

Design of beams due to Torsional Moment.

Twisting Moment



تصميم الكمرات على عزم الإلتواء

نسألكم الدعاء

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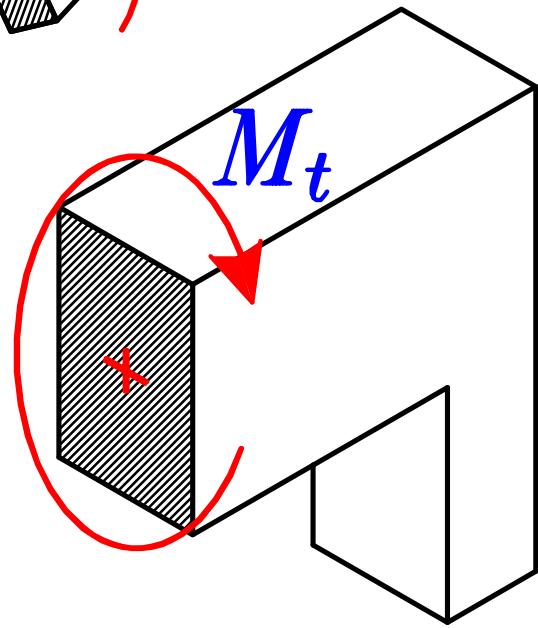
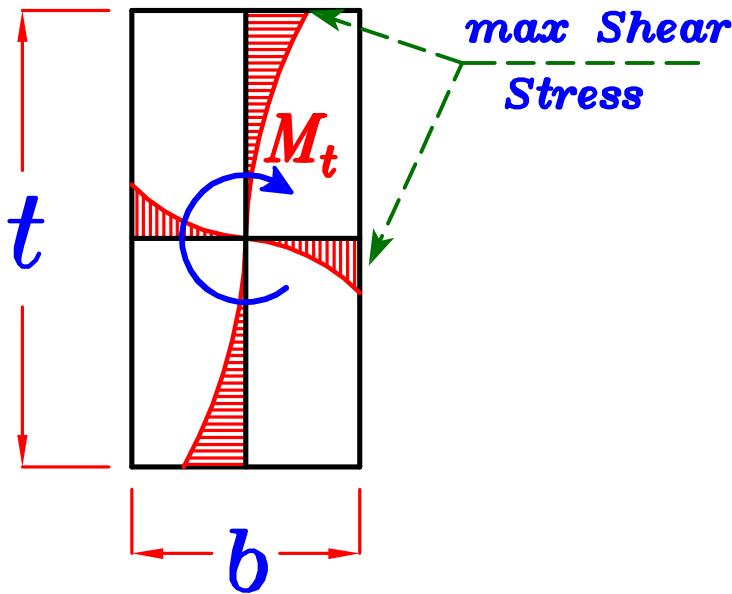
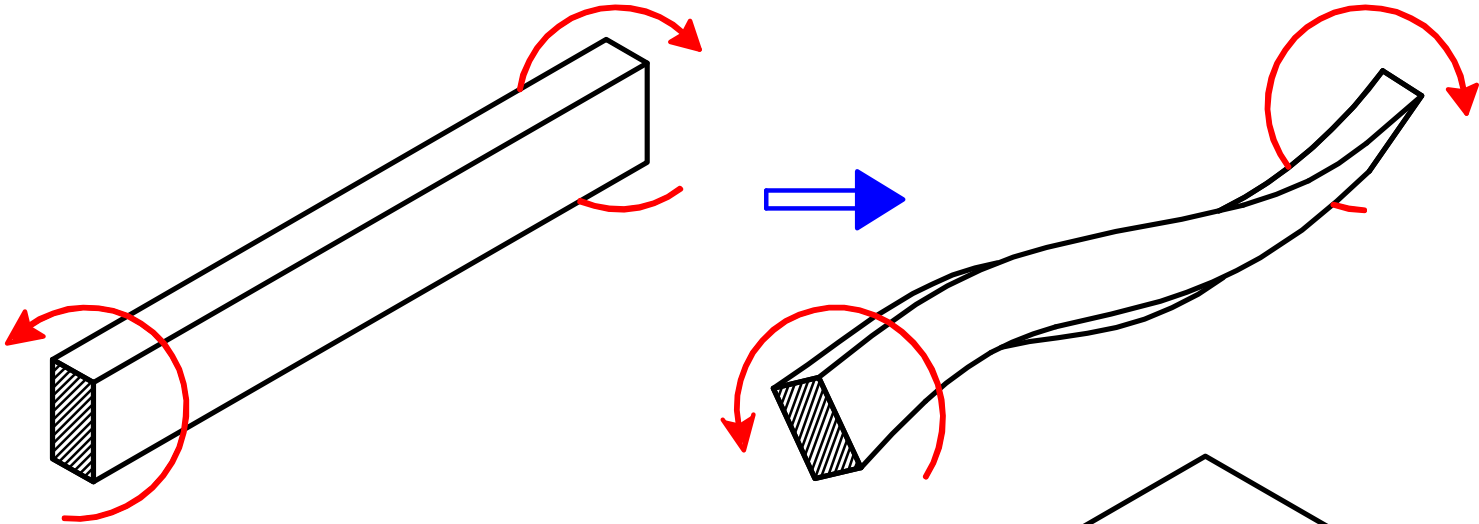
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Torsion Introduction.



عزم الإلتواء

عزم الإلتواء هو عزم ينتج عندما تدور الكمره حول محورها .



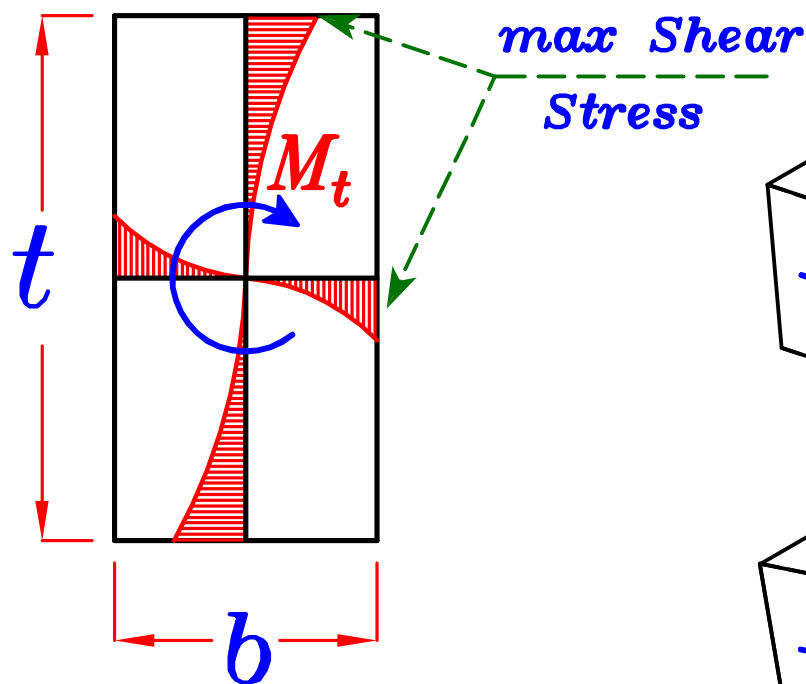
تنتج إجهادات القص *Shear Stresses* على الكمرات عن طريق

① *Shear Force* (Q) ----- قوى القص

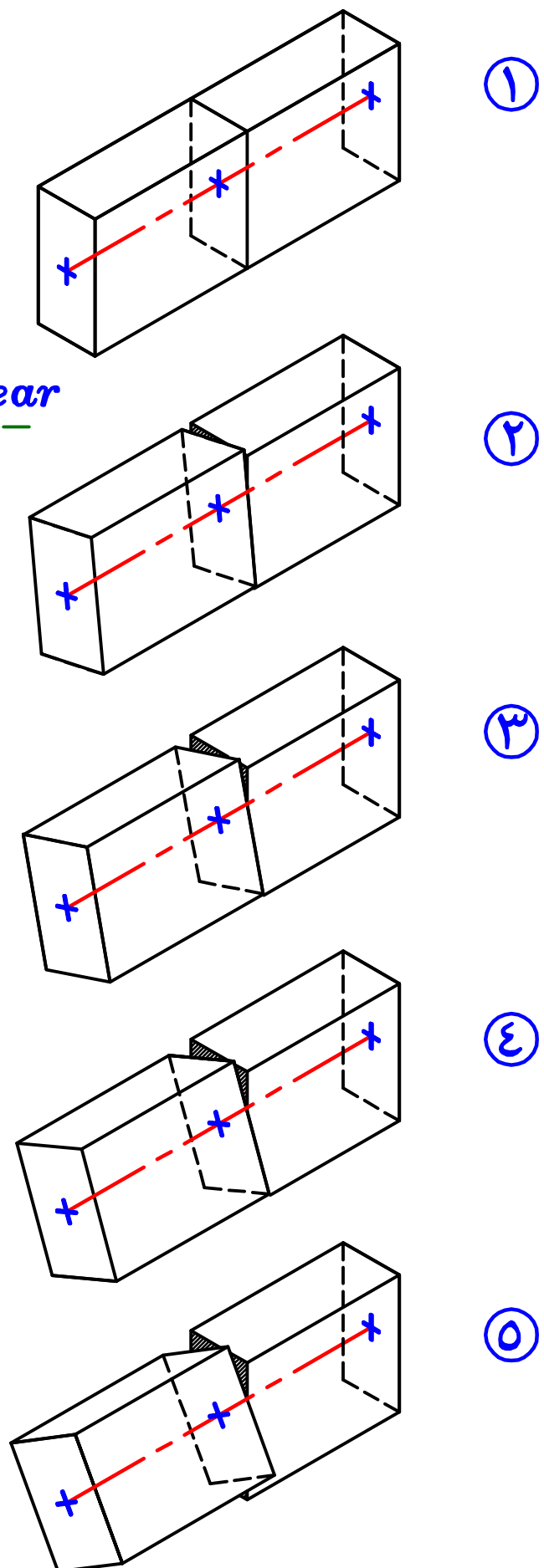
② *Torsional Moment* (M_t) ----- عزم الإلتواء

لذا سيتم تصميم الكانات لتحمل كلا من *Shear Force* & *Torsional Moment*

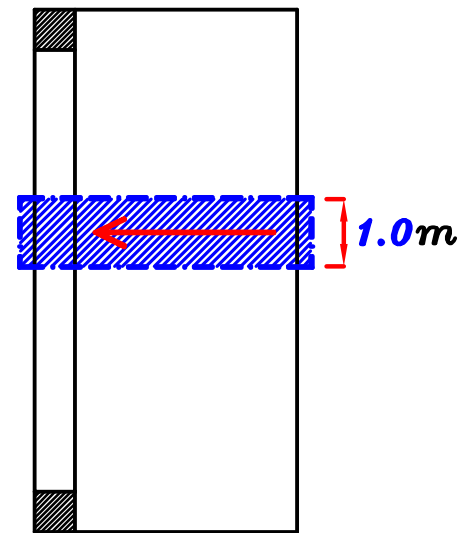
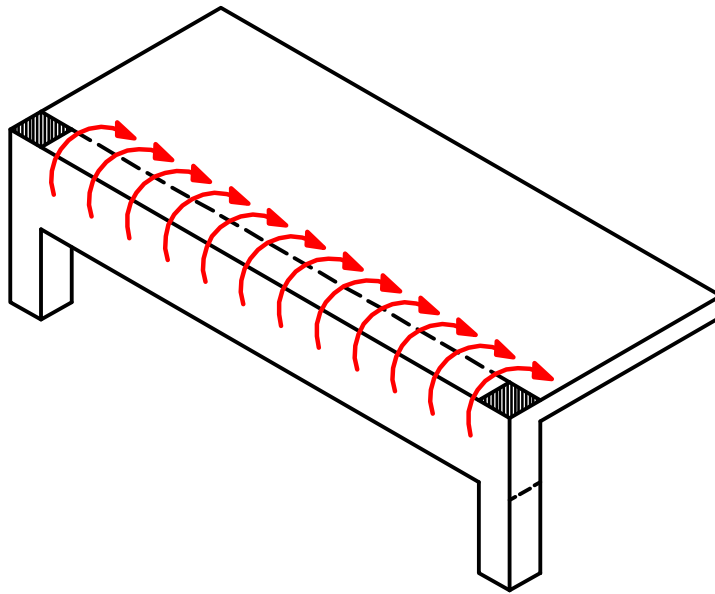
يتسبب الـ **Torsional Moment** في وجود اجهادات قص ناتجة عن دوران الكمره حول محورها .
و ذلك بسبب الاحتكاك الناتج بين سطحين متلامسين احدهما يدور بزاويه أكثر من الاخر .



Shear Stress Distribution

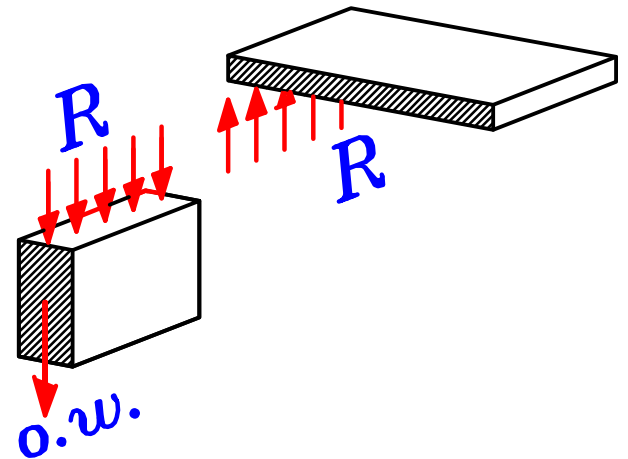
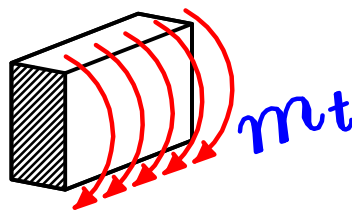
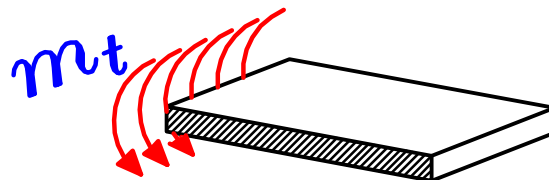
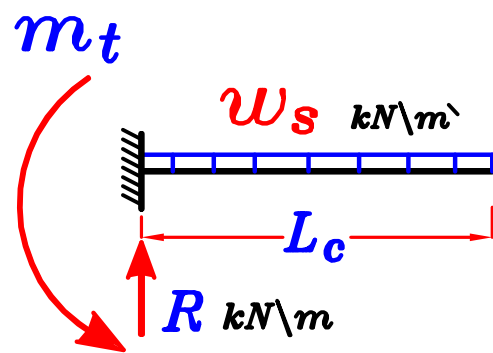
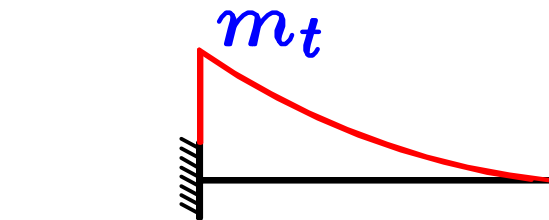


تكون الكمره عليهما **Torsion** منتظم عندما تكون شريحه البلاطه محموله على كمره واحده .

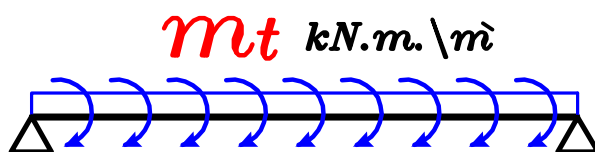


نأخذ شريحه فى البلاطه فى اتجاه ال **Load** عرضها **1.0m** - ١، م

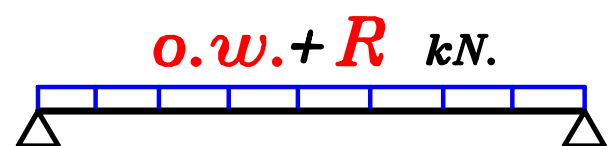
Take a strip in the slab at the load direction with width **1.0m**



Beam.



Torsional Moment

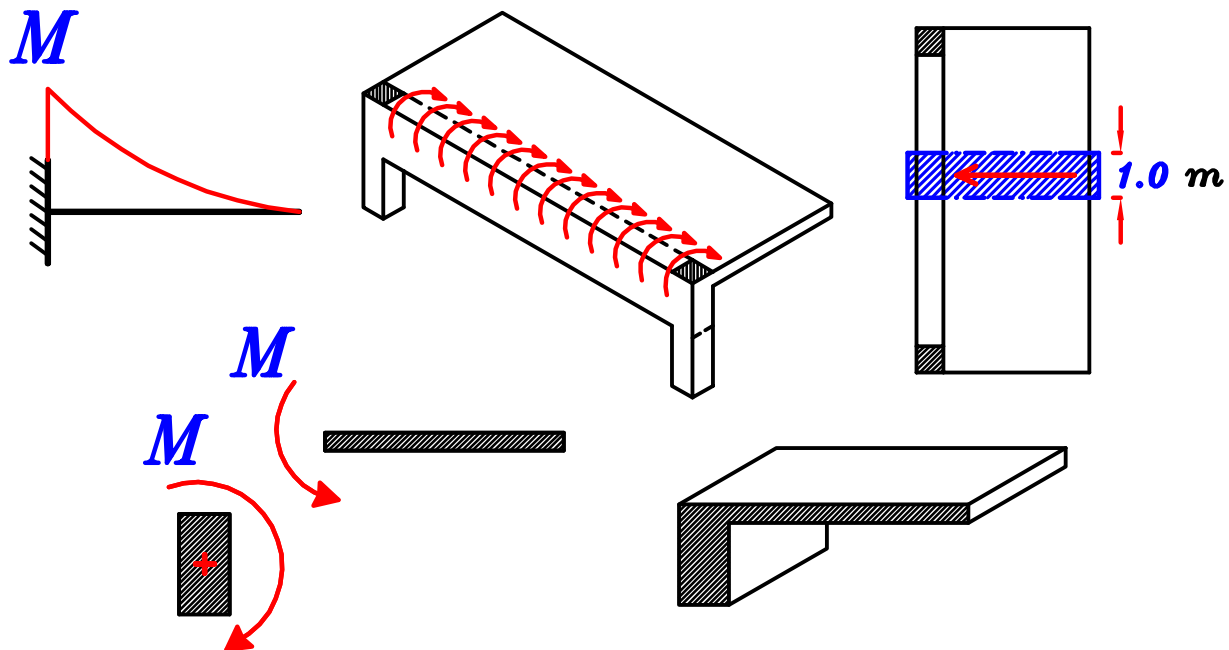


Load

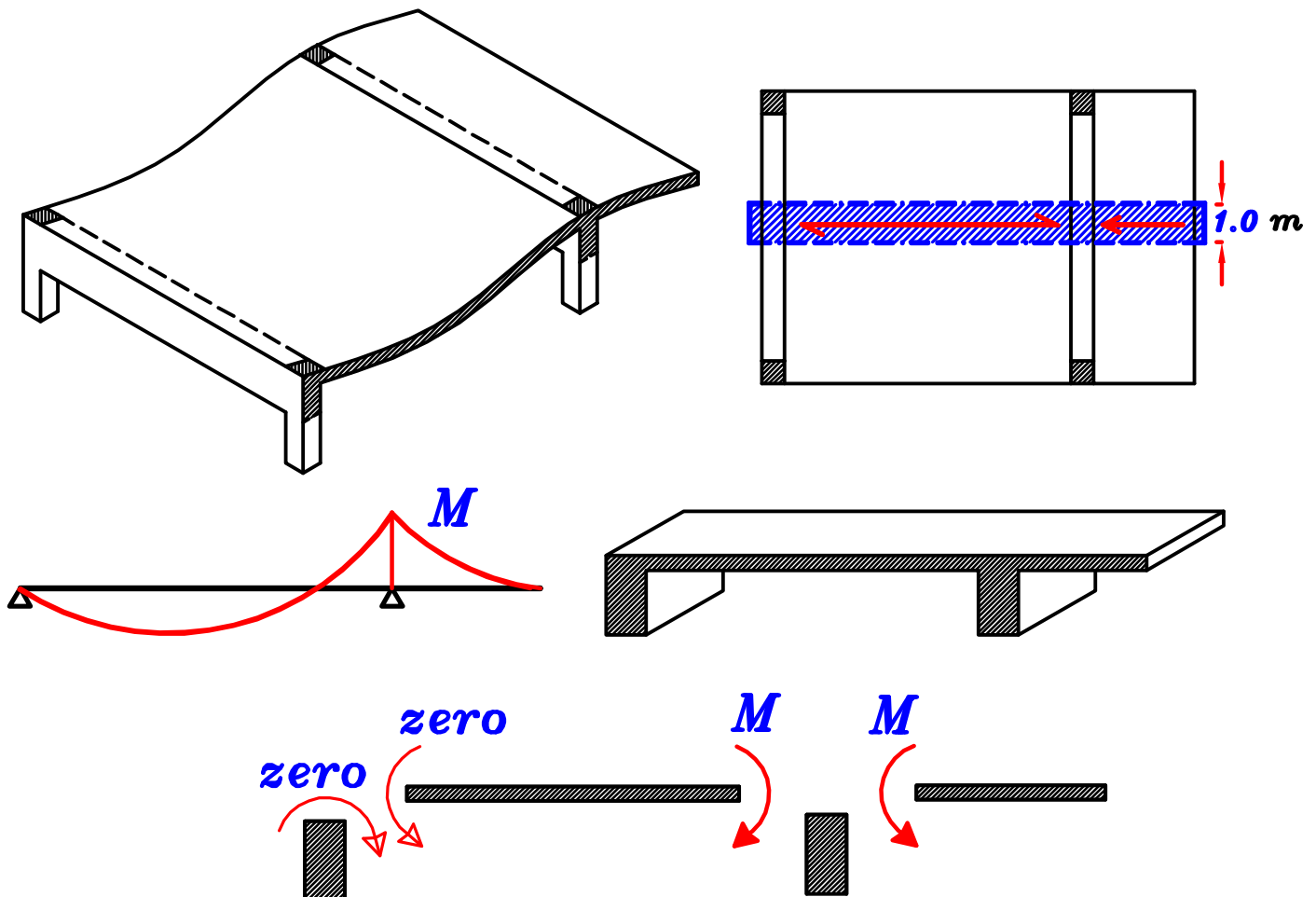
ملحوظه .

ينتقل عزم الالتواء من البلاطة الى الكمره

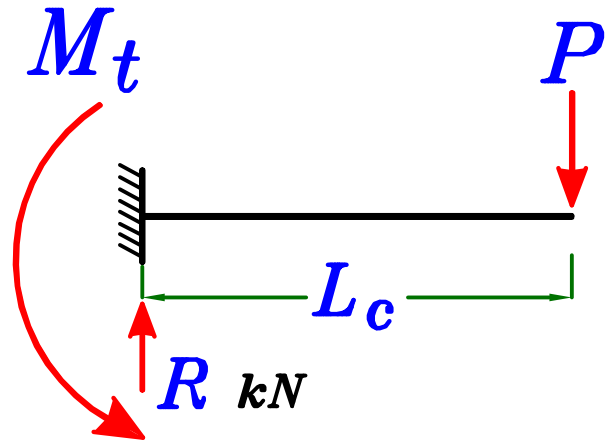
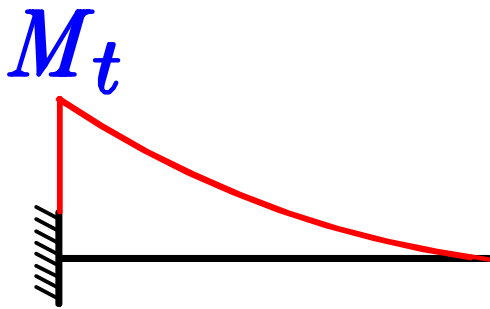
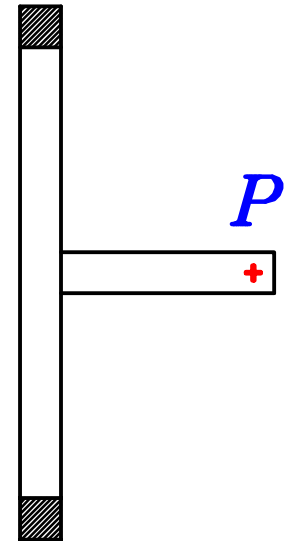
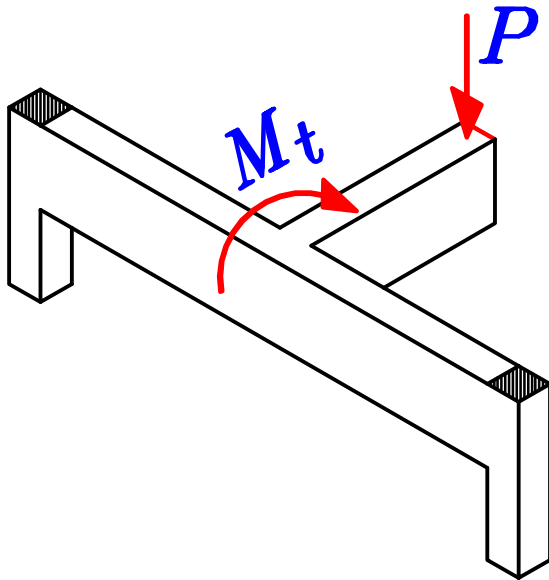
اذا كانت شريحه البلاطة محموله على كمره واحده فقط فيعمل **Torsion** على الكمره .



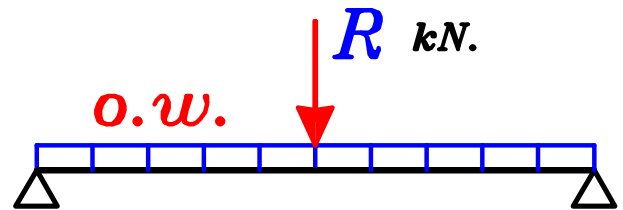
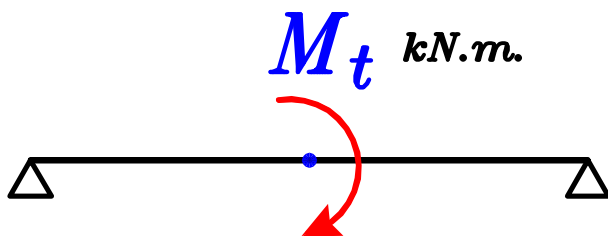
اذا كانت شريحه البلاطة محموله على اكثر من كمره لن يكون هناك **Torsion** لان العزم سينتقل من البلاطة للبلاطة المجاوره مباشره .



تكون الكمره عليها **Torsion** مركز عندما تكون كمره محموله على كمره واحده فقط.



Beam.

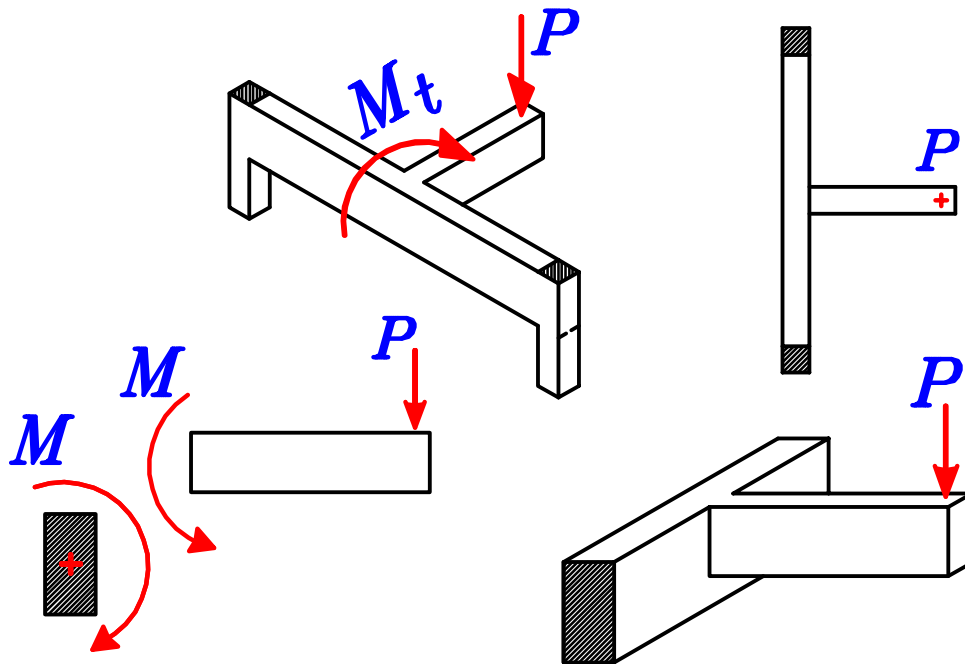


Torsional Moment

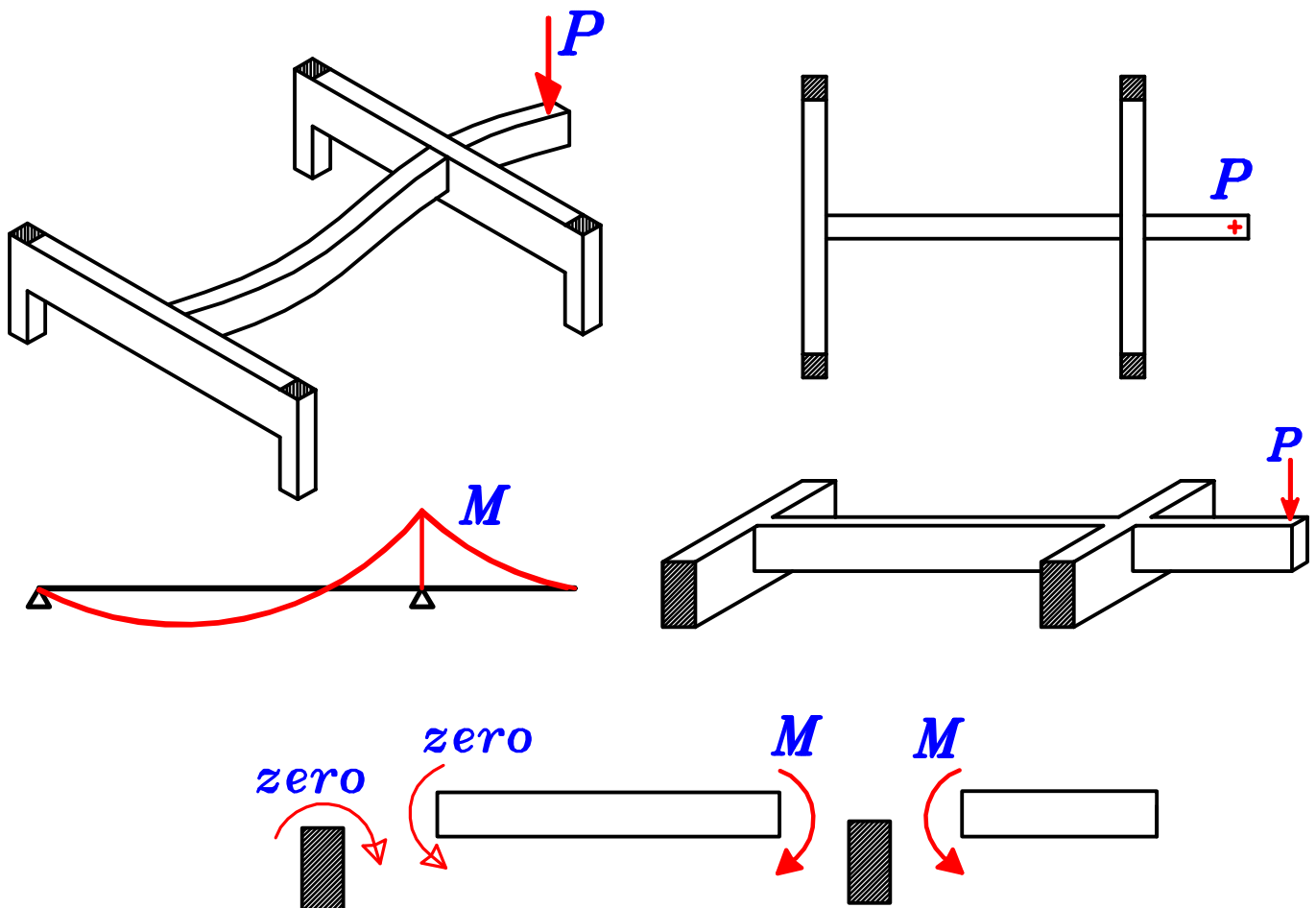
Load

ملحوظه .

ينتقل عزم الالتواء من الكمره الـ **Cantilever** الى الكمره التي تحملها .
لان الكمره الـ **Cantilever** محموله على كمره واحده فقط .



اذا كانت الكمره محموله على اكثر من كمره سينتقل العزم من الـ **cantilever** الى الباقيه التاليه دون ان ينقل **torsion** على الكمره الحامله .

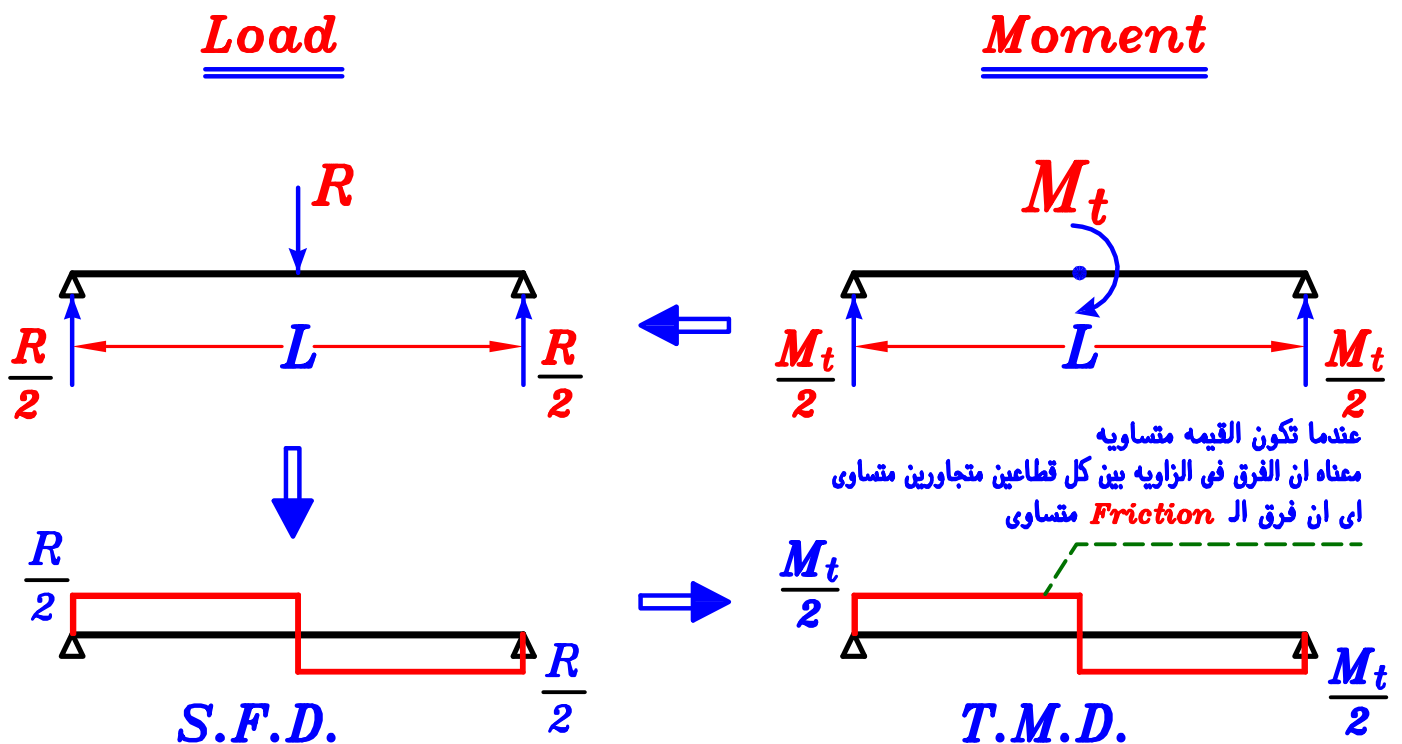
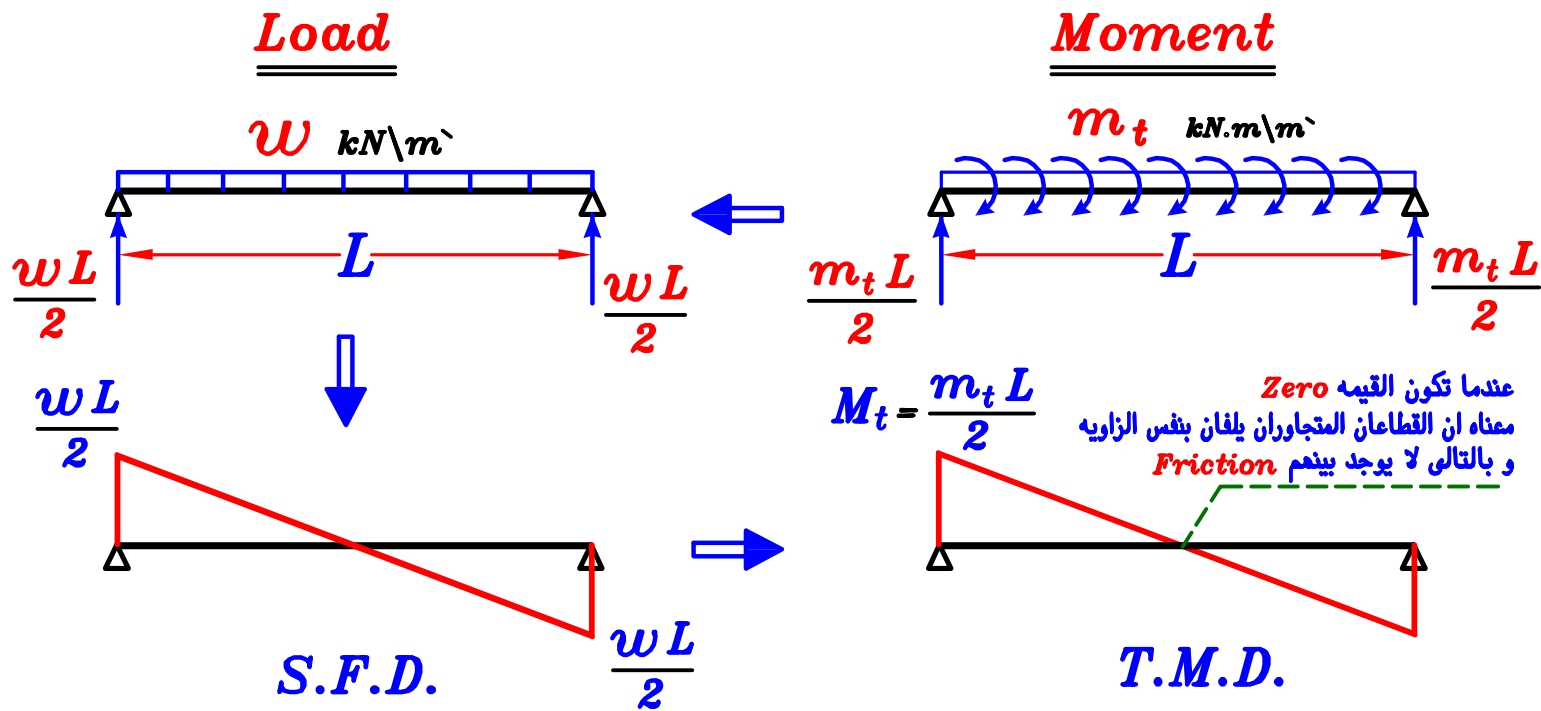


How to Draw Torsional Moment Diagram (T.M.D.)

