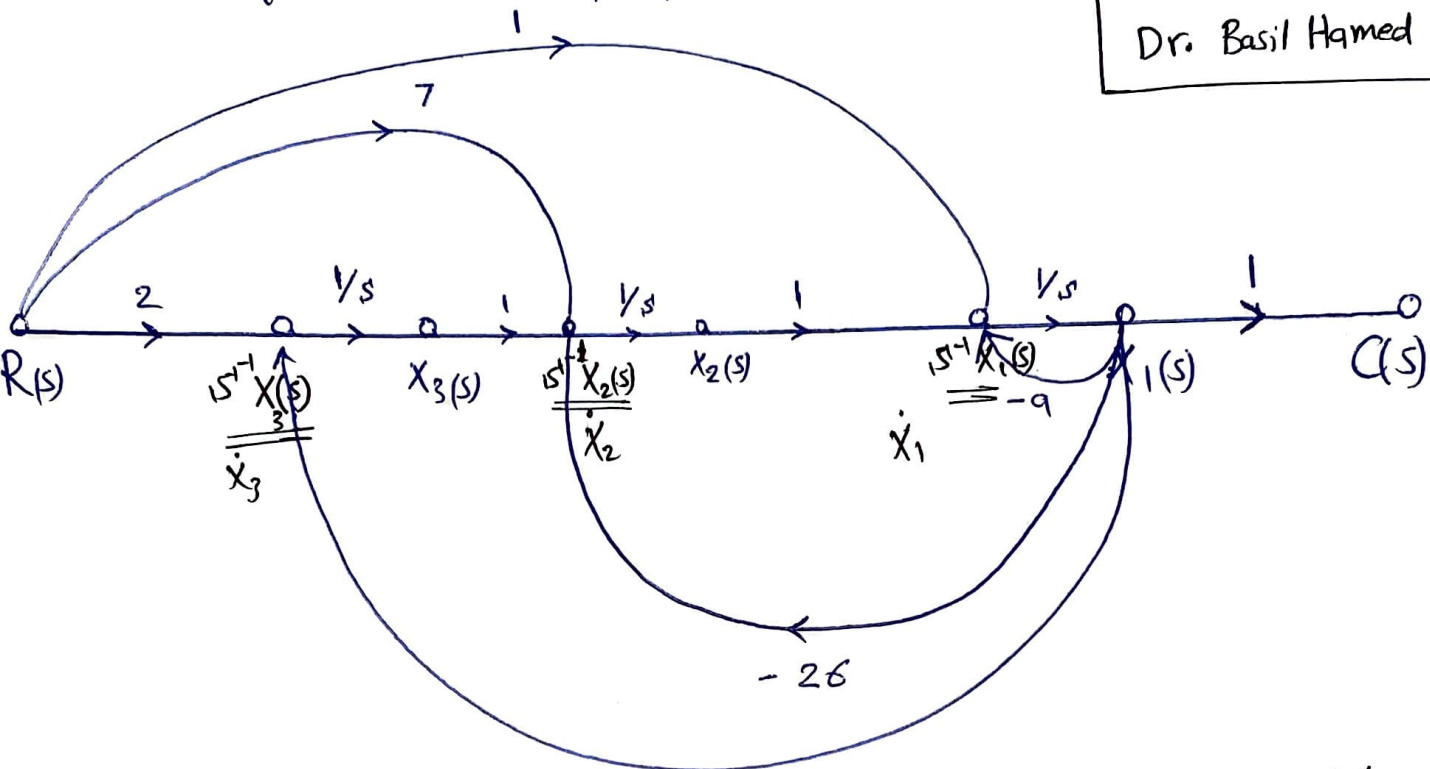


* given the following SFG, do the following:

