

Electrical Machines

Chapter 0 Review of AC Circuits

Instructor

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Electrical Machines

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Lecture 2

Chapter Focus:

- DC basics → single-phase AC → three-phase AC
- Sinusoidal waveforms, phasors and impedance.
- AC power and power factor
- Short formulas, clear examples, key ideas only
- Preparation for transformers and AC machines.

Key Relationships:

$$V(t) = V_m \cos(\omega t + \varphi)$$

$$V = Z I$$

$$P = V_{rms} I_{rms} \cos(\theta)$$

$$P_{3ph} = \sqrt{3} V_L I_L \cos(\theta)$$

DC Circuits → Single-Phase AC → Three-Phase AC → Electrical Machines

Main Definitions

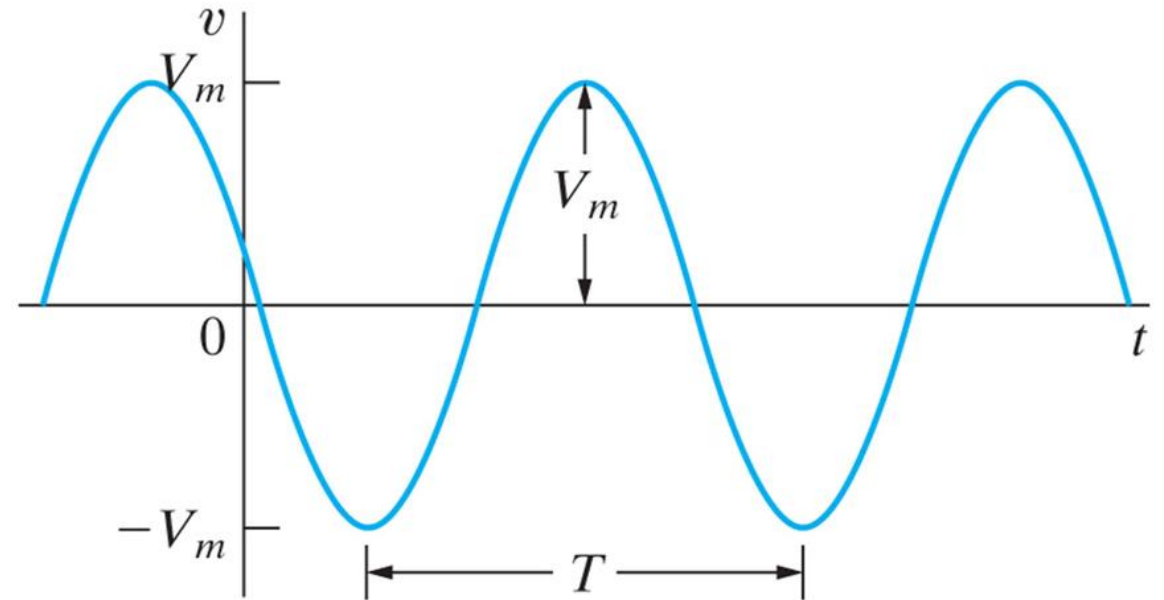
$$V(t) = V_m \cos(\omega t + \varphi)$$

T = period

$$f = 1/T \text{ (Hz)}$$

$$\omega = 2\pi f = 2\pi/T \text{ (rad/s)}$$

φ = phase angle



$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

A sinusoid is defined by magnitude, frequency, and phase.

Example 1

A sinusoidal current has a maximum amplitude of **20 A**. The current passes through one complete cycle in **1 ms**. The magnitude of the current at zero time is **10 A**.

- a) What is the frequency of the current in hertz?
- b) What is the frequency in radians per second?
- c) Write the expression for $i(t)$ using the cosine function. Express φ in degrees.
- d) What is the rms value of the current?

Answer

a) $f = 1/T = 1000 \text{ Hz}$

b) $\omega = 2\pi f = 2000\pi \text{ rad/s}$

c) $i(t) = I_m \cos(\omega t + \varphi)$

$$10 = 20 \cos\varphi \Rightarrow \varphi = 60^\circ \rightarrow i(t) = 20 \cos(2000\pi t + 60^\circ) \text{ A}$$

d) $I_{rms} = \frac{I_m}{\sqrt{2}} = 14.14 \text{ A}$

Example 2

A sinusoidal voltage is given by the expression: $v(t) = 300 \cos(120\pi t + 30^\circ)$

- a) What is the period of the voltage in ms?
- b) Write the magnitude of the voltage at $t=2.778$ ms?
- c) What is the rms value of the voltage?

Answer

a) $\omega = 120\pi \text{ rad/s}$

$$f = \omega / 2\pi = 60 \text{ Hz} \rightarrow T = 1/f = 16.667 \text{ ms}$$

b) $v(2.778 \text{ ms}) = 300 \cos(120\pi \times 2.778 \times 10^{-3} + 30^\circ) = 0 \text{ V}$

c) $V_{rms} = \frac{300}{\sqrt{2}} = 212.13 \text{ V}$

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Lecture 3

Phasor Basics

Time domain: $V(t) = V_m \cos(\omega t + \varphi)$

$V_m e^{j\varphi} \rightarrow$ *frequency domain*

$V_m e^{j\varphi} \rightarrow$ exponential form

$V = V_m \cos(\varphi) + jV_m \sin(\varphi) \rightarrow$ rectangular form

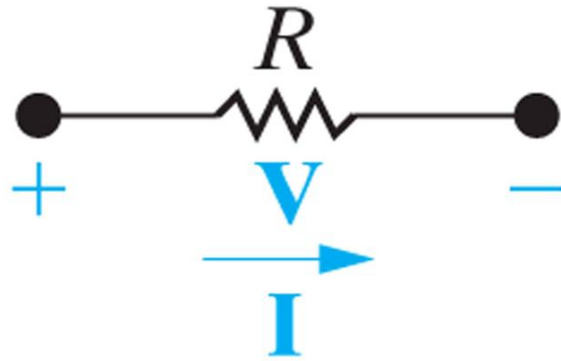
$V = V_m \angle \varphi \rightarrow$ angle notation form (Phasor form)

Example: $100 \angle -26^\circ \leftrightarrow 100 \cos(\omega t - 26^\circ)$

Phasors keep magnitude and phase in one compact quantity.

Use phasors only for sinusoidal steady-state signals of the same angular frequency ω .

9 V-I Relationship for a Resistor

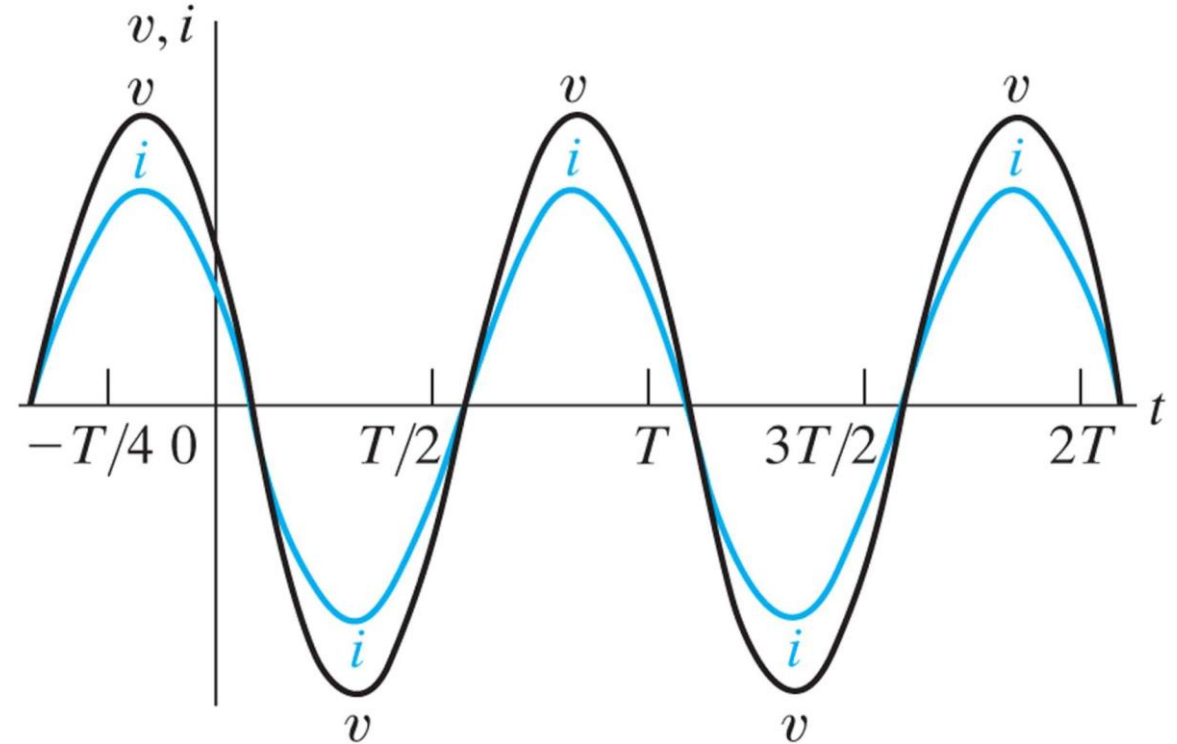


Resistor

$$Z_R = R$$

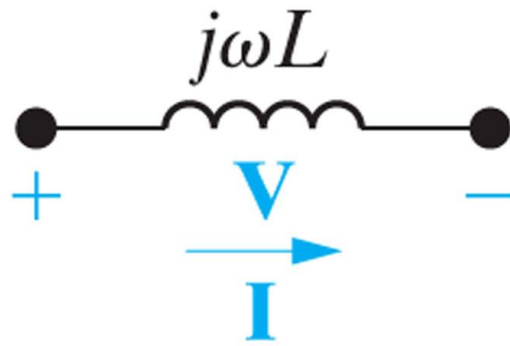
$$V = R I$$

v and i are in phase



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Purely resistive load $\rightarrow$ power factor = 1