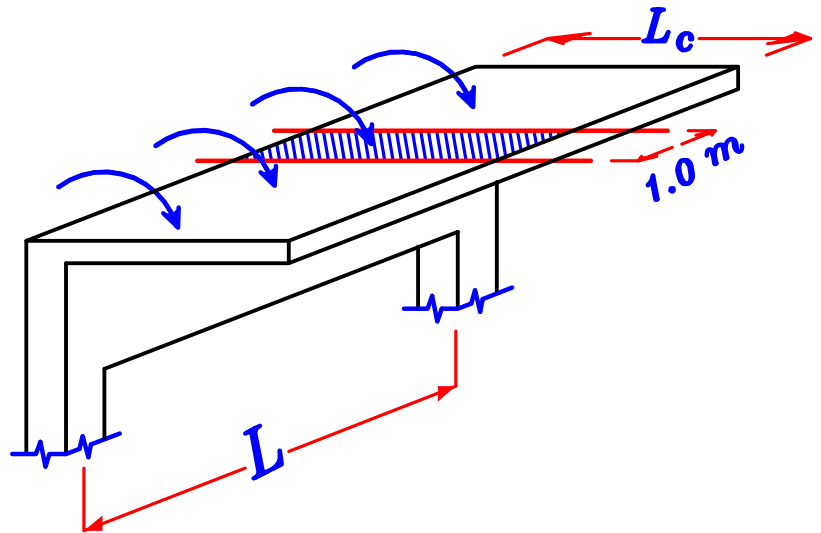
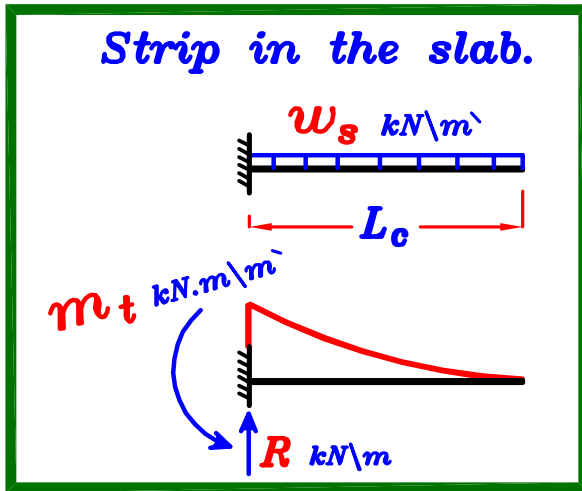
لرسم ال **Torsional Moment Diagram (T.M.D.)** يرسم بنفس طريقة رسم ال **S.F.D.**

لذا عند رسم ال **T.M.D.** نفرض أن ال **Moment** عبارة عن **Load** و نرسم

ال **S.F.D.** لهذا ال **Load** فيكون هو نفس شكل ال **T.M.D.** لل **Moment**

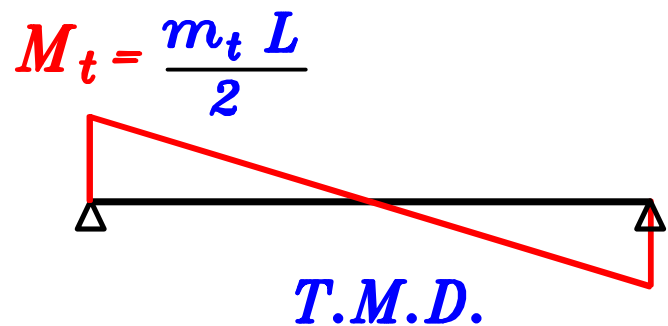
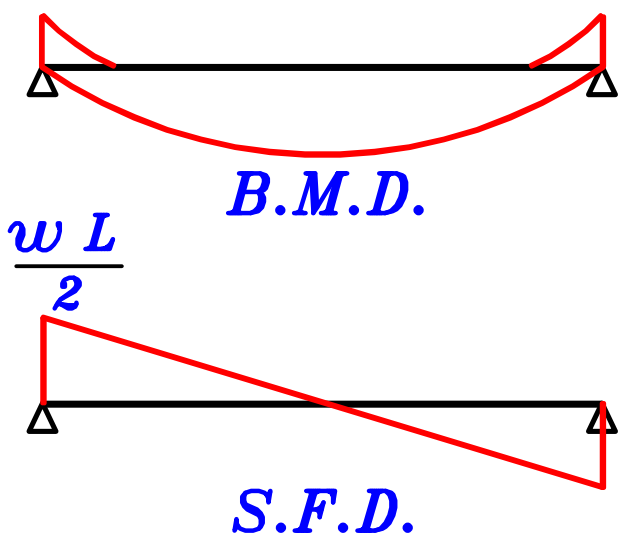
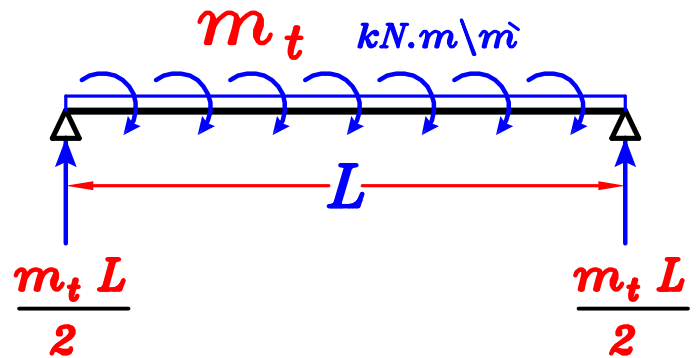
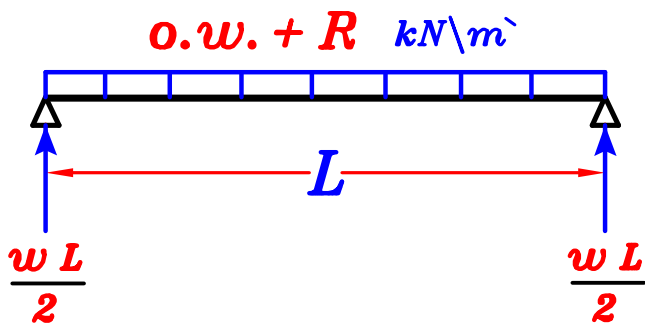


Example.

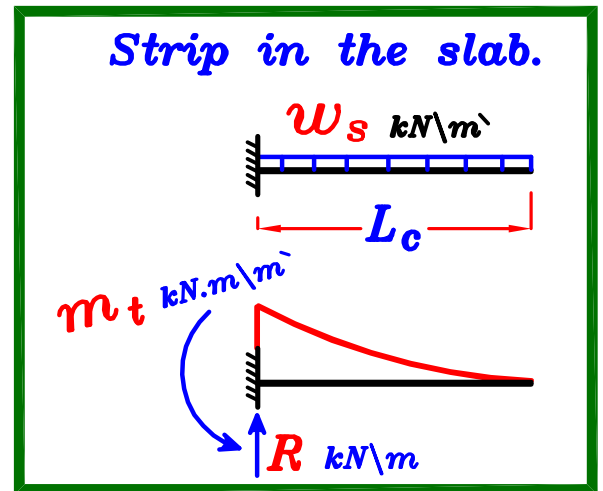
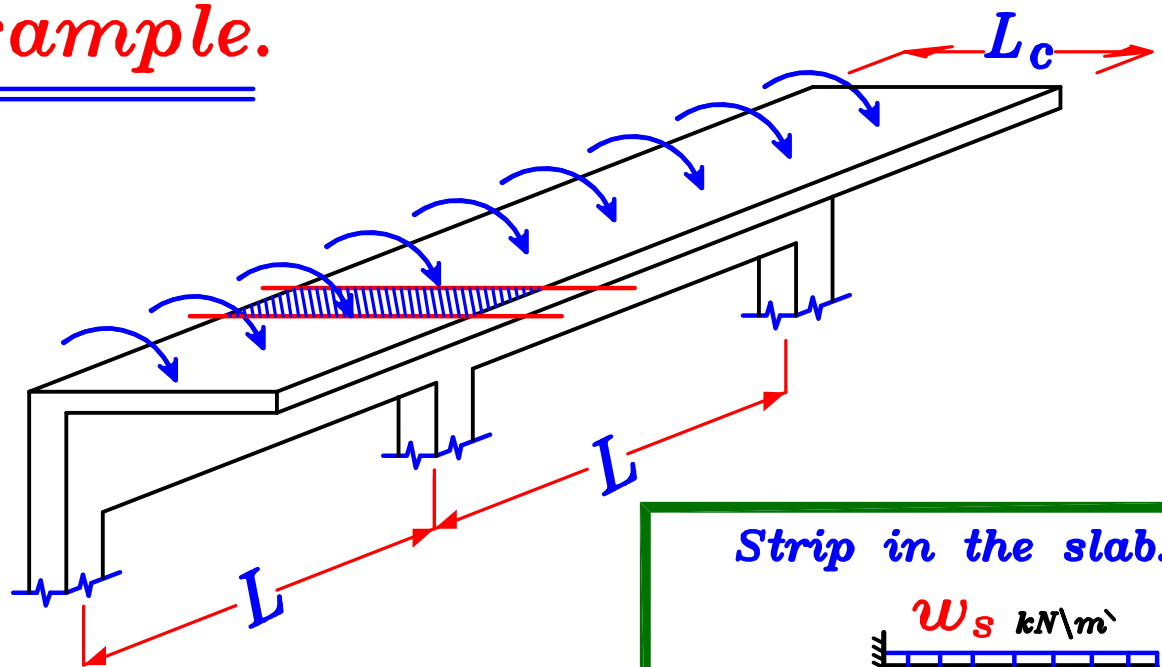


Load

Moment

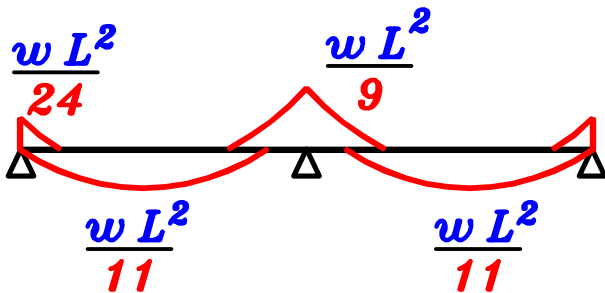
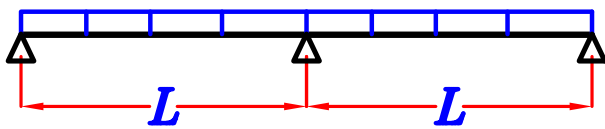


Example.



Load

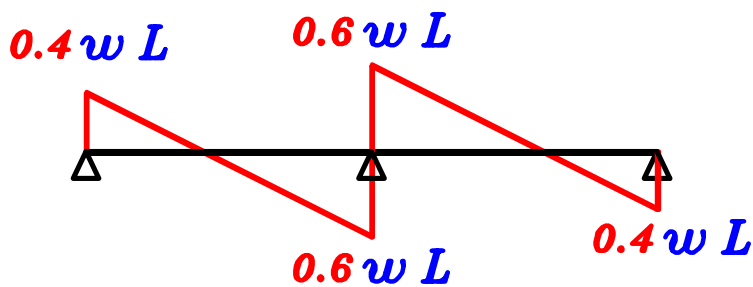
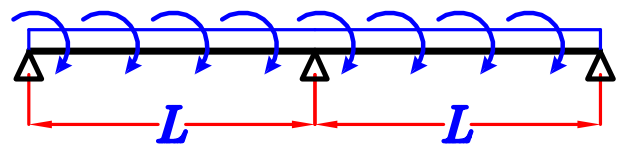
$$w = o.w. + R \quad kN/m$$



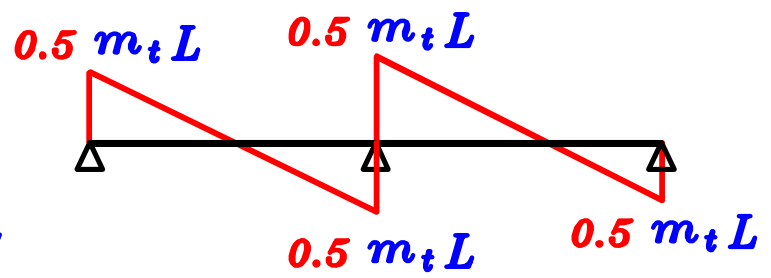
B.M.D.

Moment

$$m_t \quad kN.m/m$$

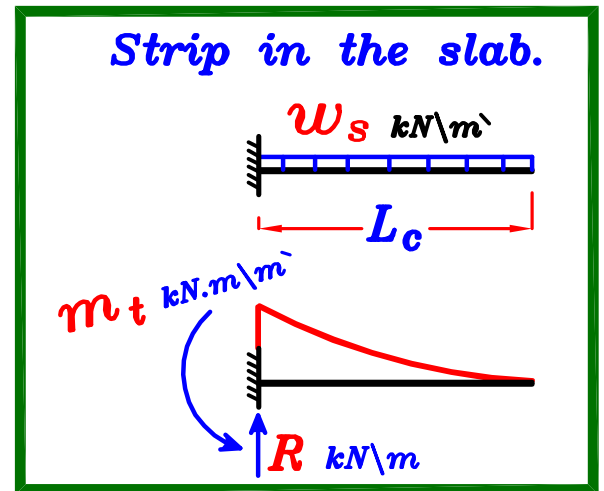
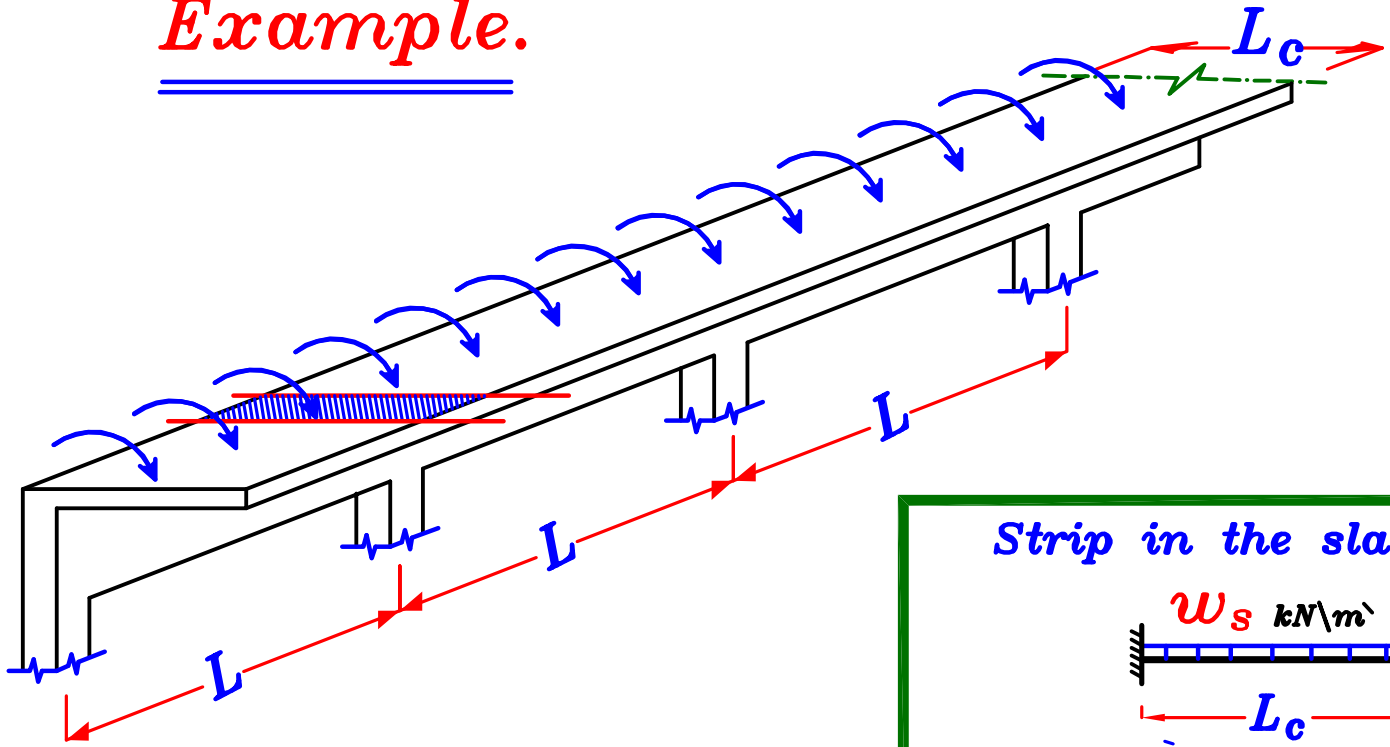


S.F.D.

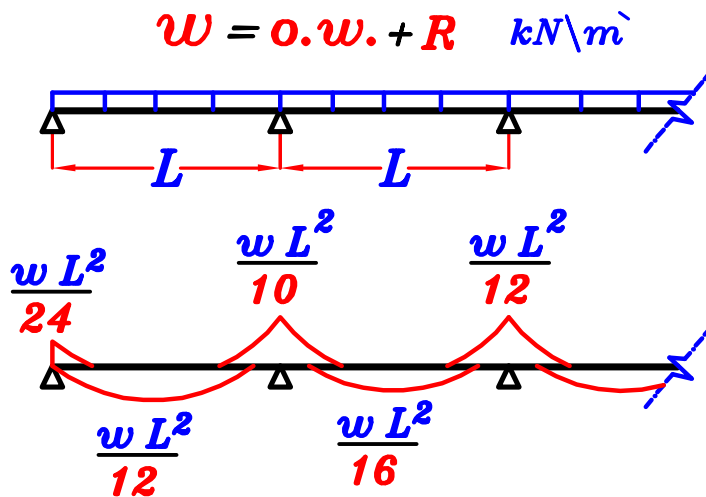


T.M.D.

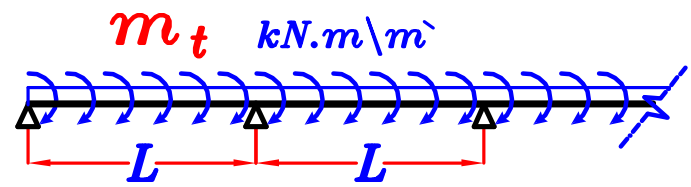
Example.



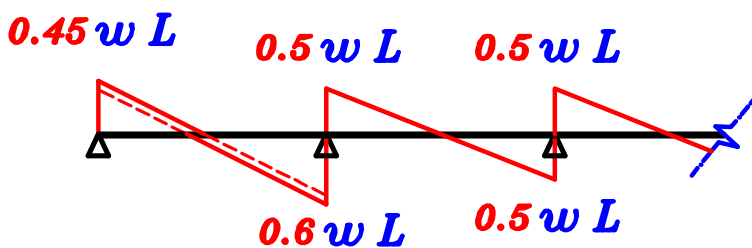
Load



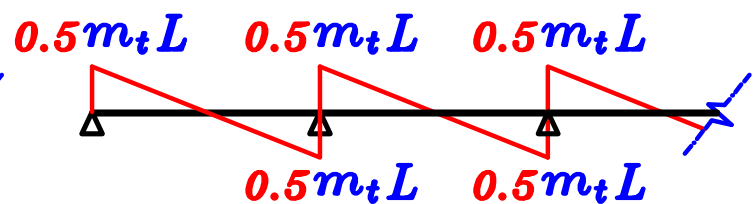
Moment



B.M.D.

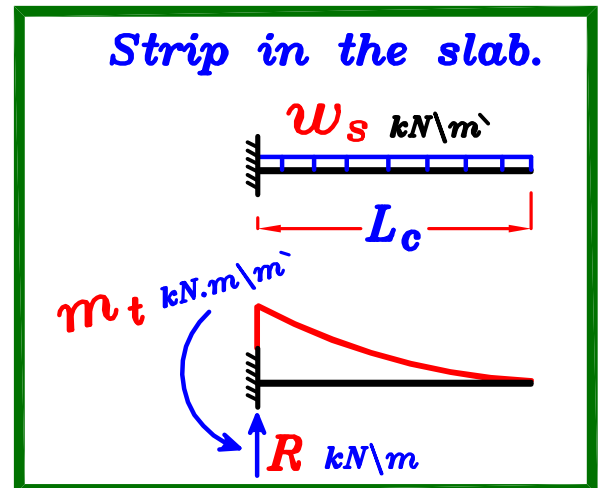
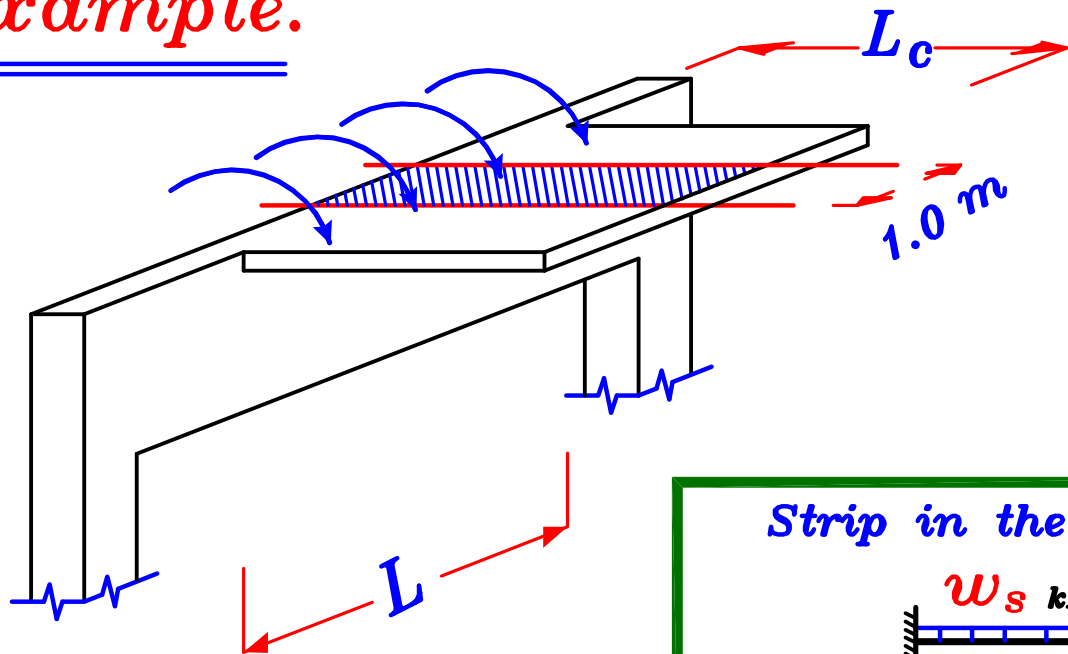


S.F.D.

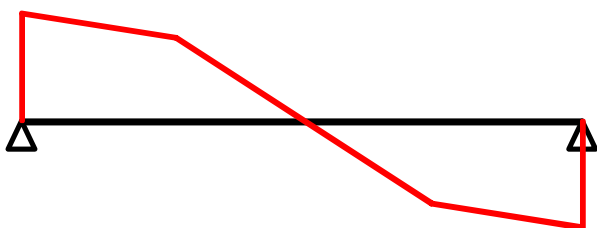
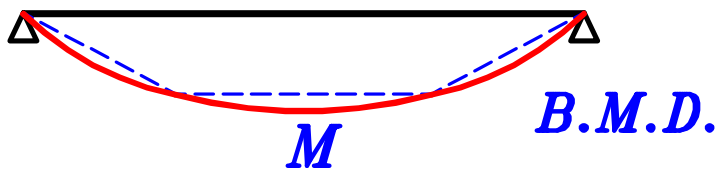
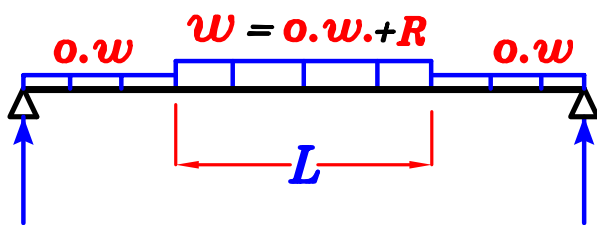


T.M.D.

Example.

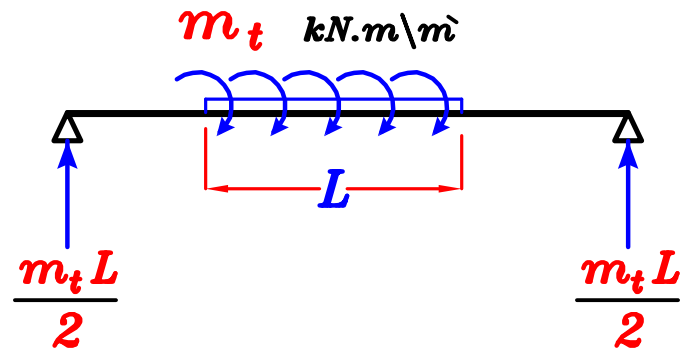


Load

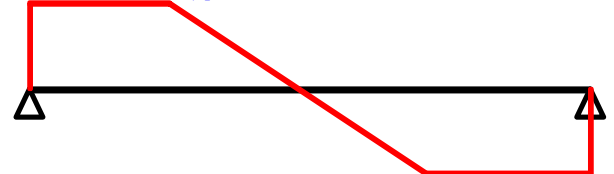


S.F.D.

Moment

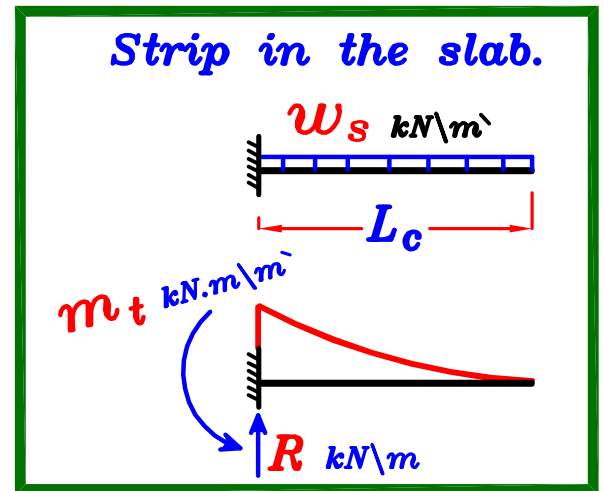
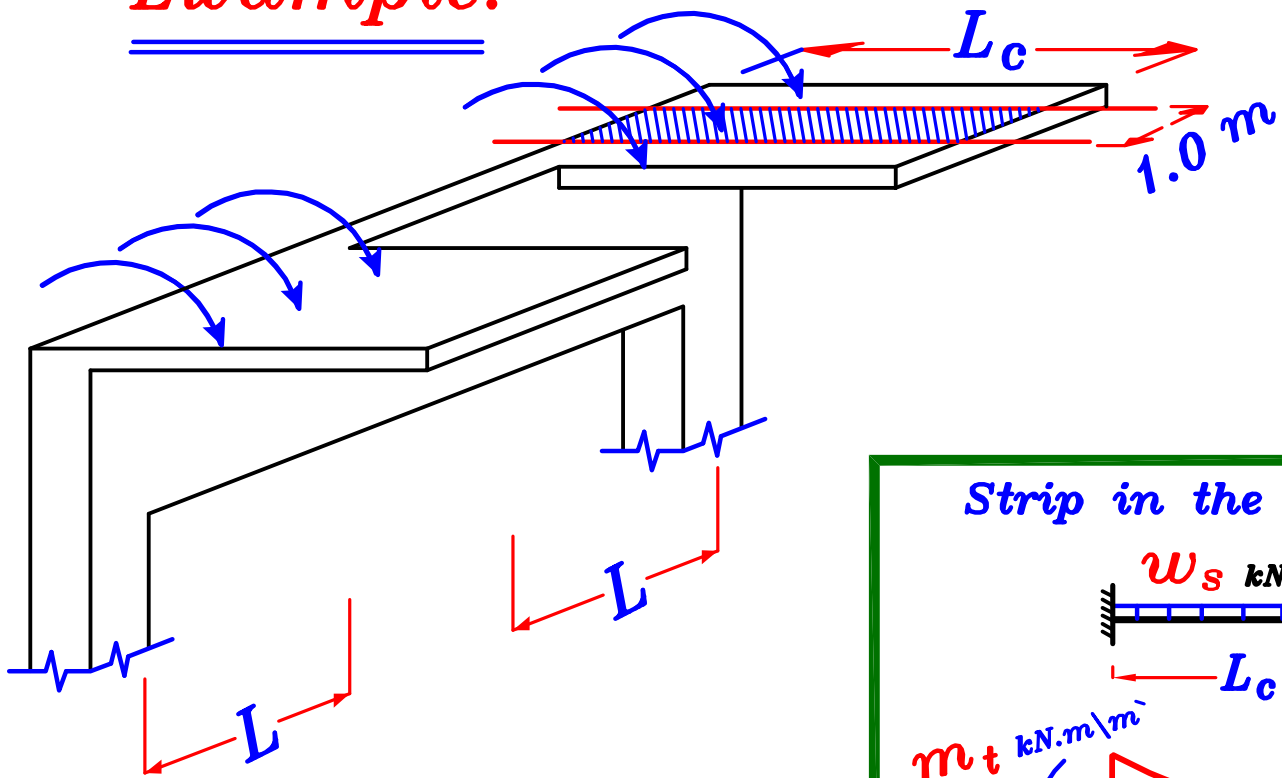


$$M_t = \frac{m_t L}{2}$$

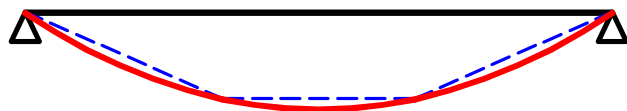
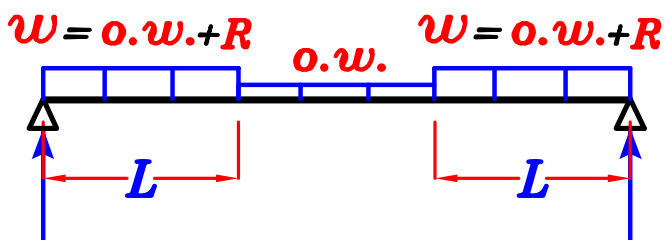


T.M.D.

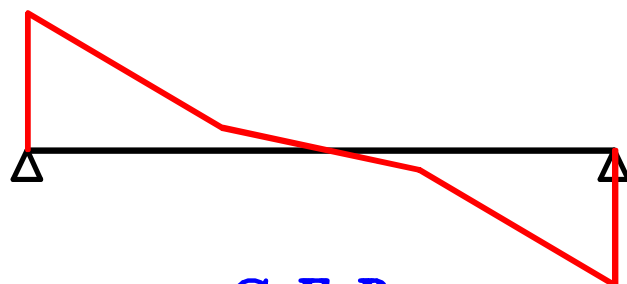
Example.



Load

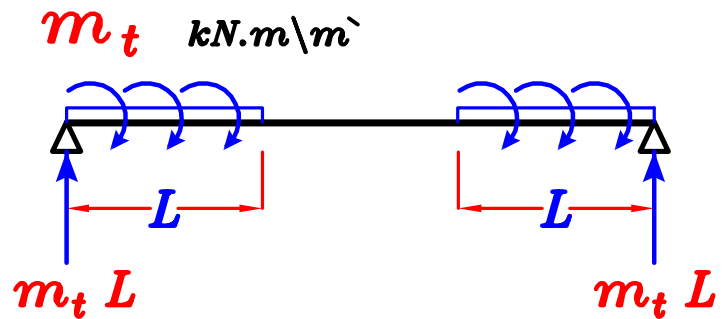


M



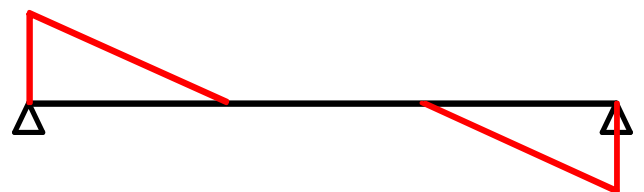
S.F.D.

Moment



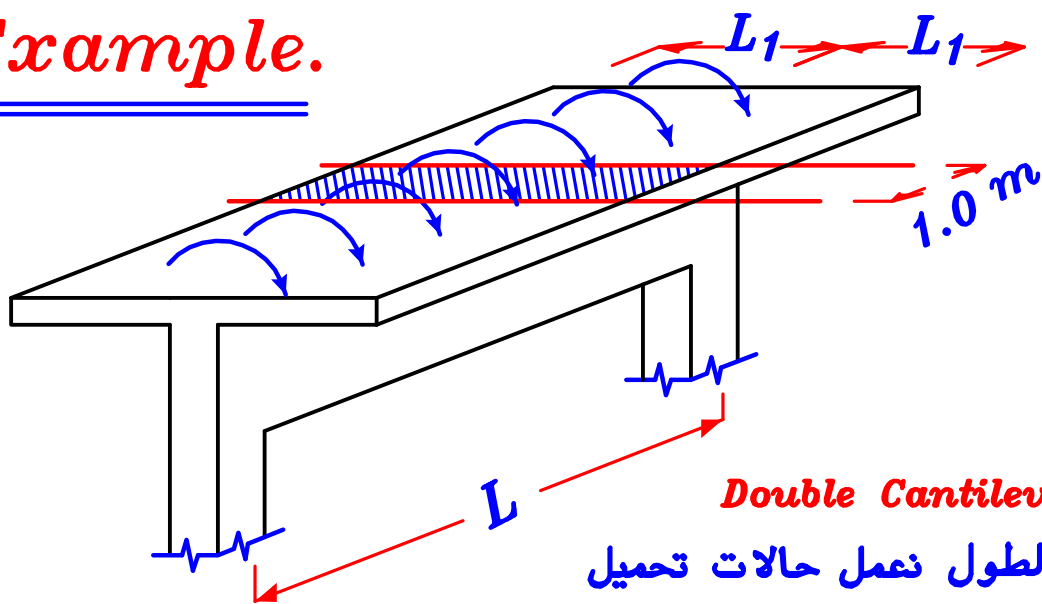
B.M.D.

$$M_t = m_t L$$



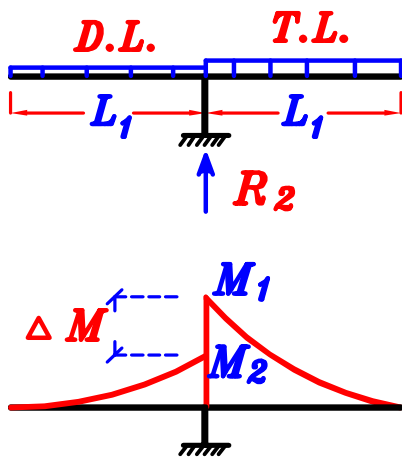
T.M.D.

Example.

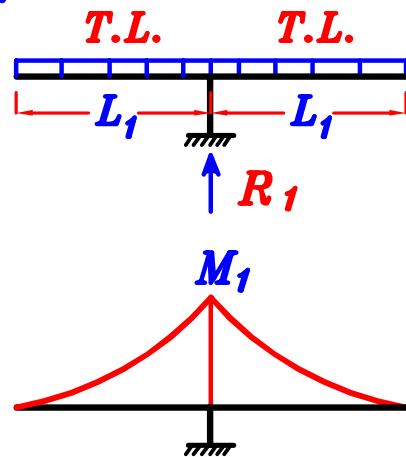


عند وجود **Double Cantilever** متساويه فى الطول نعمل حالات تحميل فنضع **T.L.** على أى منهما

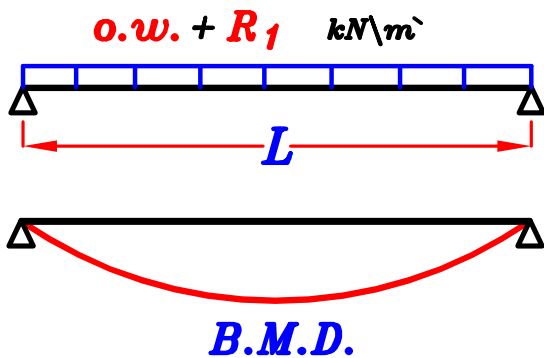
Strip in the slab.



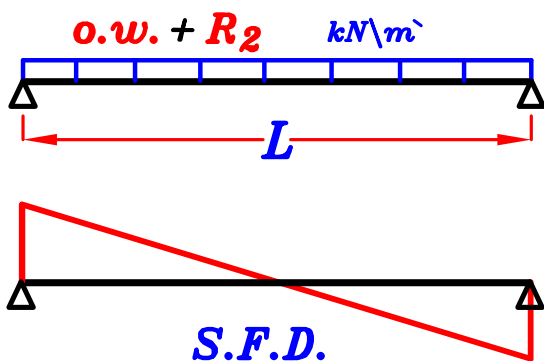
$$\Delta M = M_1 - M_2$$



Load

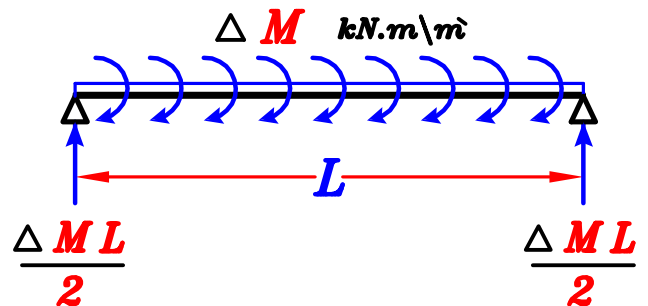


B.M.D.



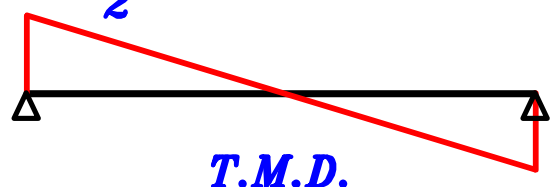
S.F.D.

Moment



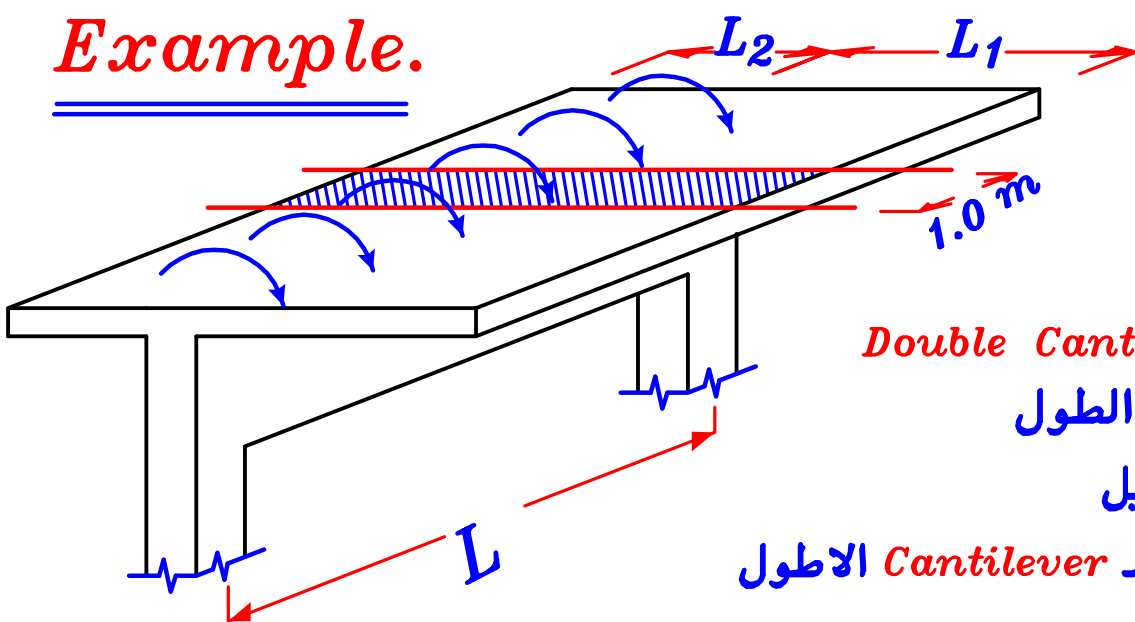
فى التصميم يجب التصميم على ال **Shear** و ال **Torsion** من نفس حاله التحميل

$$M_t = \frac{\Delta M L}{2}$$



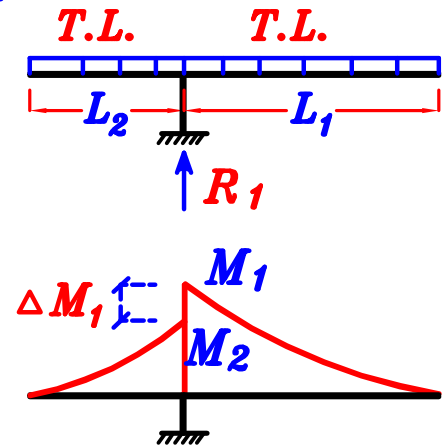
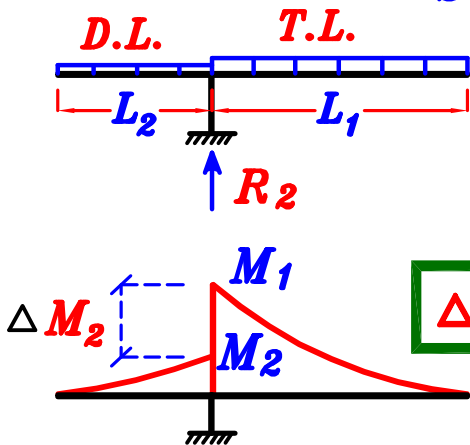
T.M.D.