- Find the state space model
- Find the transfer function

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$$C(s) = X_1(s)$$

$$\dot{X}_1(s) = -9X_1 + X_2 + R$$

$$\dot{X}_2(s) = -26X_1 + X_3 + 7R$$

$$\dot{X}_3 = -24X_1 + 2R$$

the state space model

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} -9 & 1 & 0 \\ -26 & 0 & 1 \\ -24 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix} R$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

now

$$T(s) = \frac{C(s)}{R(s)} = C(sI - A)^{-1} B =$$

$$\frac{s^2 + 7s + 2}{s^3 + 9s^2 + 26s + 24}$$

by using algebra fx 2.0 plus

منهج مانشون (Manson's rule) (منهج مانشون)

By using Mason's Rule :

of forward paths = 3

$$\Rightarrow P_1 = \frac{2}{s^3}, \quad P_2 = \frac{7}{s^2}, \quad P_3 = \frac{1}{s}$$

of loops = 3

$$\Rightarrow L_1 = \frac{-9}{s}, \quad L_2 = \frac{-26}{s^2}, \quad L_3 = \frac{-24}{s^3}$$

$$\Rightarrow \Delta = 1 - (L_1 + L_2 + L_3)$$

$$\Rightarrow \Delta_1 = \Delta_2 = \Delta_3 = 1$$

$$\underline{\underline{\text{now}}} \quad T(s) = \frac{1}{\Delta} \sum_{k=1}^3 P_k \Delta_k$$

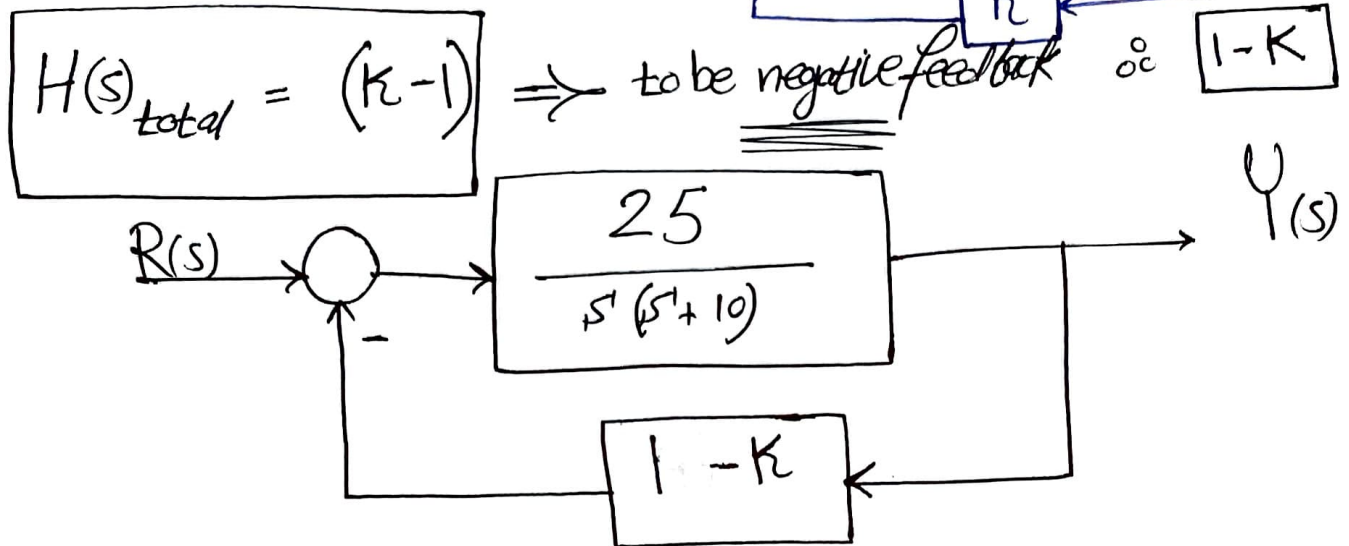
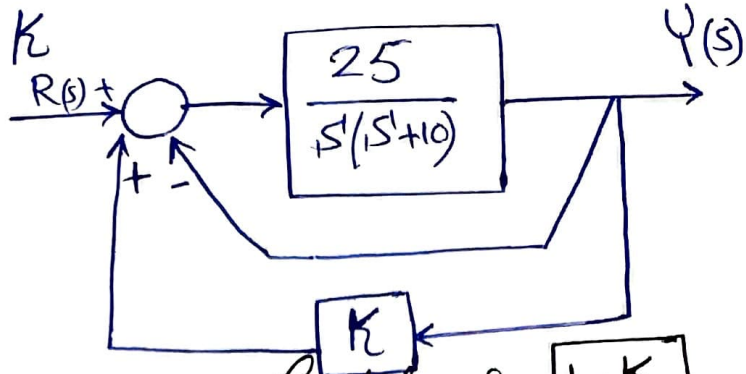
by using algebra fx 2.0 plus

