

Electrical Machines

Chapter 1 Introduction to Machinery Principles

Instructor

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Electrical Machines

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Lecture 7

1

**Rotational
motion**

2

**Torque, work,
power**

3

**Newton's law of
rotation**

4

Magnetic field

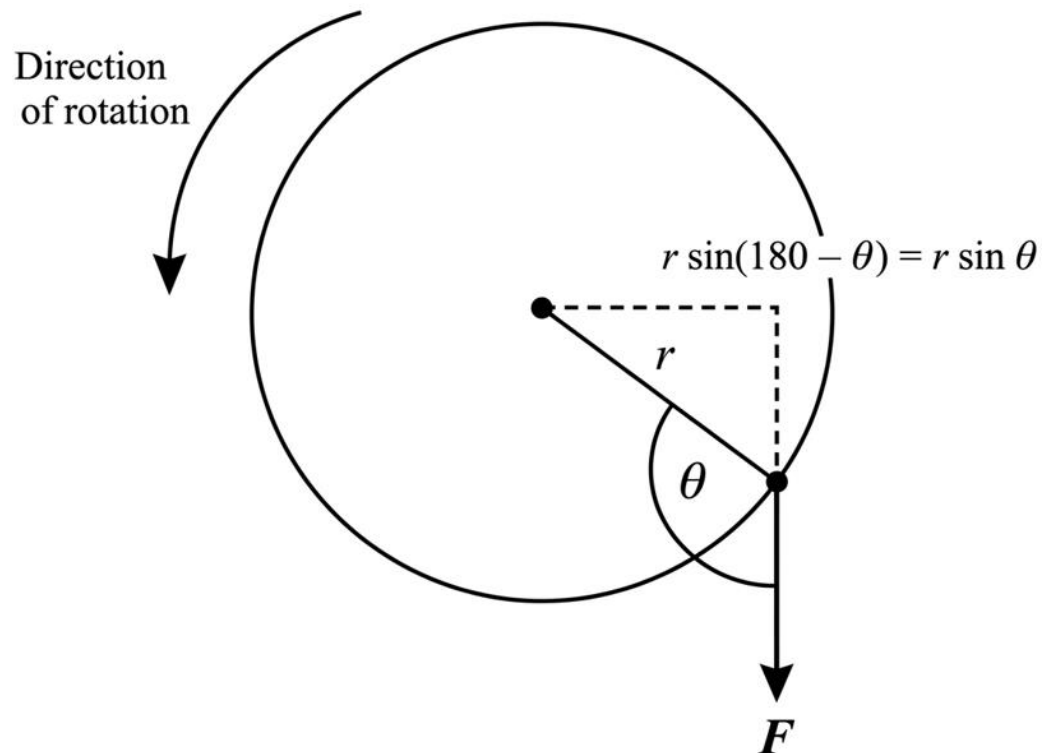
5

Magnetic circuits

6

Air-gap flux

This part builds the mechanical and magnetic foundation for electrical machines.



Rotational Kinematics

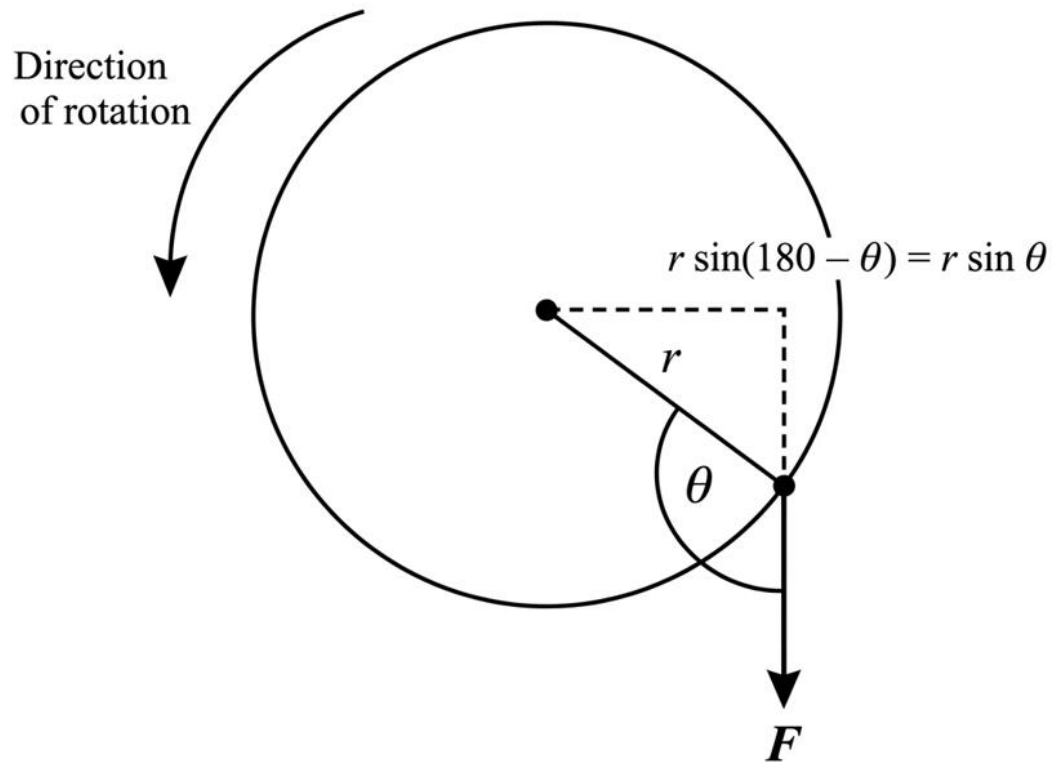
θ = angular position (rad or deg)