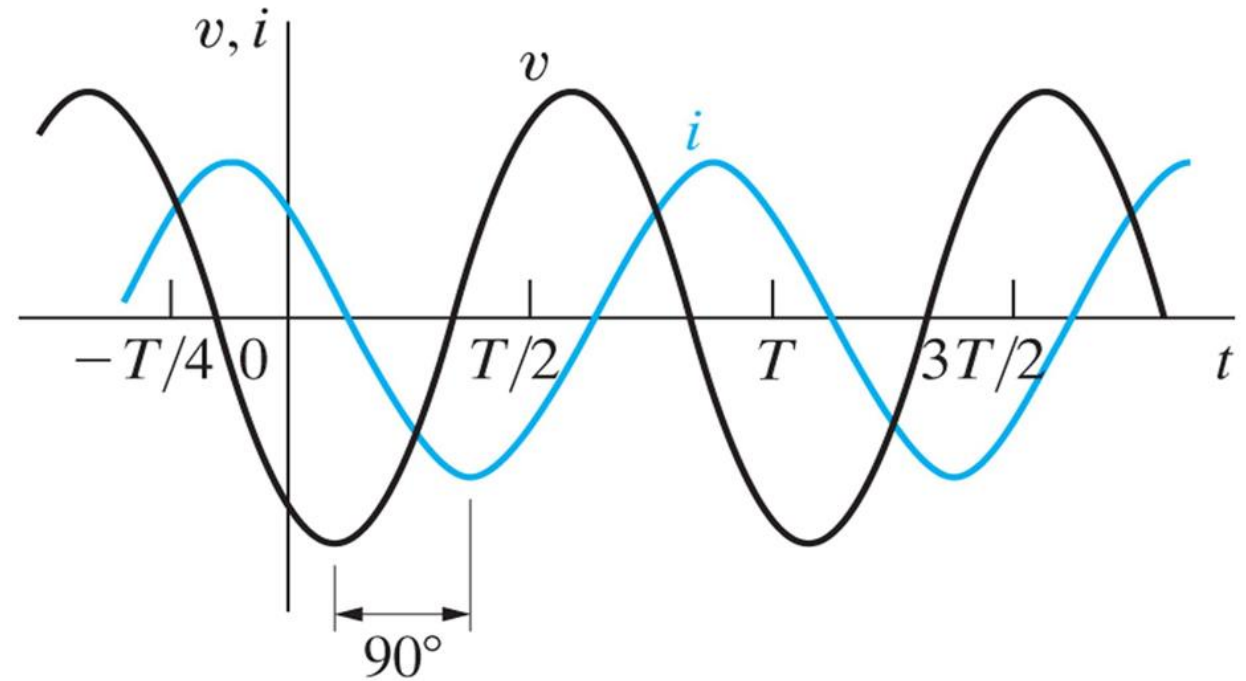


Inductor

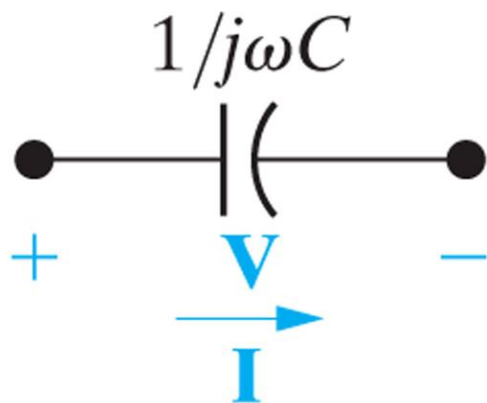
$$Z_L = j\omega L$$

$$V = Z_L I = j\omega L I$$

i lags v by 90°



Purely inductive load $\rightarrow$ lagging power factor

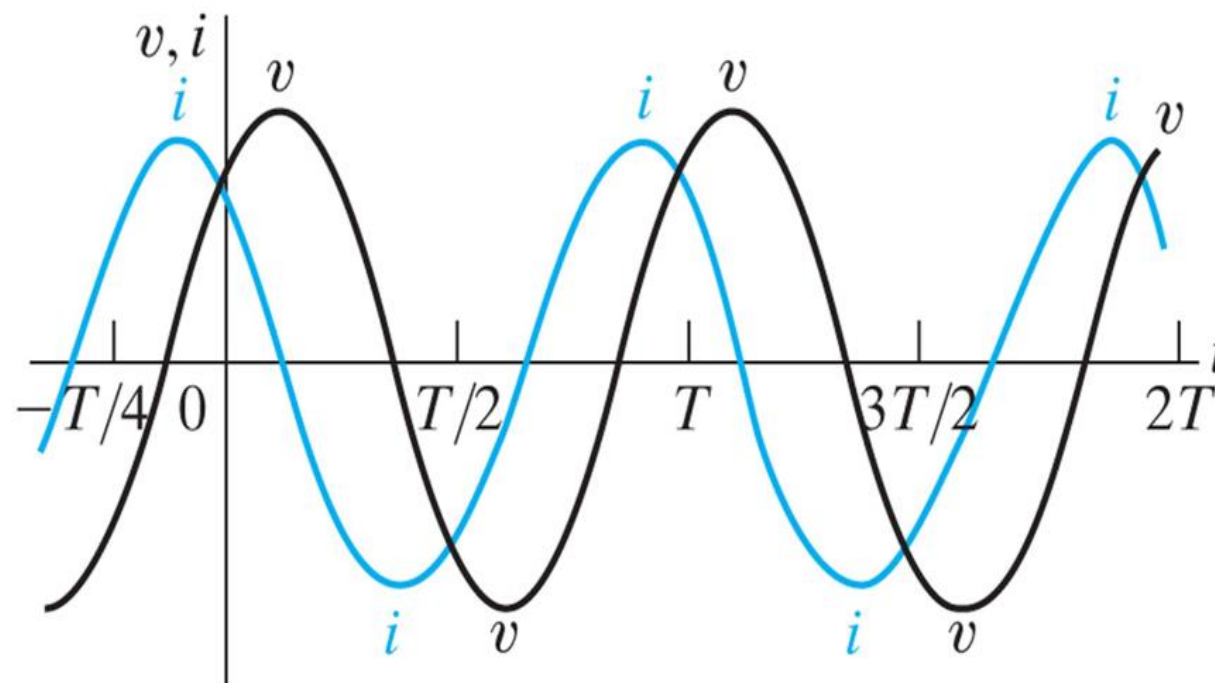


Capacitor

$$Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

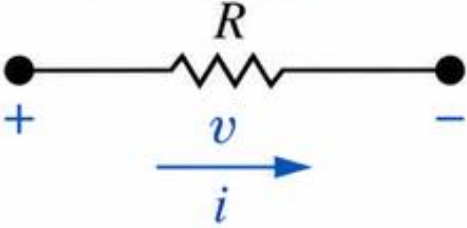
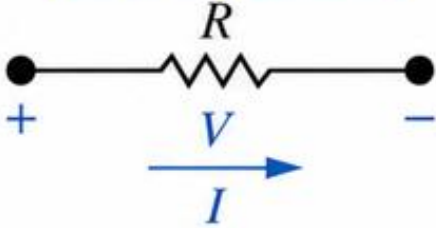
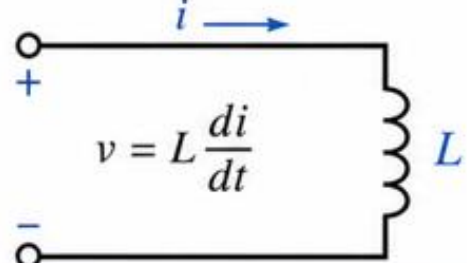
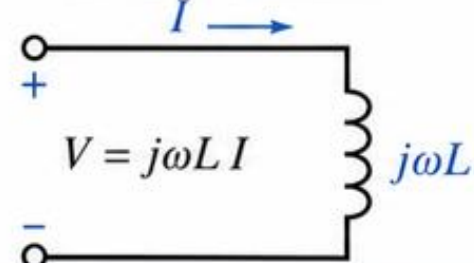
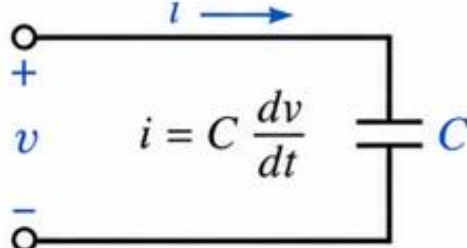
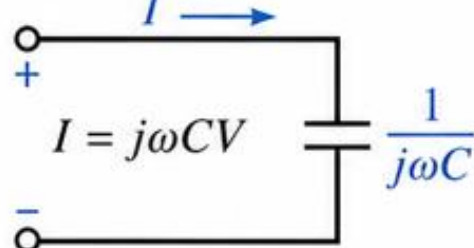
$$V = Z_C I = \frac{-j}{\omega C} I$$

i leads v by 90°



Purely capacitive load $\rightarrow$ leading power factor

Transformation of Circuit Elements from Time Domain to Frequency Domain

1 Resistor R	Time domain  $v = R i$ → Frequency domain Frequency domain  $V = R I$ Impedance: $Z = R$
2 Inductor L	Time domain  $v = L \frac{di}{dt}$ → Frequency domain Frequency domain  $V = j\omega L I$ Impedance: $Z = j\omega L$
3 Capacitor C	Time domain  $i = C \frac{dv}{dt}$ → Frequency domain Frequency domain  $I = j\omega C V$ Impedance: $Z = \frac{1}{j\omega C}$

AC Ohm's Law

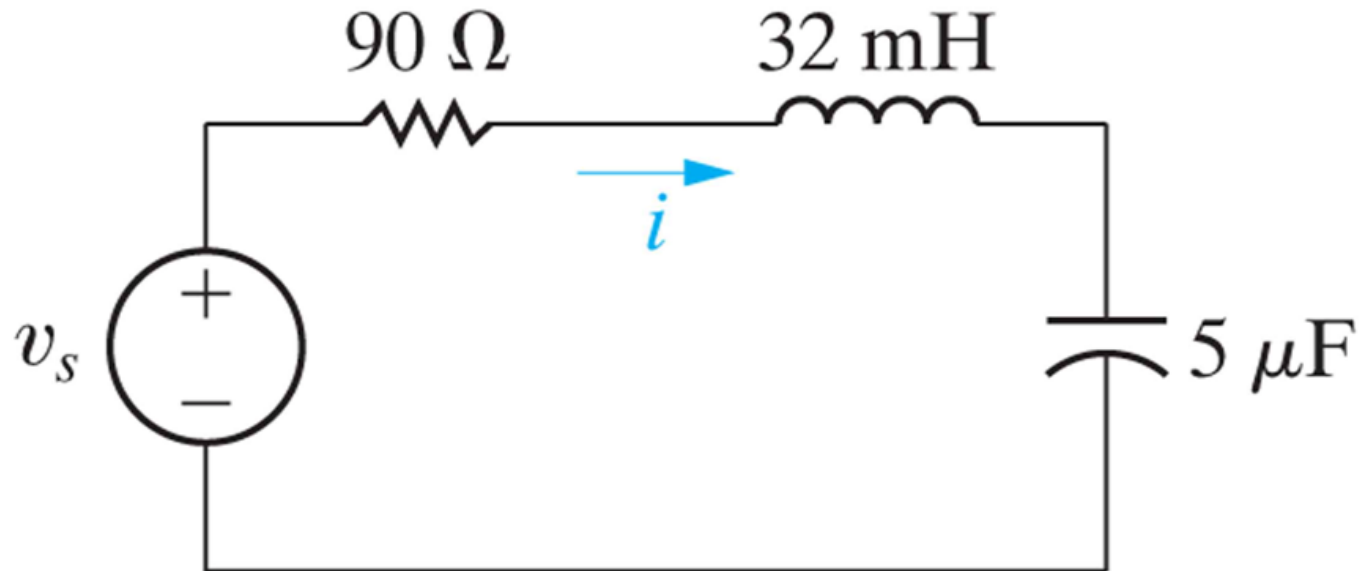
$$V = ZI \quad Z = R + jX \quad X_L = \omega L \quad X_C = \frac{-1}{\omega C}$$

Circuit Element	Impedance	Reactance
Resistor	R	—
Inductor	$j\omega L$	ωL
Capacitor	$j(-1/\omega C)$	$-1/\omega C$

Example 3

A 90Ω resistor, a 32 mH inductor, and a $5\text{ }\mu\text{F}$ capacitor are connected in series across the terminals of a sinusoidal voltage source. The steady-state expression for the source voltage v_s is: $750 \cos(5000t + 30^\circ)\text{ V}$

- a) Construct the frequency-domain equivalent circuit.
- b) Calculate the steady-state current i by the phasor method.



Example 3 — Solution

Answer

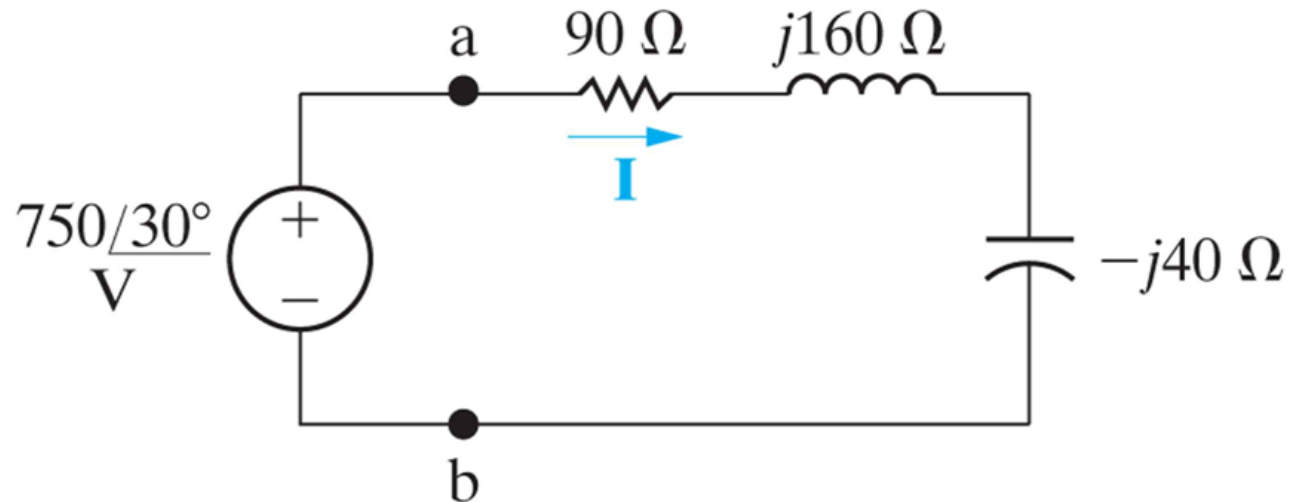
$$\omega = 5000 \text{ rad/s}$$

$$Z_L = j\omega L = j160 \, \Omega, \quad Z_C = \frac{-j}{\omega C} = -j40 \, \Omega$$

$$Z_{eq} = 90 + j120 = 150 \angle 53.13^\circ \, \Omega$$

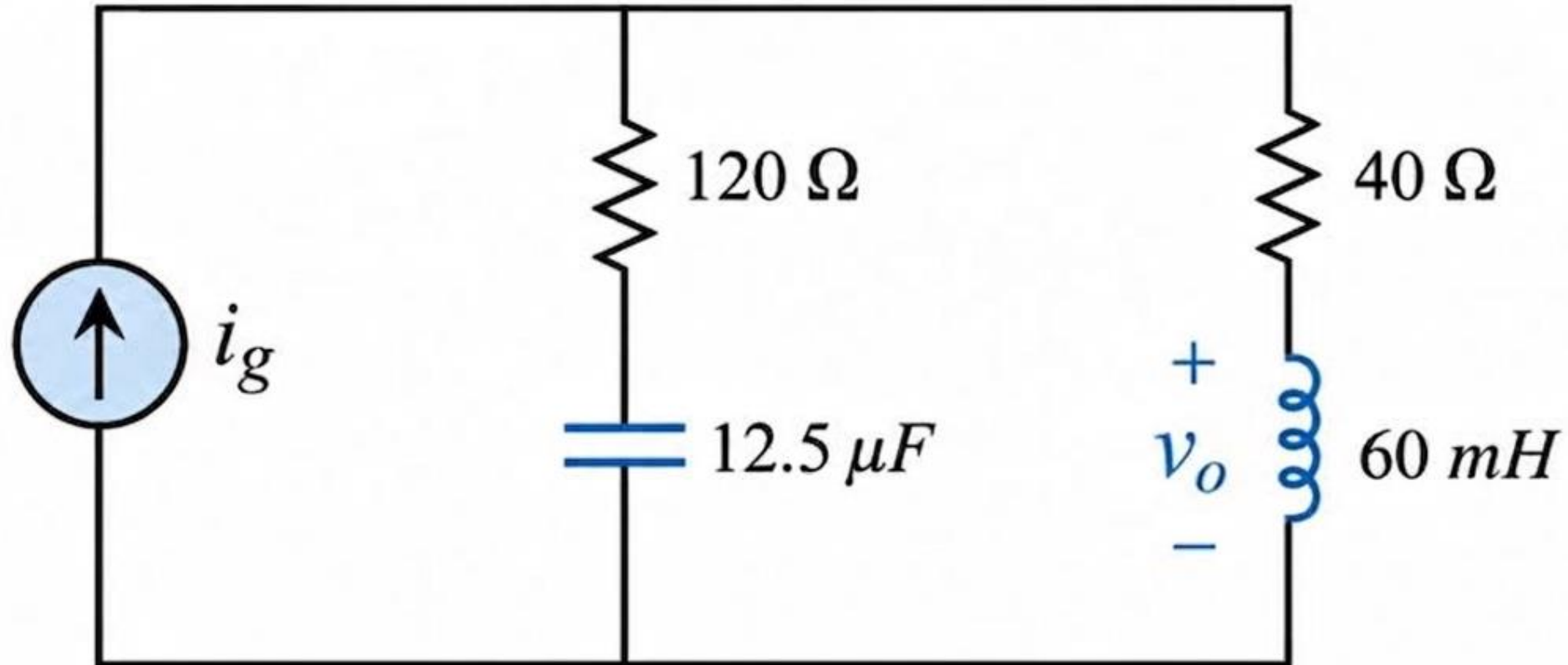
$$V = 750 \angle 30^\circ \text{ (peak-form phasor)}$$

$$I = \frac{V}{Z_{eq}} = 5 \angle -23.13^\circ \text{ A}$$



Example 4

For the circuit shown, a current source $i_g = 500 \cos(2000t)$ mA is applied. Find v_o in the time domain.