$$T(s) = \frac{s^2 + 7s + 2}{s^3 + 9s^2 + 26s + 24}$$

#

oo

* determine the value of K that gives $\zeta = \frac{1}{2}$ for the closed loop response



∞ the closed loop transfer function :

$$\frac{Y(s)}{R(s)} = \frac{25}{s^2 + 10s + (25 - 25K)} \quad (*)$$

∞ generally, $T(s) = \frac{K_{dc} \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (**)$

by comparing (*) with (**)

∞ $2\zeta\omega_n = 10 \Rightarrow$ but $\zeta = 0.5 \Rightarrow \infty$ $\omega_n = 10$

∞ $\omega_n^2 = 25 - 25K \Rightarrow 100 = 25 - 25K$

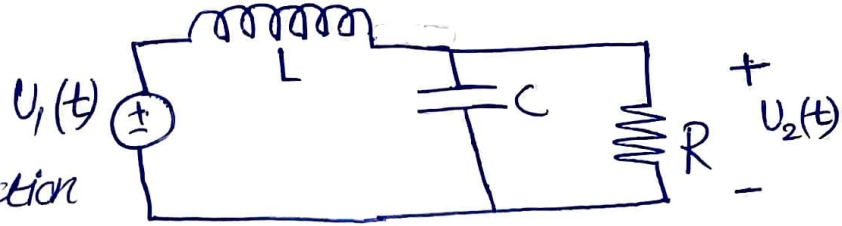
∞ $K = -3$

#

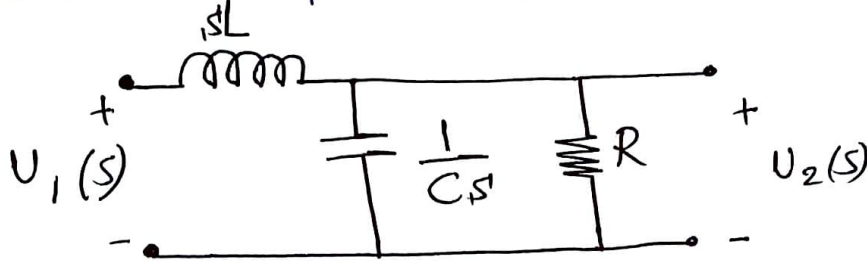
* For the circuit :

[a] Find the transfer function

$$U_2(s)/U_1(s)$$



[b] If $R=L=C=1$, find the forced response when $U_1(t) = 5$



$$U_R(s) = \frac{\frac{1}{sC} \parallel R}{\frac{1}{sC} \parallel R + sL} \quad U_1(s) = U_2(s)$$

$$\lim_{\infty} \frac{U_2(s)}{U_1(s)} = \frac{\frac{R}{sRC + 1}}{\frac{R}{s^2RC + 1} + sL} = \frac{R}{RLC s^2 + L s + R}$$

$$\lim_{\infty} \text{the transfer function} = \frac{1/LC}{s^2 + \frac{1}{RC} s + \frac{1}{LC}}$$