Example.



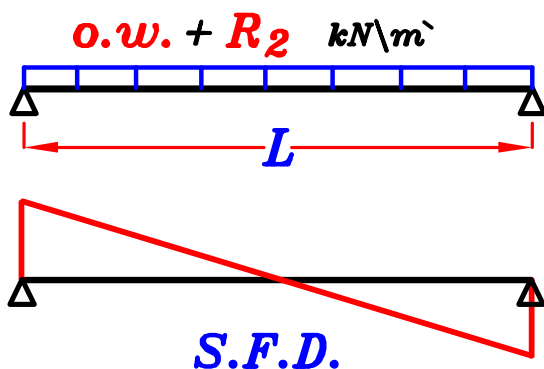
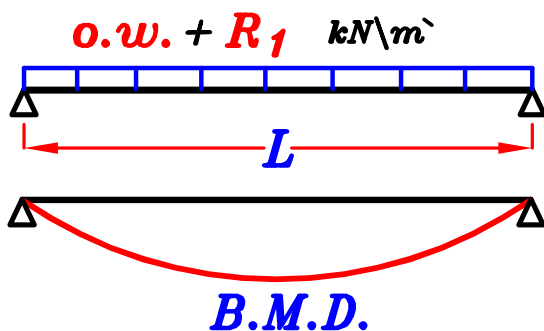
عند وجود **Double Cantilever** غير متساويه فى الطول
نعمل حالات تحميل
فنضع **T.L.** على ال **Cantilever** الاطول

Strip in the slab.

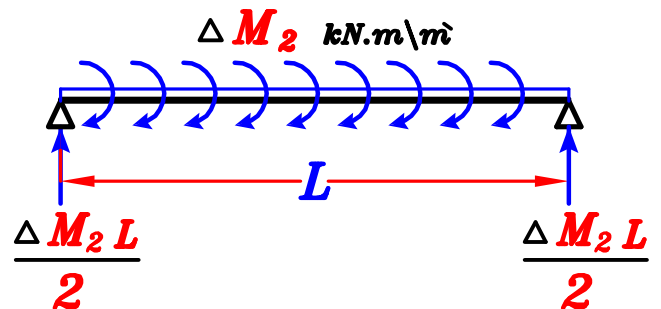


$$\Delta M = M_1 - M_2$$

Load

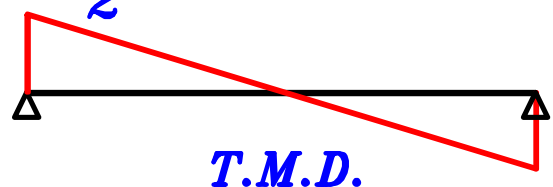


Moment



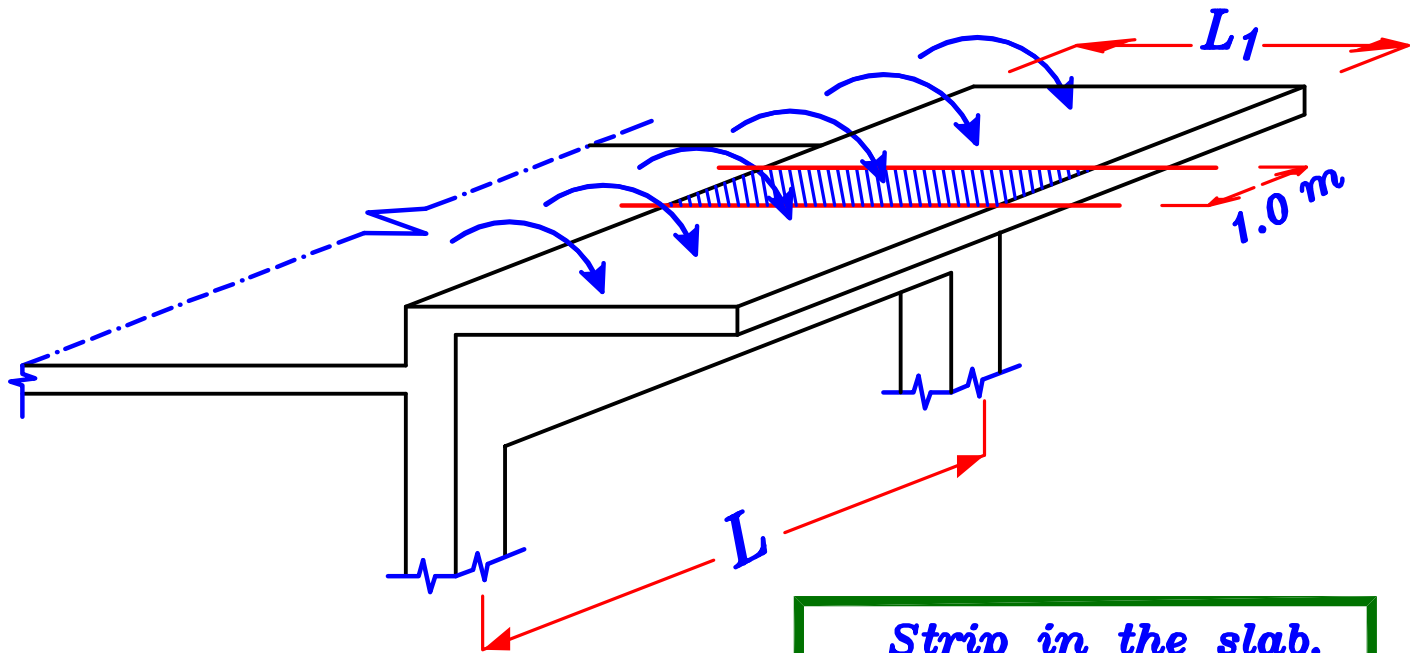
فى التصميم يجب التصميم على
ال **Shear** و ال **Torsion** من نفس حاله التحميل

$$M_t = \frac{\Delta M_2 L}{2}$$

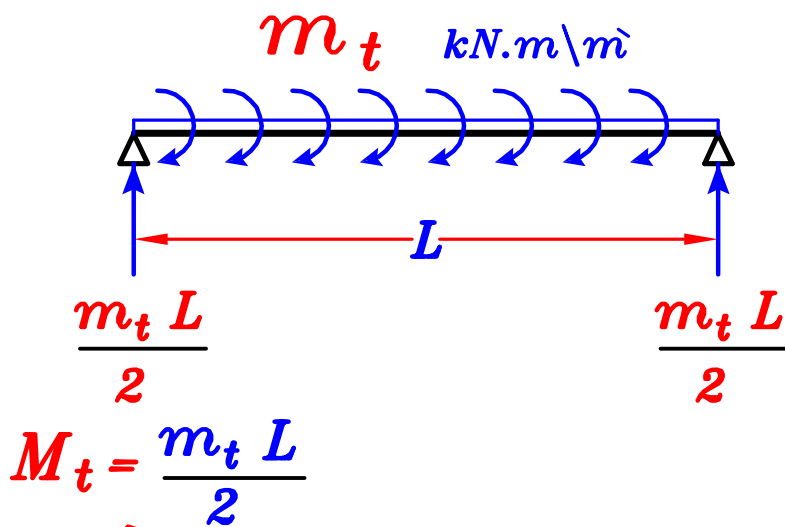


T.M.D.

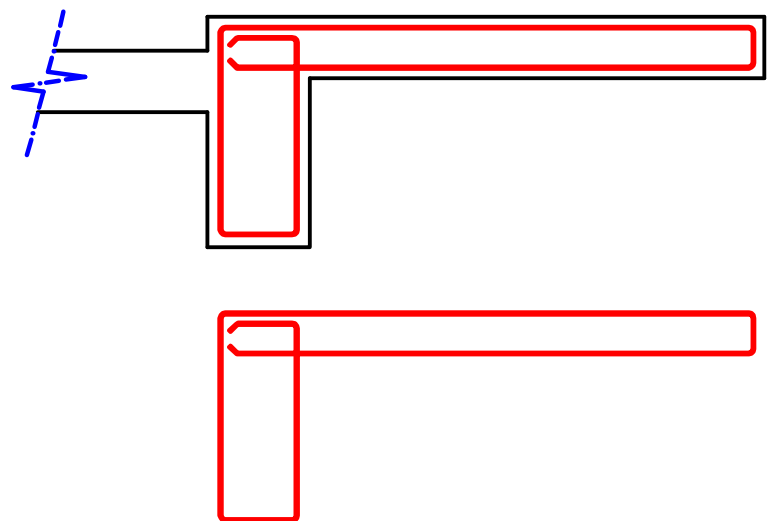
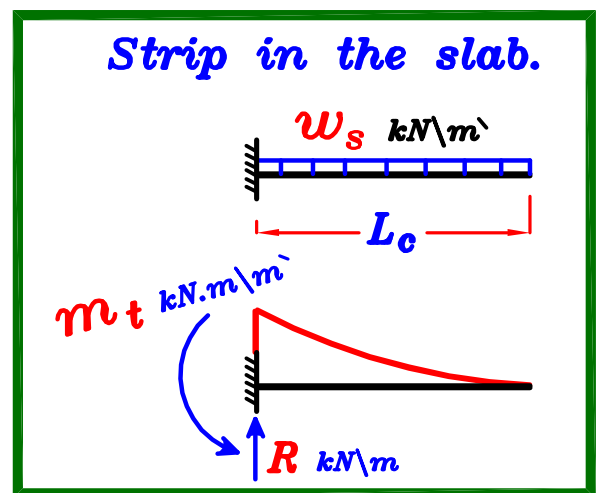
Example.



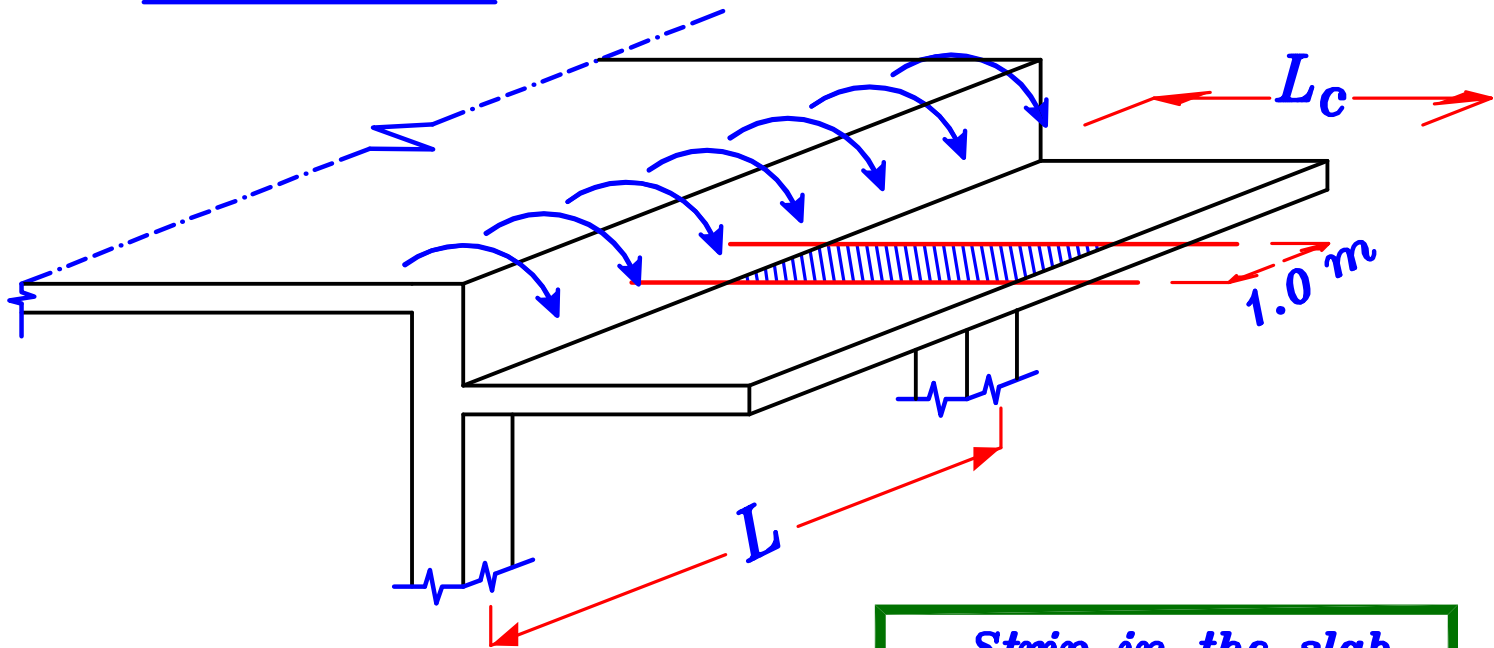
Moment



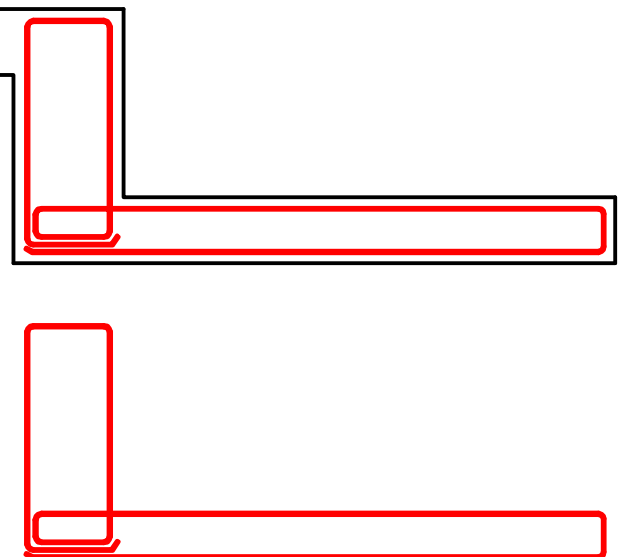
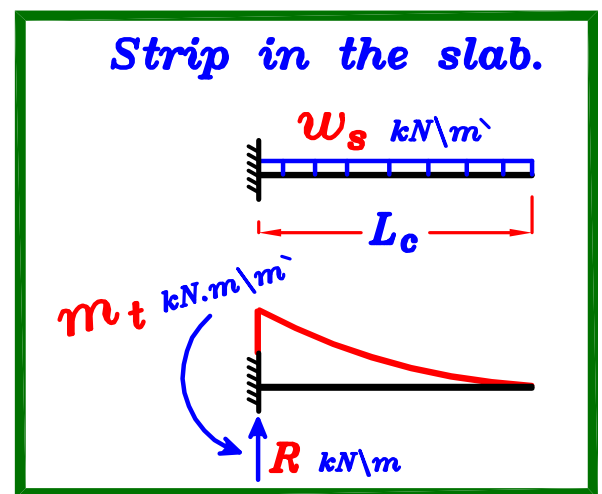
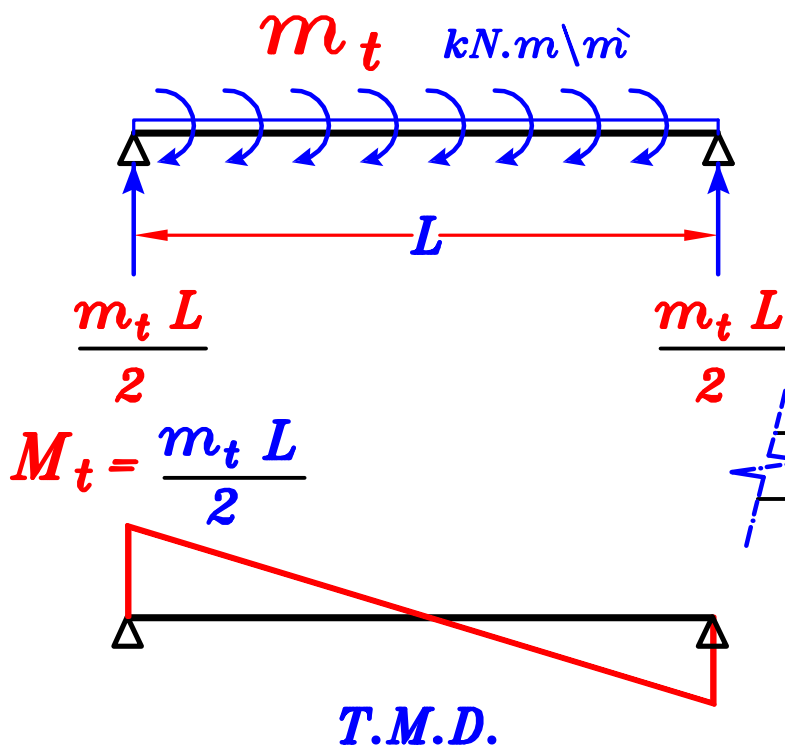
T.M.D.



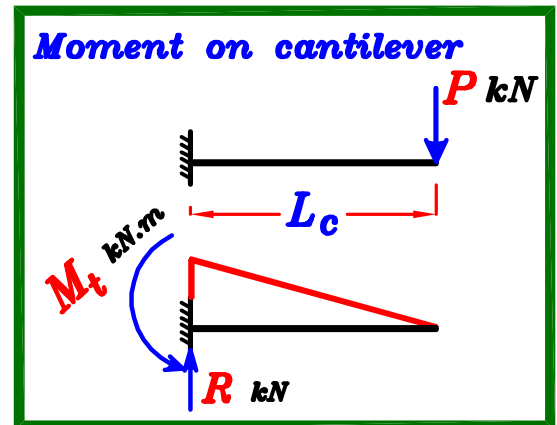
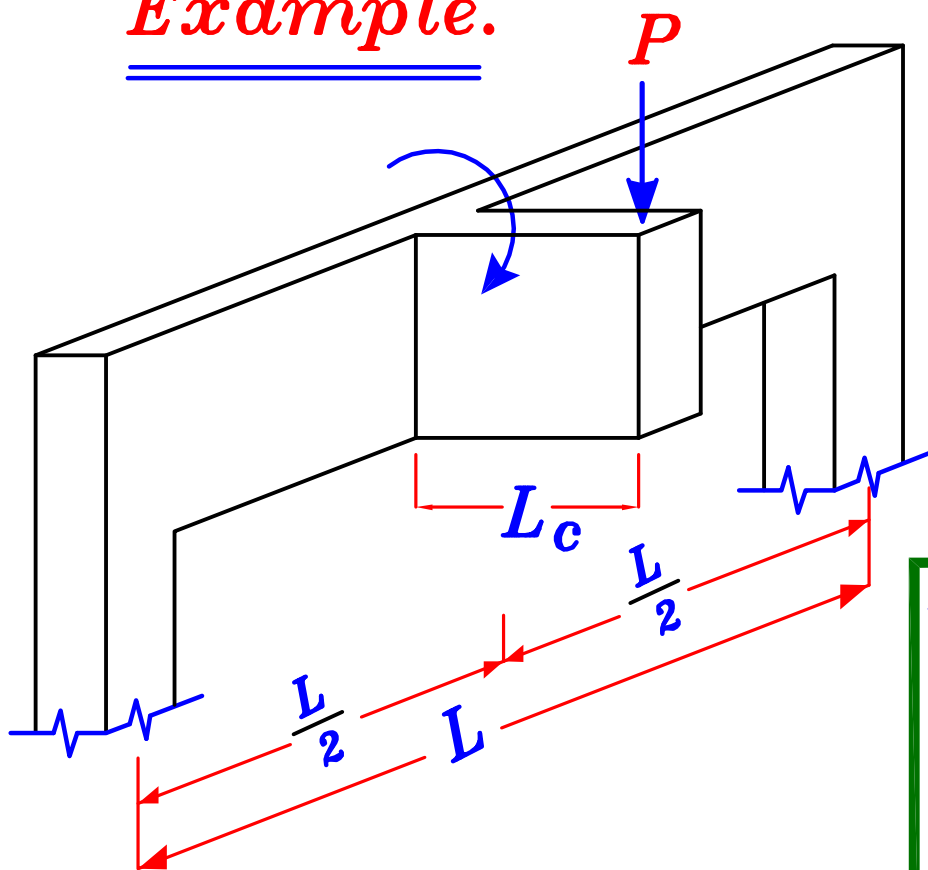
Example.



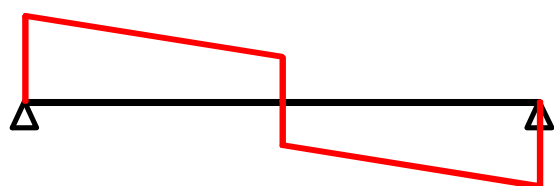
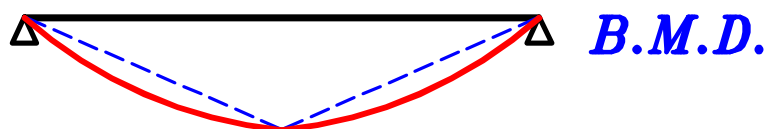
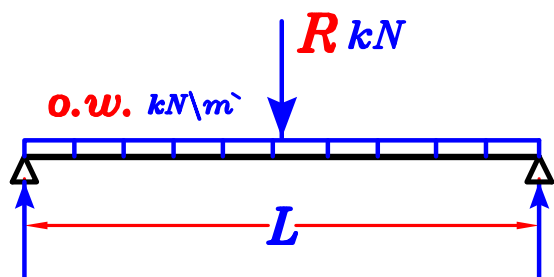
Moment



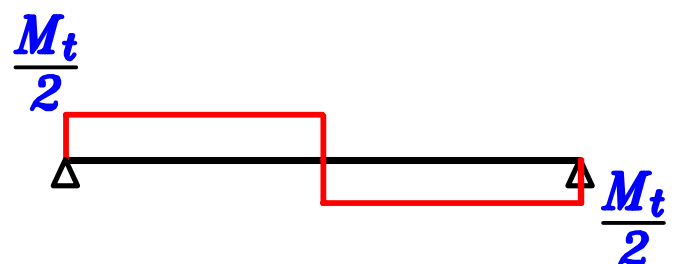
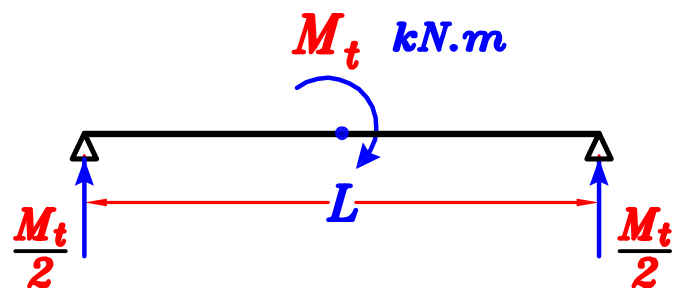
Example.



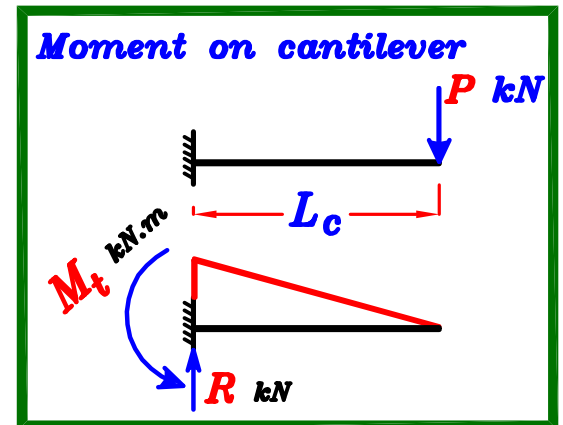
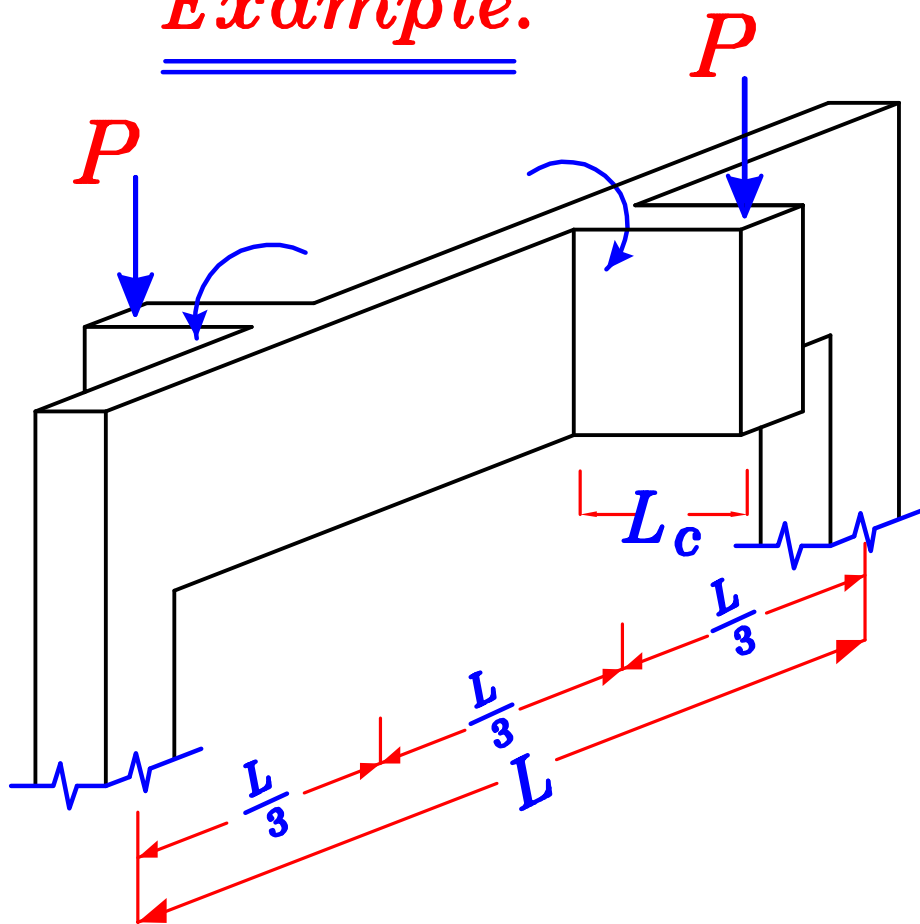
Load



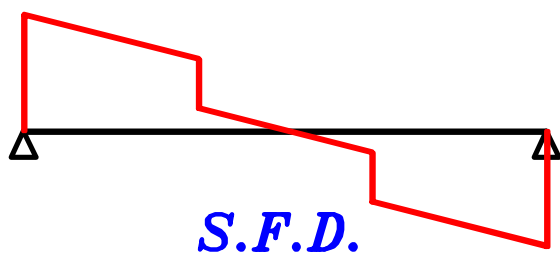
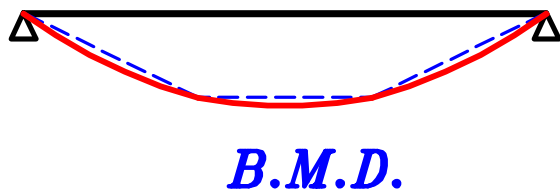
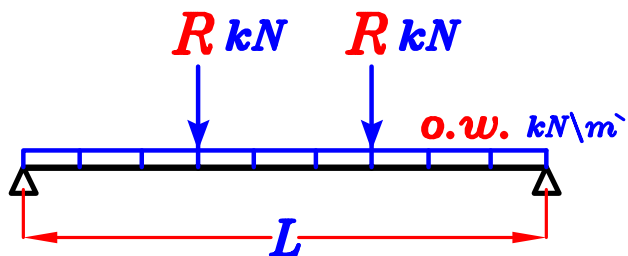
Moment



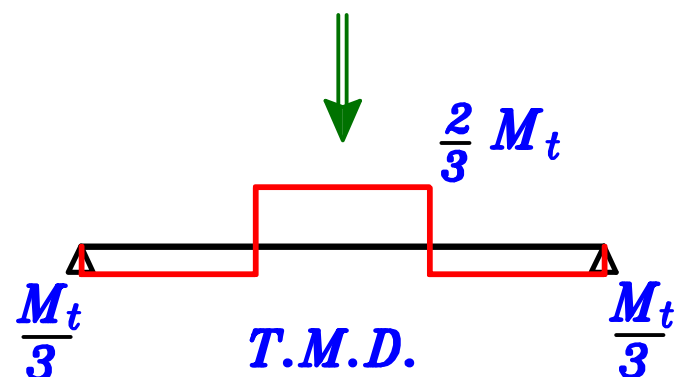
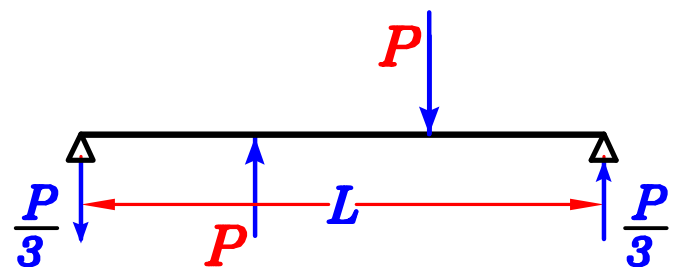
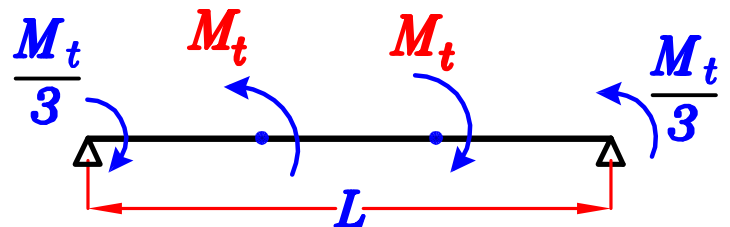
Example.



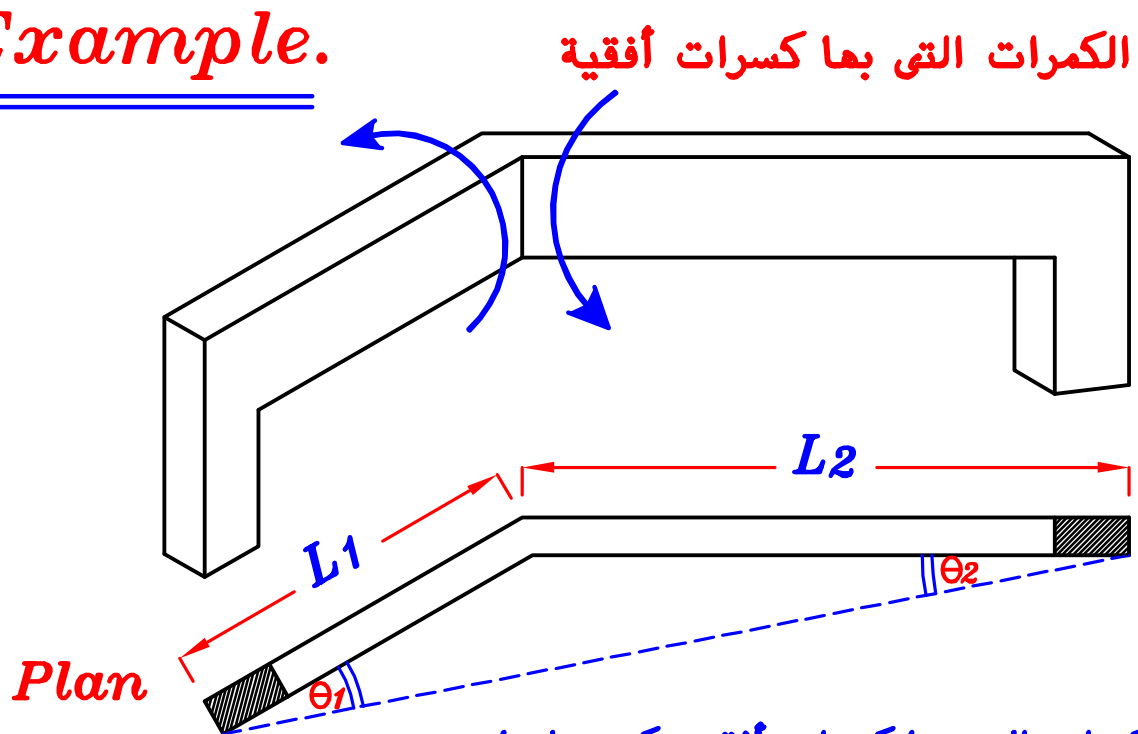
Load



Moment



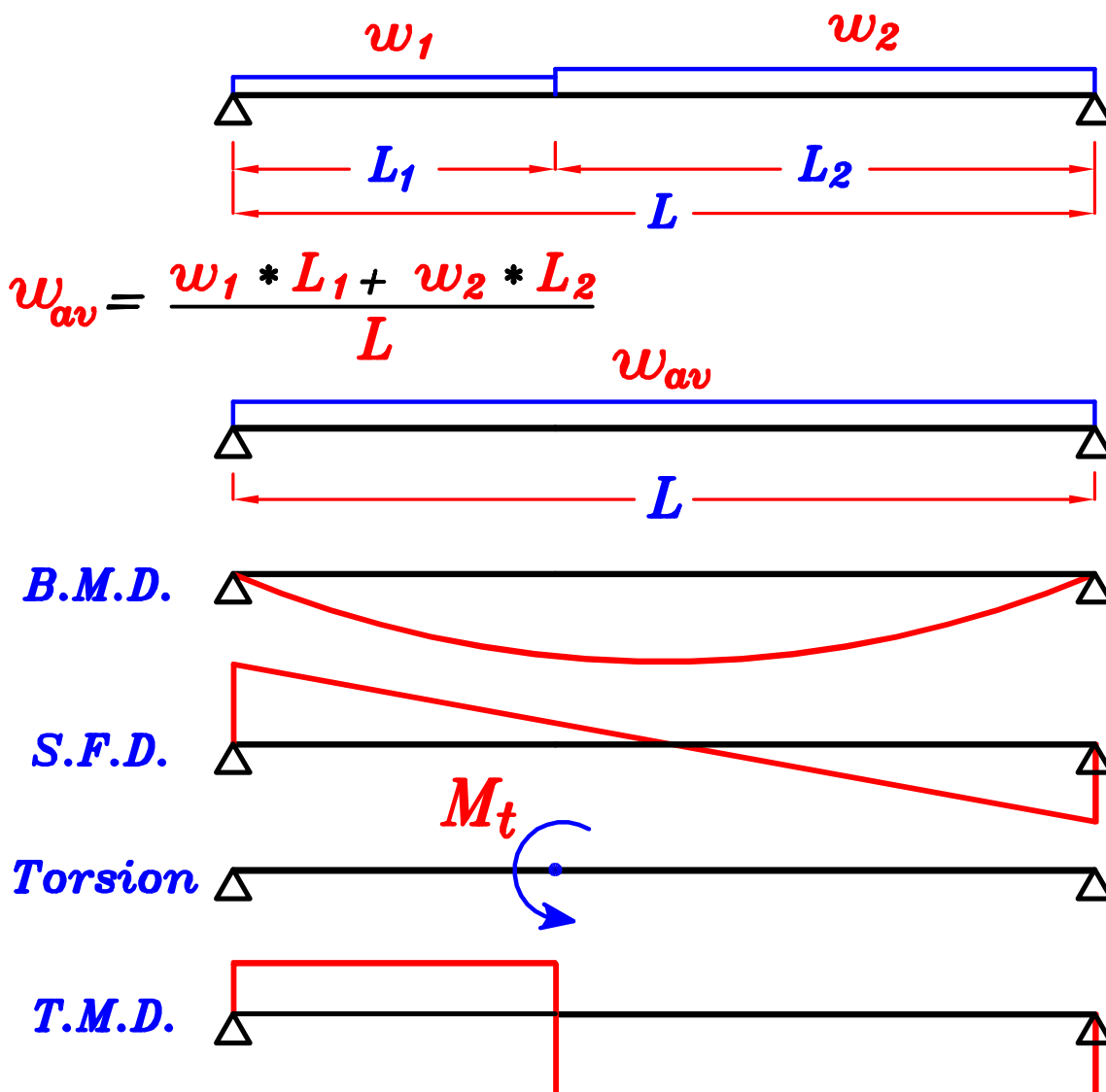
Example.



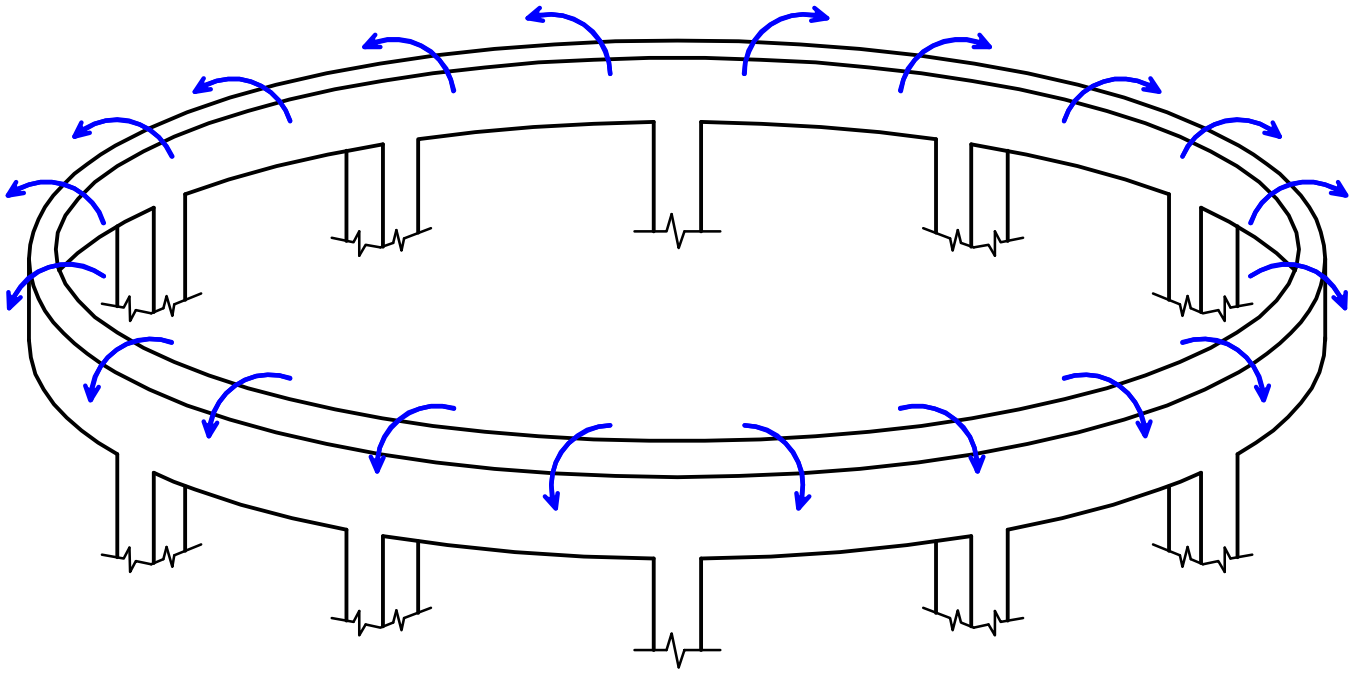
الكمرات التي بها كسرات أفقية يكون عليها

Concentrated torsional Moment عند الكسره الأفقيه

لحل هذه الكمره يجب أن نفردها (لا نسقطها على الأفقى مثل الكسره الرأسية)



Circular Beams.