Example 4 — Solution

- Convert the circuit to frequency domain.

$$Z_L = j\omega L$$

$$= j \times 2000 \times 60 \times 10^{-3} = j120 \, \Omega$$

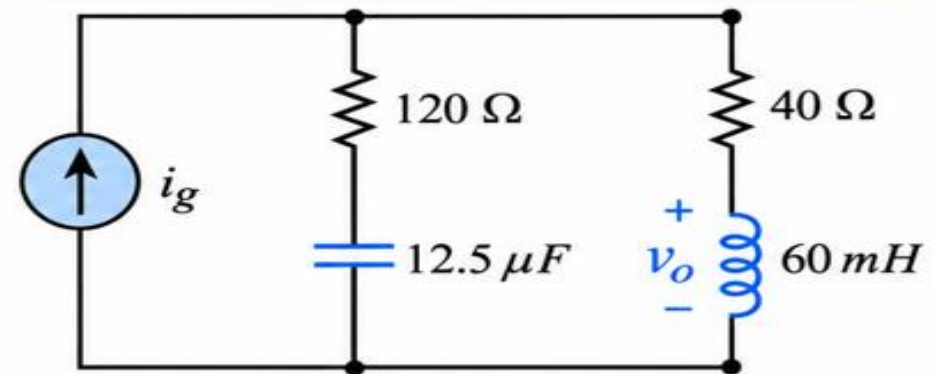
$$Z_C = -\frac{j}{\omega C}$$

$$= -\frac{j}{2000 \times 12.5 \times 10^{-6}} = -j40 \, \Omega$$

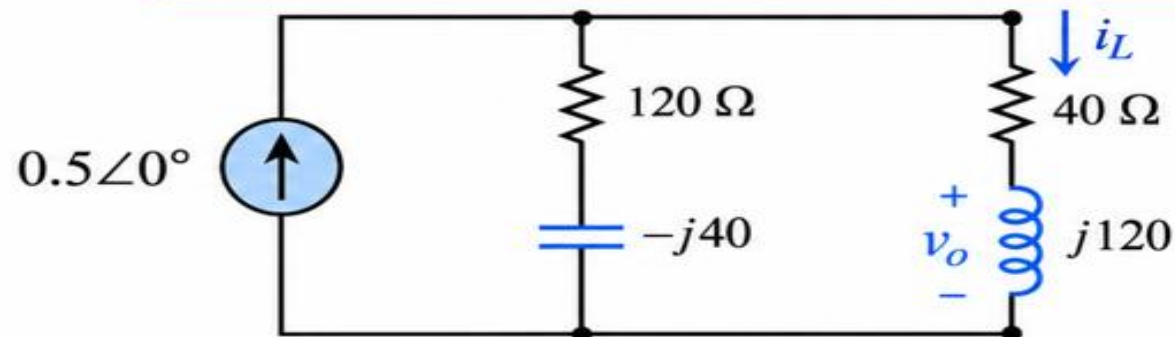
$$i_g = 500 \cos(2000t) \text{ mA} \Rightarrow 500 \times 10^{-3} \angle 0^\circ$$

$$= 0.5 \angle 0^\circ \text{ A}$$

Original circuit (time domain)



Equivalent circuit (frequency domain)



Example 4 — Solution Continued

- By current divider

$$I_L = \frac{120 - j40}{(120 - j40) + (40 + j120)} \times 0.5 \angle 0^\circ$$

$$I_L = 0.25 - j0.25 \text{ A}$$

$$\begin{aligned} V_o &= j120 \times I_L \\ &= j120 \times (0.25 - j0.25) \\ &= 30 + j30 = 42.43 \angle 45^\circ \text{ V} \end{aligned}$$

$$v_o(t) = 42.43 \cos(2000t + 45^\circ) \text{ V}$$

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Lecture 4

Instantaneous Power

$$p(t) = v(t) i(t)$$

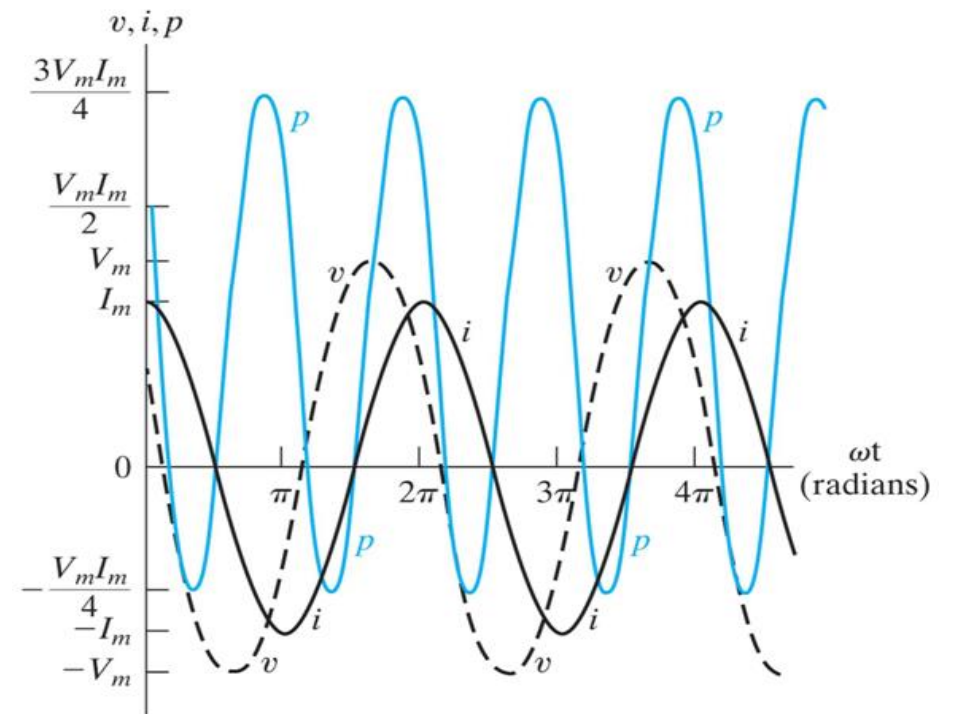
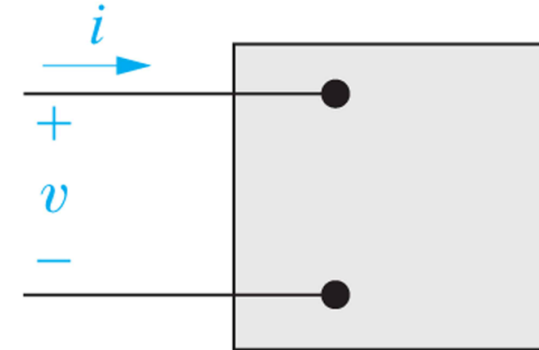
$$\text{Let: } v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$\varphi = \theta_v - \theta_i$$

$$p(t) = \frac{V_m I_m}{2} \cos \varphi + \frac{V_m I_m}{2} \cos(2\omega t + \varphi)$$

Power contains an average part and a double-frequency part.



Core Relations

$$p(t) = \frac{V_m I_m}{2} \cos(\varphi) + \frac{V_m I_m}{2} \cos(\varphi) \cos(2\omega t) - \frac{V_m I_m}{2} \sin(\varphi) \sin(2\omega t)$$

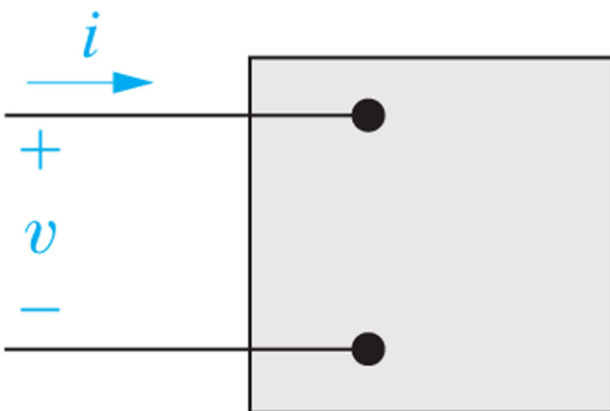
$$p(t) = P + P \cos(2\omega t) - Q \sin(2\omega t)$$