for $C=R=L$ we get $\frac{U_2(s)}{U_1(s)} = \frac{1}{s^2 + s + 1}$

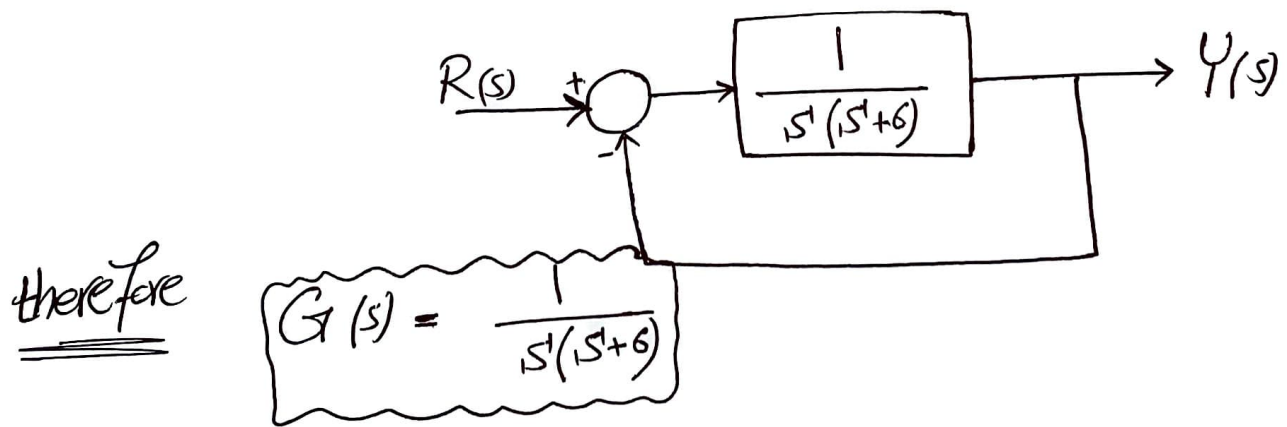
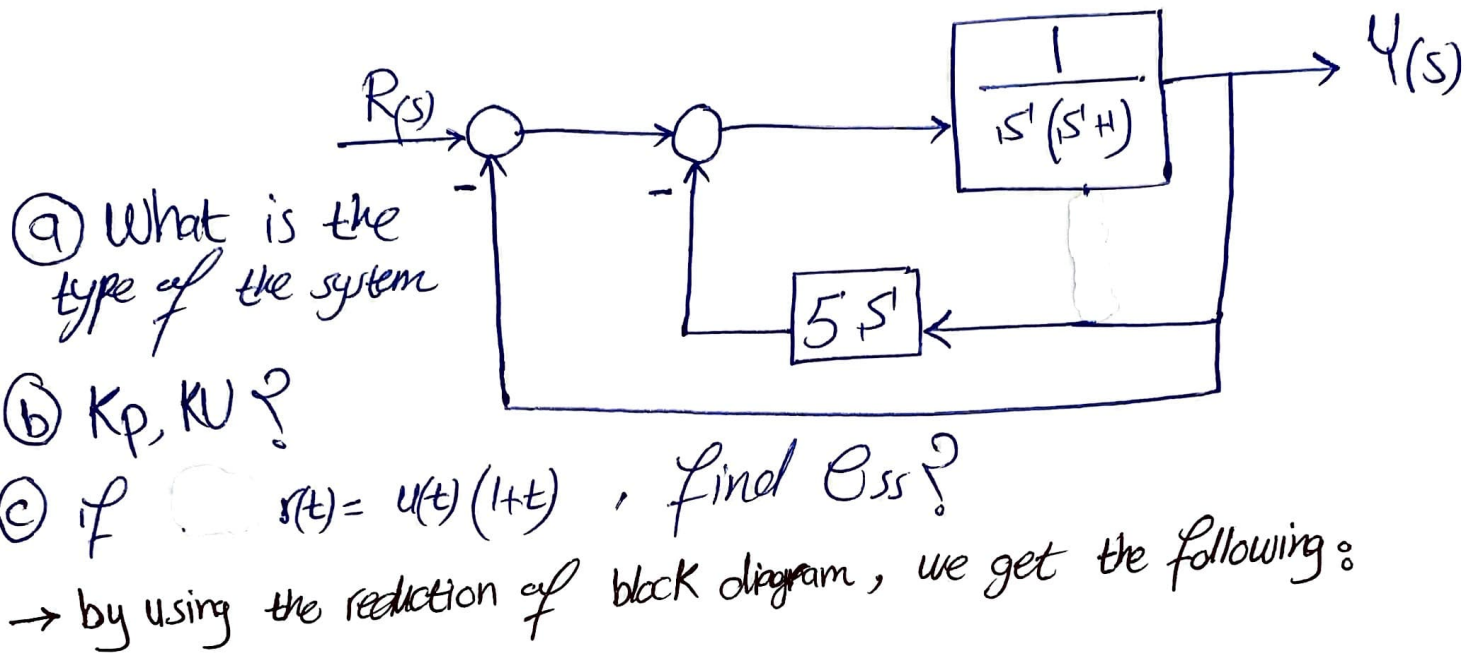
therefore