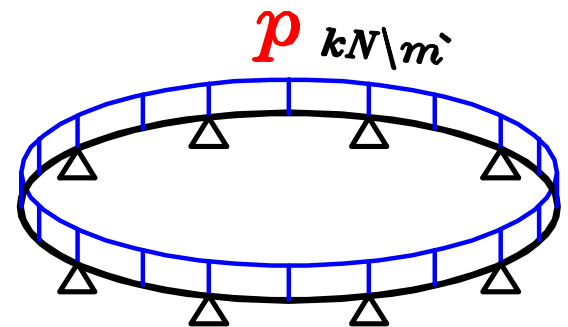
To Get S.F. , B.M. , T.M.

$$P = p \ 2\pi r$$

$$M = r^2 p \left(\frac{\pi \cos \phi}{n \sin \phi_0} - 1 \right)$$

$$M_t = -r^2 p \left(\frac{\pi \sin \phi}{n \sin \phi_0} - \phi \right)$$

$$Q = -r p \phi$$



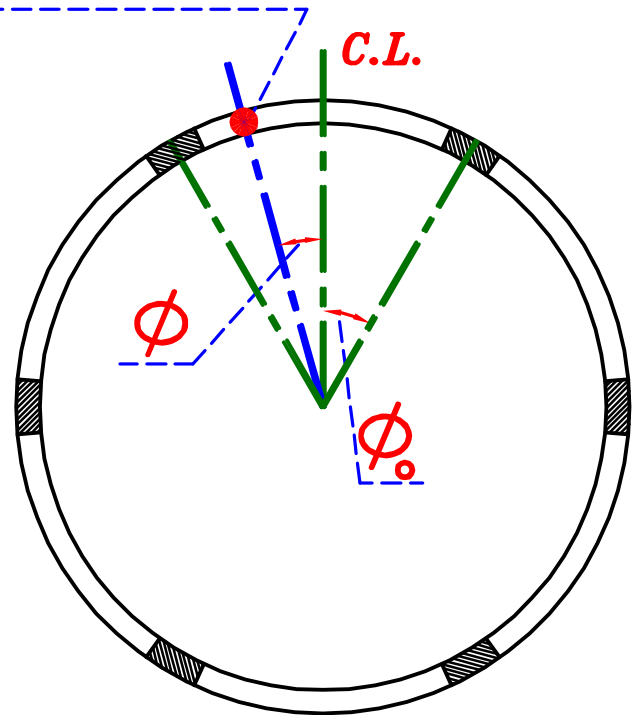
r = The Radius

n = No. of supports

النقطة التي نحسب عندها ال B.M.

هى نصف الزاوية بين عمودين متتالين
2 Successive Supports ϕ_o

$$\phi_o = \frac{\pi}{n}$$



هى الزاوية بين النقطة المدروسة و
خط المنتصف بين ال 2 Supports ϕ

To Get max. +ve Moment. Take. $\phi = \text{Zero}$

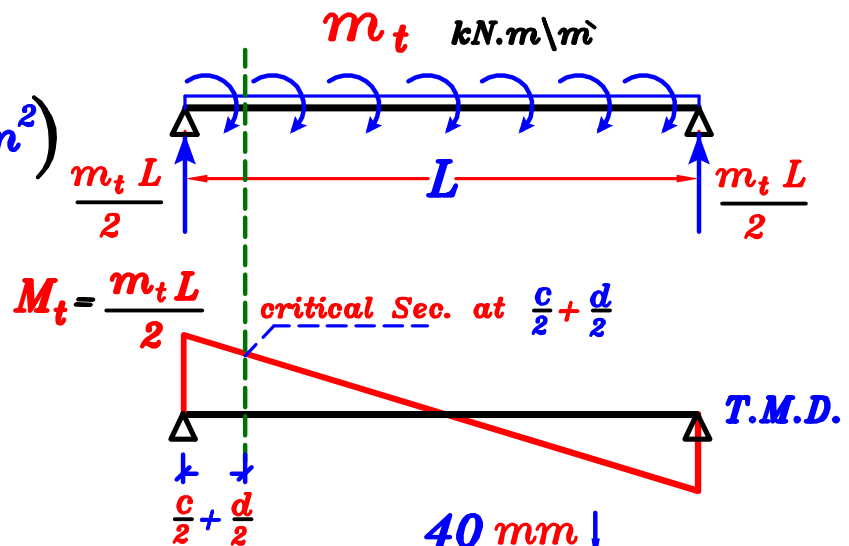
To Get max. -ve Moment. Take. $\phi = \phi_o$

OR we can use Table.

No. of Columns	Load on each Column	Max. Shearing Force	Max. Bending Moment		Max. Torsional Moment
			at C.L. of Span	Over C.L. of Column	
n	R	$Q_{max.}$	$M + Ve$	$M - Ve$	M_t
4	$P/4$	$P/8$	$.0176 Pr$	$-.0322 Pr$	$.0053 Pr$
6	$P/6$	$P/12$	$.0075 Pr$	$-.0148 Pr$	$.0015 Pr$
8	$P/8$	$P/16$	$.0042 Pr$	$-.0083 Pr$	$.0006 Pr$
10	$P/10$	$P/20$	$.0032 Pr$	$-.0052 Pr$	$.0004 Pr$
12	$P/12$	$P/24$	$.0019 Pr$	$-.0037 Pr$	$.0002 Pr$

Shear Stresses due to Torsional moment. (q_{tu})

$$q_{tu} = \frac{M_{tu}}{2 A_o t_e} \quad (N/mm^2)$$



Where:

* q_{tu} = Shear Stress due to Torsional Moment. (N/mm^2)

* M_{tu} = Torsional Moment at Critical Sec. ($N.mm$)

* $y_1 = t - 2 \text{ Cover} \approx t - 80 \text{ mm}$

* $x_1 = b - 2 \text{ Cover} \approx b - 80 \text{ mm}$

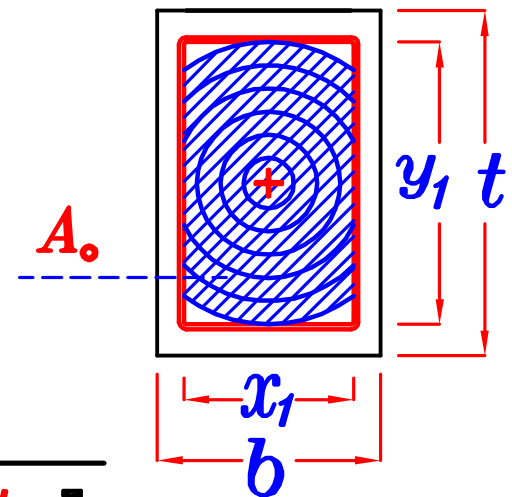
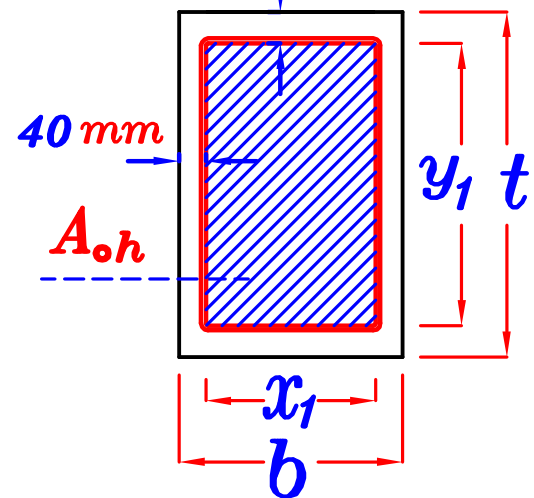
* $A_{oh} = x_1 * y_1 \quad (mm^2)$

* $A_o \approx 0.85 A_{oh} \approx 0.85 (x_1 * y_1) \quad (mm^2)$

* $P_h = 2 (x_1 + y_1) \quad (mm)$ ----- محيط الكانة

* $t_e = \frac{A_{oh}}{P_h} = \frac{x_1 * y_1}{2 (x_1 + y_1)} \quad (mm)$

∴ $q_{tu} = \frac{M_{tu}}{2 A_o t_e} = \frac{M_{tu}}{2 * 0.85 (x_1 * y_1) \left[\frac{x_1 * y_1}{2 (x_1 + y_1)} \right]}$

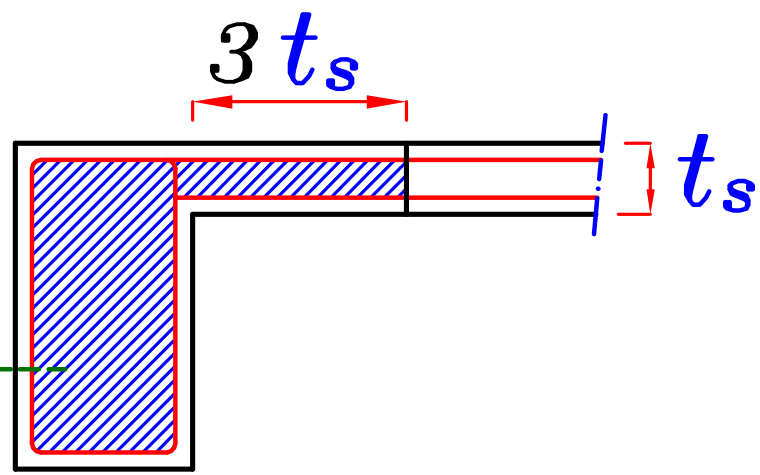


$$\therefore q_{tu} = \frac{M_{tu} (x_1 + y_1)}{0.85 (x_1^2 * y_1^2)} \quad (N/mm^2) \quad \text{للقطاعات المستطيلة فقط}$$

Special Shapes.

$$q_{tu} = \frac{M_{tu}}{2 A_o t_e}$$

A_{oh}



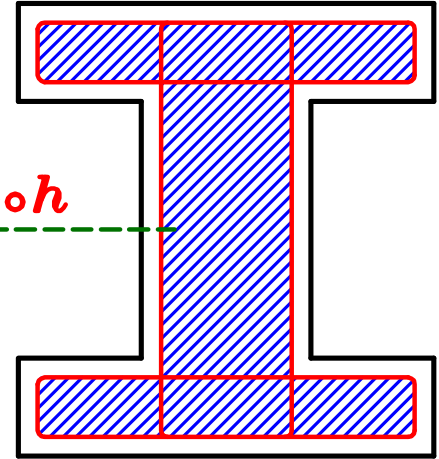
A_{oh} = Area of Stirrups

$A_o \approx 0.85 A_{oh}$

P_h = Outer Perimeter of Stirrups

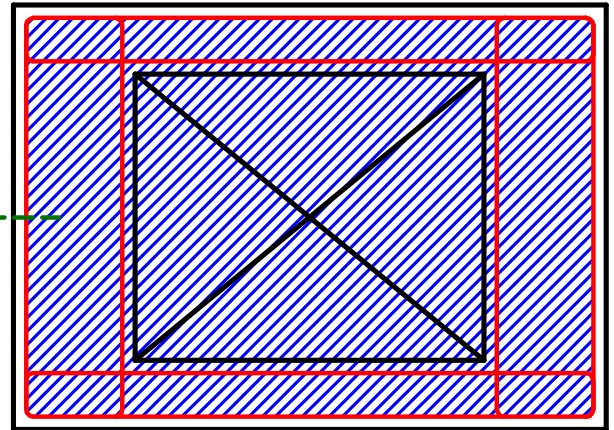
المحيط الخارجى للكانات

A_{oh}

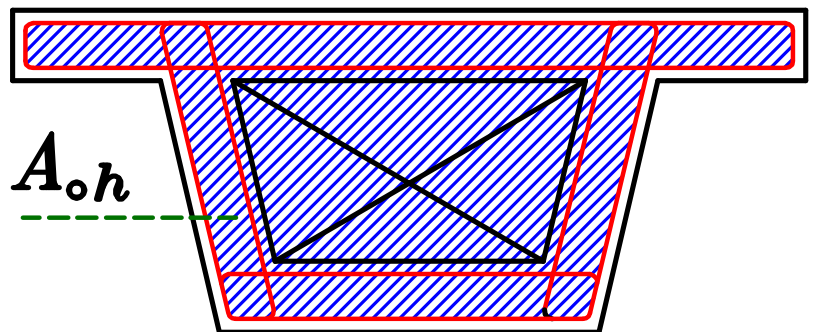


$$t_e = \frac{A_{oh}}{P_h} = \frac{\text{المساحة الكلية للكانات}}{\text{المحيط الخارجى للكانات}} \quad \text{نسبة}$$

A_{oh}



A_{oh}



Check *Shear + Torsion.*

Actual Stresses due to Shear Force. q_u

$$q_u = \frac{Q}{bd}$$

Actual Stresses due to Torsional Moment. q_{tu}

$$q_{tu} = \frac{M_{tu}}{2A_o t_e} = \frac{M_{tu}(x_1 + y_1)}{0.85(x_1^2 + y_1^2)}$$

min. allowable stresses due to Shear q_{cu}

$$q_{cu} = (0.24) \sqrt{\frac{F_{cu}}{\delta_c}}$$

min. allowable stresses due to Torsion q_{tu}

$$q_{t_{min}} = (0.06) \sqrt{\frac{F_{cu}}{\delta_c}}$$

max. allowable shear stresses $q_{u_{max}}$

$$q_{u_{max}} = (0.70) \sqrt{\frac{F_{cu}}{\delta_c}}$$

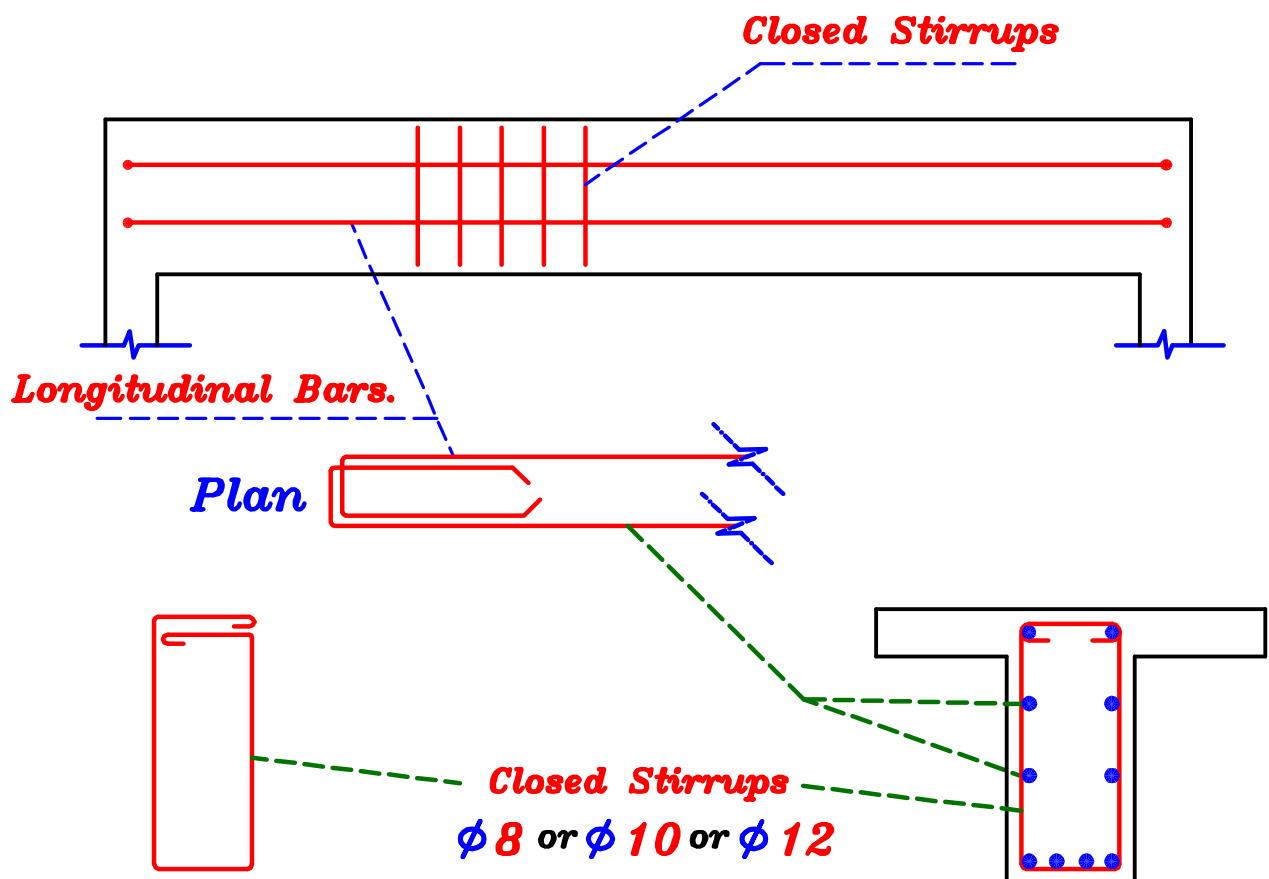
IF $\sqrt{q_u^2 + q_{tu}^2} > q_{u_{max}} \rightarrow \text{Increase Dimensions}$

$$IF \sqrt{q_u^2 + q_{tu}^2} \leq q_{u \max}$$

	q_u	q_{tu}	$RFT.$
①	$q_u < q_{cu}$	$q_{tu} < q_{t \min}$	Use Stirrups $5 \phi 8 \setminus m$
②	$q_u > q_{cu}$	$q_{tu} < q_{t \min}$	Use RFT. to resist $(q_u - \frac{q_{cu}}{2})$
③	$q_u < q_{cu}$	$q_{tu} > q_{t \min}$	Use RFT. to resist (q_{tu})
④	$q_u > q_{cu}$	$q_{tu} > q_{t \min}$	Use RFT. to resist $(q_u - \frac{q_{cu}}{2}) + (q_{tu})$

How to Resist Torsion ??

- ① Closed Stirrups. ① كانات مغلقة .
- ② Longitudinal Bars. ② أسياخ طوليه .



Case ③

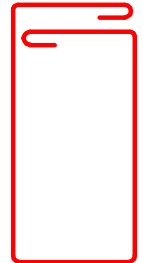
$$q_u < q_{cu}, \quad q_{tu} > q_{tmin}$$

Use Shear RFT. to resist Shear Stresses $\left(q_{tu} - \frac{q_{tmin}}{2}\right)$ applied From Torsional moment.

① Closed Stirrups.

$$A_{str} = \frac{M_{tu} S_t}{(1.7) x_1 y_1 \left(\frac{F_y}{\delta_s}\right)}$$

Code
Page (4-35)



Closed Stirrup

Where:

* A_{str} مساحة مقطع سيخ الكانه

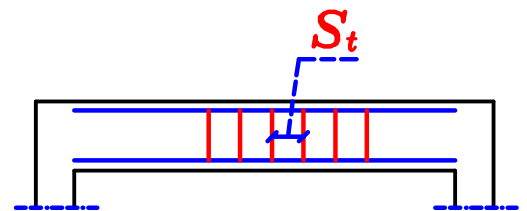

ملحوظه .

- ممكن إستخدام كانات حتى $\phi 12$ فى ال **Torsion**
- كانات ال **Torsion** تكون الكانات الخارجيه فقط .

* S_t المسافه الطويله بين كانات ال **Torsion**

$$S_t = (100 \text{ mm} \rightarrow 200 \text{ mm})$$

ϕ	A_{str}
$\phi 8$	50.3 mm ²
$\phi 10$	78.5 mm ²
$\phi 12$	113 mm ²



In the equation choose $\phi \rightarrow A_{str} = \checkmark$

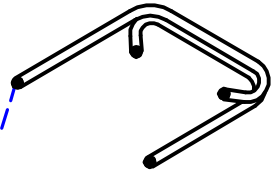
Then get $S_t = (100 \text{ mm} \rightarrow 200 \text{ mm})$

$$\left\{ \begin{array}{l} \text{IF } S_t \geq 200 \text{ mm} \xrightarrow{\text{use}} 5 \phi \checkmark \setminus m \\ \text{IF } S_t < 100 \text{ mm} \text{ Choose bigger } \phi \\ \text{IF } 100 \text{ mm} < S_t < 200 \text{ mm} \xrightarrow{\text{Get}} N_{\underline{O}} \text{ of stirrups } \setminus m = \frac{1000}{S_t} \end{array} \right.$$

② Longitudinal Bars.

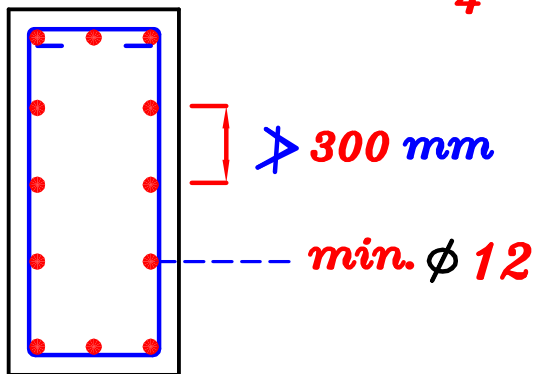
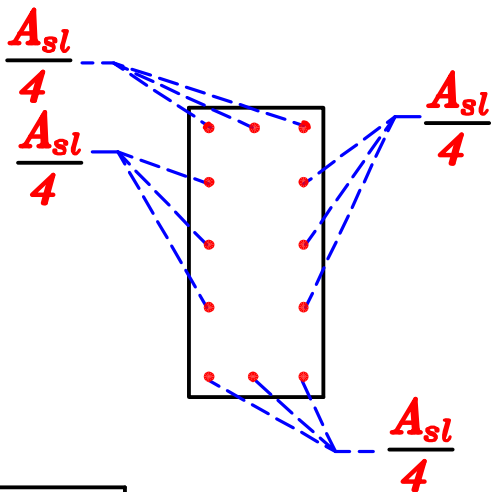
$$A_{sl} = 2 A_{str} \frac{(x_1 + y_1)}{S_t} \left(\frac{F_{y_{str.}}}{F_{y_{L.b.}}} \right)$$

Where:

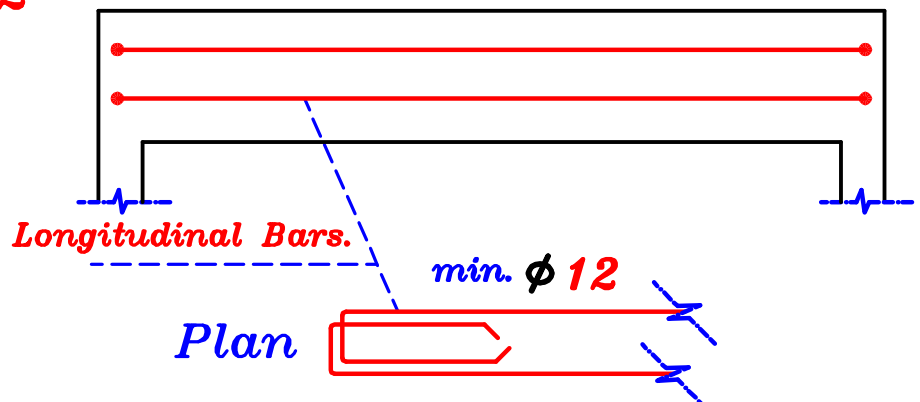
- * A_{sl} مجموع مساحة مقطع الأسيخ الطويله كلها.
- * A_{str} مساحة مقطع سيخ الكانه. 
- * $F_{y_{str.}} = F_y$ For stirrups. $\approx 240 \text{ N/mm}^2$
- * $F_{y_{L.b.}} = F_y$ For Longitudinal bars. $\approx 360 \text{ N/mm}^2$

ملاحظات.

- توزيع الأسيخ على محيط القطاع بانتظام.



- المسافه بين الأسيخ لا تزيد عن ٣٠٠ مم .
- أقل قطر للسيخ $\phi 12$



Case ④

$$q_u > q_{cu}, q_{tu} > q_{tmin}$$

Use Shear RFT. to resist Shear Stresses (q_{tu}) applied From Torsional moment.

+ Shear RFT. to resist Shear Stresses ($q_u - \frac{q_{cu}}{2}$) applied From Shear Force.

① Closed Stirrups.

@ Torsion.

$$A_{str} = \frac{M_{tu} S_t}{(1.7) x_1 y_1 \left(\frac{F_y}{\delta_s} \right)}$$

$$A_{str} = \checkmark * S \quad \text{--- ①}$$

كانات خارجيه فقط

A_{str} هي مساحه سيخ الكانه الخارجيه التي نحتاجها لمقاومه ال **Torsion** فقط.

② Shear.

$$q_u - \frac{q_{cu}}{2} = \frac{n A_s (F_y / \delta_s)}{b S_s}$$

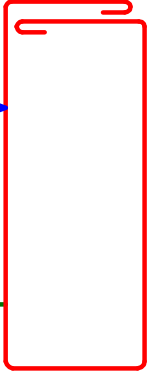
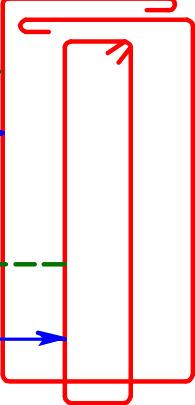
$$A_s = \checkmark * \frac{S}{n} \quad \text{--- ②}$$

كانات خارجيه و ممكن كانات داخليه

A_s هي مساحه مقطع سيخ واحد من الكانه الخارجيه أو الداخليه التي نحتاجها لمقاومه ال **Shear** فقط.

- نبدأ أولاً بفرض أن عدد فروع الكانه يساوى فرعين $n = 2$ و عدد الكانات في المتر = من (0 ← 10) أسياخ/م ثم نحسب $S = \frac{1000}{No \text{ of st. } / m}$ ثم نحسب A_{str} ، A_s و تكون مساحه السيخ من الكانات الخارجيه $A_{s_{outer}} = A_{str} + A_s$
- فاذا كانت $A_{s_{outer}} \leq 113 \text{ mm}^2$ أى أن $\phi_{outer} \leq \phi_{12}$ فنختار $\phi_{outer} = \phi_8 \text{ or } \phi_{10} \text{ or } \phi_{12}$ و لا توجد كانات داخليه
- أما اذا كانت $A_{s_{outer}} > 113 \text{ mm}^2$ فنختار عدد كانات أكثر في المتر أو نأخذ $n = 4$ ثم تحديد قيمه A_s و تحديد قيمه $A_{s_{outer}} = A_{str} + A_s$
- و تحديد قيمه $A_{s_{inner}} = A_s$

ترتيب اختيار الفروض ل n, S

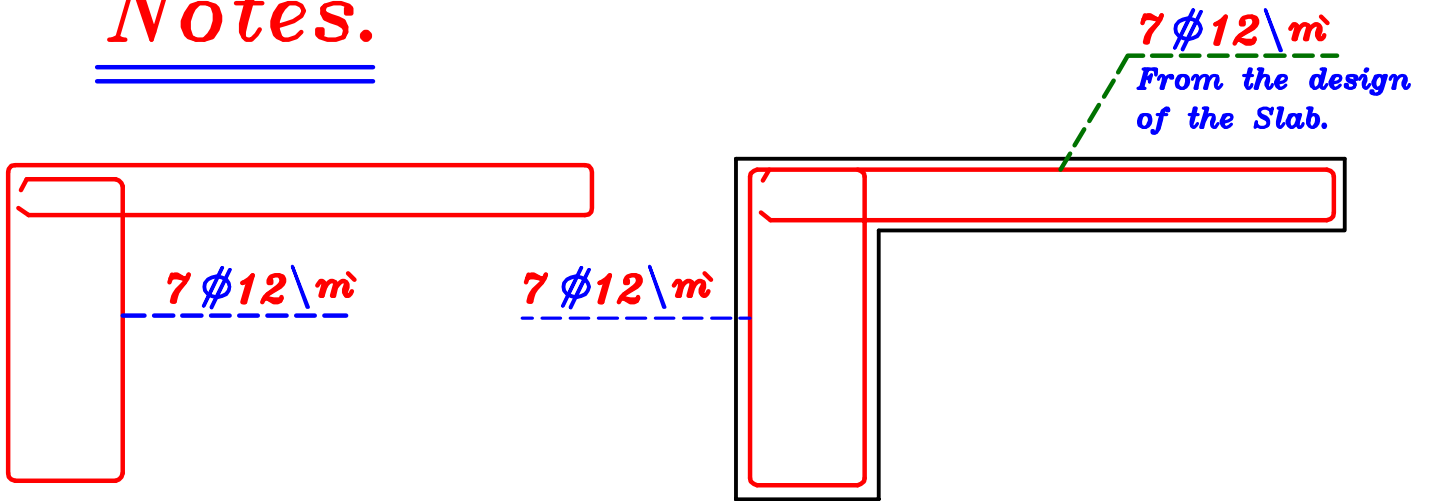
Assumption No.	n	No. of stirrups\m	$S_s = S_t$ (mm)	
1	2	5.0	$\frac{1000}{5.0}$	الكانات الخارجيه تقاوم Shear+Torsion ϕ_{Outer} 
2	2	6.0	$\frac{1000}{6.0}$	
3	2	7.0	$\frac{1000}{7.0}$	
4	2	8.0	$\frac{1000}{8.0}$	
5	2	9.0	$\frac{1000}{9.0}$	
6	2	10	$\frac{1000}{10}$	
7	4	5.0	$\frac{1000}{5.0}$	ϕ_{Outer} الكانات الخارجيه تقاوم Shear+Torsion ϕ_{Inner} الكانات الداخليه تقاوم فقط Shear 
8	4	6.0	$\frac{1000}{6.0}$	
9	4	7.0	$\frac{1000}{7.0}$	
10	4	8.0	$\frac{1000}{8.0}$	
11	4	9.0	$\frac{1000}{9.0}$	
12	4	10	$\frac{1000}{10}$	

② Longitudinal Bars.

Torsion.

$$A_{sl} = 2 A_{str} \frac{(x_1 + y_1)}{S_t} \left(\frac{F_{y_{str.}}}{F_{y_{L.b.}}} \right)$$

Notes.



إذا كان تسليح البلاطه $7 \phi 12 \backslash m$ ← الكانات $7 \phi 12 \backslash m$ سنأخذ عدد الكانات الخارجيه $V =$ كانات فى المتر و فرعان للكانه و $F_y = 360 \text{ N/mm}^2$ لا نأخذ كانات داخلية .

ثم نعمل *Check* إذا كانت $A_{s_{outer}} = \phi 12 = 113 \text{ mm}^2$ تكفى لتحمل ال *Shear Stress* أم أننا سنحتاج إلى إستخدام كانات داخلية .

For Torsion Eqn. $A_{str} = \frac{M_{tu} S_t}{(1.7) x_1 y_1 \left(\frac{F_y}{\delta_s} \right)} \rightarrow A_{str} = \checkmark$

$S_t = \frac{1000}{7.0}$, $F_y = 360 \text{ N/mm}^2$

For Shear Eqn. $q_u - \frac{q_{ou}}{2} = \frac{n A_s (F_y \delta_s)}{b S_s} \rightarrow A_s = \checkmark$

$n = 2$, $S_s = \frac{1000}{7.0}$, $F_y = 360 \text{ N/mm}^2$

After we get $A_{s_{outer}}$, A_{str} chek $A_{s_{outer}} = A_{str} + A_s \leq 113 \text{ mm}^2$

* IF $A_{s_{outer}} \leq 113 \text{ mm}^2 \therefore \text{O.K.}$

تسليح البلاطه هو نفس تسليح الكانه الخارجيه

* IF $A_{s_{outer}} > 113 \text{ mm}^2$

$\therefore$ use another RFT. system

or use choose another ϕ_{Inner}

نختار كانات داخلية عددها V فى المتر

Example.

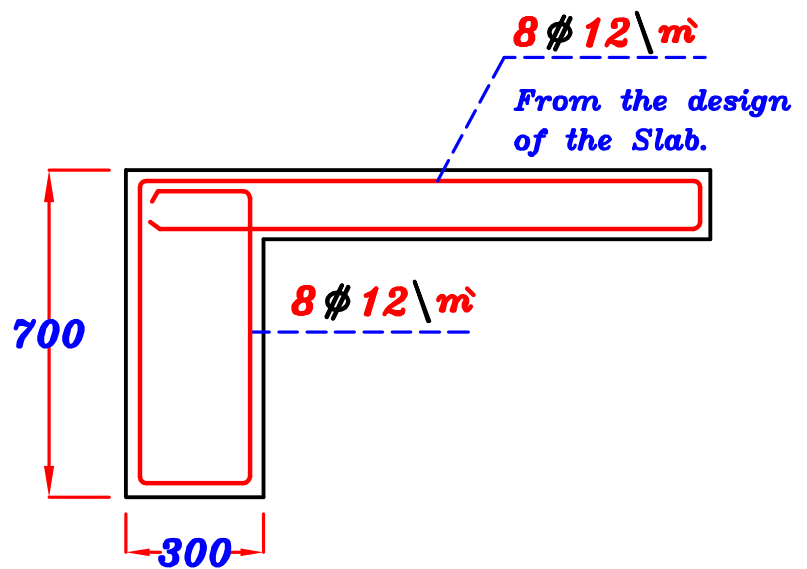
$$F_{cu} = 25 \text{ N/mm}^2$$

$$F_y = 360 \text{ N/mm}^2$$

$$M_{tu} = 55.8 \text{ kN.m}$$

$$q_u = 1.06 \text{ N/mm}^2$$

$$q_t = 2.38 \text{ N/mm}^2$$



RFT. of the cantilever Slab $8 \phi 12 \text{ m}$

Req.

Design For Shear & Torsion.

Solution.

$$q_{cu} = (0.24) \sqrt{\frac{25}{1.5}} = 0.98$$

$$q_{tmin} = (0.06) \sqrt{\frac{25}{1.5}} = 0.245$$

$$q_{u_{max}} = (0.7) \sqrt{\frac{25}{1.5}} = 2.85$$

$$\sqrt{q_u^2 + q_{tu}^2} = \sqrt{1.06^2 + 2.38^2} = 2.60 < q_{u_{max}} \therefore \text{o.k.}$$

$q_u > q_{cu} , q_{tu} > q_{tmin}$	$\text{resist} \left(q_u - \frac{q_{cu}}{2} \right) + (q_{tu})$
------------------------------------	--

@Torsion.

$$x_1 = 220 \text{ mm} , y_1 = 620 \text{ mm}$$

$$A_{str} = \frac{M_{tu} S_t}{(1.7) x_1 y_1 \left(\frac{F_y}{\delta_s} \right)} \therefore A_{str} = \frac{(55.8 * 10^6) S_t}{(1.7) (220) (620) (360/1.15)}$$

$$\therefore A_{str} = 0.768 S_t$$

⑥ Shear.

$$q_u - \frac{q_{cu}}{2} = \frac{n A_s (F_y \setminus \delta_s)}{b S_s} \therefore 1.06 - \frac{0.98}{2} = \frac{n A_s (360/1.15)}{(300) S_s}$$

$$\therefore A_s = 0.546 \frac{S_s}{n}$$

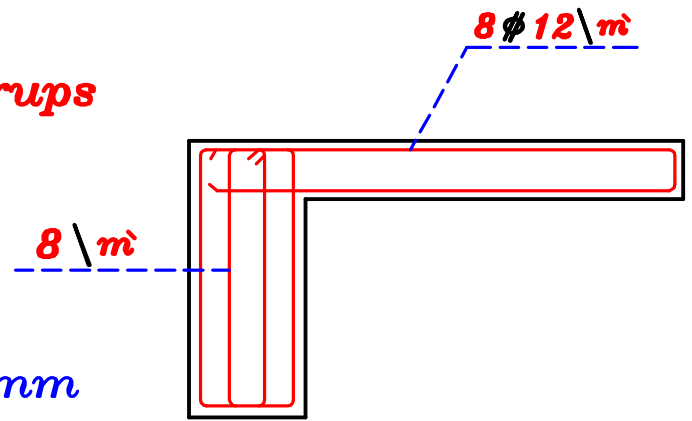
$$n = 2, S_t = S_s = \frac{1000}{8.0} = 125 \text{ mm}$$

$$\therefore A_{str} = 0.768 S_t = 0.768 * 125 = 96.0 \text{ mm}^2$$

$$A_s = 0.546 \frac{S_s}{n} = 0.546 * \frac{125}{2.0} = 34.12 \text{ mm}^2$$

$$A_{s_{outer}} = A_{str} + A_s = 96.0 + 34.12 = 130.12 > 113$$

$\therefore$ We have to use Inner Stirrups



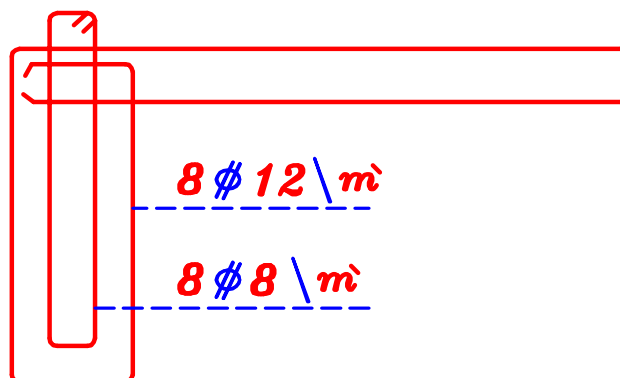
$$n = 4, S_t = S_s = \frac{1000}{8.0} = 125 \text{ mm}$$

$$\therefore A_{str} = 0.768 S_t = 0.768 * 125 = 96.0 \text{ mm}^2$$

$$A_s = 0.546 \frac{S_s}{n} = 0.546 * \frac{125}{4.0} = 17.0 \text{ mm}^2$$

$$A_{s_{outer}} = A_{str} + A_s = 96.0 + 17.0 = 113 \leq 113 \therefore \text{O.K.}$$

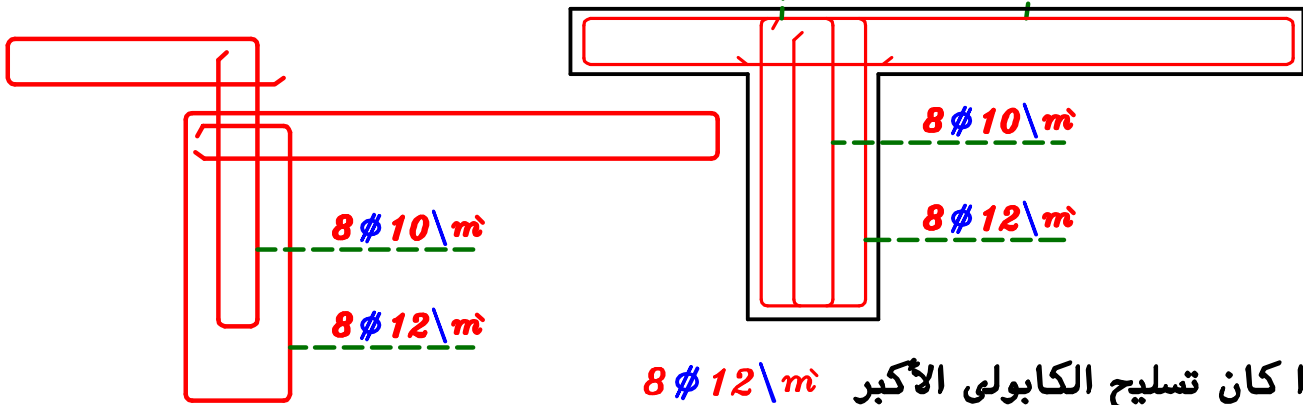
$$A_{s_{inner}} = A_s = 17.0 \text{ mm}^2 = \phi 8$$



Example.

7 ϕ 10 \ m
From the design
of the Slab.

8 ϕ 12 \ m
From the design
of the Slab.



إذا كان تسليح الكابولي الأكبر 8 ϕ 12 \ m

$$F_y = 360 \text{ N/mm}^2$$

8 ϕ 12 \ m

سنأخذ الكانات الخارجيه إذا كان تسليح الكابولي الأصغر 7 ϕ 10 \ m سوف نأخذه 8 ϕ 10 \ m و نأخذ الكانات الداخليه 8 ϕ 10 \ m لكي يكون عدد الكانات الداخليه و الخارجيه متساوي

ثم نعمل Check إذا كانت $A_{s_{inner}} \leq \phi 10 = 78.5 \text{ mm}^2$ و $A_{s_{outer}} \leq \phi 12 = 113 \text{ mm}^2$

تکفی لتحمل ال **Shear Stress**

For Torsion Eqn.

$$A_{str} = \frac{M_{tu} S_t}{(1.7) x_1 y_1 \left(\frac{F_y}{\delta_s} \right)}$$

$$A_{str} = \checkmark$$

$$F_y = 360 \text{ N/mm}^2$$

$$S_t = \frac{1000}{8.0}$$

For Shear Eqn.

$$q_u - \frac{q_{cu}}{2} = \frac{n A_s (F_y \delta_s)}{b S_s}$$

$$A_s = \checkmark$$

$$n = 4, F_y = 360 \text{ N/mm}^2, S_s = \frac{1000}{8.0}$$

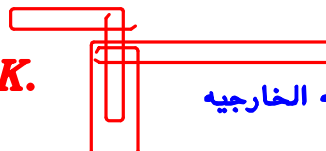
After we get A_s, A_{str} Get $A_{s_{outer}} = A_{str} + A_s \leq 113 \text{ mm}^2$

Chek

$$A_{s_{inner}} = A_s \leq 78.5 \text{ mm}^2$$

$$* \text{ IF } A_{s_{outer}} \leq 113 \text{ mm}^2$$

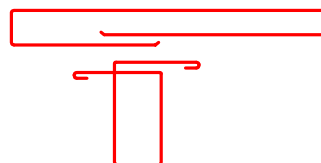
$$A_{s_{inner}} \leq 78.5 \text{ mm}^2 \therefore \text{O.K.}$$



تسليح البلاطه هو نفس تسليح الكانه الخارجيه

$$* \text{ IF } A_{s_{outer}} > 113 \text{ mm}^2$$

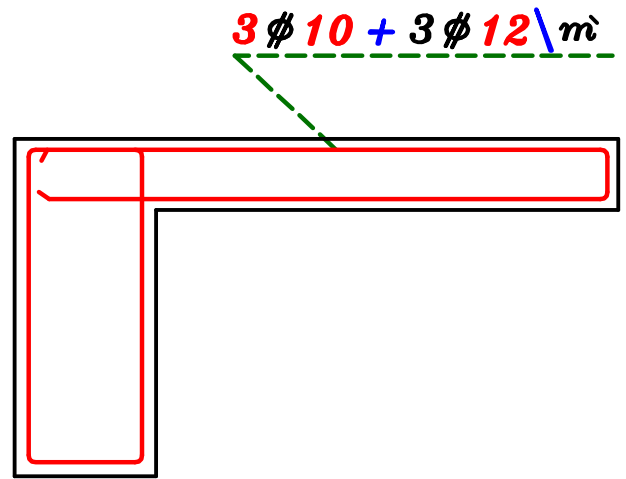
$$\text{or } A_{s_{inner}} > 78.5 \text{ mm}^2$$



ليس شرط أن يكون تسليح البلاطه هو نفس تسليح الكانه الخارجيه

$\therefore$ use another RFT. system

Example.