$$P = \frac{V_m I_m}{2} \cos \varphi = V_{rms} I_{rms} \cos(\varphi)$$

$$Q = \frac{V_m I_m}{2} \sin(\varphi) = V_{rms} I_{rms} \sin(\varphi)$$

$P \rightarrow$ average (real) power

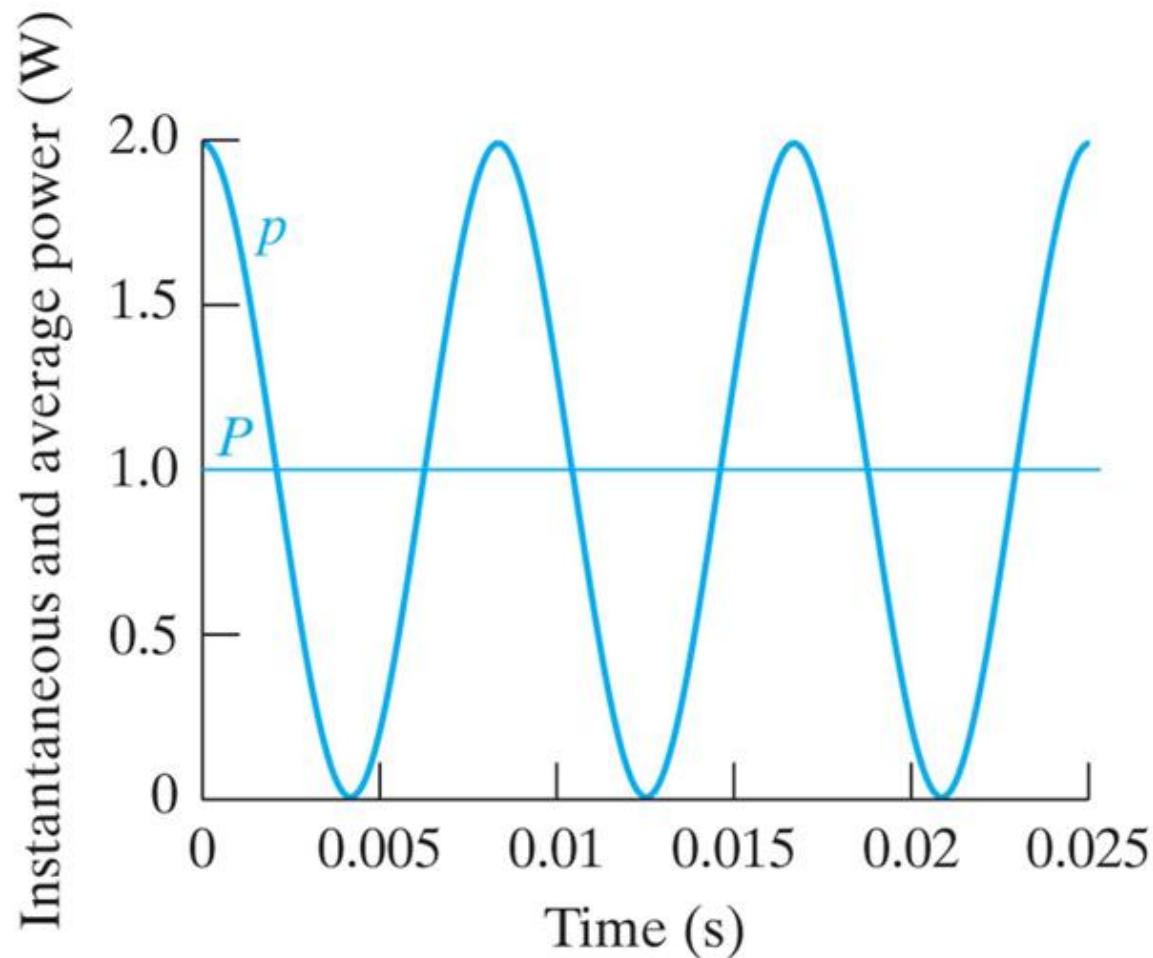
$Q \rightarrow$ reactive power

**Result**

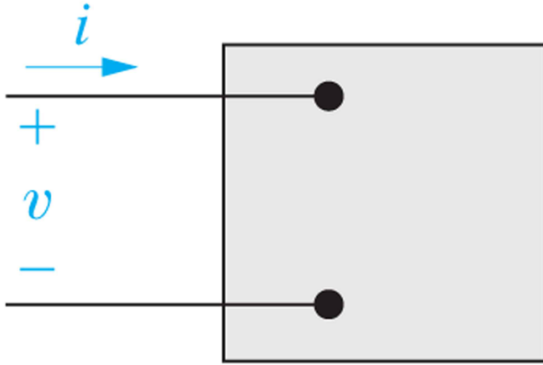
$$\theta_v = \theta_i$$

$$Q = 0$$

$$p(t) = P + P \cos(2\omega t)$$



Average power is positive and constant: P



Result

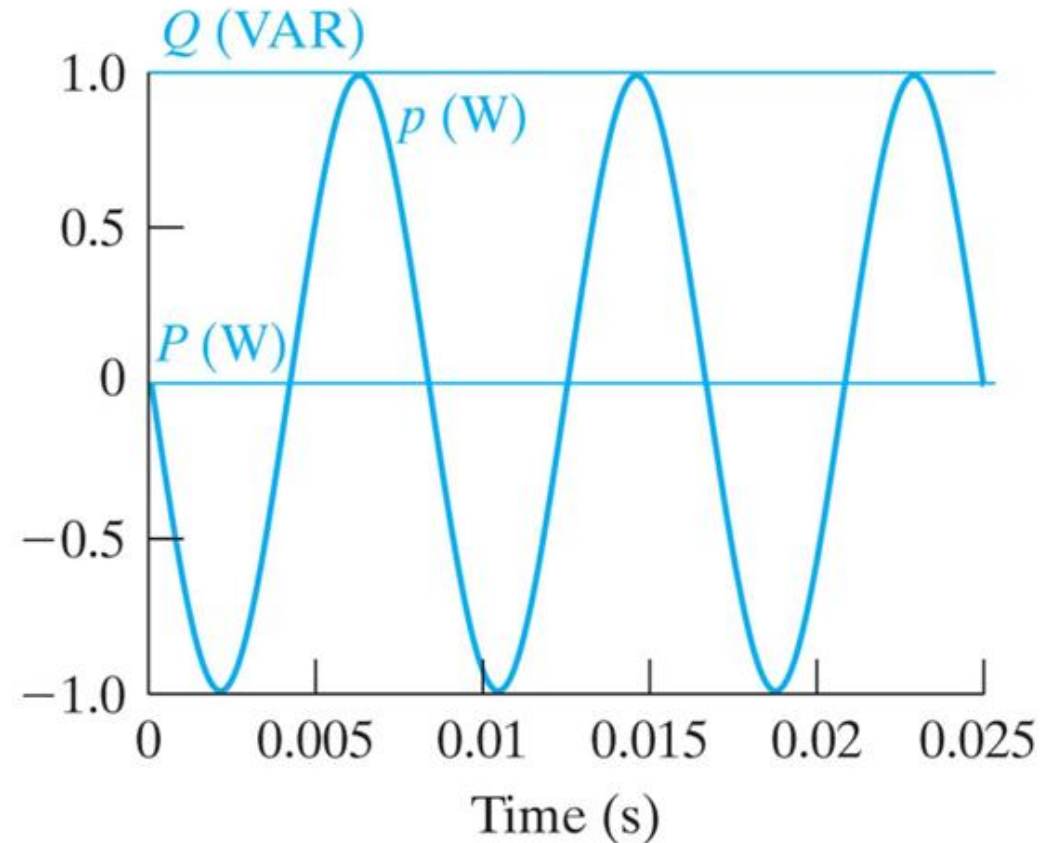
$$\theta_v - \theta_i = 90^\circ$$

$$P = 0$$

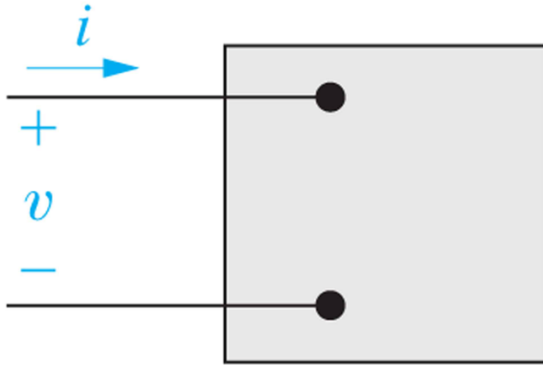
$$p(t) = -Q \sin(2\omega t)$$

$$Q > 0 \text{ (inductive)}$$

Instantaneous, average and reactive power



Average power is zero. Energy oscillates back and forth.



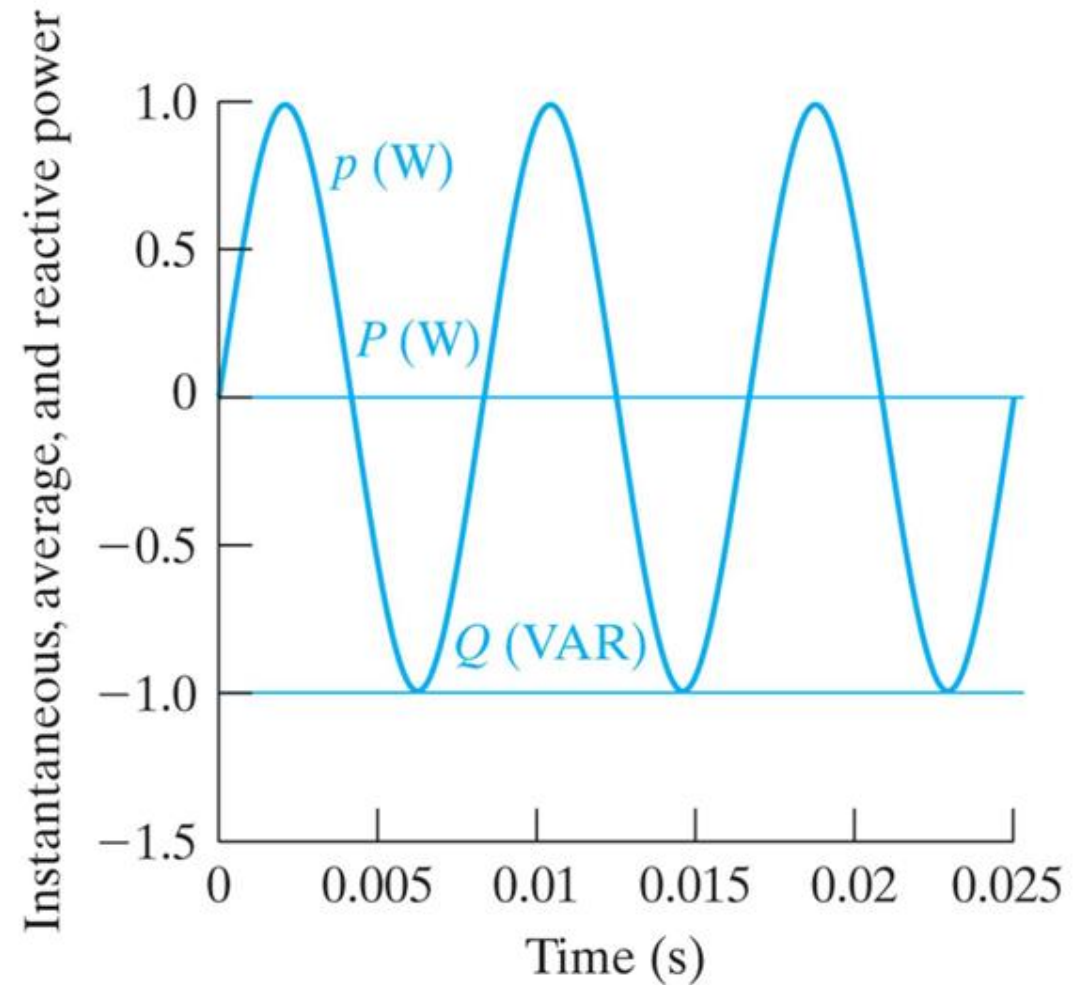
Result

$$\theta_v - \theta_i = -90^\circ$$

$$P = 0$$

$$p(t) = Q \sin(2\omega t)$$

$$Q < 0 \text{ (capacitive)}$$



Average power is zero. Energy is stored and returned.

Power Factor Rules

$\varphi = \theta_v - \theta_i \rightarrow$ power-factor angle

$\text{pf} = \cos(\varphi)$, $0 \leq |\text{pf}| \leq 1$

Inductive load $\rightarrow$ lagging pf

Capacitive load $\rightarrow$ leading pf

Pure resistor $\rightarrow \text{pf} = 1$

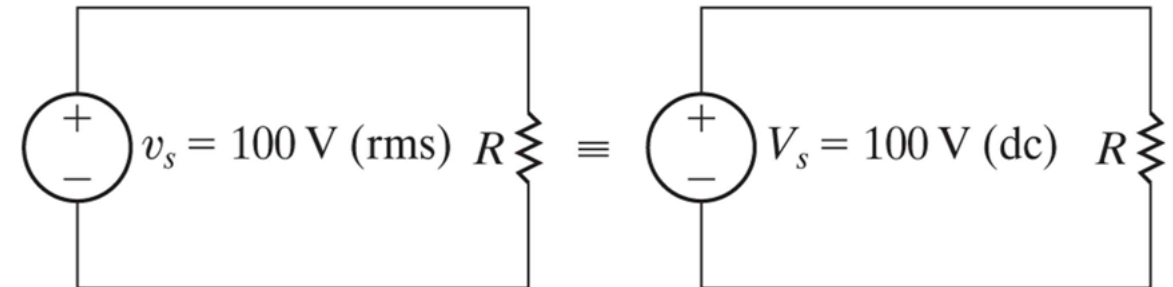
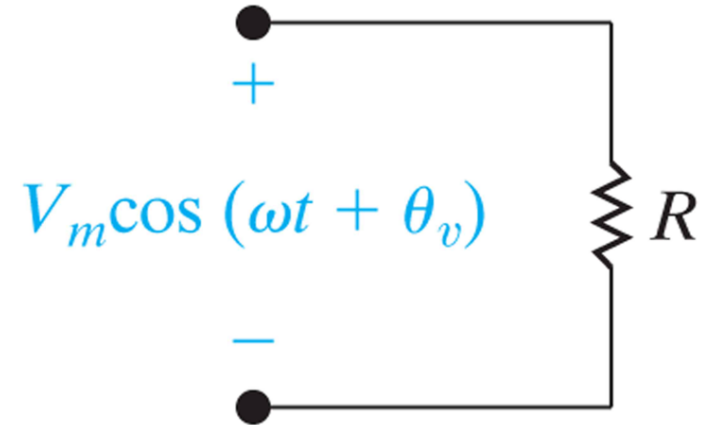
Useful Forms

$$P = \frac{V_m I_m}{2} \cos \varphi$$

$$P = V_{rms} I_{rms} \cos \varphi$$

$$Q = V_{rms} I_{rms} \sin \varphi$$

$$\text{For a resistor: } P = \frac{V_{rms}^2}{R} = I_{rms}^2 R$$



A 100 V rms source produces the same heating as 100 V dc in R.

Example 5

- a) A sinusoidal voltage having a maximum amplitude of 625 V is applied to the terminals of a 50 Ω resistor. Find the average power delivered to the resistor.
- b) Repeat (a) by first finding the current in the resistor.

Method 1: Using V_{rms}

$$V_{rms} = \frac{625}{\sqrt{2}} = 441.94 \text{ V}$$

$$P = \frac{V_{rms}^2}{R} = \frac{441.94^2}{50} = 3906.25 \text{ W}$$

Method 2: Using I_{rms}

$$I_m = \frac{V_m}{R} = \frac{625}{50} = 12.5 \text{ A}$$

$$I_{rms} = \frac{12.5}{\sqrt{2}} = 8.84 \text{ A}$$

$$P = I_{rms}^2 R$$

$$P = (8.84)^2 \times 50 = 3906.25 \text{ W}$$

Definitions

$$S = P + jQ$$

$$P = \text{Re}\{S\}$$

$$Q = \text{Im}\{S\}$$

Quantity and Units

Quantity	Units
Complex power	volt-amps
Average power	watts
Reactive power	var

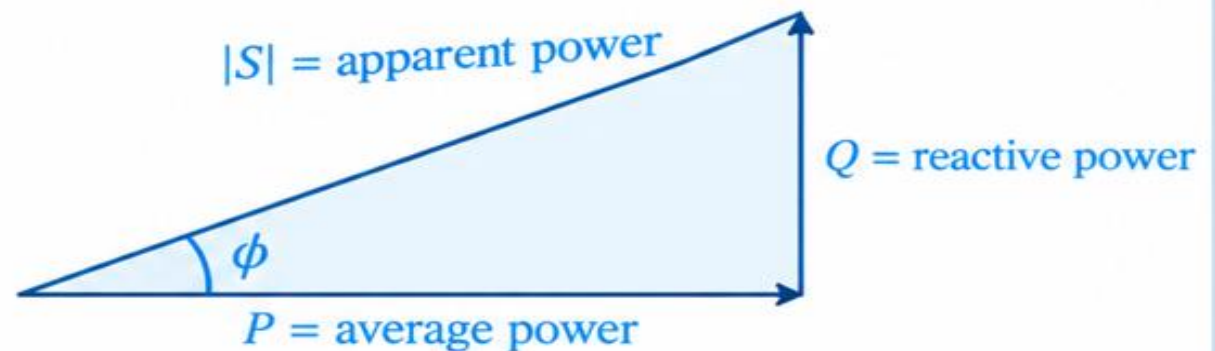
Formulas

$$|S| = \sqrt{P^2 + Q^2}$$

$$|S| = V_{rms} I_{rms}$$

$$\tan \phi = Q / P$$

Power Triangle



CORE EQUATIONS

VOLTAGE & CURRENT

$$S = V_{rms} I_{rms}^*$$

CURRENT & IMPEDANCE

$$S = I_{rms}^2 Z$$

VOLTAGE & IMPEDANCE

$$S = V_{rms}^2 / Z^*$$

POWER COMPONENTS (S = P + jQ)