$\omega = d\theta/dt$ (rad/s)

$\alpha = d\omega/dt$ (rad/s²)

$\omega = 2\pi n/60$ for n in rpm

Example: 1800 rpm $\rightarrow \omega = 188.5$ rad/s



Mechanical Power Relations

$$\tau = F \times r_{\perp}$$

$$\tau = F r \sin \theta \quad (\text{N}\cdot\text{m})$$

$$W = \int \tau d\theta$$

If τ is constant: $W = \tau \theta$

$$P = dW/dt = \tau d\theta/dt$$

$$P = \tau \omega \quad (\text{watt})$$

Linear motion

$$\mathbf{F = ma}$$

F: force (N)

m: mass (kg)

a: acceleration (m/s²)

Rotational motion

$$\boldsymbol{\tau = J\alpha}$$

τ : torque (N·m)

J: moment of inertia (kg·m²)

α : angular acceleration (rad/s²)

J describes how difficult it is to accelerate a rotating body.

Ampere's Law

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{\text{net}}$$

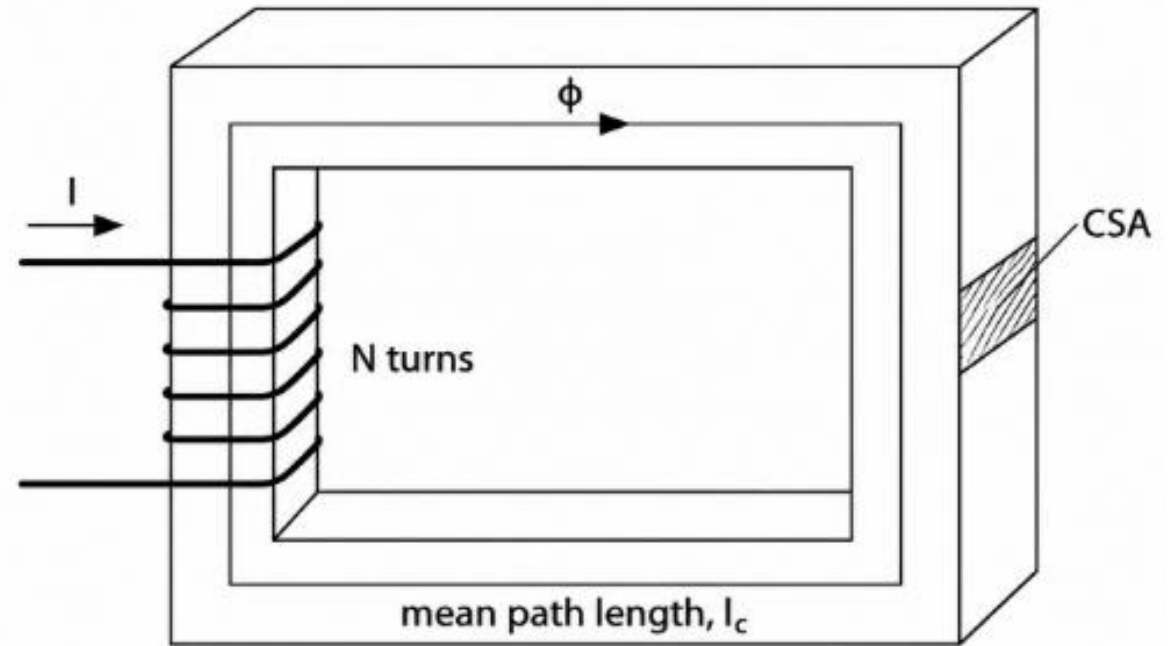
Magnetizing Field

$$H L_c = N i$$

$$H = \frac{N i}{L_c}$$

$$B = \mu H$$

$$\mu = \mu_r \mu_0$$



Current in the coil creates magnetic field intensity H and flux Φ in the core.