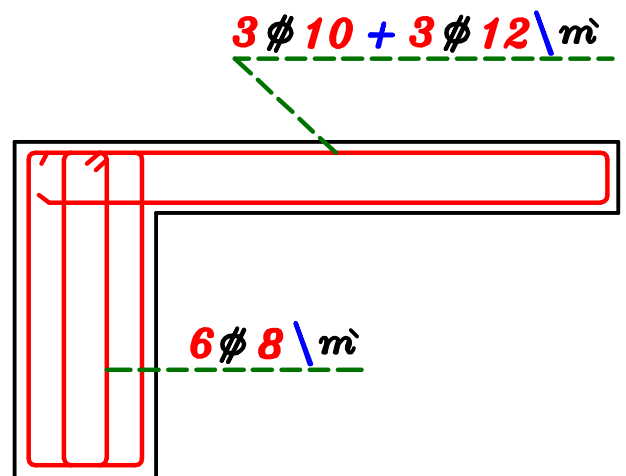
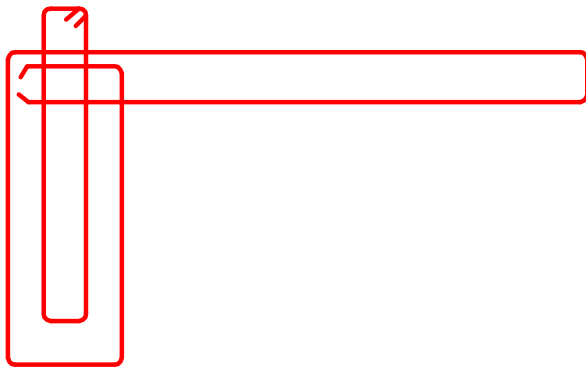


For Shear Eqn. $n = 2$, $S_s = \frac{1000}{6.0}$, $F_y = 360 \text{ N/mm}^2$

For Torsion Eqn. $S_t = \frac{1000}{6.0}$, $F_y = 360 \text{ N/mm}^2$

, Check $A_{s_{outer}} = A_s + A_{str} \leq \left(\frac{78.5 + 113}{2} \right)$

Example.



For Shear Eqn. $n = 4$, $S_s = \frac{1000}{6.0}$, $F_y = 360 \text{ N/mm}^2$

For Torsion Eqn. $S_t = \frac{1000}{6.0}$, $F_y = 360 \text{ N/mm}^2$

Check $A_{s_{inner}} = A_s \leq 50.3$

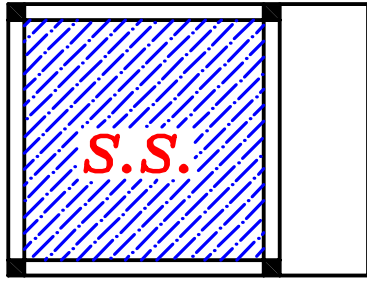
, Check $A_{s_{outer}} = A_s + A_{str} \leq \left(\frac{78.5 + 113}{2} \right)$

Special Cases.

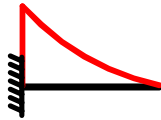
إذا وجد كابولي بجوار بلاطة الحمام

يوجد طريقتين للتسليح :

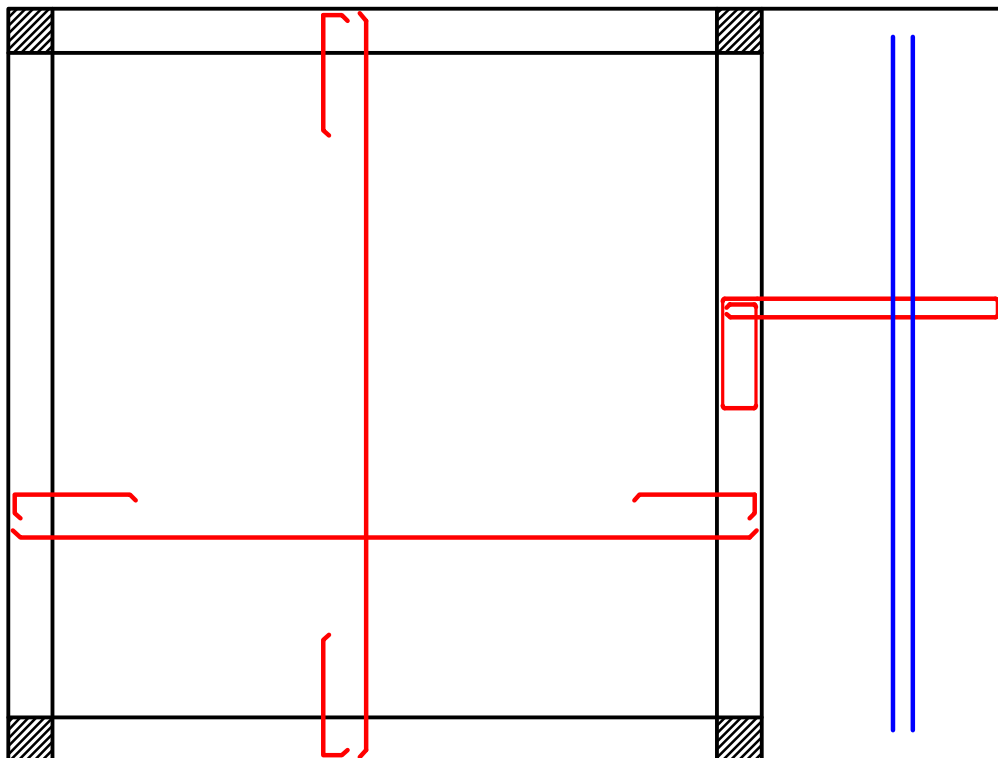
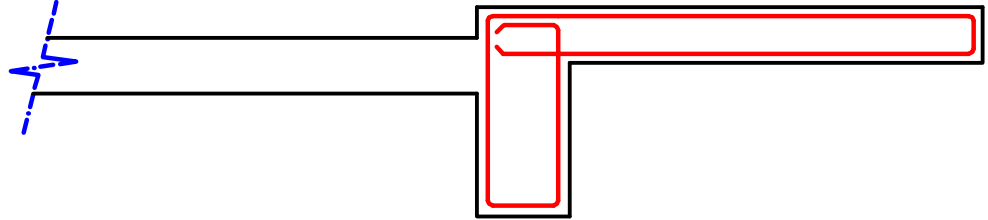
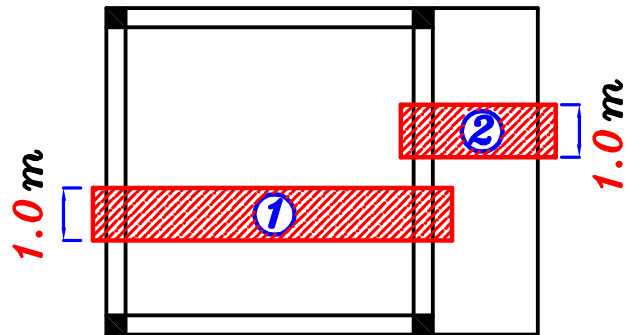
① نأخذ كل شريحة بمفردها



Strip ①

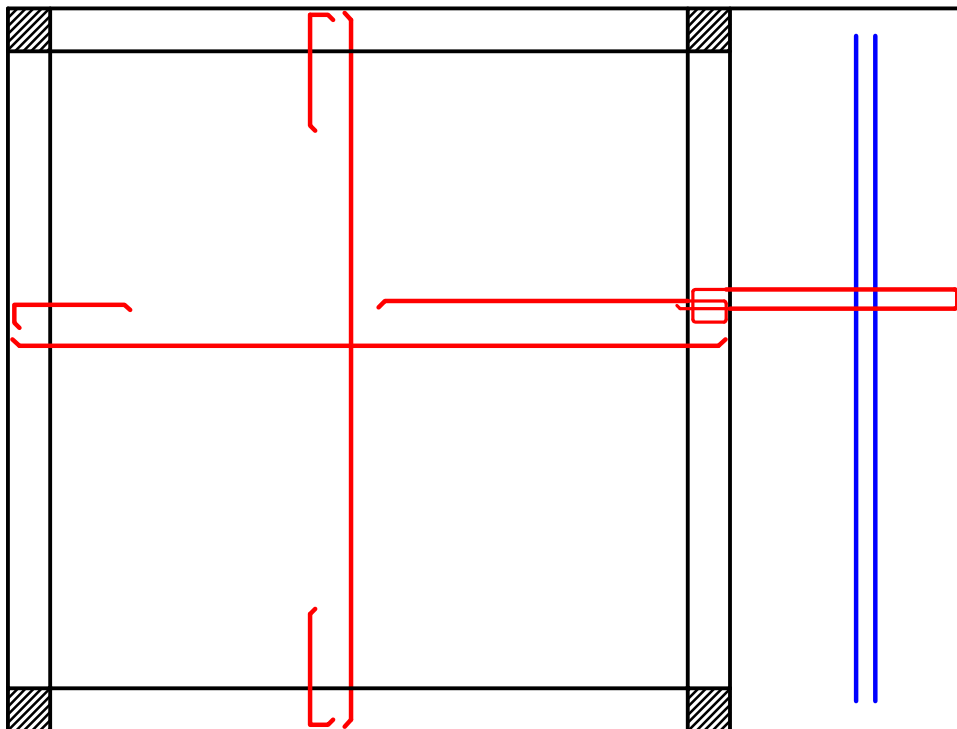
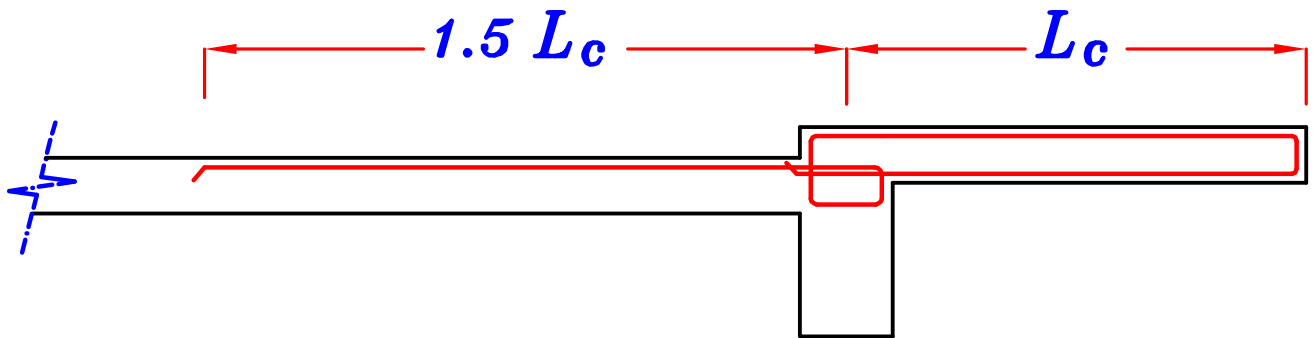
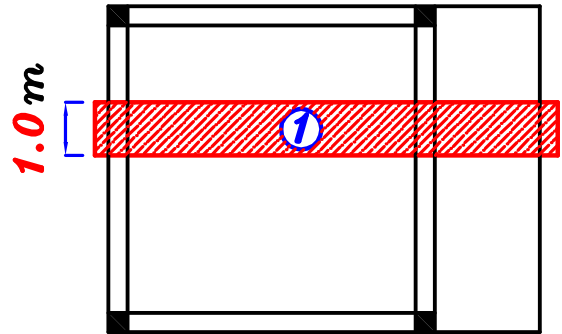
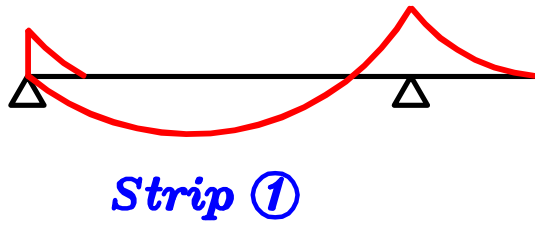


Strip ②

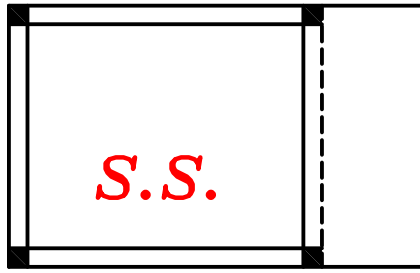


② نأخذ شريحه على البلاطه و الكابولى معا

فتكون محمولة على أكثر من *one support* فلا يتكون *Torsion* على الكمره



Special Case.



إذا وجد كابولي سفلي كما بالشكل

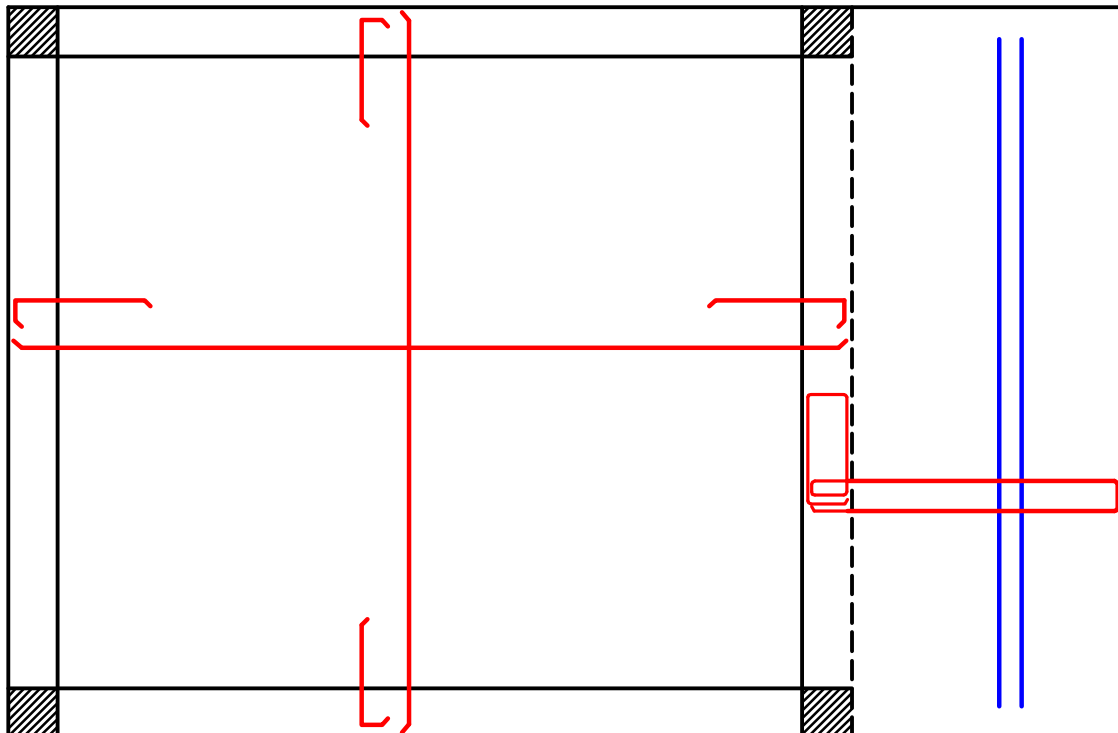
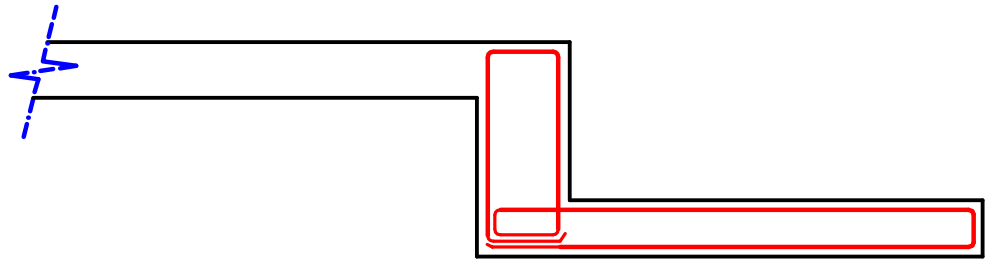
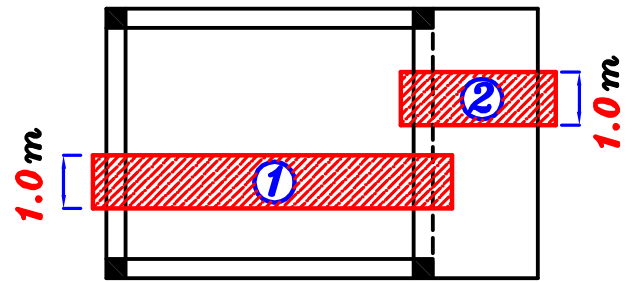
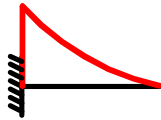
يوجد طريقتين للتسليح :

① نأخذ كل شريحة بمفردها



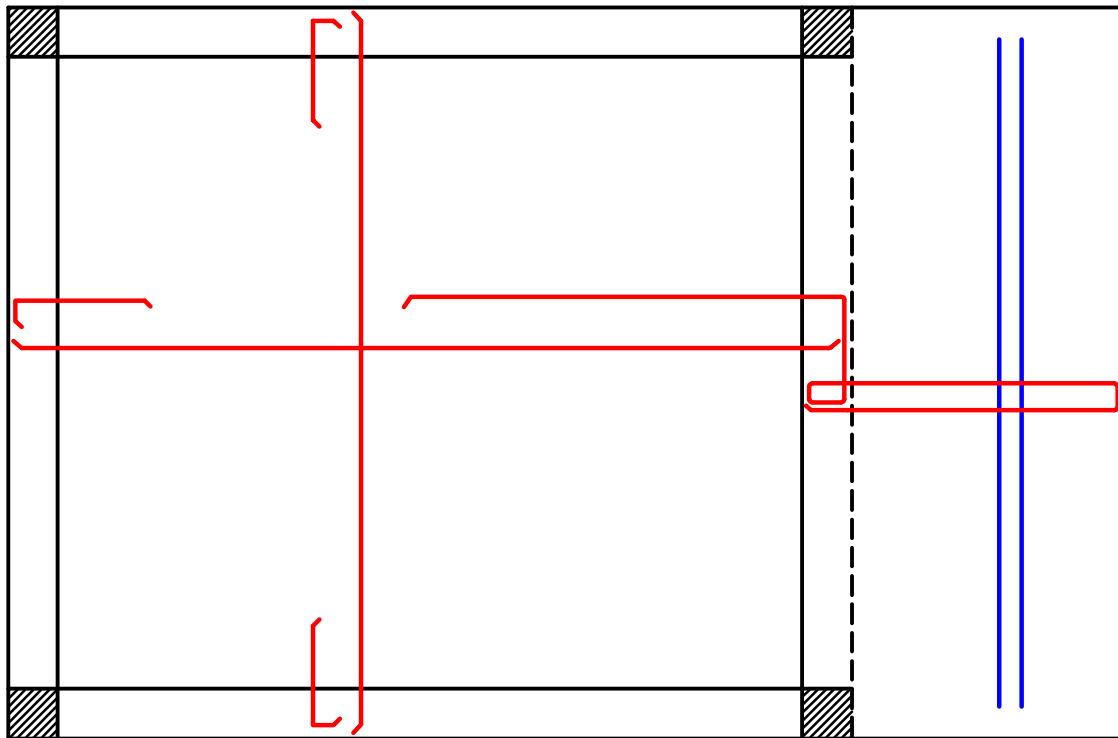
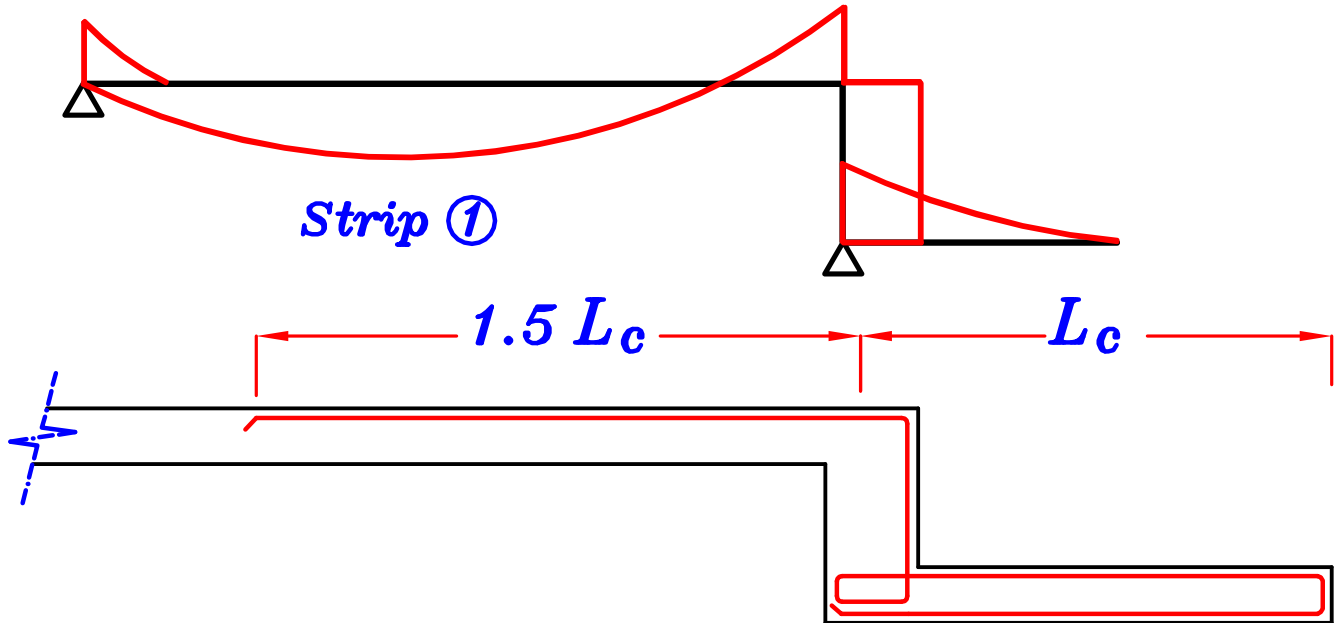
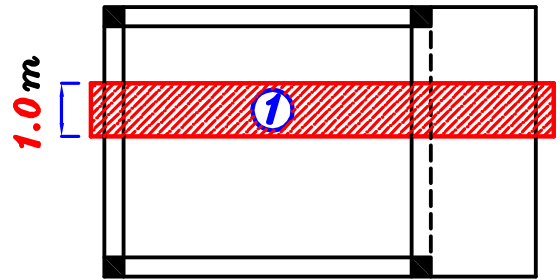
Strip ①

Strip ②



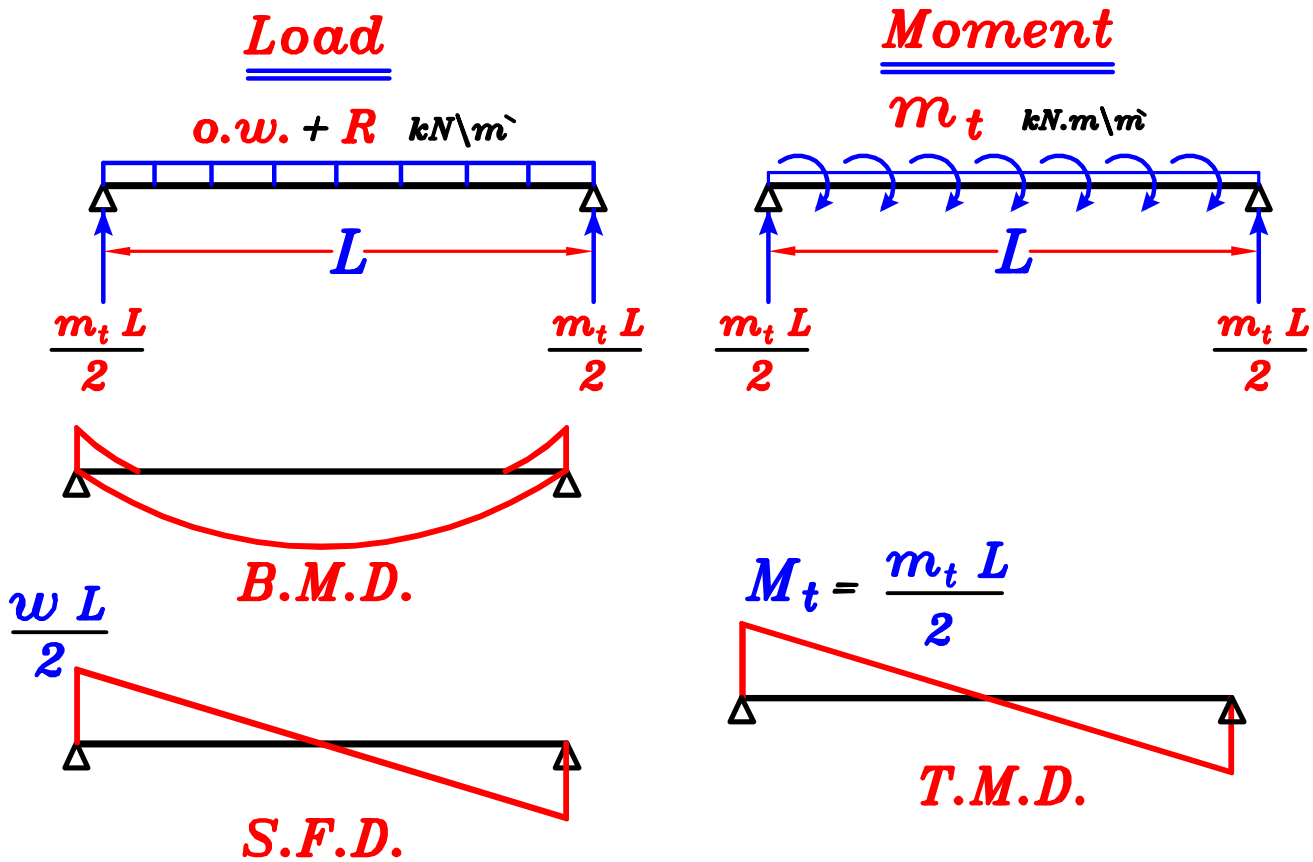
② نأخذ شريحه على البلاطه و الكابولى معا

فتكون محموله على أكثر من *one support* فلا يتكون *Torsion* على الكمره



Steps of design a Beam Subjected to Torsional Moment.

① Draw **B.M.D.** , **S.F.D.** & **T.M.D.**



② Estimate the Dimensions Of the Beam (**b, t**)

Ⓐ **b** = 300 mm OR 350 mm OR 400 mm

Ⓑ Get **t**

$$t_{tor.} \approx \frac{3 M_t}{1.6 b^2}$$

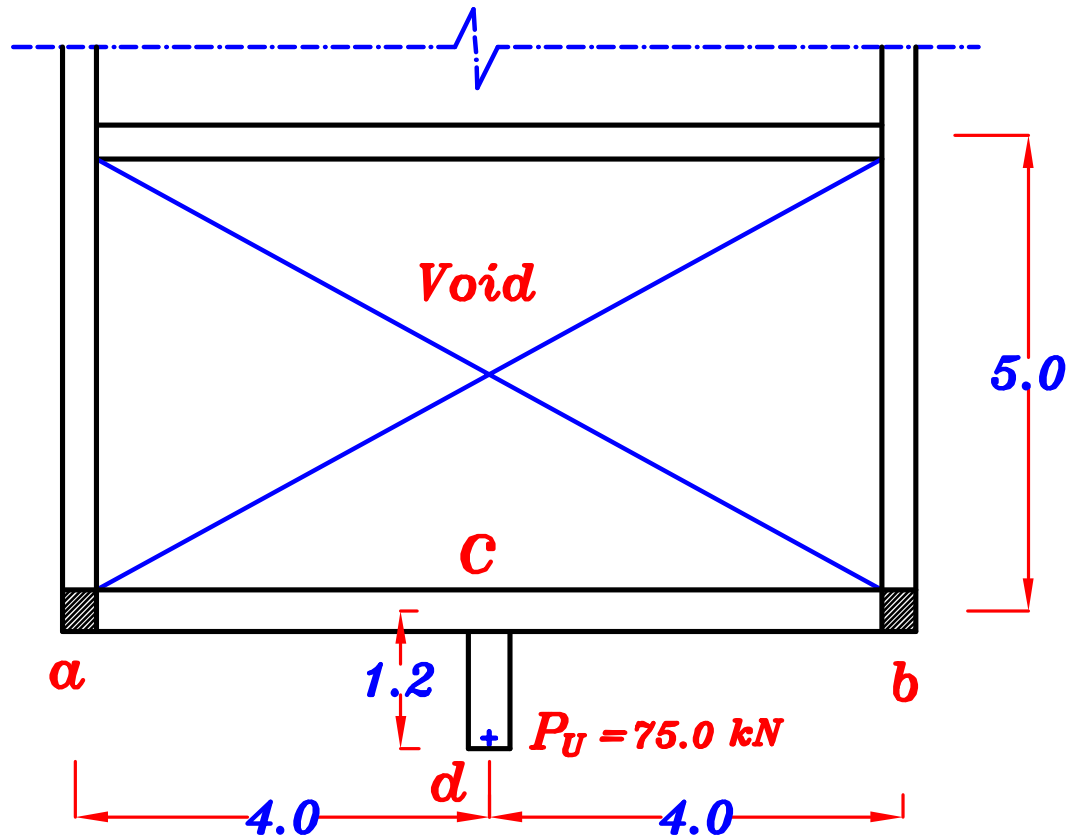
$$t_{ben.} = 3.50 \sqrt{\frac{M}{F_{cu} b}}$$

Take **t** = The bigger value of **t_{tor.}** , **t_{ben.}**

③ Design the beam according to B.M.
and get the main steel (**A_s**)

④ Design the beam according to Shear & Torsion.
and get Stirrups & Longitudinal Bars.

Example.



Data.

$$F_{cu} = 25 \text{ N/mm}^2 \quad F_y = 360 \text{ N/mm}^2$$

Req.

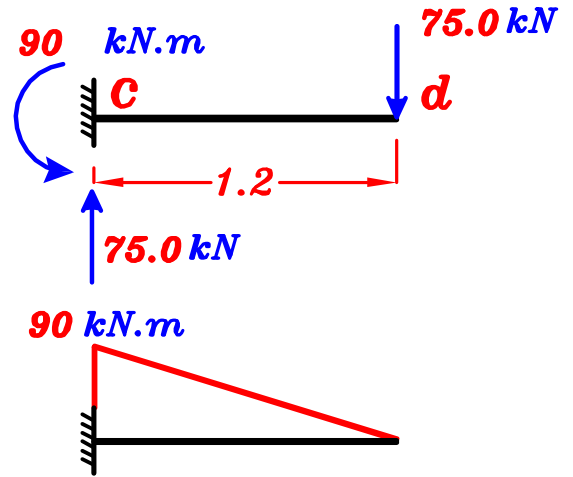
- ① Design the beam (**a b**) and Cantilever (**c d**)
(Neglecting the O.W.)
and draw The details of RFT.
- ② Get Loads on Column (**b**) at this case.
and Draw the Internal Forces Diagrams For that Column.
- ③ Draw a necessary member to make beam (**a b**)
Free of Torsion.
and Draw the Internal Forces Diagrams For the Beam at this case.

Solution.

①

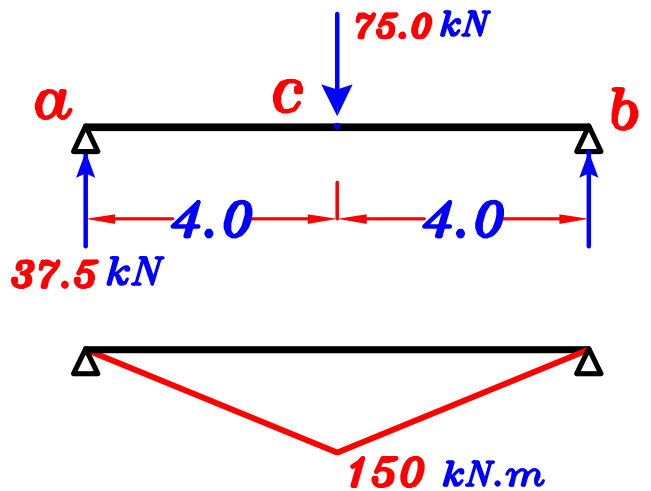
**Loads on
The Cantilever (c d)**

B.M.D. (c d)

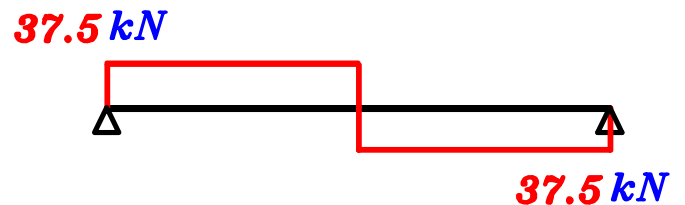


**Loads on
The Beam (a b)**

B.M.D. (a b)

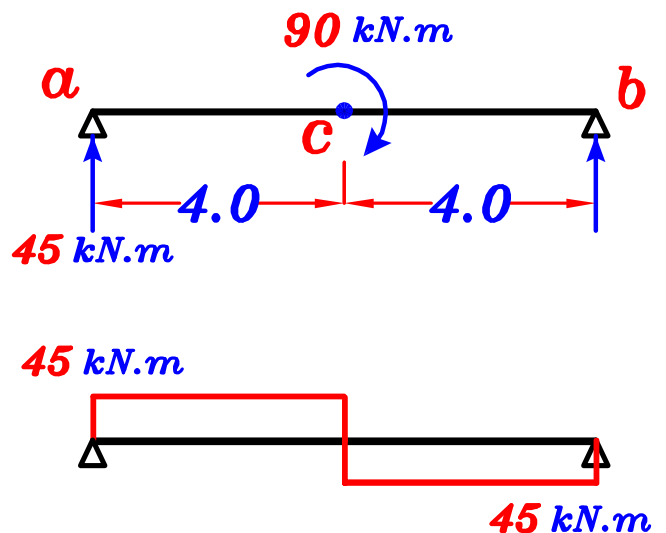


S.F.D. (a b)



**Torsional Moment on
The Beam (a b)**

T.M.D. (a b)



① Design of Cantilever (C d)

Sec. ① $M_{u.L.} = 90.0 \text{ kN.m}$ R-Sec. Take $b = 250 \text{ mm}$

- Take $C_1 = 3.50 \rightarrow J = 0.78$

$$\text{- Get } d = 3.50 \sqrt{\frac{M_{u.L.}}{F_{cu} b}} = 3.50 \sqrt{\frac{90.0 * 10^6}{25 * 250}} = 420 \text{ mm}$$

- Take $d = 450 \text{ mm}$, $t = 500 \text{ mm}$

$$\text{- Get } A_s = \frac{90.0 * 10^6}{0.78 * 360 * 420} = 763 \text{ mm}^2 \quad (4 \phi 16)$$

$$\text{- Check } A_{s_{min.}} = \frac{1.1}{F_y} b d = \frac{1.1}{360} (250) (450) = 343 \text{ mm}^2$$

$\therefore A_s > A_{s_{min.}} \therefore \text{o.k.}$

Dimensions of Beam (a b)

Take $b = 300 \text{ mm}$

$$* t_{ben.} = 3.50 \sqrt{\frac{M}{F_{cu} b}} = 3.50 \sqrt{\frac{150 * 10^6}{25 * 300}} = 494.9 \text{ mm}$$

$$* t_{tor.} \approx \frac{3 M_t}{b^2 * 1.5} = \frac{3 * 45 * 10^6}{300^2 * 1.6} = 937.5 \text{ mm} \quad t = 950 \text{ mm}$$

Design of Beam (a b)

$(300 * 950) \quad M_{u.L.} = 150 \text{ kN.m}$

$$900 = C_1 \sqrt{\frac{150 * 10^6}{25 * 300}} \rightarrow C_1 = 6.36 \rightarrow J = 0.826$$

$$A_s = \frac{150 * 10^6}{0.826 * 360 * 900} = 560.4 \text{ mm}^2$$

$$\text{- Check } A_{s_{min.}} = \frac{1.1}{F_y} b d = \frac{1.1}{360} (300) (900) = 825 \text{ mm}^2$$

$\therefore A_s < A_{s_{min.}} \therefore \text{Take } A_s = A_{s_{min.}}$

$$\left. \begin{array}{l} A_{s_{min.}} = \frac{1.1}{F_y} b d = \frac{1.1}{360} (300) (900) = 825 \\ 1.3 A_{s_{req.}} = (1.3) (560.4) = 728.5 \\ \text{st. } 360/520 \frac{0.15}{100} b d = \frac{0.15}{100} (300) (900) = 405 \end{array} \right\} 7.285 \left. \vphantom{\begin{array}{l} A_{s_{min.}} \\ 1.3 A_{s_{req.}} \\ \text{st. } 360/520} \right\} 728.5 \text{ mm}^2$$

Design the beam For Shear + Torsion.

$$q_u = \frac{Q}{bd} = \frac{37.5 \cdot 10^3}{300 \cdot 900} = 0.138$$

$$x_1 = 220 \text{ mm} , y_1 = 870 \text{ mm}$$

$$q_{tu} = \frac{M_{tu}}{2 A_o t_e} = \frac{M_{tu} (x_1 + y_1)}{0.85 (x_1^2 + y_1^2)}$$

$$q_{tu} = \frac{45 \cdot 10^6 (250 + 850)}{0.85 (250^2 + 850^2)} = 1.29$$

$$q_{cu} = (0.24) \sqrt{\frac{25}{1.5}} = 0.98$$

$$q_{t_{min}} = (0.06) \sqrt{\frac{25}{1.5}} = 0.245$$

$$q_{u_{max}} = (0.7) \sqrt{\frac{25}{1.5}} = 2.85$$

$$\sqrt{q_u^2 + q_{tu}^2} = \sqrt{0.138^2 + 1.29^2} = 1.29 < q_{u_{max}} \therefore \text{o.k.}$$

$$q_u < q_{cu} , q_{tu} > q_{t_{min}} \therefore \text{Use RFT. For Torsion only.}$$

* Stirrups.

$$\therefore A_{str} = \frac{M_{tu} S_t}{(1.7) x_1 y_1 \left(\frac{F_y}{\phi_s}\right)} \quad \therefore A_{str} = \frac{(45 \cdot 10^6) \cdot S_t}{(1.7)(220)(870) (240/1.15)}$$

$$* \text{ Take } \phi 8 \rightarrow A_{str} = 50.3 \text{ mm}^2 \quad \therefore S_t = 1.509 \cdot A_{str}$$

$$\therefore S_t = 1.509 \cdot A_{str} = 1.509 \cdot 50.3 = 75.9 \text{ mm} < 100 \text{ mm}$$

$$* \text{ Take } \phi 10 \rightarrow A_{str} = 78.5 \text{ mm}^2$$

$$\therefore S_t = 1.509 \cdot A_{str} = 1.509 \cdot 78.5 = 118.4 \text{ mm} > 100 \text{ mm} \therefore \text{o.k.}$$

$$\therefore \text{No. of stirrups/m} = \frac{1000}{S} = \frac{1000}{118.4} = 8.44 = 9.0$$

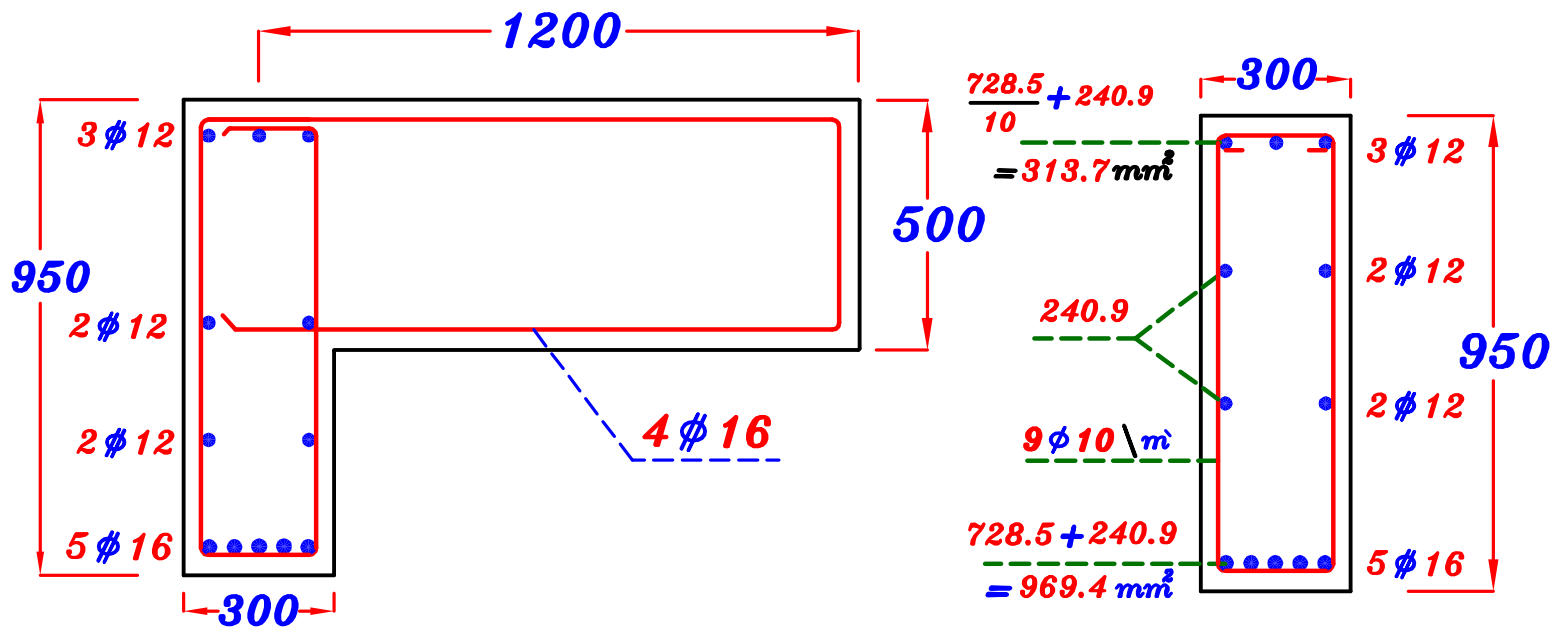
$$\therefore \text{Use Closed Stirrups } 9 \phi 10 \text{ m } 2 \text{ branches.}$$

* Longitudinal Bars.

$$\therefore A_{sl} = 2 A_{str} \frac{(x_1 + y_1)}{S_t} \left(\frac{F_{y_{str.}}}{F_{y_{L.b.}}} \right)$$

$$\therefore A_{sl} = 2 (78.5) \frac{(220 + 870)}{118.4} \left(\frac{240}{360} \right) = 963.5 \text{ mm}^2$$

$$\therefore \frac{A_{sl}}{4} = 240.9 \text{ mm}^2$$

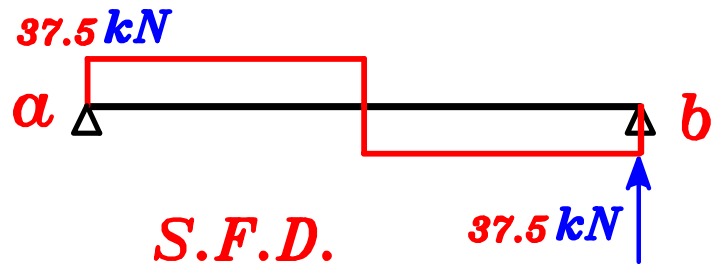


RFT. of Cantilever.