COMPONENT	FORMULA	UNIT
Real Power (P)	$P = I_{rms}^2 R = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$	W (Watts)
Reactive Power (Q)	$Q = I_{rms}^2 X = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$	VAR
Apparent Power (S)	$ S = V_{rms} I_{rms} = \sqrt{P^2 + Q^2}$	VA

QUICK DEFINITIONS

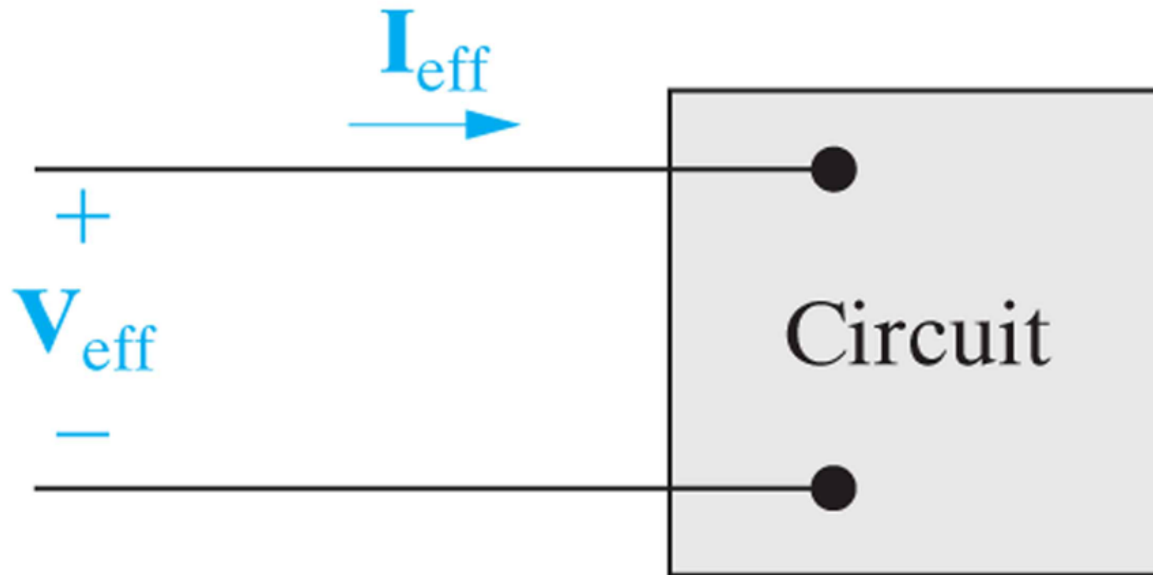
- **Complex Conjugate:** $I^* = |I| \angle -\theta$
- **Inductive Load:** $+Q$ (Absorbs Q)

- **Power Factor:** $PF = \cos(\theta_v - \theta_i) = P / |S|$
- **Capacitive Load:** $-Q$ (Supplies Q)

Example 6

An electric load operates at 240 V rms. The load absorbs an average power of 8 kW at 0.8 lagging power factor.

- Calculate the complex power of the load.
- Calculate the impedance of the load.



Example 6 — Solution

Answer

$$a) \varphi = \cos^{-1}(0.8) = 36.87^\circ$$

$$|S| = \frac{P}{pf} = \frac{8}{0.8} = 10 \text{ KVA}$$

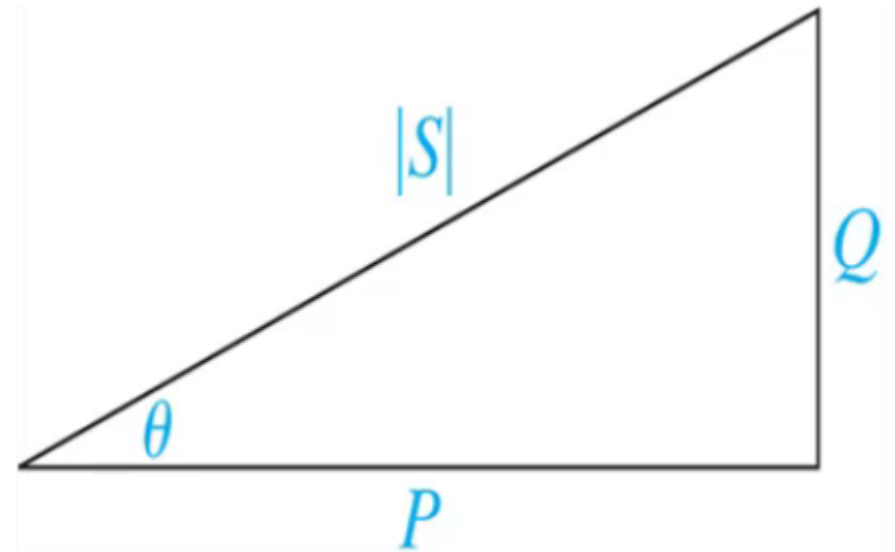
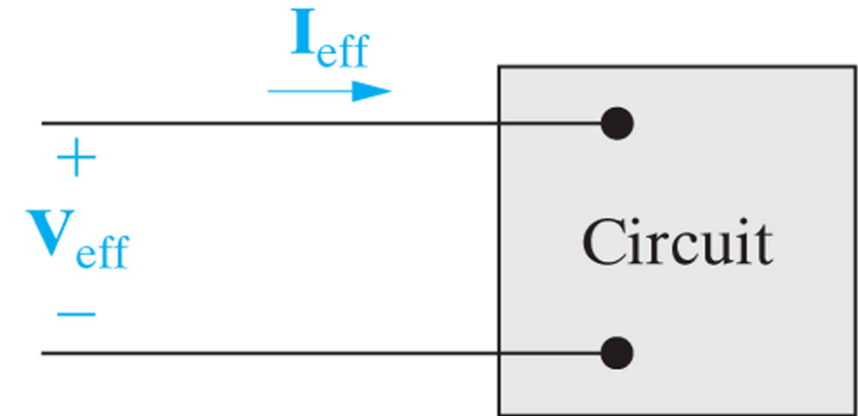
$$Q = |S| \sin(\varphi) = 10 \times 0.6 = 6 \text{ KVAR}$$

$$S = 8 + j6 \text{ KVA}$$

$$b) I = \frac{P}{V \times pf} = \frac{8000}{240 \times 0.8} = 41.67 \text{ A}$$

$$|Z| = \frac{V}{I} = \frac{240}{41.67} = 5.76 \Omega$$

$$Z = 5.76 \angle 36.87^\circ = 4.608 + j3.456 \Omega$$



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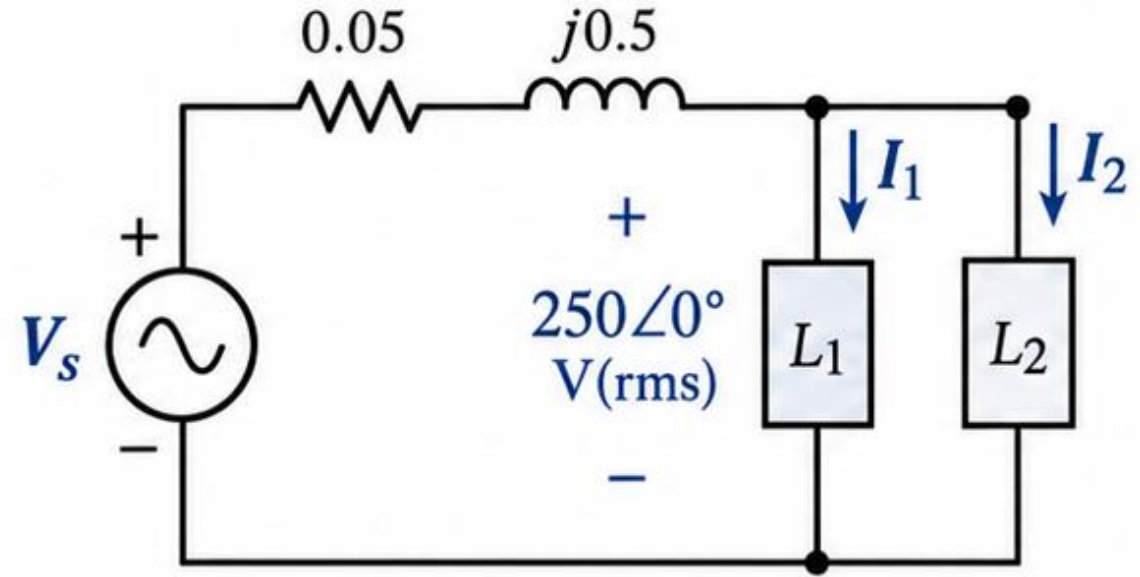
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Lecture 5

The two loads in the circuit shown can be described as follows:

Load 1: $P = 8 \text{ kW}$, 0.8 leading p.f.

Load 2: $|S| = 20 \text{ kVA}$, 0.6 lagging p.f. V_s



Determine:

- Line current
- Source voltage
- Power factor of the two loads in parallel

Example 7 — Solution Part (a)

1 For Load 1

$$P_1 = 8 \text{ kW, p.f.} = 0.8 \text{ leading}$$

$$|S_1| = \frac{P_1}{0.8} = 10 \text{ kVA}$$

$$Q_1 = |S_1| \sin(\cos^{-1}(0.8)) \\ = 10 \times 0.6 = 6 \text{ kVAR}$$

Since the power factor is leading,
reactive power is negative:

$$S_1 = (8 - j6) \text{ kVA}$$

2 For Load 2

$$|S_2| = 20 \text{ kVA, p.f.} = 0.6 \text{ lagging}$$

$$P_2 = |S_2| \times 0.6 = 12 \text{ kW}$$

$$Q_2 = |S_2| \sin(\cos^{-1}(0.6)) \\ = 20 \times 0.8 = 16 \text{ kVAR}$$

Since the power factor is lagging,
reactive power is positive:

$$S_2 = (12 + j16) \text{ kVA}$$

3 Total complex power

$$S_{total} = S_1 + S_2 \\ = (8 - j6) + (12 + j16) \\ = (20 + j10) \text{ kVA}$$

4 Use the complex power relation

$$S = VI^*$$

$$V = 250 \angle 0^\circ \text{ V (rms)}$$

$$I = \left(\frac{S}{V} \right)^* = \left(\frac{(20 + j10) \times 10^3}{250} \right)^*$$

$$I = (80 + j40)^* \\ = 80 - j40 \text{ A}$$

$$|I| = \sqrt{80^2 + (-40)^2} = 89.44 \text{ A}$$

$$\angle I = \tan^{-1} \left(\frac{-40}{80} \right) = -26.6^\circ$$

$$I = 89.44 \angle -26.6^\circ \text{ A}$$

$$\text{Line current: } I = 80 - j40 \text{ A} = 89.44 \angle -26.6^\circ \text{ A}$$

(b) Source voltage

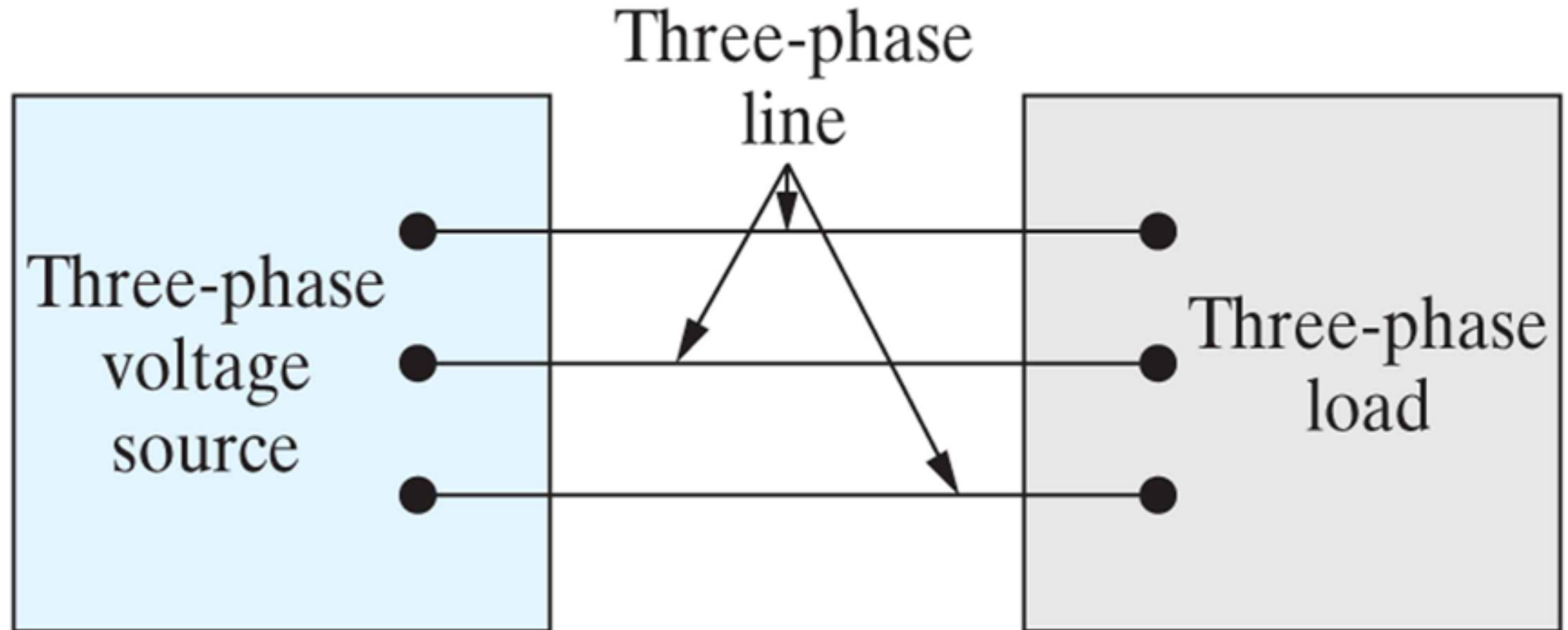
$$\begin{aligned}V_s &= (0.05 + j0.5)I + 250 \\&= (0.05 + j0.5)(80 - j40) + 250 \\&= 274 + j38 \text{ V} \\&= 276.62 \angle 7.89^\circ \text{ V}\end{aligned}$$