$$U_2(s) \Big|_{U_1(s) = \frac{5}{s}} = \frac{5}{s(s^2 + s + 1)} = \frac{A}{s} + \frac{Bs + C}{s^2 + s + 1}$$

$\uparrow$ forced response $\uparrow$ Natural response

$\lim_{\infty} A = 5$
 ∞

* For the system shown, find the steady state error of the system for inputs: step, ramp and parabola in terms of K and K_t , let $n(t) = 0$ (Assume the system is stable)



∴ type of system is one

$$K_p = \lim_{s \rightarrow 0} G(s) = \boxed{\infty}, \quad K_v = \lim_{s \rightarrow 0} s G(s) = \boxed{1/6}$$

now $R(s) = \frac{1}{s} + \frac{1}{s^2} = \frac{s+1}{s^2}$

$$e_{ss} = \lim_{s \rightarrow 0} \frac{s R(s)}{1+G(s)} = \lim_{s \rightarrow 0} \frac{(s+1)(s+6)}{s^2 + 6s + 1} = \boxed{6}$$

* Apply the Routh-Hurwitz stability criterion for the Polynomial and determine the number of poles in the number of roots RHP, LHP and jw-axis

$$P(s) = s^6 + 2s^5 + s^4 - s^2 - 2s - 1$$

we'll substitute ϵ for the missing s^3 co-factor

s^6	1	1	-1	-1
s^5	2	ϵ	-2	0
s^4	$\frac{2-\epsilon}{2}$	0	-1	0
s^3	ϵ	$\frac{-2\epsilon}{\epsilon-2}$	0	0
s^2	-1	-1	0	0
s^1	$\frac{-\epsilon^2}{\epsilon-2}$	0	0	0
s^0	-1	0	0	0

$$\left. \begin{array}{l} \frac{2-\epsilon}{2} = (+) \\ \epsilon = (+) \\ \frac{-\epsilon^2}{\epsilon-2} = (-) \end{array} \right\} \infty \quad \begin{array}{c} (+) \\ (+) \\ (+) \\ (+) \\ (-) \\ (-) \\ (-) \end{array}$$

the sign changed
once

∞ one root in
RHP

$$s' \mid \begin{array}{c} -\epsilon^2 \\ \epsilon^{-2} \end{array} \quad 0 \quad 0 \quad 0$$

an entire row of zeros

∞ Equation of the row above

$$-1 s'^2 - 1 = 0$$

$$\infty s'^2 = -1 \Rightarrow \boxed{s' = \pm j}$$

∞ two roots on the jw-axis

=====

∞ one root in RHP

two roots in jw-axis

∞ 3 roots in LHP

to check:

by solving algebra fx 2.0 plus

$$\rightarrow s_1 = s_2 = s_3 = -1$$

$$\rightarrow s_4 = 1$$

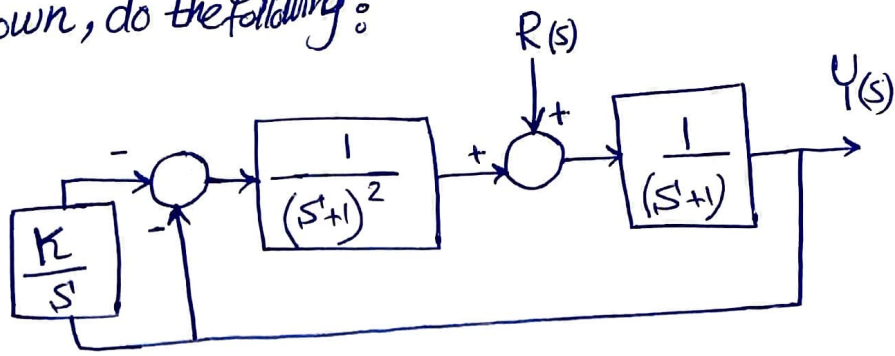
$$\rightarrow s_{5,6} = \pm j$$

* For the system shown, do the following:

a) sketch the root locus

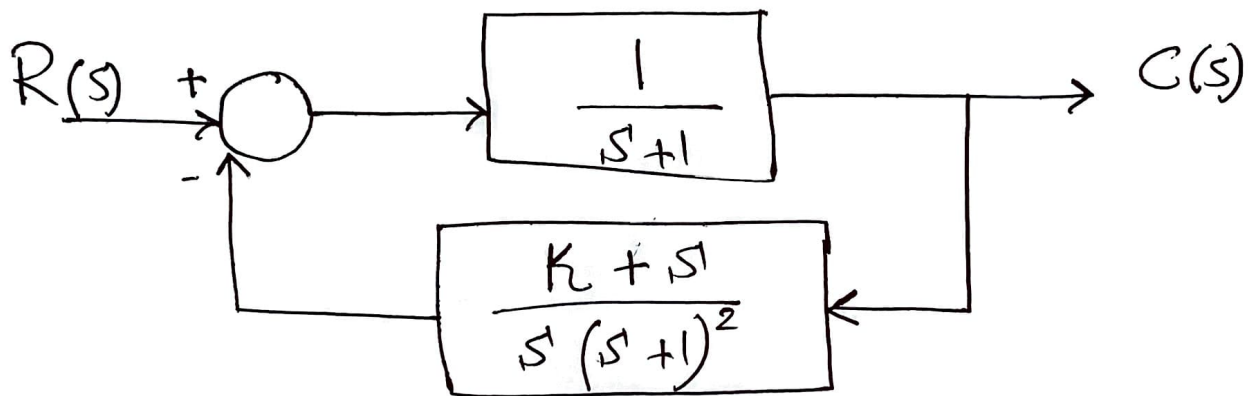
b) Find the value of K at the breakaway point

c) Find the $j\omega$ -axis crossing and the gain, K , at the crossing



$$H(s)_{\text{total}} = \left(\frac{K}{s} + 1 \right) \cdot \frac{1}{(s+1)^2}$$

∴ the system is:



$$T(s) = \frac{C(s)}{R(s)} = \frac{s(s+1)^2}{s^4 + 3s^3 + 3s^2 + 2s + K}$$

$$\therefore T(s) = \frac{N(s)}{D(s)}$$

$\begin{cases} N \equiv \text{numerator} \\ D \equiv \text{denominator} \end{cases}$