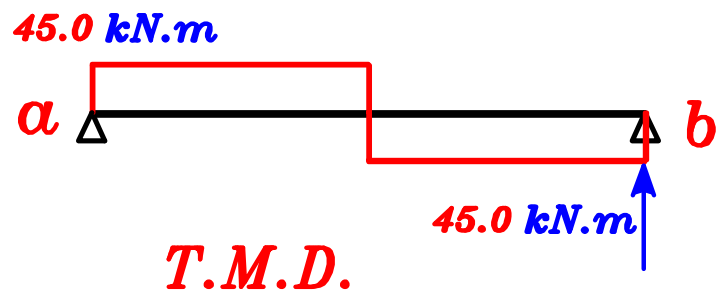
**Cross Sec.
In Beam (a b)**

② Loads on Column (b)

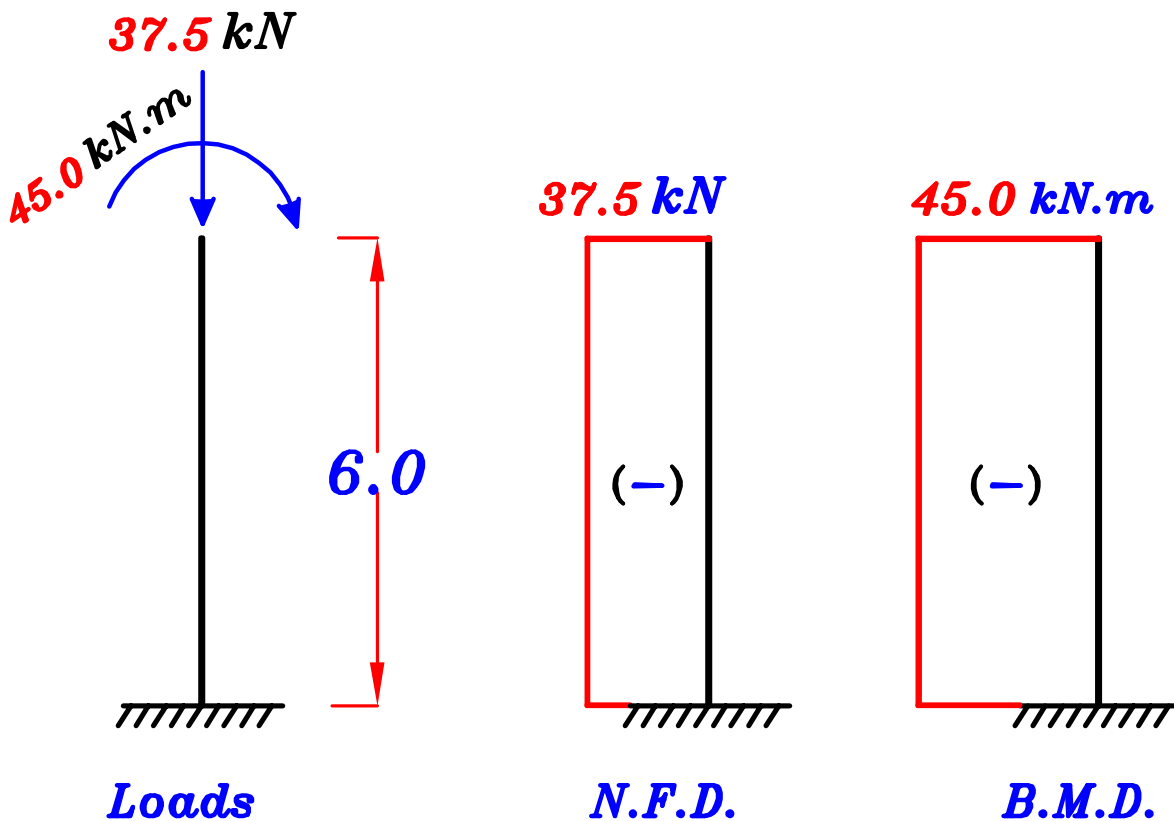
Reaction of S.F.D.
=
Load on the column.



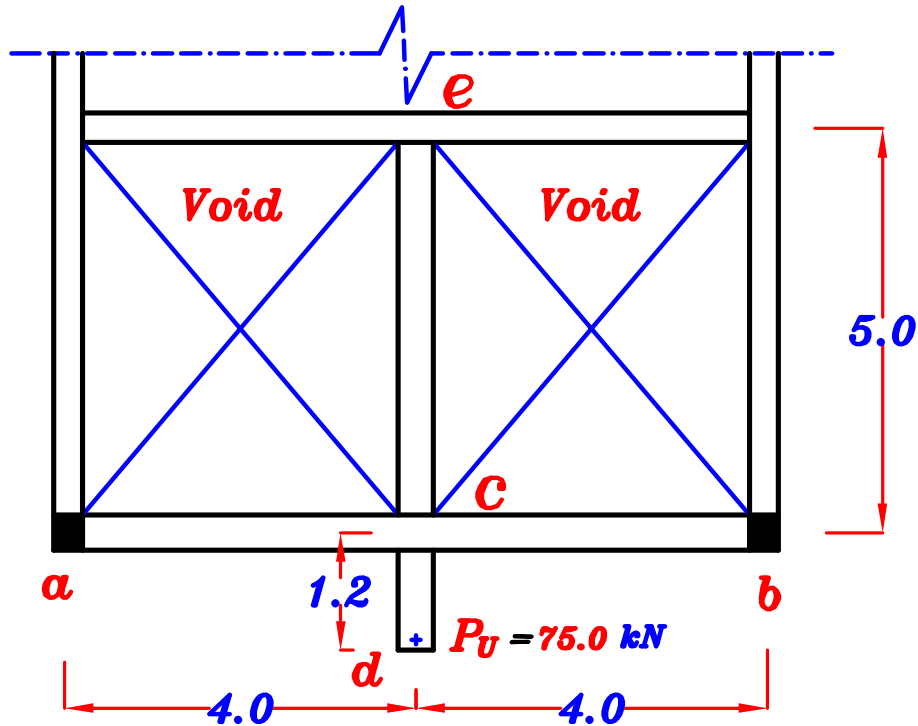
Reaction of T.M.D.
=
Moment on the column.



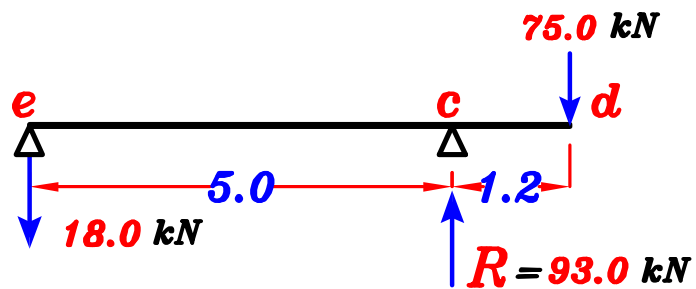
Internal Forces Diagrams on Column (b).



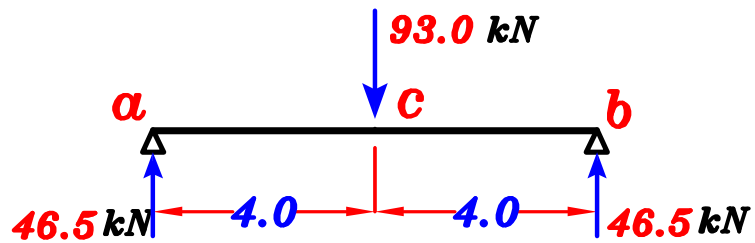
- ③ To make the Beam (a b) Free of Torsion.
We can put a member at (c e) as shown.



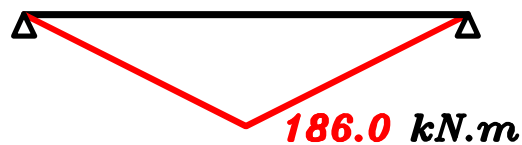
Beam (e c d)



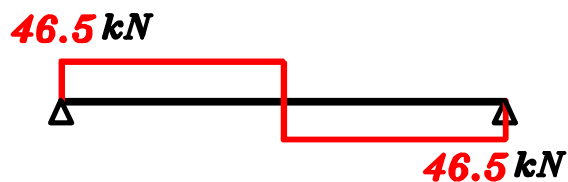
Beam (a c b)



B.M.D.



S.F.D.



T.M.D.



Example.

Short Column.

Data.

$$F_{cu} = 25 \text{ N/mm}^2$$

$$F_y = 360 \text{ N/mm}^2$$

$$F.C. = 1.50 \text{ kN/m}^2$$

$$L.L. = 2.0 \text{ kN/m}^2$$

The Column is Short column.

Req.

- ① Complete design For Slab, Beam & Column.
- ② Draw the details of RFT.

Solution.

Loads on the Slab.

$$t_s = \frac{L_c}{10} = \frac{1500}{10} = 150 \text{ mm}$$

$$t_s = 150 \text{ mm}$$

$$(w_s)_{U.L.} = 1.4 (t_s \delta_c + F.C.) + 1.6 (L.L.)$$

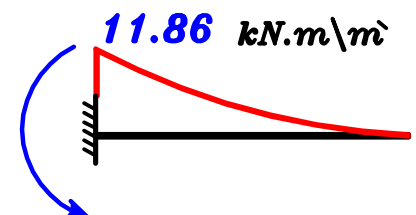
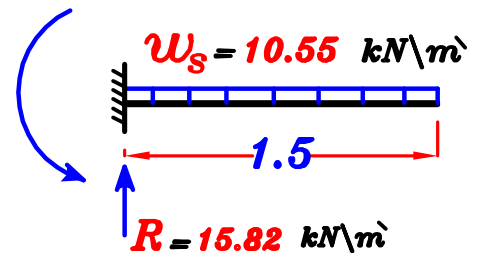
$$(w_s)_{U.L.} = 1.4 (0.15 * 25 + 1.5) + 1.6 (2.0) = 10.55 \text{ kN/m}^2$$

$$R = 10.55 * 1.5 = 15.82 \text{ kN/m}$$

$$M = \frac{10.55 * 1.5^2}{2} = 11.86 \text{ kN.m/m}$$

Strip in the slab.

$$11.86 \text{ kN.m/m}$$



Design the Slab.

Sec. ①

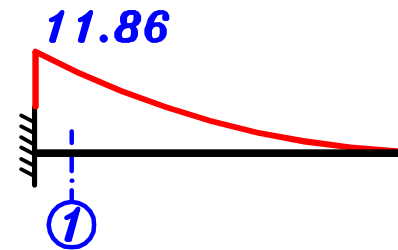
$$M_{U.L.} = 11.86 \text{ kN.m/m}$$

$$, t_s = 150 \text{ mm} , d = 130 \text{ mm}$$

$$130 = C_1 \sqrt{\frac{11.86 \cdot 10^6}{25 \cdot 1000}} \longrightarrow C_1 = 5.96 \longrightarrow J = 0.826$$

$$A_s = \frac{11.86 \cdot 10^6}{0.826 \cdot 360 \cdot 130} = 306.8 \text{ mm}^2/\text{m}$$

5 ϕ 10 \text{m}

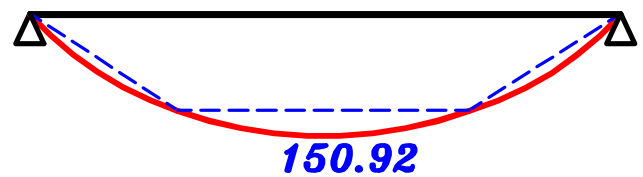
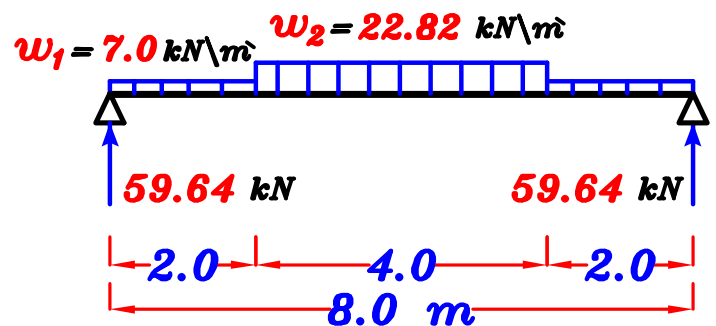
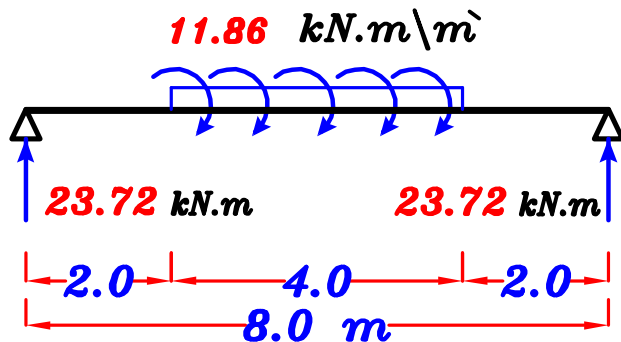


Loads on the Beam.

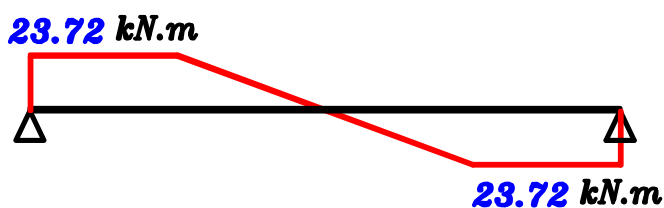
$$w_1 = 0.w_{(beam)} \approx 7.0 \text{ kN/m} \quad (U.L.)$$

$$w_2 = 0.w_{(beam)} + \text{Load From Slab} = 7.0 + 10.55 \cdot 1.5 = 22.82 \text{ kN/m}$$

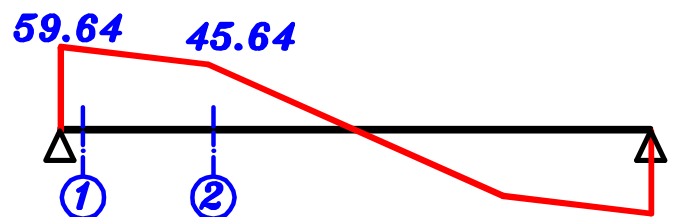
$$M = 11.86 \text{ kN.m}$$



B.M.D.



T.M.D.



S.F.D.

Dimensions of the Beam.

Take $b = 300$ mm

$$* t_{ben.} = 5.0 \sqrt{\frac{M}{F_{cu} b}} = 5.0 \sqrt{\frac{150.92 * 10^6}{25 * 300}} = 709 \text{ mm (L-sec.)}$$

$$* t_{tor.} \approx \frac{3 M_t}{b^2 * 1.6} = \frac{3 * 23.72 * 10^6}{300^2 * 1.6} = 494 \text{ mm} \quad \boxed{t = 750 \text{ mm}}$$

Design the beam For bending. (300 * 750)

$$M_{u.L.} = 150.92 \text{ kN.m (L-sec.)}$$

$$B = \left\{ \begin{array}{l} \text{C.L.-C.L.} = 1.5 \text{ m} = 1500 \text{ mm} \\ 6 t_s + b = 6 * 150 + 300 = 1200 \\ K \frac{L}{10} + b = 1.0 * \frac{8000}{10} + 300 = 1100 \text{ mm} \end{array} \right\} \quad \boxed{B = 1100 \text{ mm}}$$

$$700 = C_1 \sqrt{\frac{150.92 * 10^6}{25 * 1100}} \rightarrow C_1 = 9.44 \rightarrow J = 0.826$$

$$A_s = \frac{150.92 * 10^6}{0.826 * 360 * 700} = 725 \text{ mm}^2$$

$$- \text{Check } A_{s_{min}} = \frac{1.1}{F_y} b d = \frac{1.1}{360} (300) (700) = 641.6 \text{ mm}^2$$

Design the beam For Shear + Torsion.

$$\text{Sec. ① } q_u = \frac{Q}{b d} = \frac{59.64 * 10^3}{300 * 700} = 0.284$$

$$x_1 = 220 \text{ mm} , y_1 = 670 \text{ mm}$$

$$q_{tu} = \frac{23.72 * 10^6 (220 + 670)}{0.85 (220^2 + 670^2)} = 1.14$$
$$q_{tu} = \frac{M_{tu}}{2 A_o t_e} = \frac{M_{tu} (x_1 + y_1)}{0.85 (x_1^2 + y_1^2)}$$

$$q_{cu} = (0.24) \sqrt{\frac{25}{1.5}} = 0.98$$

$$q_{t_{min}} = (0.06) \sqrt{\frac{25}{1.5}} = 0.245$$

$$q_{u_{max}} = (0.7) \sqrt{\frac{25}{1.5}} = 2.85$$

$$\sqrt{q_u^2 + q_{tu}^2} = \sqrt{0.284^2 + 1.14^2} = 1.17 < q_{u_{max}} \therefore \text{o.k.}$$

$$\boxed{q_u < q_{cu} , q_{tu} > q_{t_{min}}} \therefore \text{Use RFT. For Torsion only.}$$

* Stirrups.

$$\therefore A_{str} = \frac{M_{tu} S_t}{(1.7) x_1 y_1 \left(\frac{F_y}{\delta_s} \right)} \quad \therefore A_{str} = \frac{(23.72 \cdot 10^6) \cdot S_t}{(1.7)(220)(670) (240/1.15)}$$

$$\therefore S_t = 2.204 \cdot A_{str}$$

* Take $\phi 8 \rightarrow A_{str} = 50.3 \text{ mm}^2$

$$\therefore S_t = 2.204 \cdot A_{str} = 2.204 \cdot 50.3 = 110.86 \text{ mm} > 100 \text{ mm} \therefore \text{o.k.}$$

$$\therefore \text{No. of stirrups/m} = \frac{1000}{S} = \frac{1000}{110.86} = 9.02 = 10$$

$\therefore$ Use Closed Stirrups $10 \phi 8 \text{ /m}$ 2 branches.

* Longitudinal Bars.

$$A_{sl} = 2 A_{str} \frac{(x_1 + y_1)}{S_t} \left(\frac{F_{y_{str.}}}{F_{y_{L.b.}}} \right) = 2 (50.3) \left(\frac{220 + 670}{110.86} \right) \left(\frac{240}{360} \right)$$

$$\therefore A_{sl} = 538.4 \text{ mm}^2 \quad \therefore \frac{A_{sl}}{4} = 134.6 \text{ mm}^2$$

Check IF the Stirrups 5 #10 \ m is safe.

Sec. ②

$$q_u = \frac{Q}{bd} = \frac{45.64 \cdot 10^3}{300 \cdot 700} = 0.217$$

$$x_1 = 220 \text{ mm} , y_1 = 670 \text{ mm}$$

$$q_{tu} = \frac{M_{tu}}{2 A_o t_e} = \frac{M_{tu} (x_1 + y_1)}{0.85 (x_1^2 + y_1^2)}$$

$$q_{tu} = \frac{23.72 \cdot 10^6 (220 + 670)}{0.85 (220^2 + 670^2)} = 1.14$$

$$q_{cu} = (0.24) \sqrt{\frac{25}{1.5}} = 0.98$$

$$q_{t \min} = (0.06) \sqrt{\frac{25}{1.5}} = 0.245$$

$$q_{u \max} = (0.7) \sqrt{\frac{25}{1.5}} = 2.85$$

$$\sqrt{q_u^2 + q_{tu}^2} = \sqrt{0.217^2 + 1.14^2} = 1.16 < q_{u \max} \therefore \text{o.k.}$$

$$q_u < q_{cu} , q_{tu} > q_{t \min} \therefore \text{Use RFT. For Torsion only.}$$

* Stirrups.

$$\therefore A_{str} = \frac{M_{tu} S_t}{(1.7) x_1 y_1 \left(\frac{F_y}{\delta_s} \right)} \therefore A_{str} = \frac{(23.72 \cdot 10^6) \cdot S_t}{(1.7)(220)(670)(360/1.15)}$$

$$\therefore S_t = 3.307 \cdot A_{str}$$

* The given #10 $\rightarrow A_{str} = 78.5 \text{ mm}^2$

$$\therefore S_t = 3.307 \cdot A_{str} = 3.307 \cdot 78.5 = 259.6 \text{ mm} > 100 \text{ mm} \therefore \text{o.k.}$$

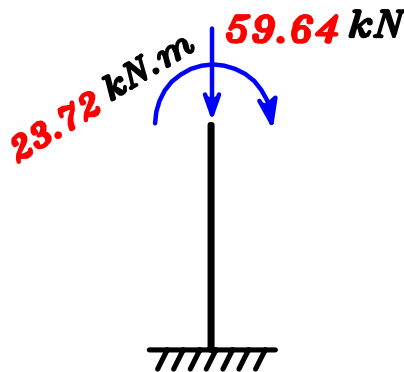
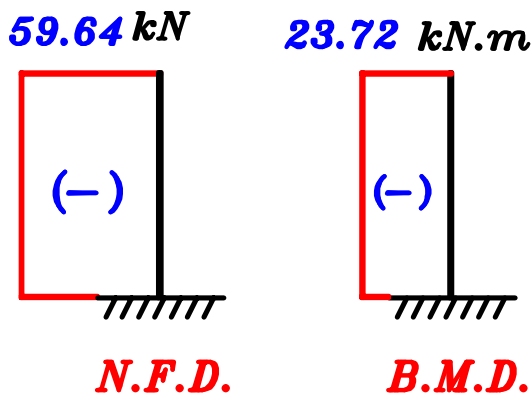
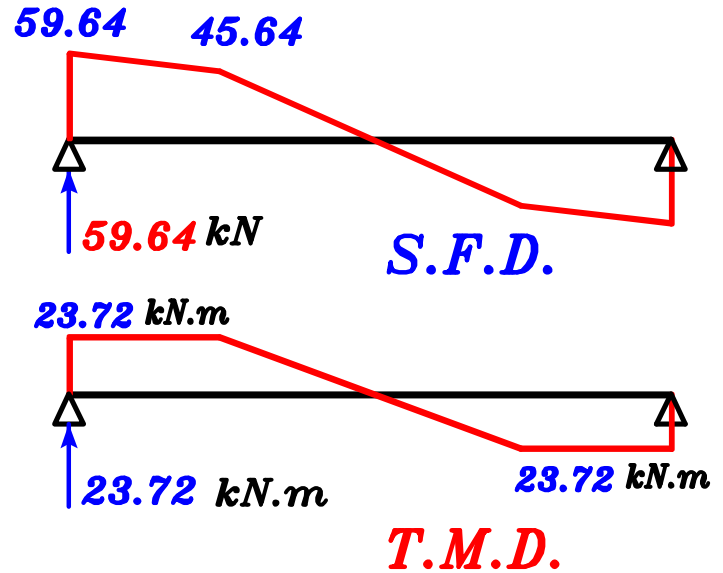
$$\therefore \text{No. of stirrups required \ m} = \frac{1000}{S} = \frac{1000}{259.6} = 3.85 < 5.0$$

$\therefore$ the stirrups 5 #10 \ m is enough.

Design of the column.

Reactions of the S.F.D.
=
Load on the column.

Reactions of the T.M.D.
=
Moment on the column.



Take the Column. (300*300)

$$M_{U.L.} = 23.72 \text{ kN.m} \quad N_{U.L.} = 59.64 \text{ kN}$$

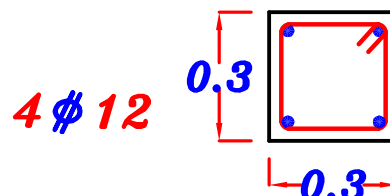
$$e = \frac{M}{N} = \frac{23.72}{59.64} = 0.397 \text{ m} \quad \therefore \frac{e}{t} = \frac{0.397}{0.30} = 1.325 > 0.5 \xrightarrow{\text{use}} e_s$$

$$e_s = e + \frac{t}{2} - c = 0.397 + \frac{0.30}{2} - 0.05 = 0.497 \text{ m}$$

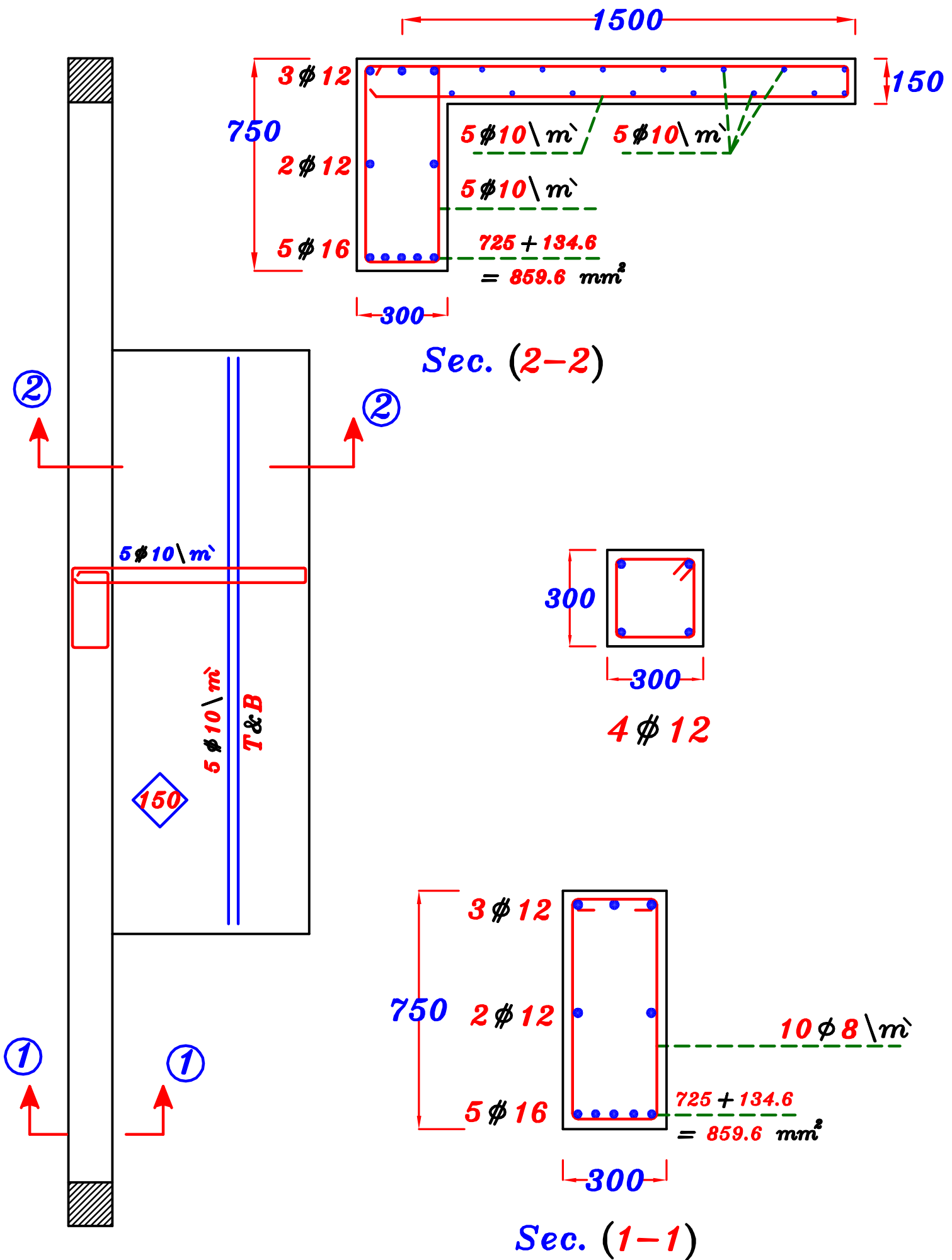
$$M_s = N * e_s = 59.64 * 0.497 = 29.64 \text{ kN.m}$$

$$\therefore 250 = C_1 \sqrt{\frac{29.64 * 10^6}{25 * 300}} \rightarrow C_1 = 3.97 \rightarrow J = 0.802$$

$$\therefore A_s = \frac{M_s}{J F_y d} - \frac{N_{U.L.}}{(F_y \setminus \delta_s)} = \frac{29.64 * 10^6}{0.802 * 360 * 250} - \frac{59.64 * 10^3}{(360 \setminus 1.15)} = 220.1 \text{ mm}^2$$



4 Φ 12



Example.

Data.

$$F_{cu} = 25 \text{ N/mm}^2$$

$$F_y = 360 \text{ N/mm}^2$$

$$F.C. = 2.5 \text{ kN/m}^2$$

$$L.L. = 1.0 \text{ kN/m}^2 \text{ (HL. Projection)}$$

$$\text{Spacing} = 5.0 \text{ m}$$

Req.

- ① Complete design of the slab, beam & the column.
- ② Draw the details of RFT in plan & cross-sections.

Solution.

$$\text{For } (S_1) \quad t_s = \frac{L_c}{10} = \frac{2200}{10} = 220 \text{ mm}$$

$$t_{s1} = 220 \text{ mm}$$

$$\text{For } (S_2) \quad t_s = \frac{L_c}{10} = \frac{1100}{10} = 110 \text{ mm}$$

$$t_{s2} = 120 \text{ mm}$$

Case of Total Load. (T.L.)

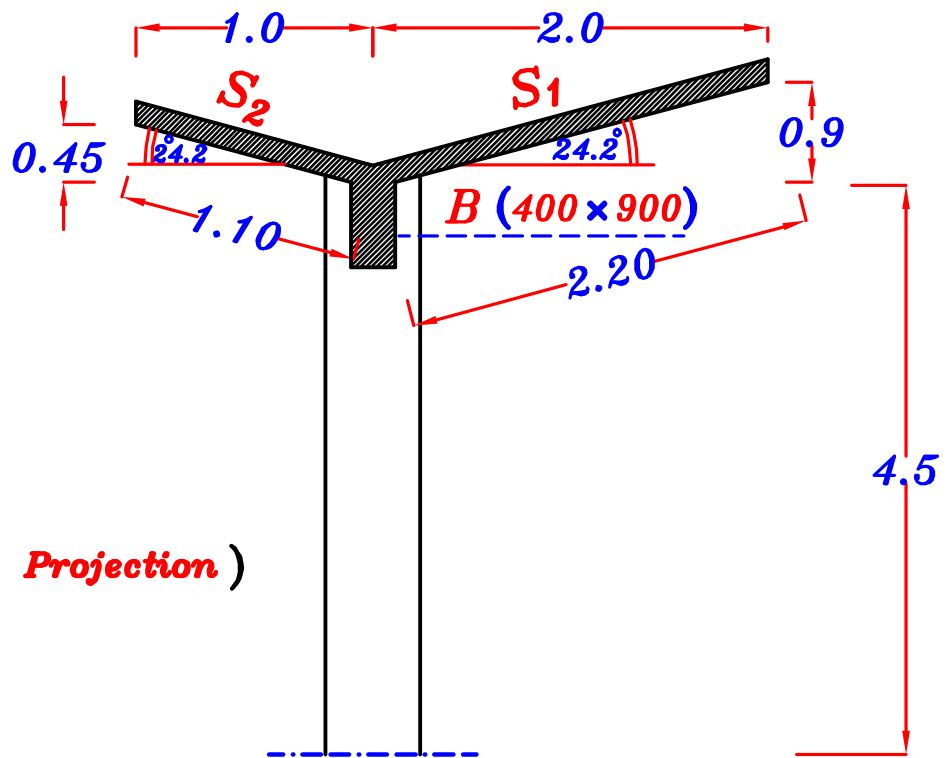
$$(w_{s1}) \quad T = 1.4 (0.22 * 25 + 2.5) + 1.6 (1.0) \cos 24.2^\circ = 12.65 \text{ kN/m}^2$$

$$(w_{s2}) \quad T = 1.4 (0.12 * 25 + 2.5) + 1.6 (1.0) \cos 24.2^\circ = 9.15 \text{ kN/m}^2$$

Case of Dead Load. (D.L.)

$$(w_{s1}) \quad D = 0.9 (0.22 * 25 + 2.5) = 7.20 \text{ kN/m}^2$$

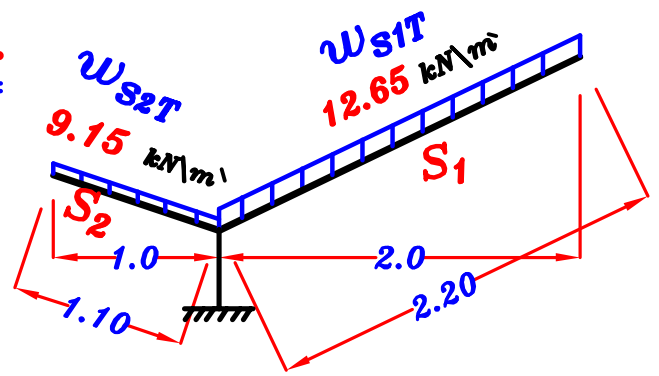
$$(w_{s2}) \quad D = 0.9 (0.12 * 25 + 2.5) = 4.95 \text{ kN/m}^2$$



Loads to design the Slabs.

$$M_1 = 12.65 * 2.20 * \frac{2.0}{2} = 27.83 \text{ kN.m/m}$$

$$M_2 = 9.15 * 1.10 * \frac{1.0}{2} = 5.03 \text{ kN.m/m}$$



Design of Slab. (S1)

Sec. ①

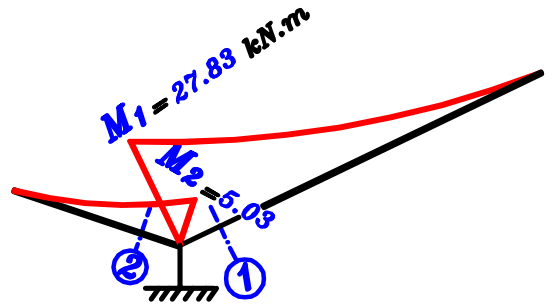
$$M_{U.L.} = 27.83 \text{ kN.m/m}$$

$$, t_s = 220 \text{ mm} , d = 200 \text{ mm}$$

$$200 = C_1 \sqrt{\frac{27.83 * 10^6}{25 * 1000}} \longrightarrow C_1 = 5.99 \longrightarrow J = 0.826$$

$$A_s = \frac{27.83 * 10^6}{0.826 * 360 * 200} = 467.9 \text{ mm}^2/\text{m}$$

5 ϕ 12 \text{ / m}



Design of Slab. (S2)