(c) Power factor of the two parallel loads

$$\begin{aligned}S_{total} &= (20 + j10) \text{ kVA} \\ \theta &= \tan^{-1}\left(\frac{Q}{P}\right) = \tan^{-1}\left(\frac{10}{20}\right) \\&= 26.56^\circ \\ \text{p.f.} &= \cos \theta = \cos 26.56^\circ \\&= 0.8944 \text{ lagging}\end{aligned}$$

→ **Source voltage:** $V_s = 274 + j38 \text{ V} = 276.62 \angle 7.89^\circ \text{ V}$

→ **Power factor:** 0.8944 lagging



Positive phase sequence (abc)

$$V_a = V_m \angle 0^\circ$$

$$V_b = V_m \angle -120^\circ = V_m \angle 240^\circ$$

$$V_c = V_m \angle -240^\circ = V_m \angle 120^\circ$$

Negative phase sequence (acb)

$$V_a = V_m \angle 0^\circ$$

$$V_c = V_m \angle -120^\circ = V_m \angle 240^\circ$$

$$V_b = V_m \angle -240^\circ = V_m \angle 120^\circ$$

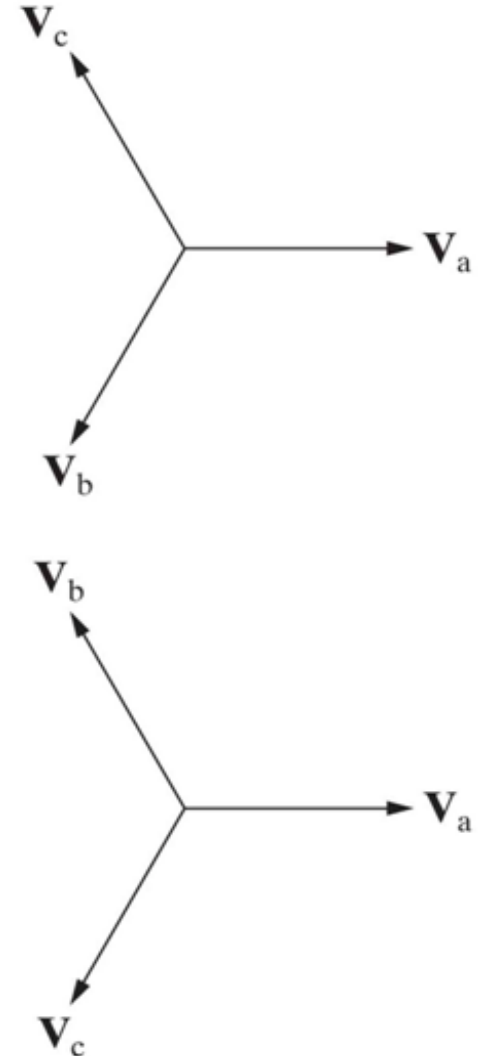
Definition

Balanced set:

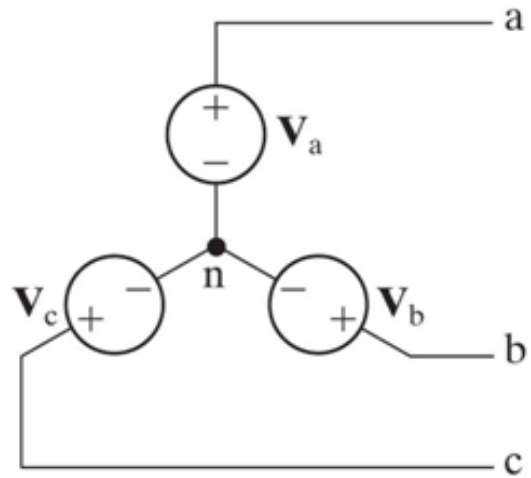
Equal magnitudes

Phase displacement = 120°

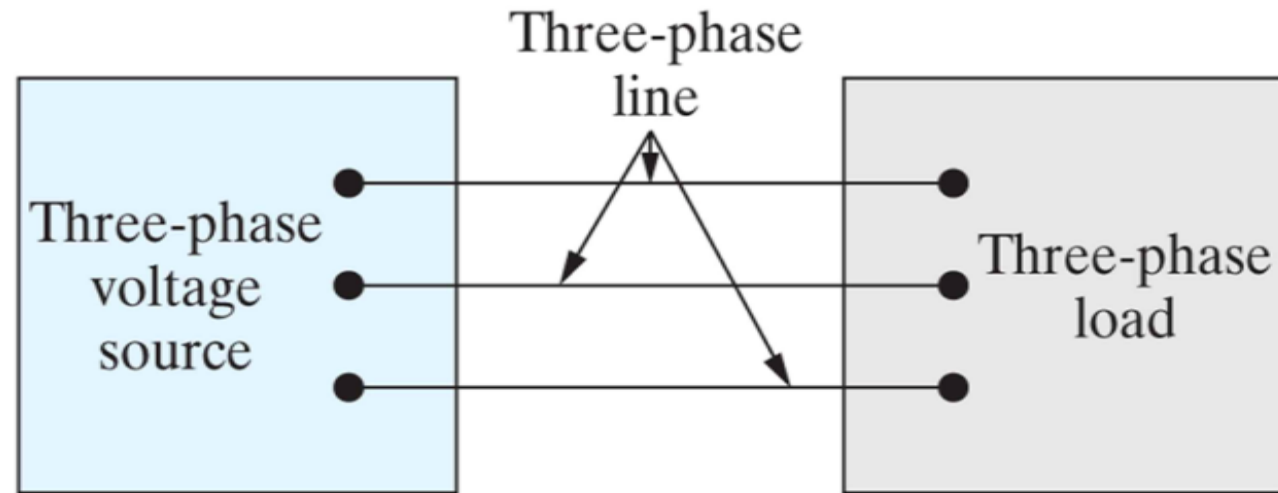
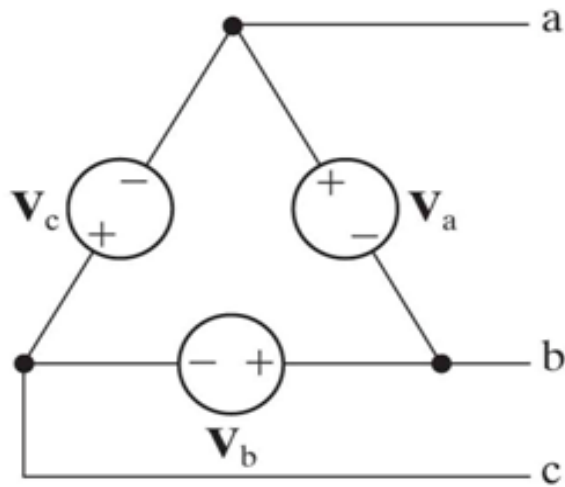
$$\text{Also: } V_a + V_b + V_c = 0$$



Wye (Y) source



Delta (Δ) source



Connections

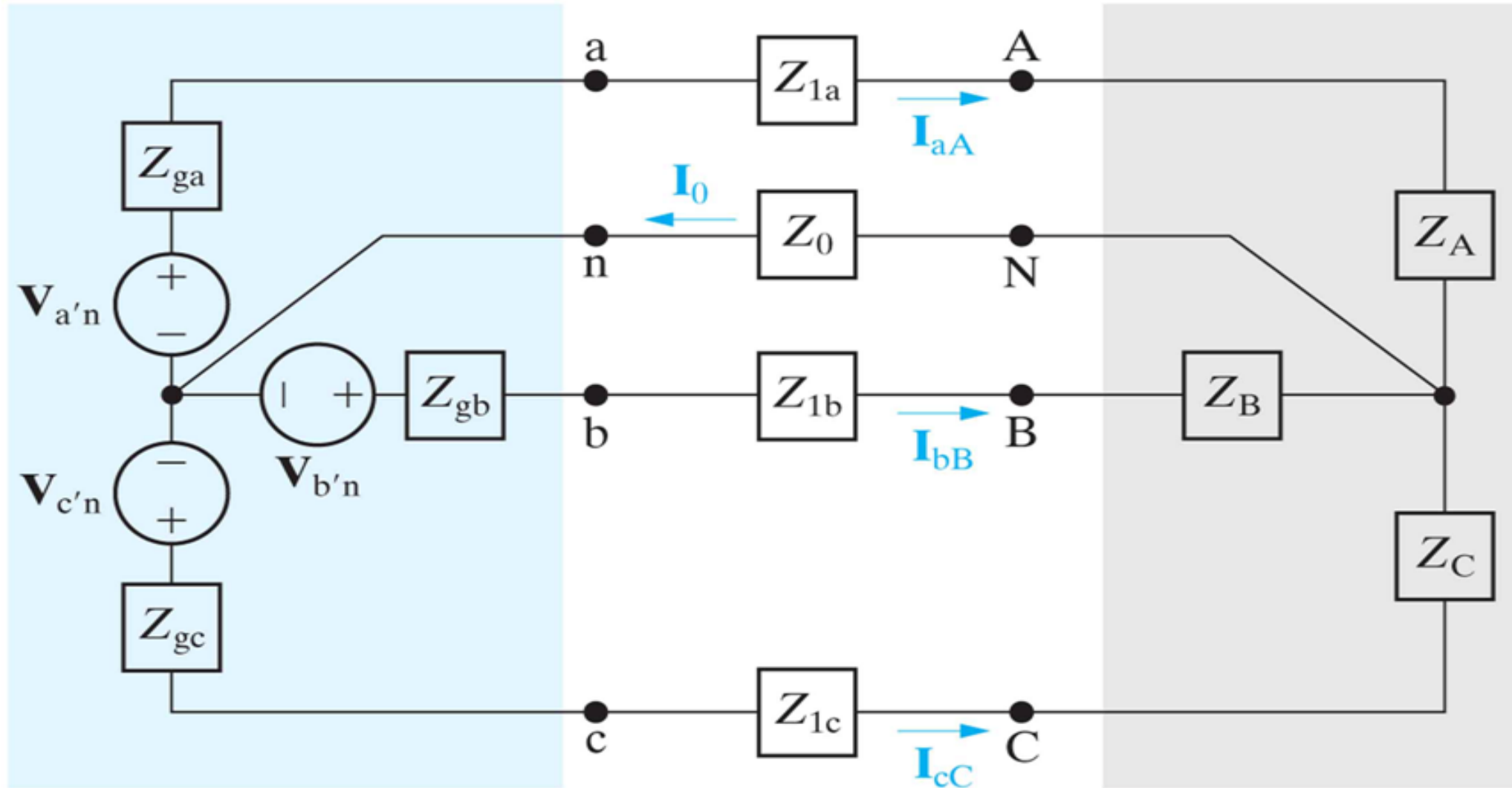
Common source/load combinations:

Y-Y

Y- Δ

Δ -Y

Δ - Δ



Key Relations

For a balanced system:

$$Z_{\phi} = Z_{\phi A} = Z_{\phi B} = Z_{\phi C}$$

$$Z_{\phi} = Z_g + Z_{Line} + Z_L$$

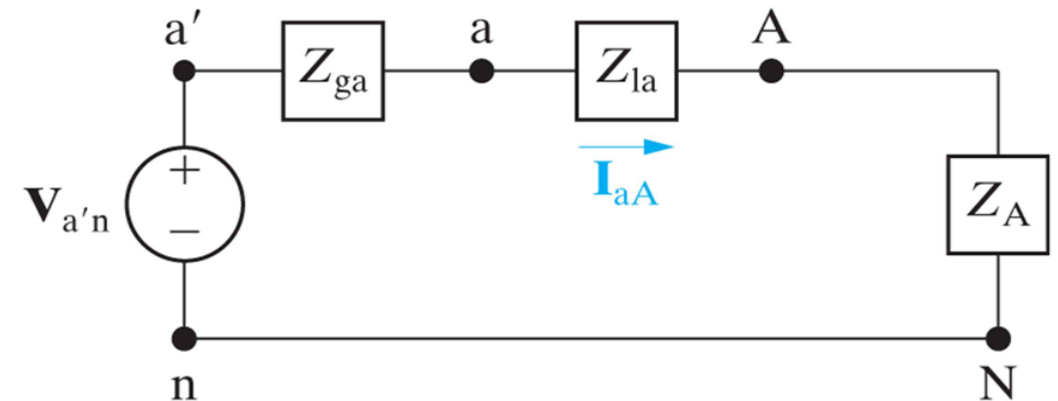
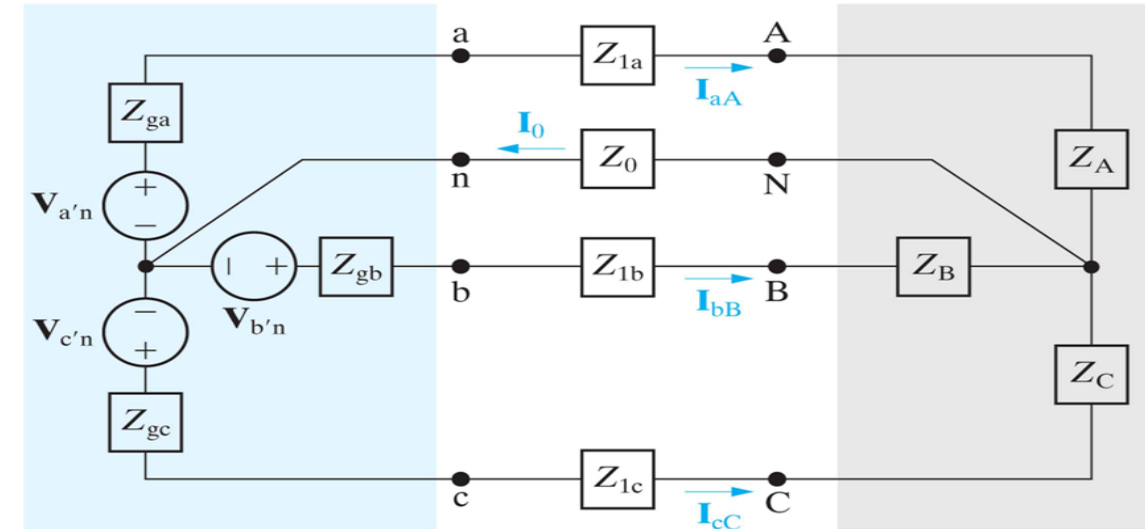
$$I_{aA} = V_{a'n} / Z_{\phi}$$

$$I_{bB} = V_{b'n} / Z_{\phi}$$

$$I_{cC} = V_{c'n} / Z_{\phi}$$

$$V_N = 0 \quad (\text{balanced neutral})$$

Solve one phase, then shift by $\pm 120^\circ$



A balanced three-phase Y-connected generator with positive sequence has an impedance of $0.2+j0.5 \Omega/\phi$ and an internal voltage of $120 \text{ V}/\phi$. The generator feeds a balanced three-phase Y-connected load having an impedance of $39+j28 \Omega/\phi$. The impedance of the line connecting the generator to the load is $0.8+j1.5 \Omega/\phi$. The a-phase internal voltage of the generator is specified as the reference phasor.

- a) Construct the a-phase equivalent circuit of the system.
- b) Calculate the three-line currents I_{aA} , I_{bB} , and I_{cC} .
- c) Calculate the three phase voltages at the load, V_{AN} , V_{BN} , and V_{CN} .
- d) Calculate the line voltages, V_{AB} , V_{AC} , and V_{CA} , at the terminals of the load.
- e) Calculate the phase voltages at the generator terminals, V_{an} , V_{bn} , and V_{cn} .
- f) Calculate the line voltages at the generator terminals, V_{ab} , V_{bc} , and V_{ca} .

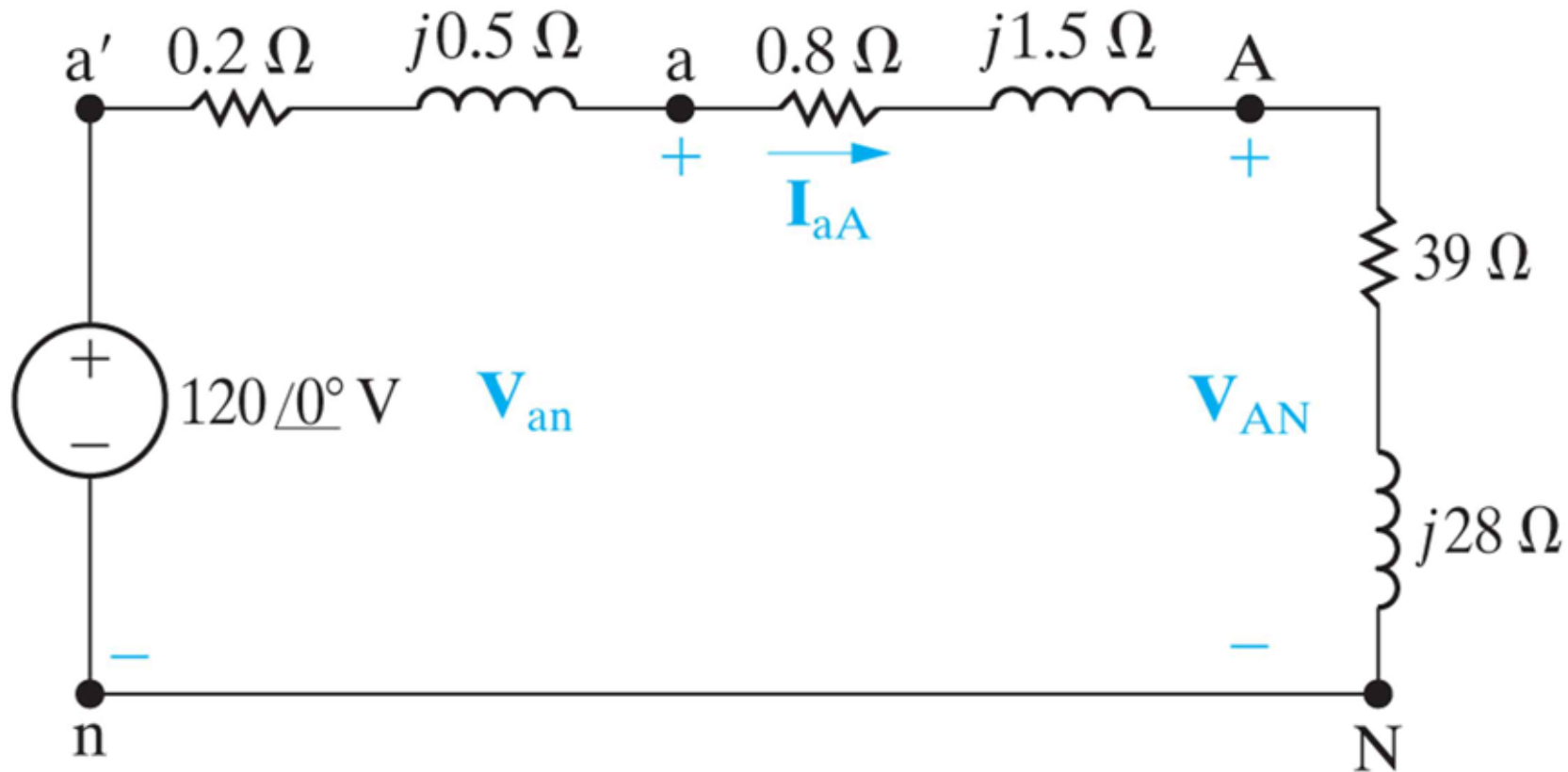
Example 8 — Solution

Given Data

Parameter	Value
Generator internal impedance	$Z_g = 0.2 + j0.5 \, \Omega/\text{phase}$
Line impedance	$Z_{\text{line}} = 0.8 + j1.5 \, \Omega/\text{phase}$
Load impedance	$Z_L = 39 + j28 \, \Omega/\text{phase}$
Generator internal phase voltage	$E_{\text{an}} = 120 \angle 0^\circ \, \text{V}$
Sequence	Positive sequence (abc)

a) a-Phase Equivalent Circuit

The per-phase circuit contains the generator internal impedance, line impedance, and load impedance in series.



b) Three Line Currents

$$Z_T = Z_g + Z_{\text{line}} + Z_L$$

$$Z_T = (0.2 + j0.5) + (0.8 + j1.5) + (39 + j28)$$

$$Z_T = 40 + j30 = 50 \angle 36.87^\circ \quad \Omega$$

$$I_{aA} = E_{an} / Z_T$$

$$I_{aA} = (120 \angle 0^\circ) / (50 \angle 36.87^\circ)$$

$$I_{aA} = 2.4 \angle -36.87^\circ \quad \text{A}$$

For a positive-sequence abc system:

$$I_{bB} = 2.4 \angle -156.87^\circ \quad \text{A}$$

$$I_{cC} = 2.4 \angle 83.13^\circ \quad \text{A}$$

c) Phase Voltages at the Load

$$V_{AN} = I_{aA} Z_L$$

$$|Z_L| = \sqrt{39^2 + 28^2} = 48.01 \quad \Omega$$

$$\angle Z_L = \tan^{-1}(28/39) = 35.68^\circ$$

$$V_{AN} = (2.4 \angle -36.87^\circ)(48.01 \angle 35.68^\circ)$$

$$V_{AN} = 115.225 \angle -1.193^\circ \quad \text{V}$$

$$V_{BN} = 115.225 \angle -121.193^\circ \quad \text{V}$$

$$V_{CN} = 115.225 \angle 118.807^\circ \quad \text{V}$$

d) Line Voltages at the Load

For a balanced positive-sequence Y system, each line voltage has magnitude $\sqrt{3}$ times the corresponding phase voltage and leads it by 30° .

$$V_{AB} = \sqrt{3} V_{AN} \angle 30^\circ$$

$$V_{AB} = 199.576 \angle 28.807^\circ \text{ V}$$

$$V_{BC} = 199.576 \angle -91.193^\circ \text{ V}$$

$$V_{CA} = 199.576 \angle 148.807^\circ \text{ V}$$

Note: If the required quantity is V_{AC} instead of V_{BC} , then $V_{AC} = 199.576 \angle -31.193^\circ \text{ V}$.

e) Phase Voltages at the Generator Terminals

$$V_{an} = E_{an} - I_{aA} Z_g$$

$$I_{aA} = 2.4 \angle -36.87^\circ = 1.92 - j1.44 \text{ A}$$

$$I_{aA} Z_g = (1.92 - j1.44)(0.2 + j0.5)$$

$$I_{aA} Z_g = 1.104 + j0.672 \text{ V}$$

$$V_{an} = 120 - (1.104 + j0.672)$$

$$V_{an} = 118.896 - j0.672 = 118.898 \angle -0.324^\circ \text{ V}$$

$$V_{bn} = 118.898 \angle -120.324^\circ \text{ V}$$

$$V_{cn} = 118.898 \angle 119.676^\circ \text{ V}$$

f) Line Voltages at the Generator Terminals

$$V_{ab} = \sqrt{3} V_{an} \angle 30^\circ$$

$$V_{ab} = 205.937 \angle 29.676^\circ \text{ V}$$

$$V_{bc} = 205.937 \angle -90.324^\circ \text{ V}$$

$$V_{ca} = 205.937 \angle 149.676^\circ \text{ V}$$

Electrical Machines

Chapter 0 Review of AC Circuits

Instructor

Eng. Nael S. Dokhan

MSc – Electrical Engineering

Lecture 6

Balanced Wye Connected Load

Total Power

$$|V_\phi| = |V_{AN}| = |V_{BN}| = |V_{CN}|$$

$$|V_L| = \sqrt{3} |V_\phi|$$

$$|I_\phi| = |I_{aA}| = |I_{bB}| = |I_{cC}| = |I_L|$$

$$P_\phi = |V_\phi| |I_\phi| \cos \theta_\phi$$

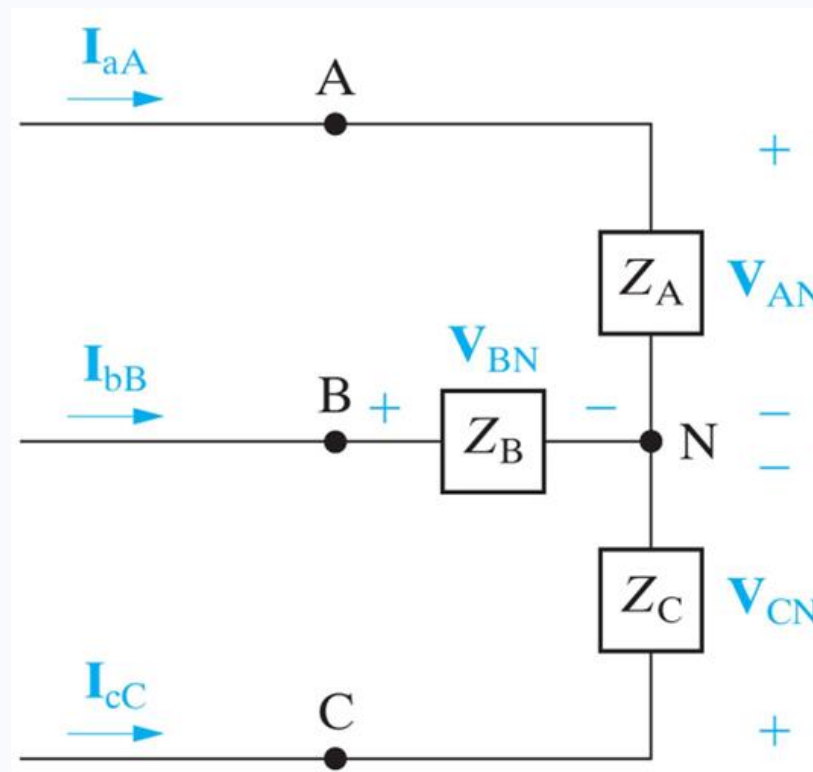
$$P_T = 3 P_\phi = 3 |V_\phi| |I_\phi| \cos \theta_\phi$$