Notes

∴

$$\underline{\underline{D(s) = s^4 + 3s^3 + 3s^2 + 2s + K}}$$

recall,

$$\frac{C(s)}{R(s)} = \frac{K G(s)}{1 + K G(s) H(s)} = \frac{N(s)}{D(s)}$$

$$\infty D(s) = 1 + K G(s) H(s)$$

now $s^4 + 3s^3 + 3s^2 + 2s + K = 1 + \frac{K}{s^4 + 3s^3 + 3s^2 + 2s}$

Therefore $K G(s) H(s) = \frac{K}{s^4 + 3s^3 + 3s^2 + 2s}$ } open loop gain

∞ Poles : $\left\{ 0, -2, \frac{-1 \pm j\sqrt{3}}{2} \right\}$ } poles & zeros

zeros: $\{ \emptyset \}$

$$\alpha = 4 - 0 = 4$$

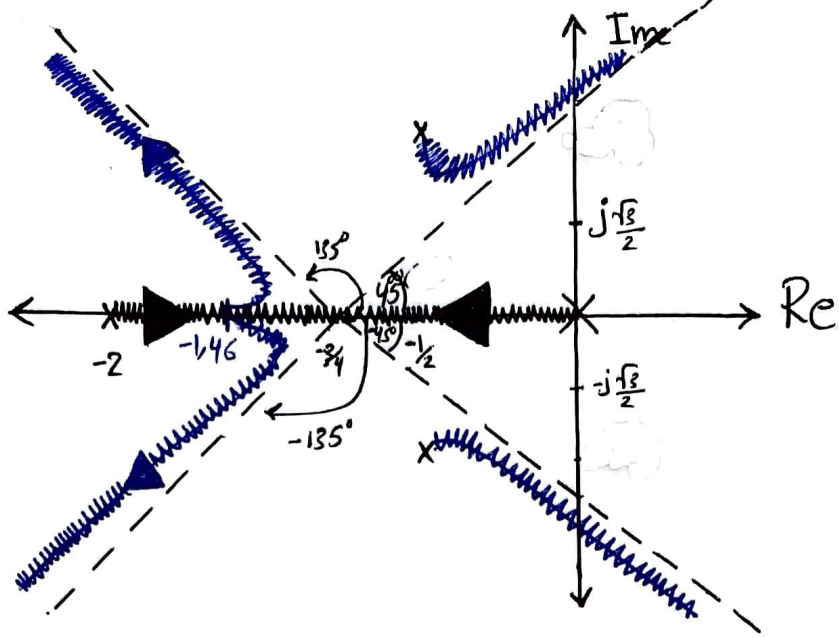
$$\sigma_m = \frac{\sum \text{Poles} - \sum \text{zeros}}{\alpha} = \boxed{-\frac{3}{4}}$$

$$\theta_m = r \frac{180}{\alpha}, \quad r = \pm 1, \pm 3$$

$$\infty \theta_m = \pm 45^\circ, \pm 135^\circ$$

sketch of root locus / رسم

الرسم تقريبي



من MATLAB لي
هههه!!

now we need to find the breakaway points :-

$$\frac{d}{ds} G(s)H(s) = 0$$

$$\infty - \frac{(4s^3 + 9s^2 + 6s + 2)}{(s^4 + 3s^3 + 3s^2 + 2s)^2} = 0$$

by solving $-(4s^3 + 9s^2 + 6s + 2) = 0$ for s

$$\infty s = -1,455410041$$

$$-0,397294974 \pm j 0,4309321229$$

RL تقريبي

X مرفوض

Breaking away point is $s \approx -1,46$

الآن نقوم بتدويرها مع s -plane ونأخذ الرسم بالانزلة

نتبع

→ K at the breakaway point is:

$$K = \frac{-1}{G(s)H(s)} \bigg|_{s=-1.46} = \boxed{-1,317944713}$$

→ jw-axis crossing :-

$$\therefore T(s) = \frac{s^1 (s^1 + 1)^2}{s^4 + 3s^3 + 3s^2 + 2s + K}$$

By using Routh-Hurwitz stability criterion to find the range of K that gives a stable system and the value of K where the system becomes marginally stable

s^4	1	3	K
s^3	3	2	0
s^2	$7/3$	K	0
s^1	$\frac{14-9K}{14}$	0	0
s^0	K	0	0

$$\boxed{K > 0}$$

$$14 - 9K > 0$$

$$\therefore \boxed{\frac{14}{9} > K}$$

∴ range of K for a stable system is:

$$\boxed{0 < K < 14/9}$$

→ system will margin at $\boxed{K = \frac{14}{9}}$

the equation of the row above s^1 will be the following auxiliary equation:

$$\frac{7}{3} s^2 + \frac{14}{9} = 0 \Rightarrow \boxed{s = \pm j \frac{\sqrt{6}}{3}}$$

... on 12/5

Utt. JGR 1999
for.