Sec. ②

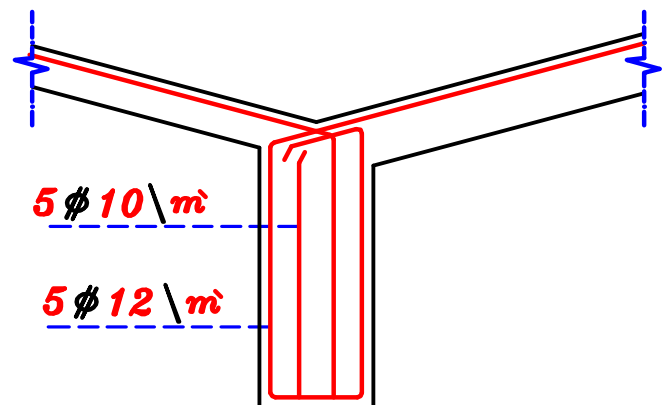
$$M_{U.L.} = 5.03 \text{ kN.m/m}$$

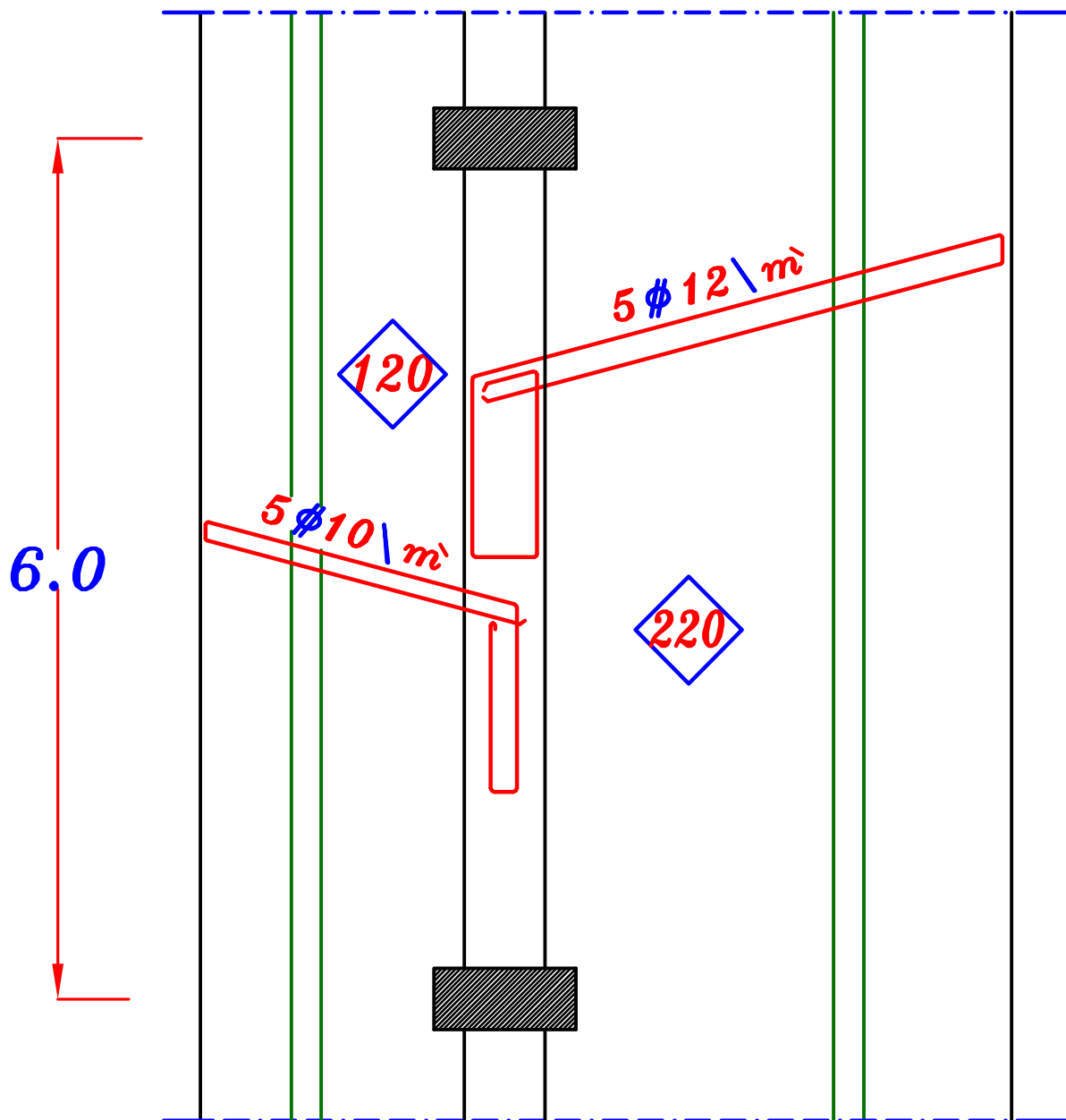
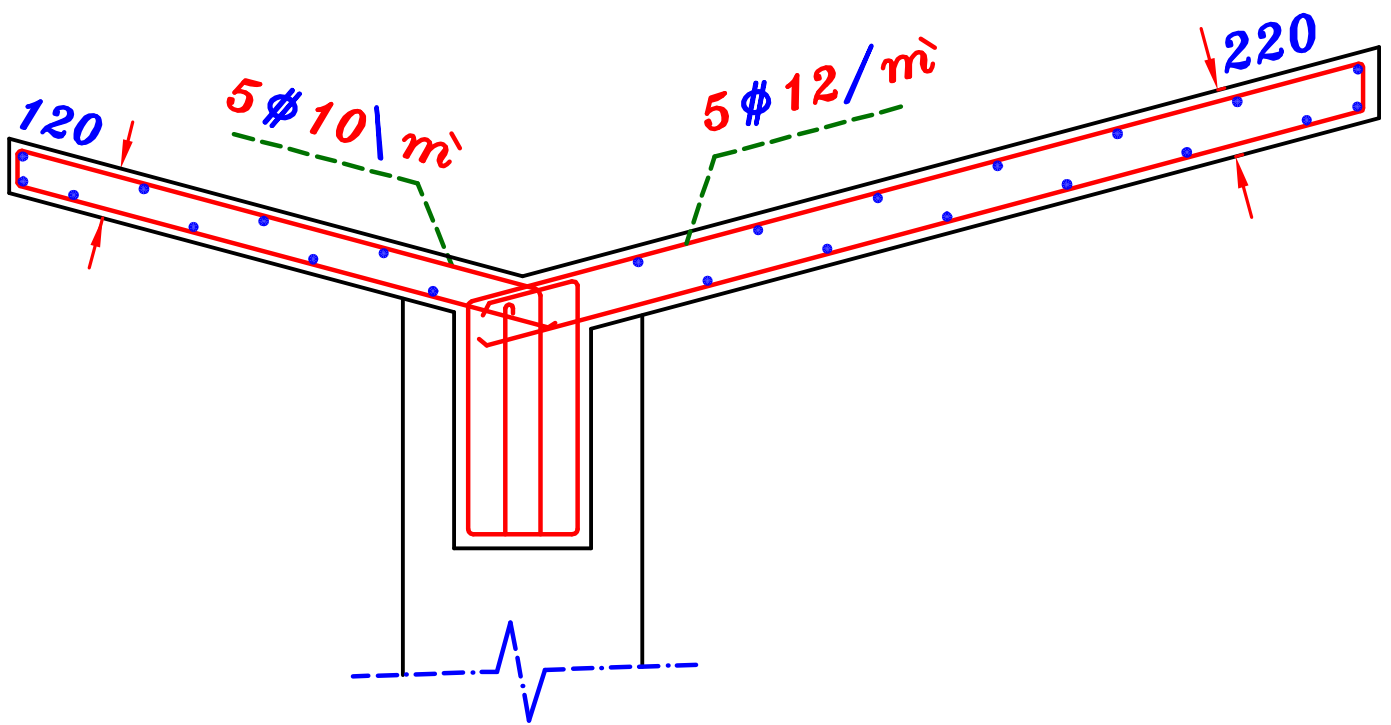
$$, t_s = 120 \text{ mm} , d = 100 \text{ mm}$$

$$100 = C_1 \sqrt{\frac{5.03 * 10^6}{25 * 1000}} \longrightarrow C_1 = 7.05 \longrightarrow J = 0.826$$

$$A_s = \frac{5.03 * 10^6}{0.826 * 360 * 100} = 169.1 \text{ mm}^2/\text{m}$$

5 ϕ 10 \text{ / m}



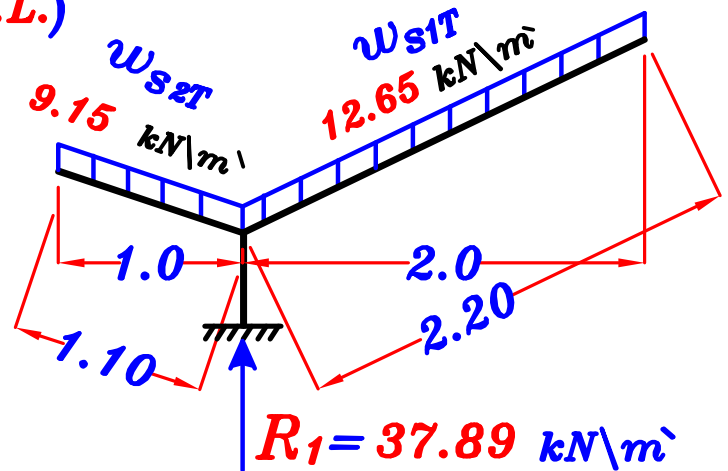


Design of the beam.

1- Draw B.M.D. , S.F.D. & T.M.D.

To Draw B.M.D. For the Beam.

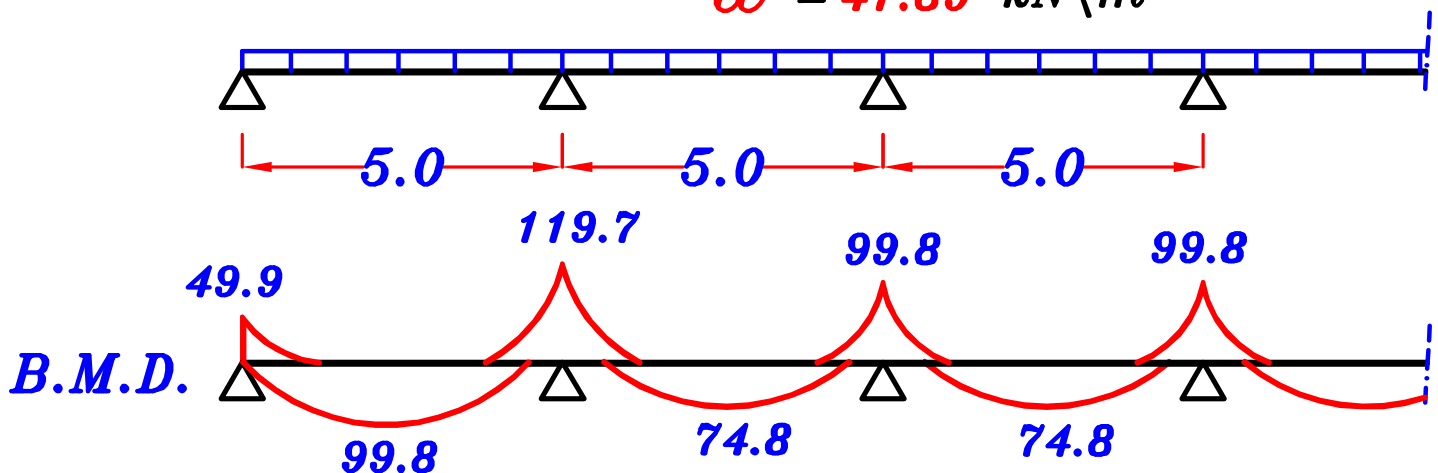
$$o.w._{(beam)} \approx 10 \text{ kN/m} \quad (U.L.)$$



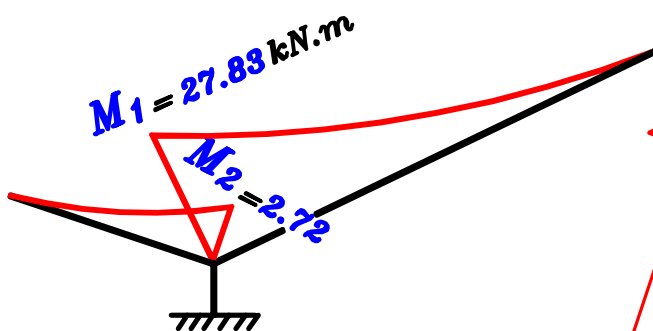
$$W = o.w._{(beam)} + R_1$$

$$= 10 + 37.89 = 47.89 \text{ kN/m}$$

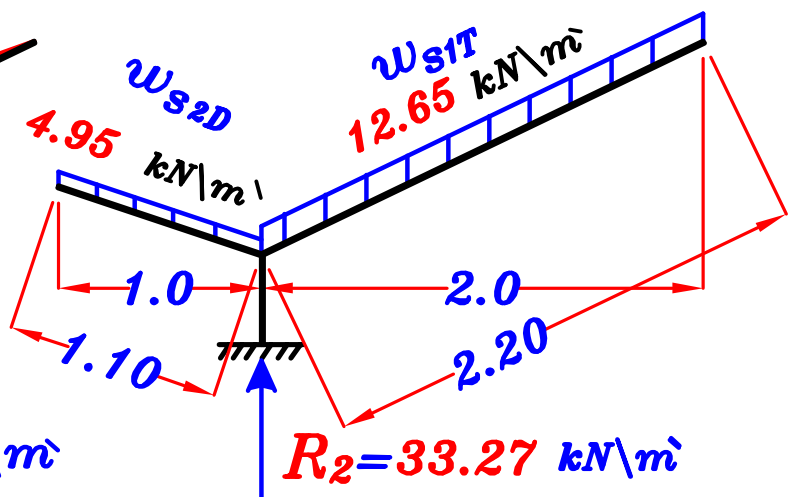
$$W = 47.89 \text{ kN/m}$$



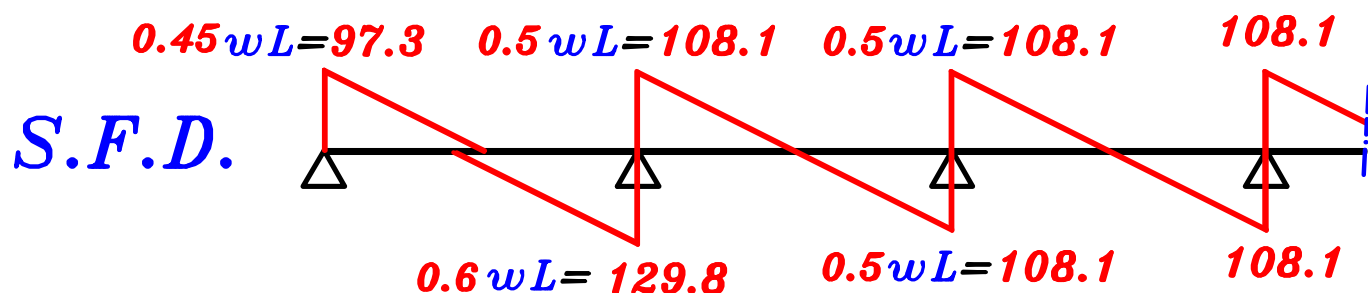
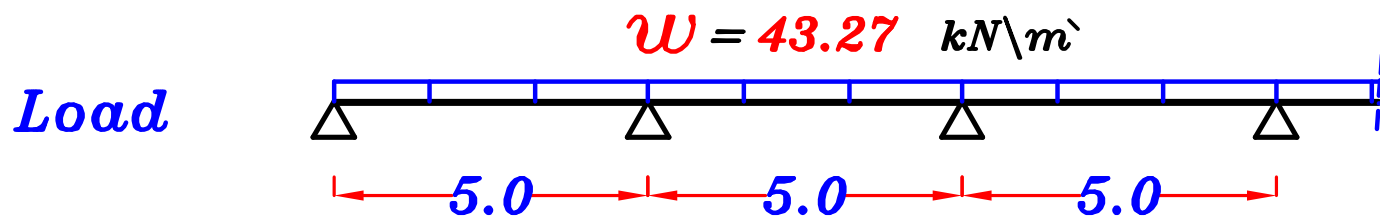
To Draw S.F.D. & T.M.D.. For the Beam.



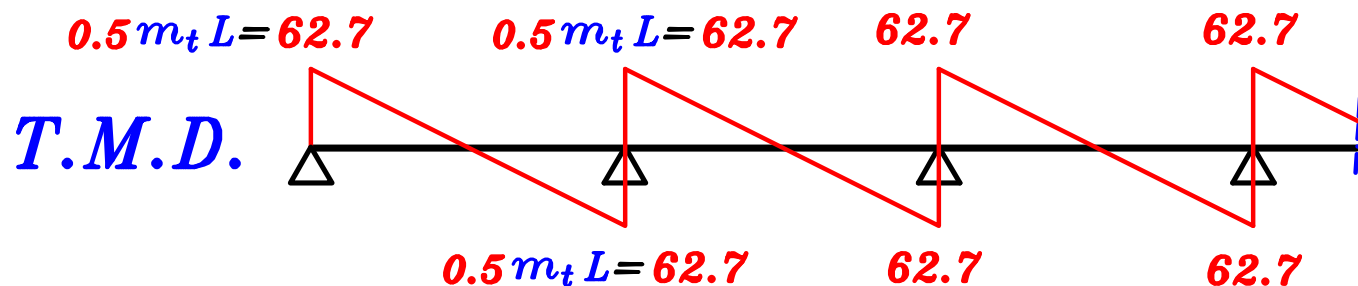
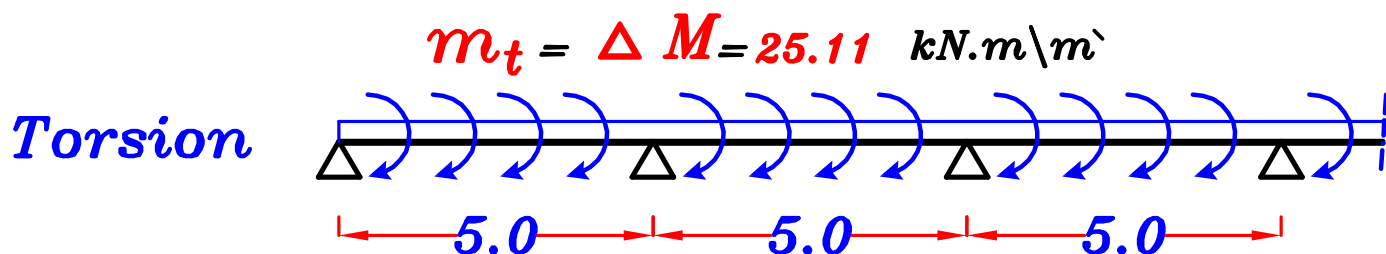
$$\Delta M = 25.11 \text{ kN.m/m}$$



$$w = 0.w_{(beam)} + R_2 = 10 + 33.27 = 43.27 \text{ kN/m}$$



$$\Delta M = M_1 - M_2 = 27.83 - 2.72 = 25.11 \text{ kN.m/m}$$



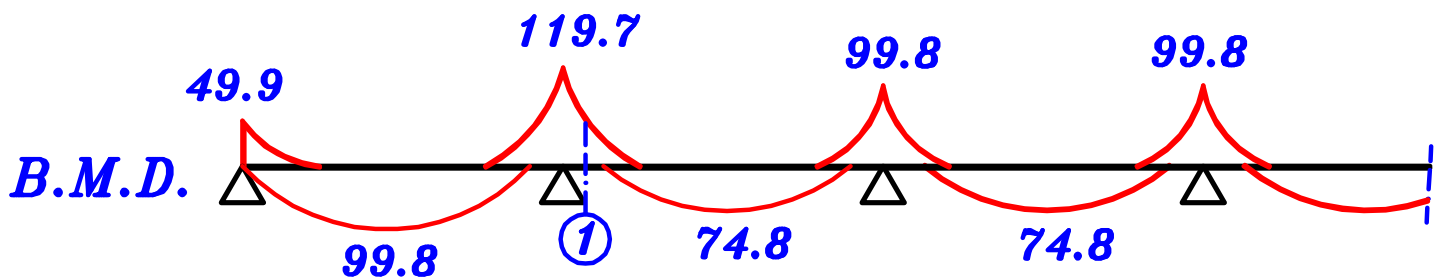
Dimensions of the Beam.

Take $b = 400 \text{ mm}$

$$* t_{\text{ben.}} = 3.50 \sqrt{\frac{M}{F_{cu} b}} = 3.50 \sqrt{\frac{119.7 \cdot 10^6}{25 \cdot 400}} = 494.9 \text{ mm}$$

$$* t_{\text{tor.}} \approx \frac{3 M_t}{1.6 \cdot b^2} = \frac{3 \cdot 62.7 \cdot 10^6}{1.6 \cdot 400^2} = 734.7 \text{ mm} \quad \boxed{t = 750 \text{ mm}}$$

Design the beam For bending. (400*750)



All Sections will be R-Sec.

Sec ① $M_{U.L.} = 119.7 \text{ kN.m}$

$$700 = C_1 \sqrt{\frac{119.7 \cdot 10^6}{25 \cdot 400}} \rightarrow C_1 = 6.40 \rightarrow J = 0.826$$

$$A_s = \frac{119.7 \cdot 10^6}{0.826 \cdot 360 \cdot 700} = 575.0 \text{ mm}^2$$

$$- \text{Check } A_{s_{\min.}} = \frac{1.1}{F_y} b d = \frac{1.1}{360} (400) (700) = 855.5 \text{ mm}^2$$

$$\therefore A_s < A_{s_{\min.}} \therefore \text{Take } A_s = A_{s_{\min.}}$$

$$\left. \begin{aligned} A_{s_{\min.}} &= \frac{1.1}{F_y} b d = \frac{1.1}{360} (400) (700) = 855.5 \\ 1.3 A_{s_{\text{req.}}} &= (1.3) (575.0) = 747.5 \\ \text{st. } 360/520 \frac{0.15}{100} b d &= \frac{0.15}{100} (400) (700) = 420 \end{aligned} \right\} 747.5 \text{ mm}^2$$

Shear

$$q_u = \frac{Q}{bd} = \frac{129.8 * 10^3}{400 * 700} = 0.463$$

$$q_{cu} = (0.24) \sqrt{\frac{25}{1.5}} = 0.98$$

$$q_{u_{max}} = (0.7) \sqrt{\frac{25}{1.5}} = 2.85$$

$$\sqrt{q_u^2 + q_{tu}^2} = \sqrt{0.463^2 + 1.59^2} = 1.65 < q_{u_{max}} \therefore \text{o.k.}$$

Torsion

$$x_1 = 320 \text{ mm}, y_1 = 670 \text{ mm}$$

$$q_{tu} = \frac{M_{tu}}{2 A_o t_e} = \frac{M_{tu} (x_1 + y_1)}{0.85 (x_1^2 + y_1^2)}$$

$$q_{tu} = \frac{62.7 * 10^6 (320 + 670)}{0.85 (320^2 + 670^2)} = 1.59$$

$$q_{t_{min}} = (0.06) \sqrt{\frac{25}{1.5}} = 0.245$$

$$q_u < q_{cu}, q_{tu} > q_{t_{min}} \therefore \text{Use RFT. For Torsion only.}$$

Torsion.

$$\therefore A_{str} = \frac{M_{tu} S_t}{(1.7) x_1 y_1 \left(\frac{F_y}{\delta_s}\right)} \quad \therefore A_{str} = \frac{(62.7 * 10^6) * S_t}{(1.7)(320)(670) (360/1.15)}$$

$$\therefore A_{str} = \frac{S_t}{1.82}$$

$$\text{The given } S_t = \frac{1000}{5} = 200 \text{ mm}$$

$$\therefore \text{required } A_{str} = \frac{S_t}{1.82}$$

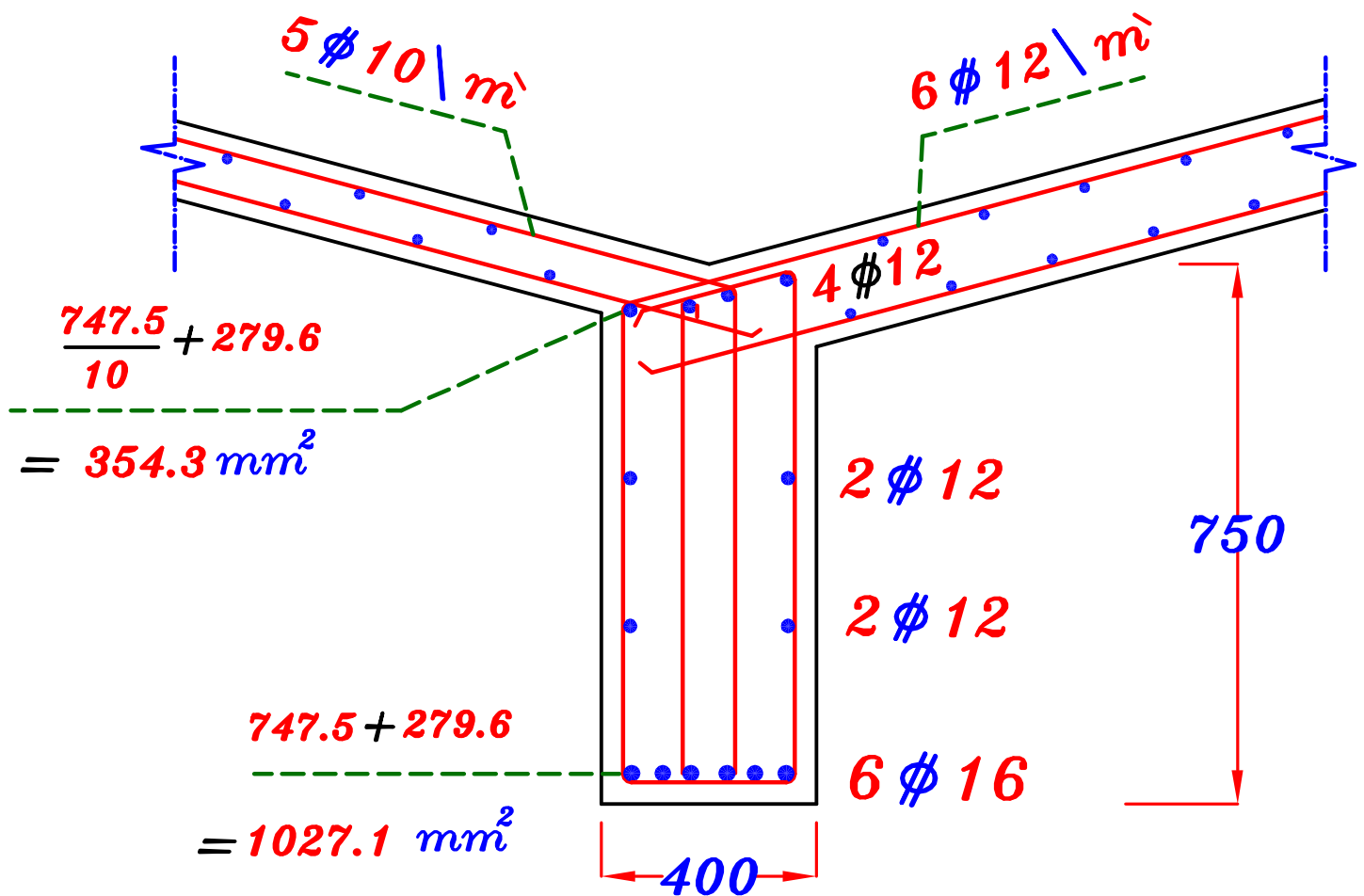
$$\therefore A_{str} = \frac{200}{1.82} = 109.9 \text{ mm}^2 < \phi 12 = 113 \therefore \text{o.k.}$$

$$\therefore \text{the stirrups } 6 \phi 12 \setminus m \text{ is enough.}$$

* Longitudinal Bars.

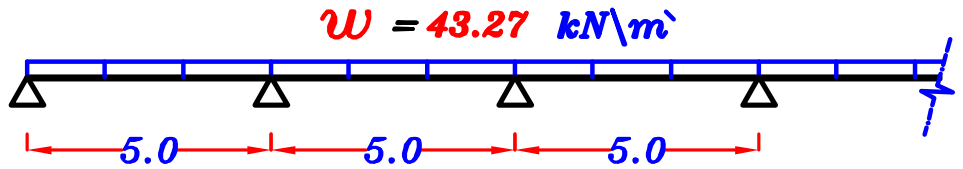
$$A_{sl} = 2 A_{str} \frac{(x_1 + y_1)}{S_t} \left(\frac{F_{y_{str.}}}{F_{y_{L.b.}}} \right) = 2 (113) \left(\frac{320 + 670}{200} \right) \left(\frac{360}{360} \right)$$

$$\therefore A_{sl} = 1118.7 \text{ mm}^2 \quad \therefore \frac{A_{sl}}{4} = 279.6 \text{ mm}^2$$

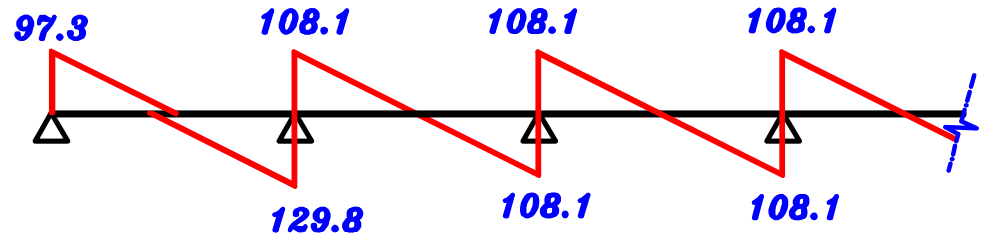


Normal Force & Bending Moment on the column.

Load



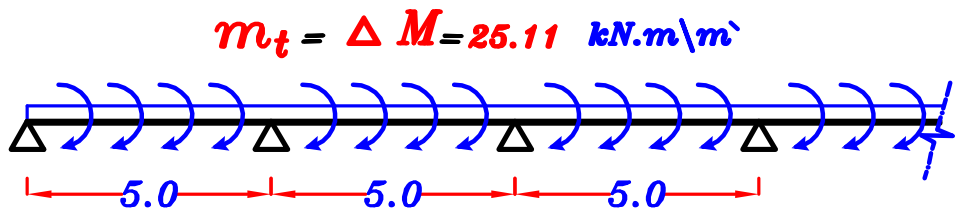
S.F.D.
For the beam.



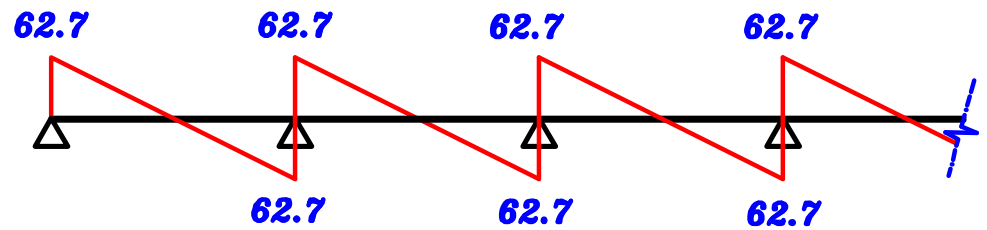
Reactions of the S.F.D.
=
Load on the column.



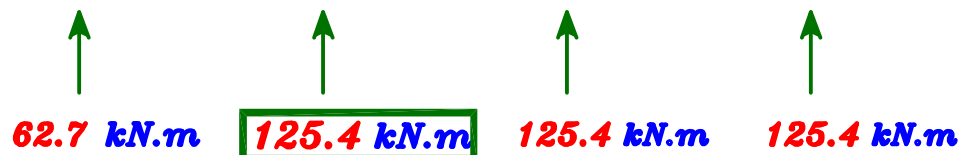
Torsion



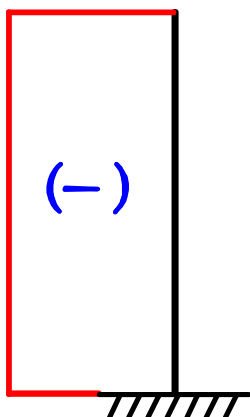
T.M.D.
For the beam.



Reactions of the T.M.D.
=
Moment on the column.

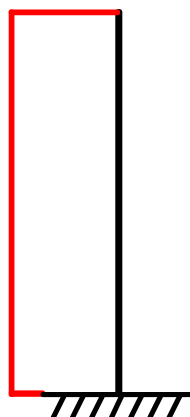


237.9 kN

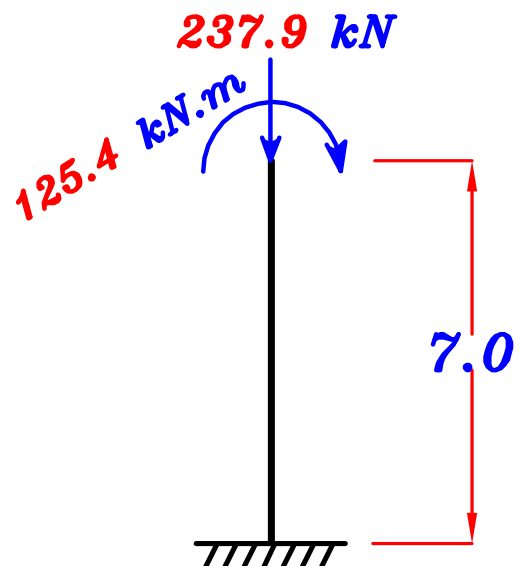


N.F.D.

125.4 kN.m



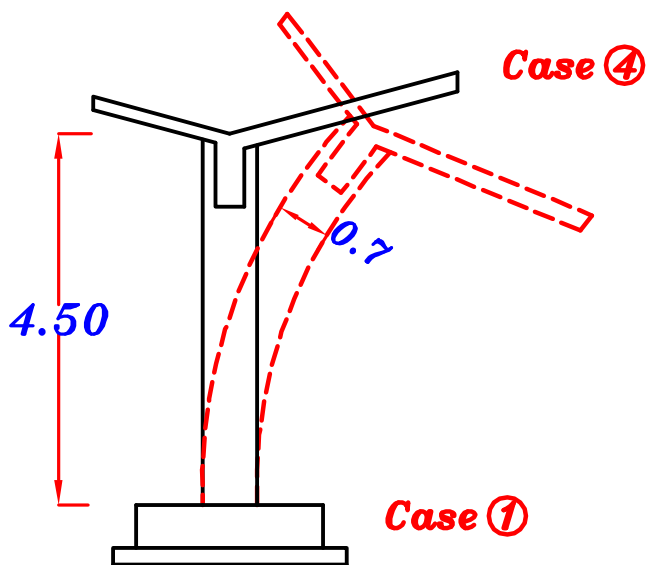
B.M.D.



Take the Column (300*700)

Check Buckling.

① In plane.



Upper Condition Case ④ }
Lower Condition Case ① } $k = 2.2$

$$H_o = 4.50 \text{ m}$$

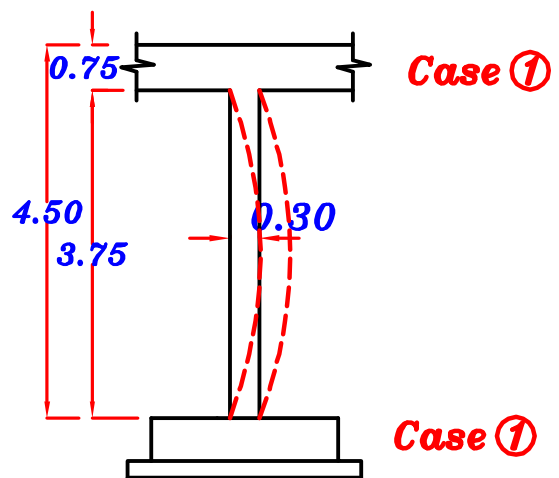
$$\lambda_b = \frac{2.2 * 4.5}{0.7} = 14.14 > 10$$

Take the bigger value of $\lambda_b = 14.14$ (In plane)

$$\delta = \frac{(\lambda_b)^2 * t}{2000} = \frac{14.14^2 * 0.7}{2000} = 0.07 \text{ m}$$

$$M_{add.} = P * \delta = 237.9 * 0.07 = 16.65 \text{ kN.m}$$

② Out of plane.

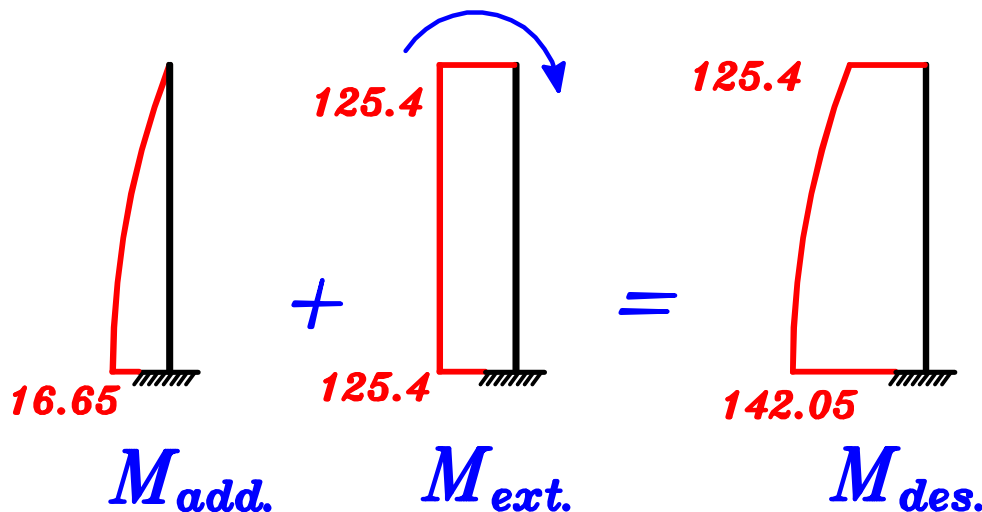


Upper Condition Case ① }
Lower Condition Case ① } $k = 1.2$

$$H_o = 3.75 \text{ m}$$

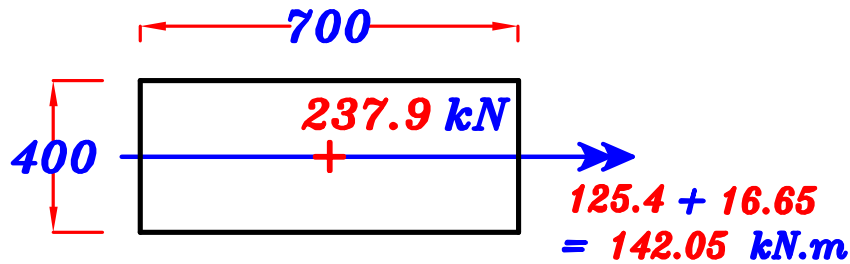
$$\lambda_b = \frac{1.2 * 3.75}{0.4} = 11.2 > 10$$

125.4 kN.m



$$\therefore M_{des.} = M_{ext.} + M_{add.}$$

$$\therefore M_{des.} = 125.4 + 16.65 = 142.05 \text{ kN.m}$$



Design the Sec.

$$e = \frac{M}{N} = \frac{142.05}{237.9} = 0.597 \text{ m} \quad \therefore \frac{e}{t} = \frac{0.597}{0.7} = 0.85 > 0.5 \xrightarrow{\text{use}} e_s$$

$$e_s = e + \frac{t}{2} - c = 0.597 + \frac{0.70}{2} - 0.05 = 0.897 \text{ m}$$

$$M_s = N * e_s = 237.9 * 0.897 = 213.4 \text{ kN.m}$$

$$\therefore 650 = C_1 \sqrt{\frac{213.4 * 10^6}{25 * 400}} \rightarrow C_1 = 4.45 \rightarrow J = 0.817$$

$$\therefore A_s = \frac{M_s}{J F_y d} - \frac{N_{u.l.}}{(F_y \setminus \delta_s)} = \frac{213.4 * 10^6}{0.817 * 360 * 650} - \frac{237.9 * 10^3}{(360 \setminus 1.15)} = 356.27 \text{ mm}^2$$

$$\text{Take } A_{s'} = A_s = 356.27 \text{ mm}^2$$

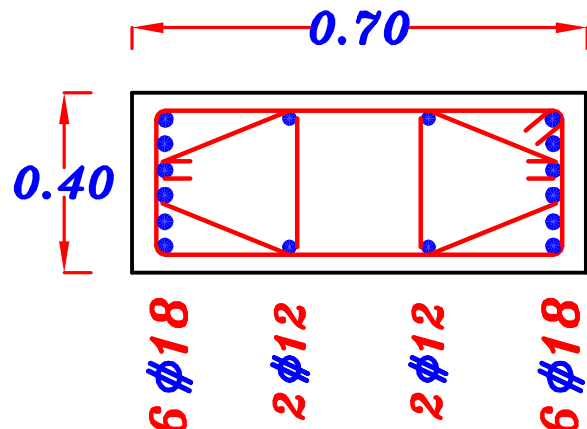
$$A_{s_{total}} = A_s + A_{s'} = 712.55 \text{ mm}^2$$

$$A_{s_{min}} = \frac{0.25 + 0.052 \lambda_{max}}{100} * b * t$$

$$= \frac{0.25 + 0.052 (14.14)}{100} * 400 * 700 = 2758.8 \text{ mm}^2 > A_{s_{total}}$$

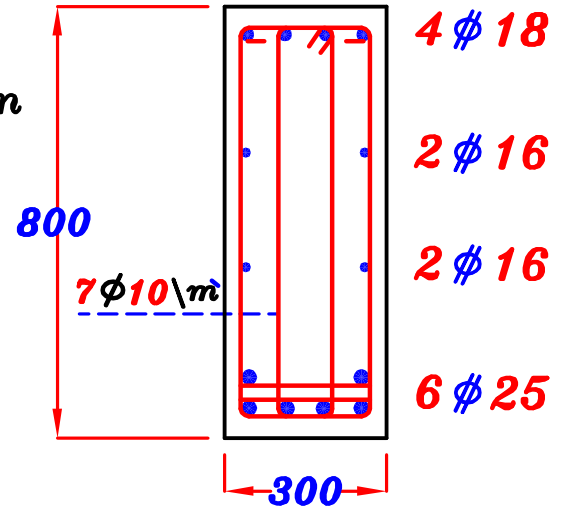
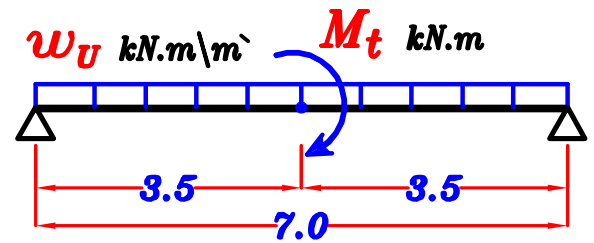
$$A_{s'} = A_s = \frac{A_{s_{min}}}{2} = 1379.4 \text{ mm}^2$$

6 ϕ 18



Example.