$$\theta_\phi = \theta_{vA} - \theta_{iA} = \theta_{vB} - \theta_{iB} = \theta_{vC} - \theta_{iC}$$

$$P_T = \sqrt{3} |V_L| |I_L| \cos \theta_\phi$$

$$Q_T = \sqrt{3} |V_L| |I_L| \sin \theta_\phi$$

$$S_T = \sqrt{3} |V_L| |I_L|$$



For Y loads: $|V_L| = \sqrt{3} |V_\phi|$
 $|I_L| = |I_\phi|$

Balanced Delta Connected Load

Total Power

$$|V_\phi| = |V_{AB}| = |V_{BC}| = |V_{CA}| = |V_L|$$

$$|I_\phi| = |I_{AB}| = |I_{BC}| = |I_{CA}|$$

$$|I_L| = \sqrt{3} |I_\phi|$$

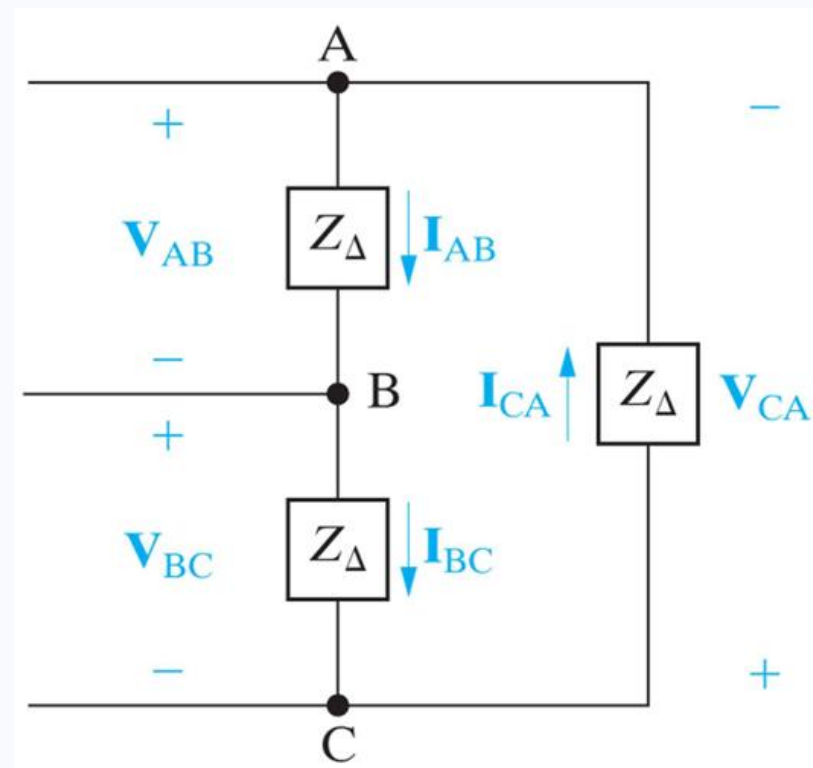
$$P_\phi = |V_\phi| |I_\phi| \cos \theta_\phi$$

$$P_T = 3 P_\phi = 3 |V_\phi| |I_\phi| \cos \theta_\phi$$

$$P_T = \sqrt{3} |V_L| |I_L| \cos \theta_\phi$$

$$Q_T = \sqrt{3} |V_L| |I_L| \sin \theta_\phi$$

$$S_T = \sqrt{3} |V_L| |I_L|$$



For Δ loads: $|I_L| = \sqrt{3} |I_\phi|$
 $|V_L| = |V_\phi|$

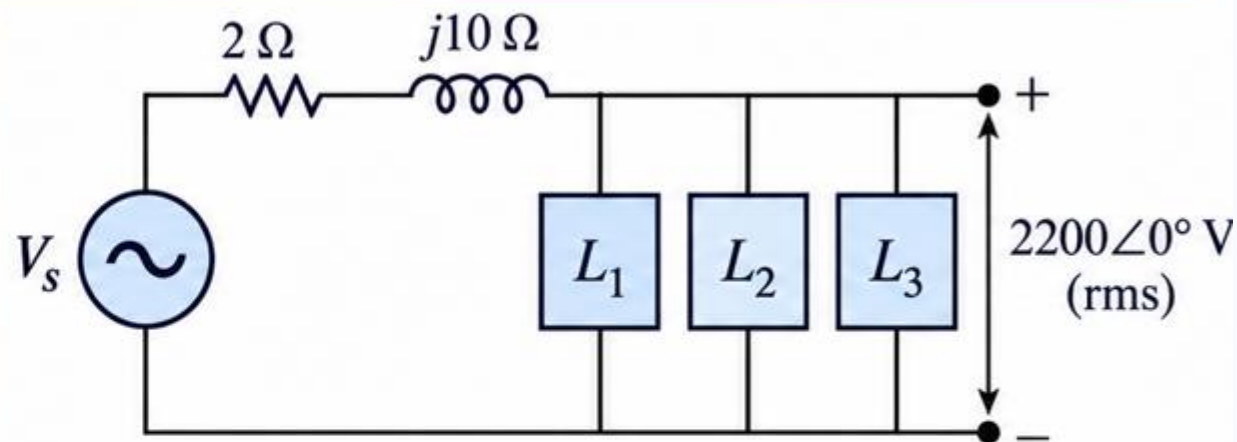
Example 9

A three-phase generator supplies a three-phase load. The loads are connected in parallel as shown in the figure.

Load 1: 150 kVA, 0.8 leading power factor

Load 2: 200 kVA, 0.6 lagging power factor

Load 3: $(100 + j20)$ kVA



Find:

- (a) Draw the power triangle for each load.
- (b) Total complex power consumed by the load.
- (c) Supply current.
- (d) Complex power of the source.

Example 9 — Solution Part (a) — Load 1

Given: $|S_1| = 150 \text{ kVA}$, p.f. = 0.8 leading

1 Real Power (P)

$$P = |S_1| \cos \theta = 150 \times 0.8 = 120 \text{ kW}$$

2 Reactive Power (Q)

$$Q = |S_1| \sin \theta = 150 \times 0.6 = 90 \text{ kVAR}$$

3 Power Factor Angle

$$\theta = \cos^{-1}(0.8) = 36.87^\circ$$

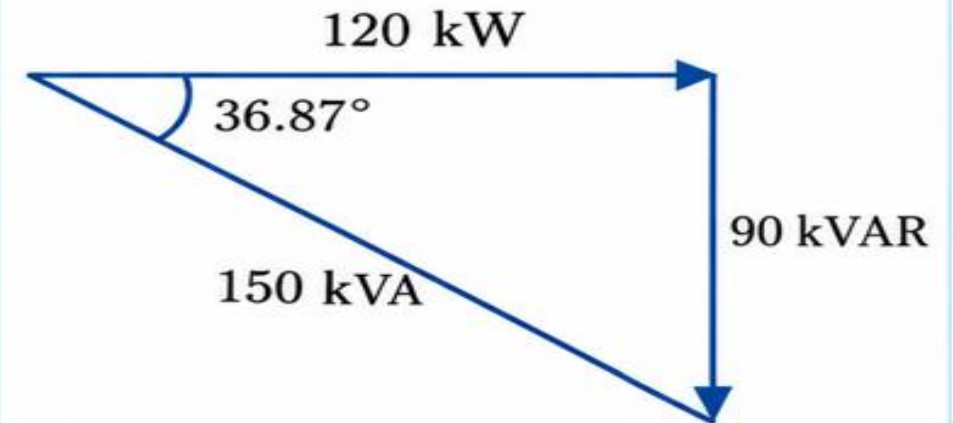
4 Sign of Reactive Power

Since the power factor is leading,
the reactive power is negative.

5 Complex Power

$$S_1 = (120 - j90) \text{ kVA}$$

Power Triangle (Leading p.f.)



Example 9 — Solution Continued Part (a) — Load 2

Given: $|S_2| = 200 \text{ kVA}$, p.f. = 0.6 lagging

1 Real Power (P)

$$P = |S_2| \cos \theta = 200 \times 0.6 = 120 \text{ kW}$$

2 Reactive Power (Q)

$$Q = |S_2| \sin \theta = 200 \times 0.8 = 160 \text{ kVAR}$$

3 Power Factor Angle

$$\theta = \cos^{-1}(0.6) = 53.13^\circ$$

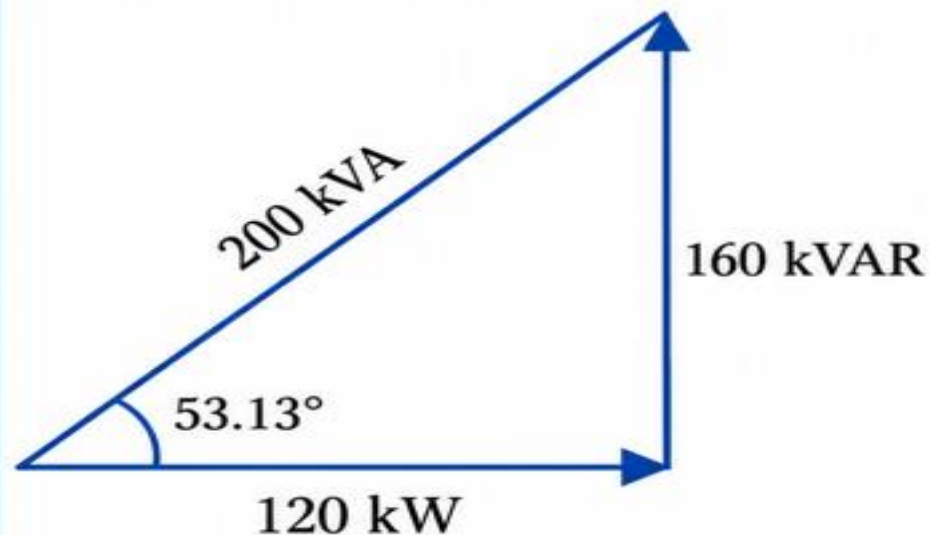
4 Sign of Reactive Power

Since the power factor is lagging, the reactive power is positive.

5 Complex Power

$$S_2 = (120 + j160) \text{ kVA}$$

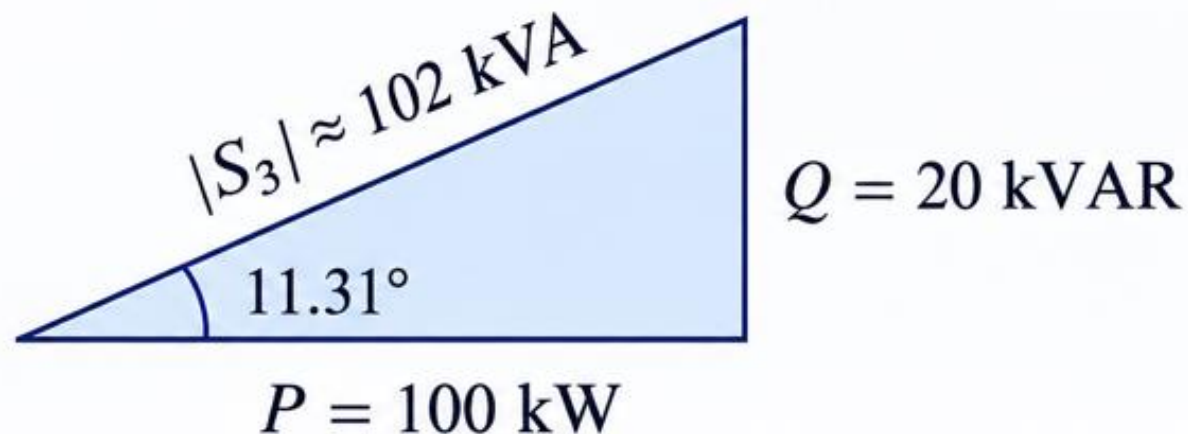
Power Triangle (Lagging p.f.)



Load 3: $S_3 = (100 + j20) \text{ kVA}$

$$|S_3| = \sqrt{100^2 + 20^2} = \sqrt{10400} = 101.98 \text{ kVA} \approx \mathbf{102 \text{ kVA}}$$

$$\theta = \tan^{-1} \left(\frac{Q}{P} \right) = \tan^{-1} \left(\frac{20}{100} \right) = \mathbf{11.31^\circ}$$



(b) Total complex power consumed by the load

$$S_{\text{total}} = S_1 + S_2 + S_3 = (340 + j90) \text{ kVA}$$

(c) Supply current

$$S_{\phi} = \frac{S_{\text{total}}}{3} = \frac{340 + j90}{3} = (113.33 + j30) \text{ kVA}$$

$$I = \left(\frac{S_{\phi}}{V} \right)^* = \left(\frac{(113.33 + j30) \times 10^3}{2200} \right)^* = 51.5 - j13.63 \text{ A}$$

$$I = 53.29 \angle -14.83^\circ \text{ A}$$

$$S_{source} = -(S_{line} + S_{load})$$

$$S_{line, 1ph} = |I|^2 Z = (53.29)^2 (2 + j10) = (5.68 + j28.4) \text{ kVA}$$

$$\begin{aligned} S_{source} &= -(3 S_{line, 1ph} + S_{load, 3ph}) \\ &= -[(3)(5.68 + j28.4) + (340 + j90)] \text{ kVA} \\ &= -(357.04 + j175.2) \text{ kVA} \end{aligned}$$

$$\boxed{S_{source} = -357.04 - j175.2 \text{ kVA}}$$

Thank you