 1.4s + 1) + k = 0$

s^3	1	1	0
s^2	1.4	k	0
s^1	$\frac{1.4 - k}{1.4}$		
s^0	k		

$$\therefore k > 0$$

$$1.4 - k > 0$$

$$k < 1.4$$

$$\therefore \underline{\underline{0 < k < 1.4}}$$

when $k = 1.4$ system is marginally stable

$$\therefore 1.4s^2 + k = 0 \Rightarrow 1.4s^2 + 1.4 = 0$$

$$s^2 = -1 \quad \therefore s_{1,2} = \pm j$$

$\therefore$ frequency of oscillation is $\pm j$

3. a) Given $G_p(s) = \frac{10}{s(s+4)}$, $H(s) = \frac{1}{s+1}$, $G_c(s) = 1$ Calculate steady-state error for step input, ramp input, and parabola input in Figure 1. (5 Points)

$$G_c G_c H = \frac{10}{s(s+4)(s+1)} \quad \text{system type is } \underline{\underline{one}}$$

$$e_{ss|step} = 0, \quad e_{ss|ramp} = 1/k_r, \quad e_{ss|parabola} = \infty$$

$$k_r = \lim_{s \rightarrow 0} s G(s) = \lim_{s \rightarrow 0} s \frac{10(s+1)}{s(s^2+5s+4)} = \frac{10}{4} = 2.5$$

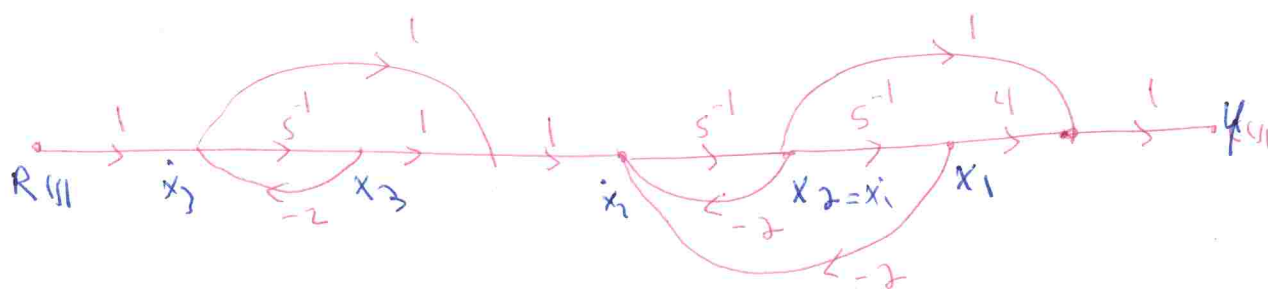
$$\therefore e_{ss} = 1/k_r = \boxed{-6/10}$$

- b) Using cascade decomposition technique, obtain the state-space model of the system $\frac{Y(s)}{R(s)} = \frac{(s+1)(s+4)}{(s+2)(s^2+2s+2)}$ (10 Points)

$$\frac{Y(s)}{R(s)} = \frac{(s+1)}{(s+2)} \cdot \frac{s+4}{s^2+2s+2} = G_1(s) G_2(s)$$

$$G_1(s) = \frac{1+s^{-1}}{1+2s^{-1}+2s^{-2}}$$

$$G_2(s) = \frac{s^{-1}+4s^{-2}}{1+2s^{-1}+2s^{-2}}$$



$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 - 2x_2 + \dot{x}_3 + x_3$$

$$\dot{x}_3 = -2x_3 + r(t)$$

$$\therefore \dot{x}_2 = -x_1 - 2x_3 - x_3 + r(t)$$

$$G'_{CSI} = \frac{\cancel{1+G} \quad G}{1+GH-G}$$

$$= \frac{\frac{10}{s(s+4)}}{1 + \frac{10}{s(s+4)} \cdot \frac{1}{s+1} - \frac{10}{s(s+4)}}$$

$$G' = \frac{10(s+1)}{s(s+4)(s+1) + 10 - 10(s+1)}$$

$$G' = \frac{10(s+1)}{s^3 + 5s^2 - 6s} = \frac{10(s+1)}{s(s^2 + 5s - 6)}$$

$$\begin{bmatrix} x_1 \\ \dot{x}_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -2 & -2 & -1 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \text{ h1d1}$$

$$y = \begin{bmatrix} 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

4. Choose the right answer;

(5 Points)

i. Which among the following is a closed-loop control system?

- a) Traffic signal.
- b) Washing machine.
- ☒ c) A man driving a car.
- d) All the above.

ii. The most important advantage of the closed-loop system over open loop system is

- a) More gain.
- b) Better stability.
- ☒ c) Greater control over system dynamics.
- d) No advantage.

iii. The time domain specification which is dependent only on, the damping ratio ζ is

- a) Rise time.
- b) Peak time.
- c) Settling time.
- ☒ d) Peak overshoot.

iv. The presence of multiple poles on the imaginary axis that the system is

- a) Stable.
- b) ☒ Unstable.
- c) Marginally stable
- d) none of the above.

v. The damping ratio ζ of a second order system whose response to unit step input is having sustained oscillation is

- ☒ a) $\zeta = 0$
- b) $\zeta > 1$
- c) $\zeta = 1$
- d) $\zeta < 1$