The Following Fig. shows a simple girder subjected to uniform distributed Factored load (w_U) including own weight and local torque moment (M_t) at mid span. It is required to calculate the max. torque moment (M_t), For the girder and its section given in the Figure.



Data.

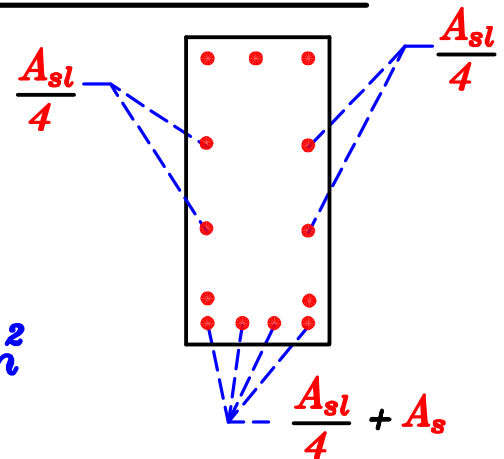
$$F_{cu} = 30 \text{ N/mm}^2$$

$$F_y = 400 \text{ N/mm}^2$$

Solution.

$$\frac{A_{sl}}{4} = 2 \phi 16$$

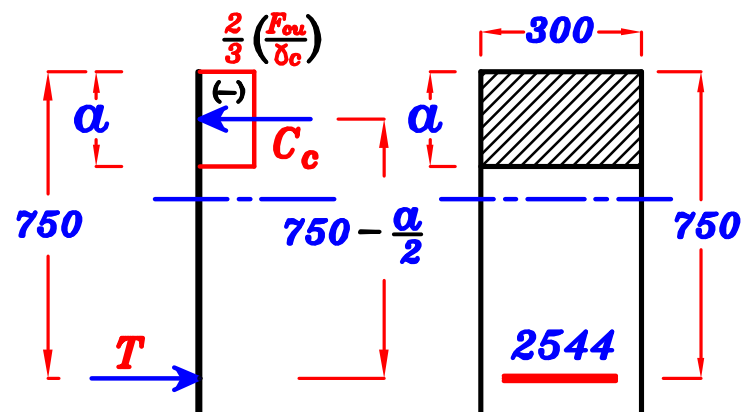
$$\begin{aligned} A_s (\text{From moment}) &= 6 \phi 25 - 2 \phi 16 \\ &= 2946 - 402 = 2544 \text{ mm}^2 \end{aligned}$$



Use First Principles to get $M_{U.L.}$

$$\begin{aligned} \frac{2}{3} \frac{F_{cu}}{\gamma_c} * \alpha * b &= A_s * \frac{F_y}{\gamma_s} \\ \frac{2}{3} \left(\frac{30}{1.5} \right) (\alpha) (300) &= (2544) \left(\frac{400}{1.15} \right) \end{aligned}$$

$$\alpha = 221.2 \text{ mm} > 0.1 d$$



$$\begin{aligned} M_{U.L.} &= \frac{2}{3} \frac{F_{cu}}{\gamma_c} \alpha b \left(d - \frac{\alpha}{2} \right) = \frac{2}{3} \left(\frac{30}{1.5} \right) (221.2) (300) \left(750 - \frac{221.2}{2} \right) \\ &= 565741120 \text{ N.mm} = 565.7 \text{ kN.m} \end{aligned}$$

$$\therefore M_{U.L.} = 565.7 = \frac{w_U (7.0)^2}{8}$$

$$\therefore w_U = 92.45 \text{ kN/m}$$

$$Q_{max} = \frac{w * L}{2} = 323.5 \text{ kN}$$

$$q_u = \frac{Q}{bd} = \frac{323.5 * 10^3}{300 * 750} = 1.437 \text{ N/mm}^2$$

$$\therefore q_{cu} = (0.24) \sqrt{\frac{30}{1.5}} = 1.07$$

$$S_s = S_t = \frac{1000}{7.0} = 142.85 \text{ mm}$$

$$q_u - \frac{q_{cu}}{2} = \frac{n A_s (F_y / \delta_s)}{b S_s} \quad \therefore 1.437 - \frac{1.07}{2} = \frac{4 * A_s (240 / 1.15)}{(300) (142.85)}$$

$$\therefore A_s = 46.3 \text{ mm}^2$$

$$\therefore A_{s_{outer}} = A_{str} + A_s = A_{str} + 46.3 = 78.5 \text{ mm}^2$$

$$\therefore A_{str} = 32.2 \text{ mm}^2$$

$$x_1 = 220 \text{ mm}, y_1 = 720 \text{ mm}$$

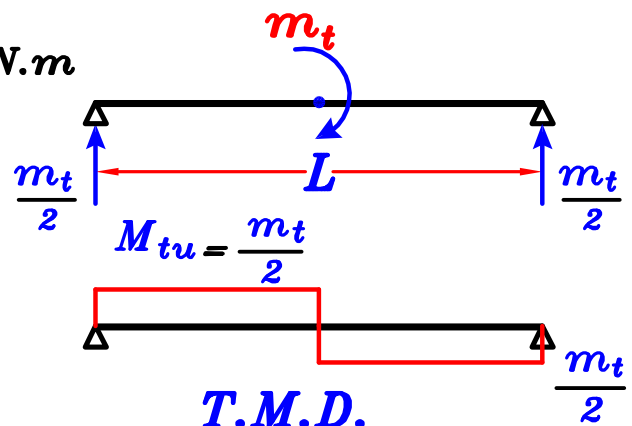
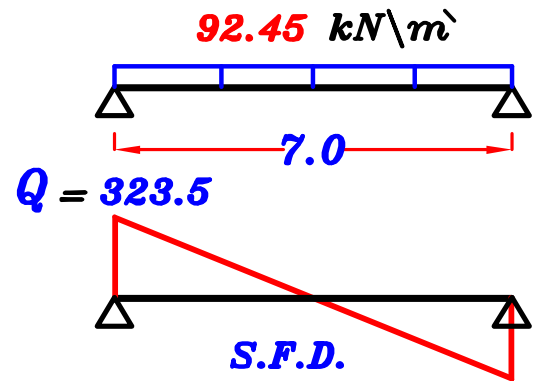
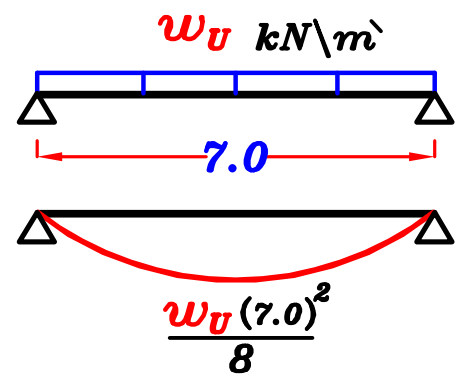
$$\therefore A_{str} = \frac{M_{tu} S_t}{(1.7) x_1 y_1 \left(\frac{F_y}{\delta_s} \right)} \quad \therefore 32.2 = \frac{M_{tu} * (142.85)}{(1.7)(220)(720) (240 / 1.15)}$$

$$\therefore M_{tu} = 12667564.6 \text{ N.mm} = 12.66 \text{ kN.m}$$

$$\therefore M_{tu} = \frac{m_t}{2}$$

$$\therefore m_t = 12.66 * 2 = 25.32 \text{ kN.m}$$

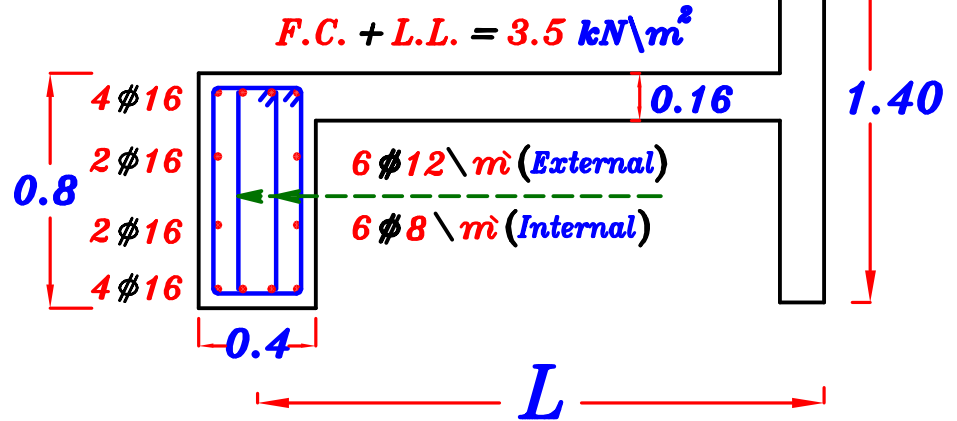
$$\therefore M_t = 25.32 \text{ kN.m}$$



Example.

$$F_{cu} = 25 \text{ N/mm}^2$$

$$F_y = 360 \text{ N/mm}^2$$



The Figure shows the cross-section dimensions and RFT. of continuous beam supported on columns spaced 6.0m . The beam has a cantilever solid slab of 0.16m thickness. It is required to calculate the safety length of the cantilever slab to satisfy the RFT. & loads shown in Fig.

$$w_s = 1.5 (0.16 * 25 + 3.50) = 11.25 \text{ kN/m}^2$$

$$\text{parapet weight} = 1.4 (0.15 * 1.4 * 1.0) * 25 = 7.35 \text{ kN/m}$$

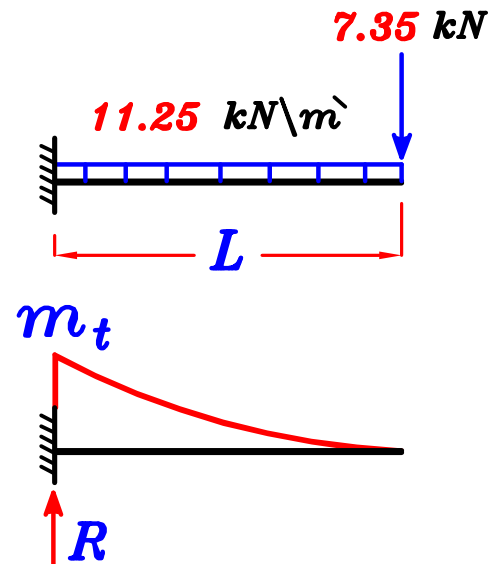
$$\text{o.w. (beam)} = 1.4 (0.4 * 0.8 * 1.0) * 25 = 11.2 \text{ kN/m}$$

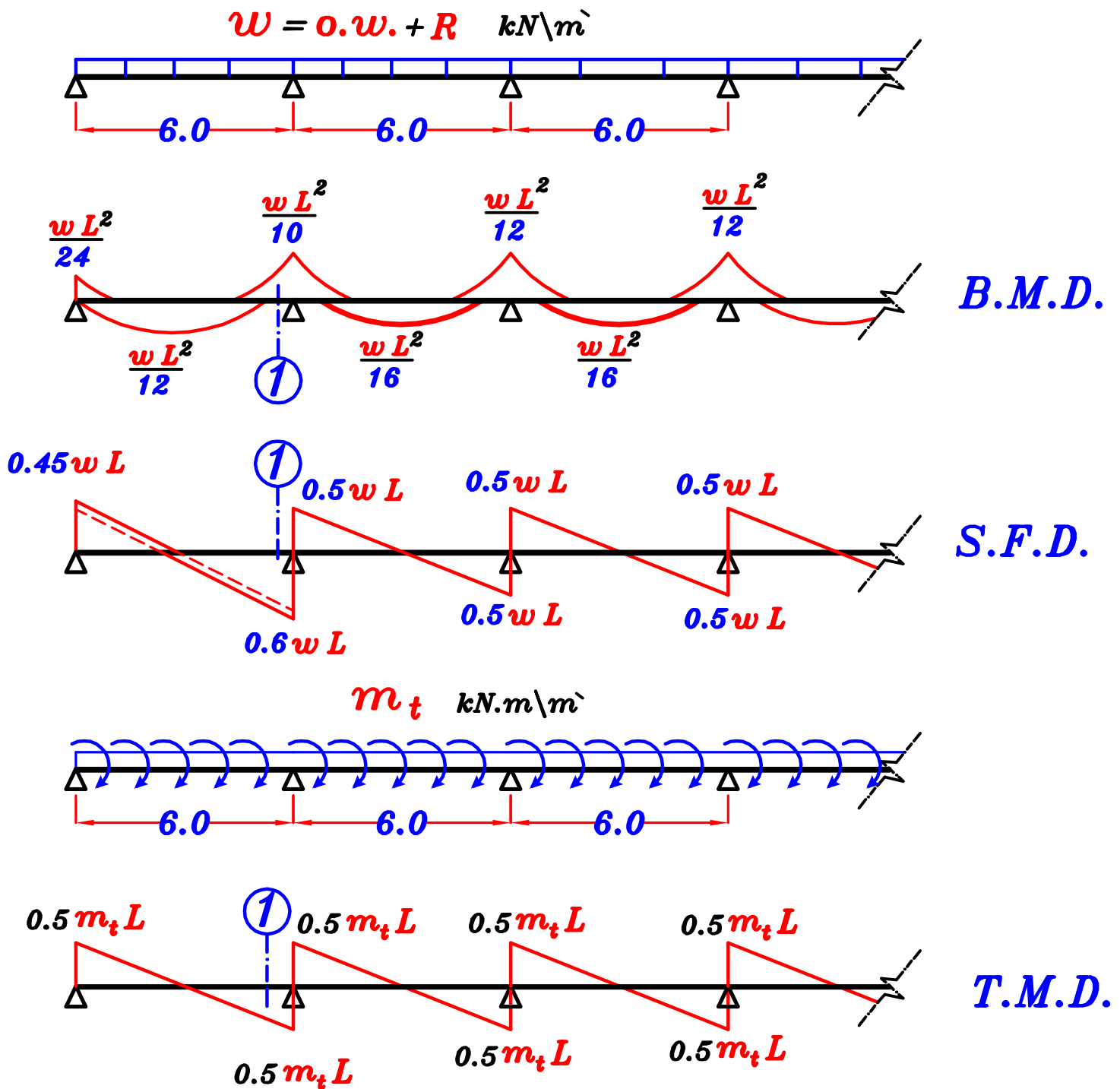
Strip in the slab.

$$R = 11.25 L + 7.35 \text{ kN}$$

$$\begin{aligned} m_t &= 7.35 L + 11.25 \frac{L^2}{2} \text{ kN.m} \\ &= 7.35 L + 5.625 L^2 \end{aligned}$$

$$\begin{aligned} w &= \text{o.w.} + R = 11.2 + (11.25 L + 7.35) \text{ kN/m} \\ &= 18.55 + 11.25 L \end{aligned}$$





For Sec. ①

$$M_{U.L.} = \frac{wL^2}{10} = \frac{(18.55 + 11.25L) * 6.0^2}{10} \quad \therefore \boxed{M_{U.L.} = 66.78 + 40.5L} \quad kN.m$$

$$Q = 0.6wL = 0.6(18.55 + 11.25L) * 6.0 \quad \therefore \boxed{Q_{U.L.} = 66.78 + 40.5L} \quad kN$$

$$M_t = 0.5m_tL = 0.5(7.35L + 5.625L^2) * 6.0$$

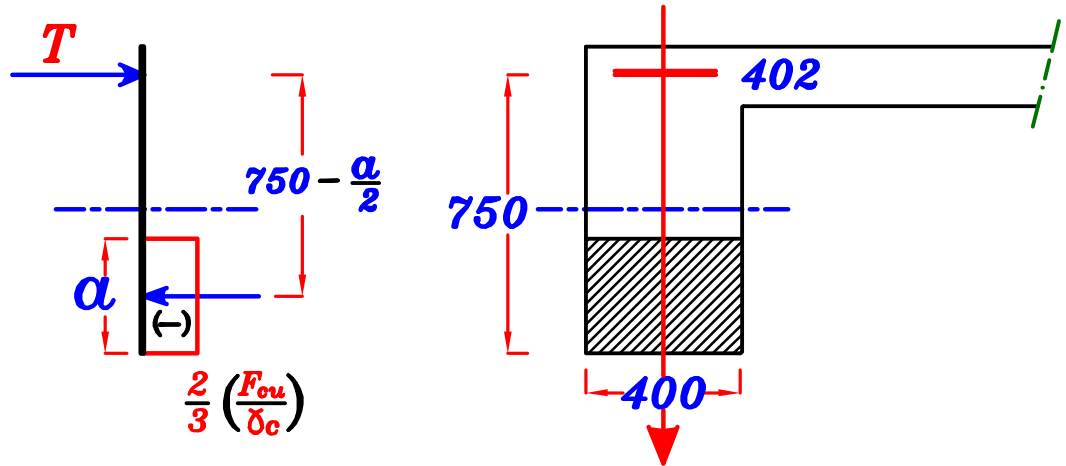
$$\therefore \boxed{M_t = 22.05L + 16.87L^2} \quad kN.m$$

For Bending Moment.

$$\frac{A_{sl}}{4} = 2 \phi 16$$

$$A_s (\text{From moment}) = 4 \phi 16 - 2 \phi 16 = 804 - 402 = 402 \text{ mm}^2$$

Neglect A_s'



Use First Principles to get $M_{U.L.}$

$$\frac{2}{3} \frac{F_{cu}}{\gamma_c} * \alpha * b = A_s * \frac{F_y}{\gamma_s}$$

$$\frac{2}{3} \left(\frac{25}{1.5} \right) (\alpha) (400) = (402) \left(\frac{360}{1.15} \right) \longrightarrow \alpha = 28.3 \text{ mm} < 0.1 d$$

$$M_{U.L.} = 0.826 A_s F_y d = 0.826 (402) (360) (750) \\ = 89654040 \text{ N.mm} = 89.65 \text{ kN.m}$$

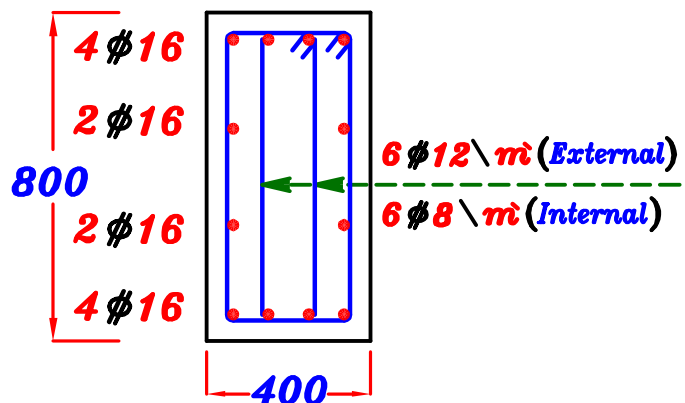
$$\therefore M_{U.L.} = 89.65 \text{ kN.m}$$

$$n = 4.0$$

$$A_{s_{inner}} = \phi 8 = A_s = 50.3 \text{ mm}^2$$

$$A_{s_{outer}} = \phi 12 = A_{str} + A_s = 113 \text{ mm}^2$$

$$\therefore A_{str} = 113 - 50.3 = 62.7 \text{ mm}^2$$



For Shear

$$q_{cu} = (0.24) \sqrt{\frac{25}{1.5}} = 0.98, \quad S_s = S_t = \frac{1000}{6.0} = 166.67 \text{ mm}$$

$$\therefore q_u - \frac{q_{cu}}{2} = \frac{n A_s (F_y \setminus \delta_s)}{b S_s}$$

$$\therefore q_u - \frac{0.98}{2} = \frac{4 (50.3) (360 \setminus 1.15)}{(400) (166.67)} \longrightarrow q_u = 1.434 \text{ N/mm}^2$$

$$\therefore q_u = \frac{Q}{b d} \quad \therefore 1.434 = \frac{Q}{(400) (750)} \longrightarrow Q = 430200 \text{ N}$$

$$\therefore \boxed{Q_{U.L.} = 430.2 \text{ kN}}$$

For Torsional Moment.

$$A_{s_{inner}} = \phi 8 = A_s = 50.3 \text{ mm}^2, \quad A_{s_{outer}} = \phi 12 = A_{str} + A_s = 113 \text{ mm}^2$$

$$\therefore A_{str} = 113 - 50.3 = 62.7 \text{ mm}^2$$

$$x_1 = 320 \text{ mm}, \quad y_1 = 720 \text{ mm} \quad S_s = S_t = \frac{1000}{6.0} = 166.67 \text{ mm}$$

$$\therefore A_{str} = \frac{M_{tu} S_t}{(1.7) x_1 y_1 (F_y \setminus \delta_s)}$$

$$\therefore 62.7 = \frac{M_{tu} \cdot (166.67)}{(1.7)(320)(720) (360/1.15)} \longrightarrow M_{tu} = 46126039 \text{ N.mm}$$

$$\therefore \boxed{M_t = 46.12 \text{ kN.m}}$$

To get the safety length of the cantilever get the least of L_1, L_2, L_3

$$- M_{U.L.} = 89.65 \text{ kN.m} = 66.78 + 40.5 L_1 \longrightarrow L_1 = 0.564 \text{ m}$$

$$- Q_{U.L.} = 430.2 \text{ kN} = 66.78 + 40.5 L_2 \longrightarrow L_2 = 8.97 \text{ m}$$

$$- M_t = 46.12 \text{ kN.m} = 22.05 L_3 + 16.87 L_3^2 \longrightarrow L_3 = 1.12 \text{ m}$$

$$\therefore \boxed{L = 0.564 \text{ m}}$$

Example.

For The given Figure of structural plan.

It is required to :

Draw B.M.D. , S.F.D. & T.M.D. For the main Girder (G)

Data.

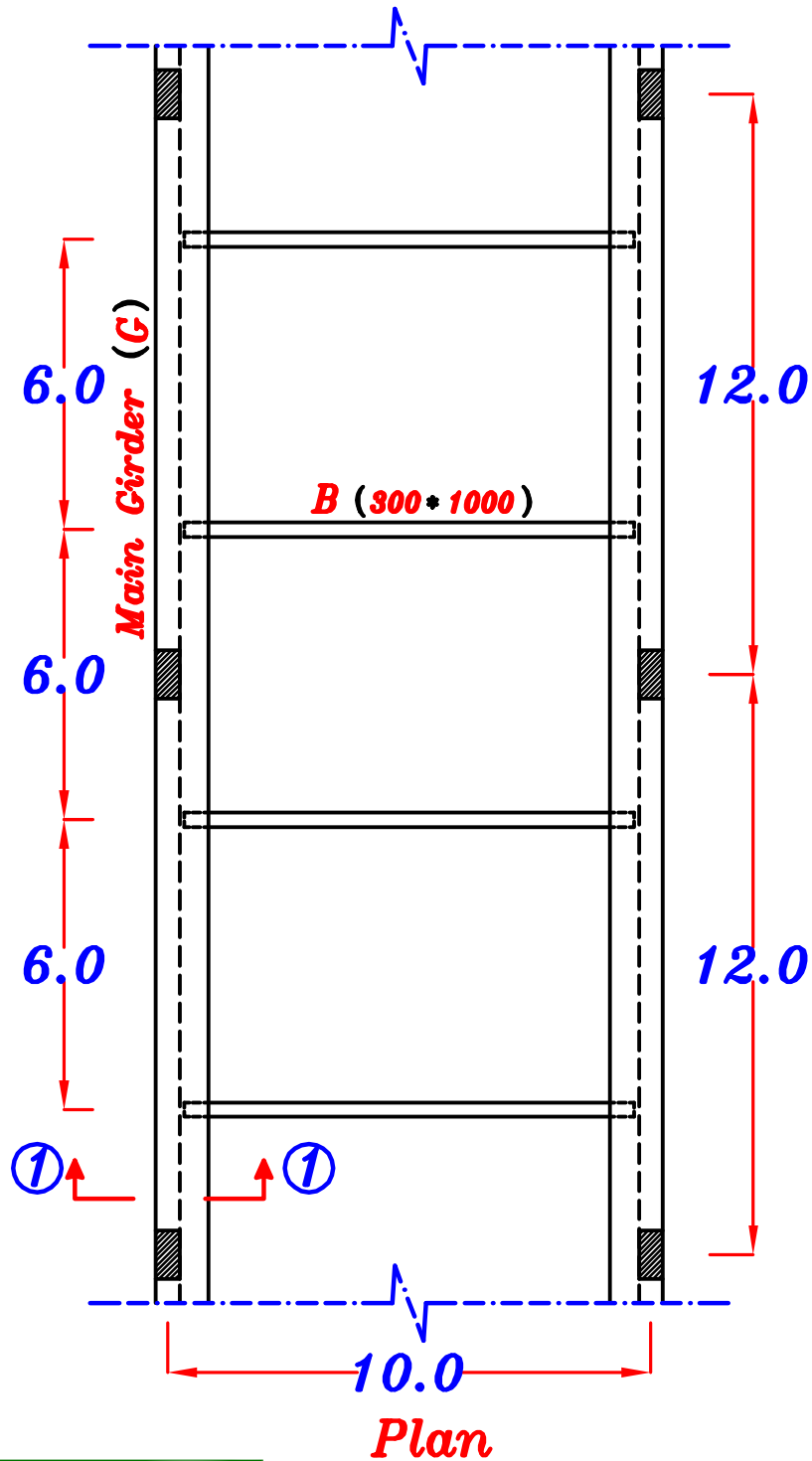
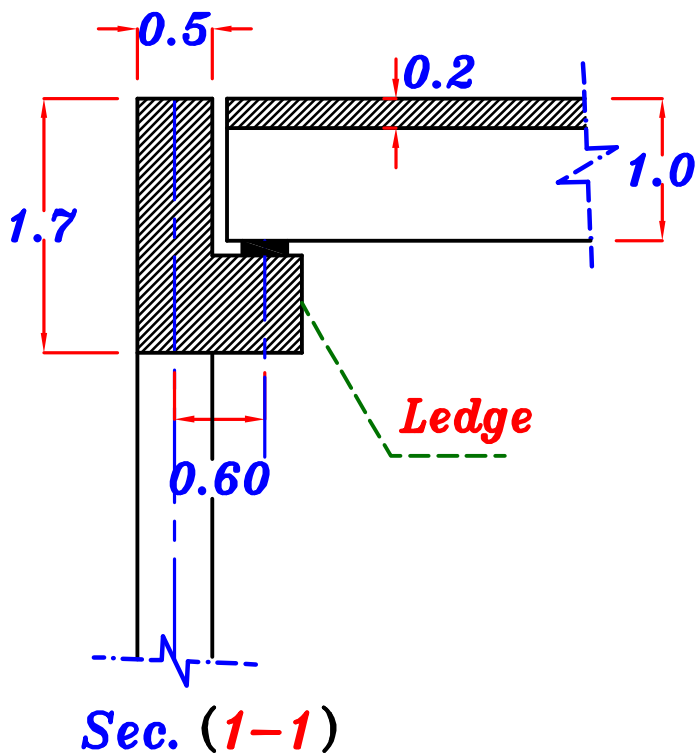
$$F_{cu} = 40 \text{ N/mm}^2$$

$$F_y = 400 \text{ N/mm}^2$$

$$F.C. = 3.0 \text{ kN/m}^2$$

$$L.L. = 5.0 \text{ kN/m}^2$$

$$t_s = 200 \text{ mm}$$



Neglect effect of Ledge إهمل تأثير النتوء
Design of Ledge is not required

Solution.

$$w_s = 1.4 (t_s \delta_c + F.C.) + 1.6 (L.L.)$$

$$w_s = 1.4 (0.20 * 25 + 3.0) + 1.6 (5.0) = 19.2 \text{ kN/m}^2$$

$$O.W. = 1.4 (b * t * \delta_c) = 1.4 (0.3 * 1.0 * 25) = 10.5 \text{ kN/m}$$

(Beam)

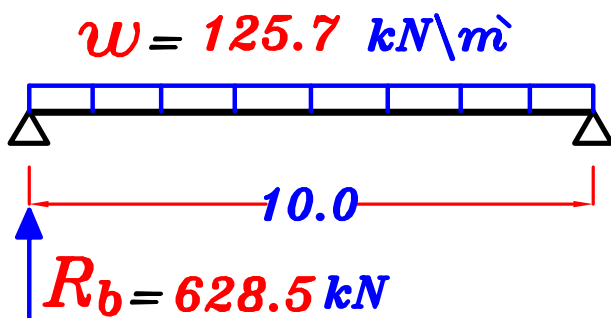
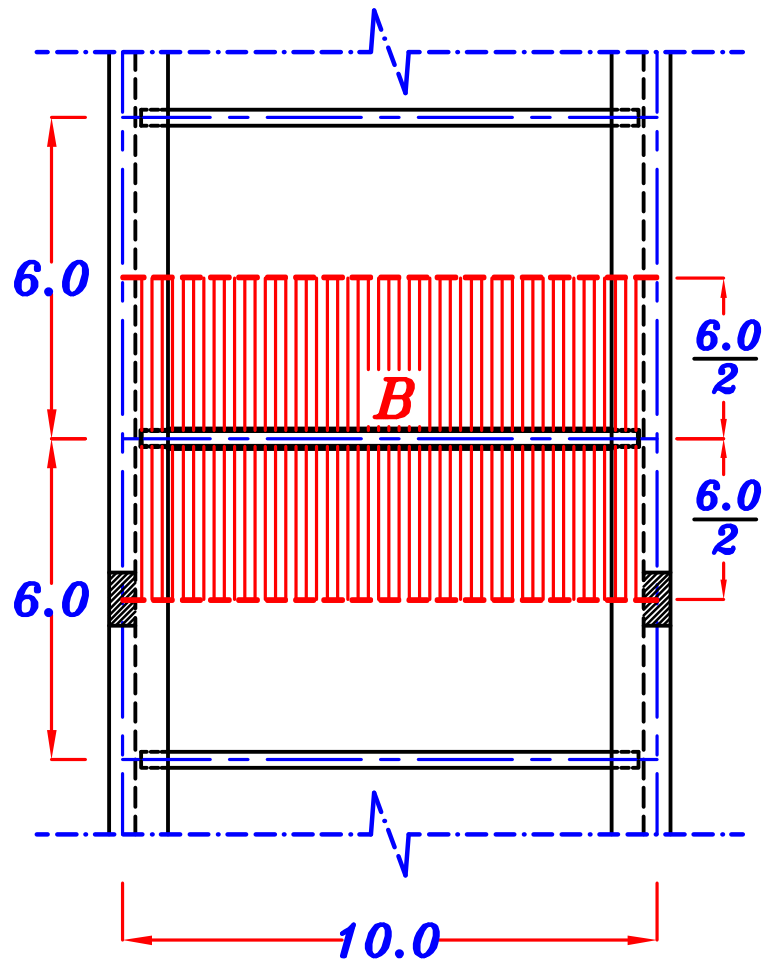
$$O.W. = 1.4 (b * t * \delta_c) = 1.4 (0.5 * 1.7 * 25) = 29.75 \text{ kN/m}$$

(Girder)

Neglect effect of Ledge إهمل تأثير النتوء

Beam.

$$\begin{aligned} W(\text{Beam}) &= O.W. + 2 w_s \frac{L_s}{2} \\ &= 10.5 + 2 (19.2) \frac{6.0}{2} \\ &= 125.7 \text{ kN/m} \end{aligned}$$

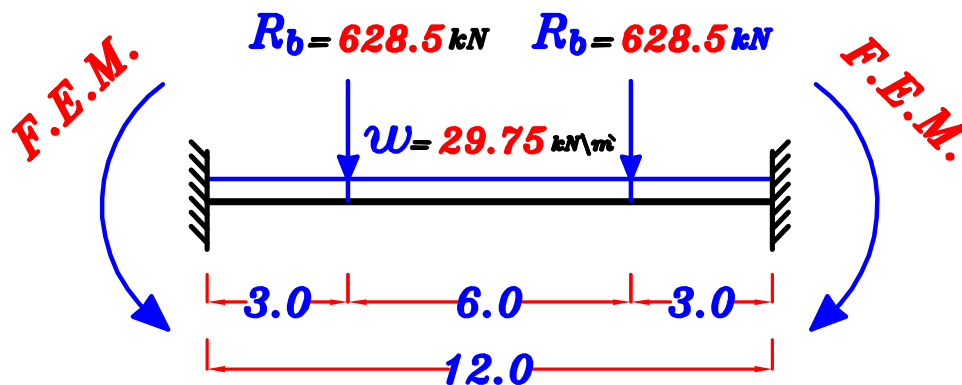
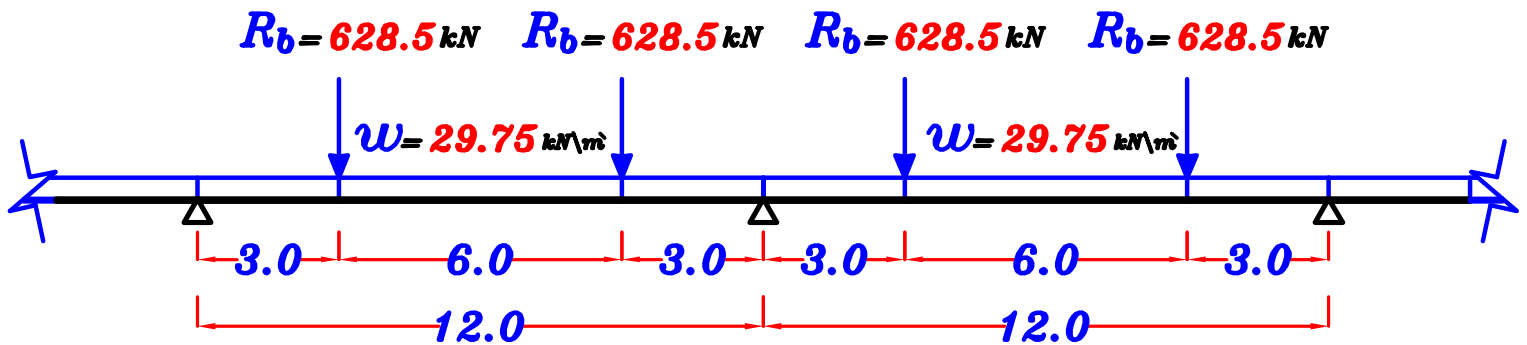


$$R_b = 628.5 \text{ kN}$$

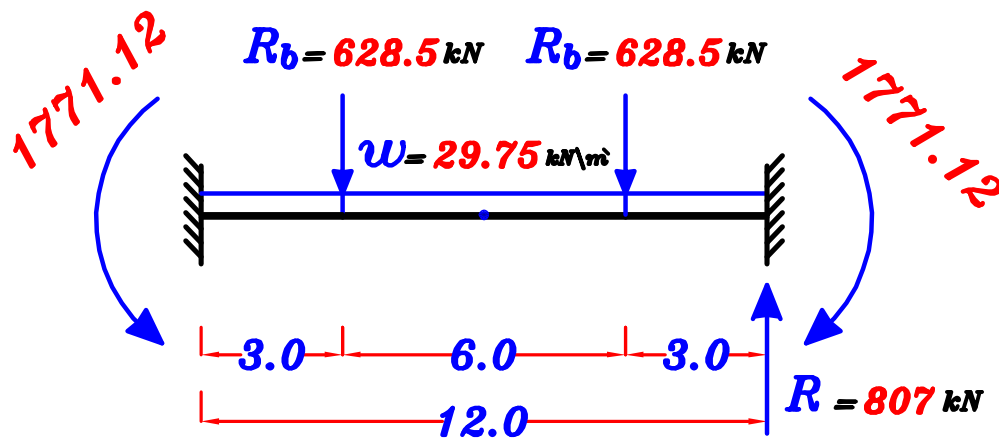
Main Girder.

$$W(\text{Girder}) = O.W. = 29.75 \text{ kN/m}$$

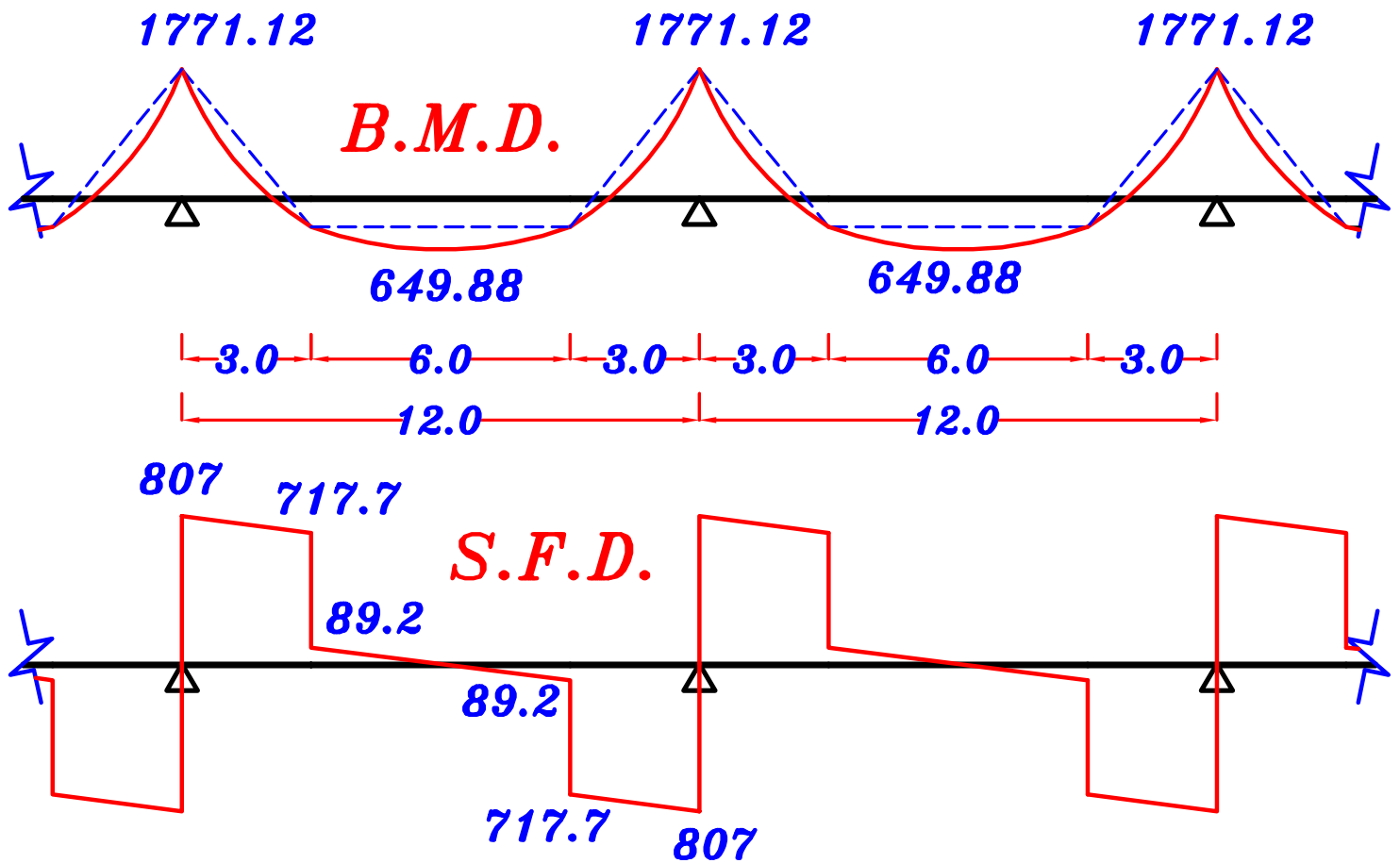
Loads on Girder.



$$\begin{aligned} \text{Fixed End Moment} &= \frac{wL^2}{12} + \frac{Pa(b)^2}{L^2} + \frac{Pb(a)^2}{L^2} \\ &= \frac{29.75 * 12^2}{12} + \frac{628.5 * 3 * 9^2}{12^2} + \frac{628.5 * 9 * 3^2}{12^2} = 1771.12 \text{ kN.m} \end{aligned}$$



$$\begin{aligned} B.M. (\text{at mid. span}) &= 807 (6.0) - 1771.12 - 29.75 * 6.0 (3.0) - 628.5 (3.0) \\ &= 649.88 \text{ kN.m} \end{aligned}$$



Torsion on Girder.

$$M_t = 628.5 * 0.60 = 337.1 \text{ kN.m}$$