H**Field intensity**

A/m

B**Flux density**

T

 Φ **Magnetic flux**

Wb

 μ **Permeability**

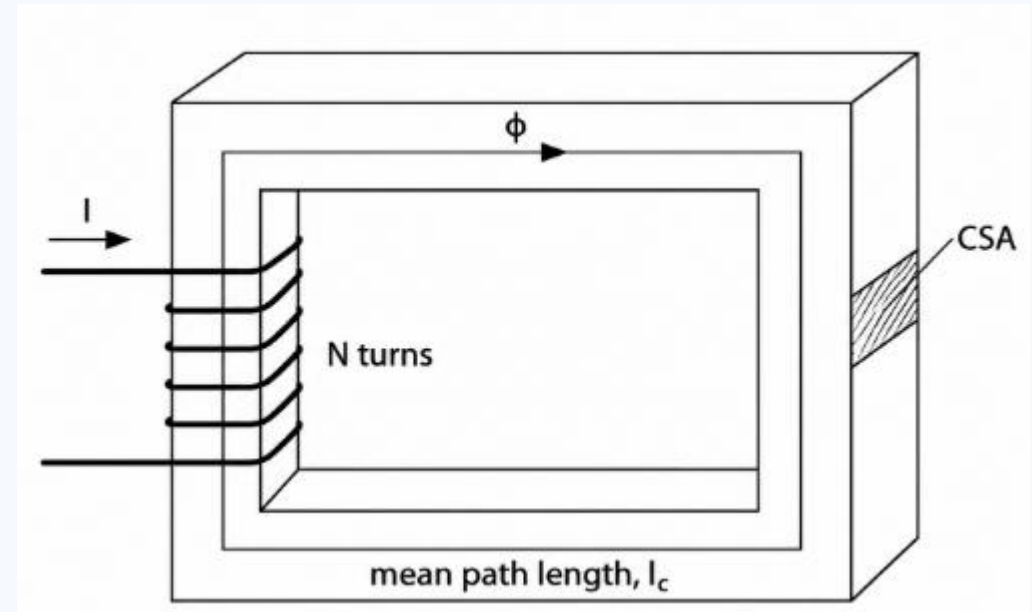
H/m

Flux Relations

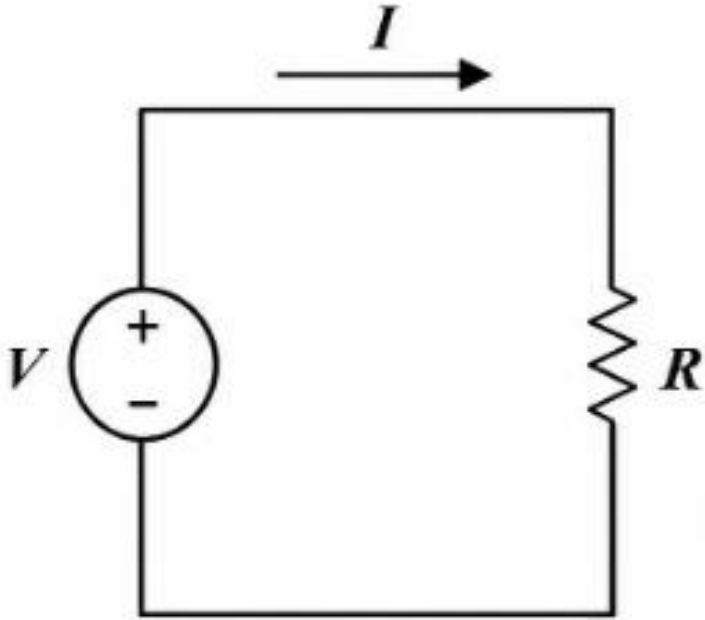
$$\Phi = \int B \, dA$$

For uniform B: $\Phi = B A$

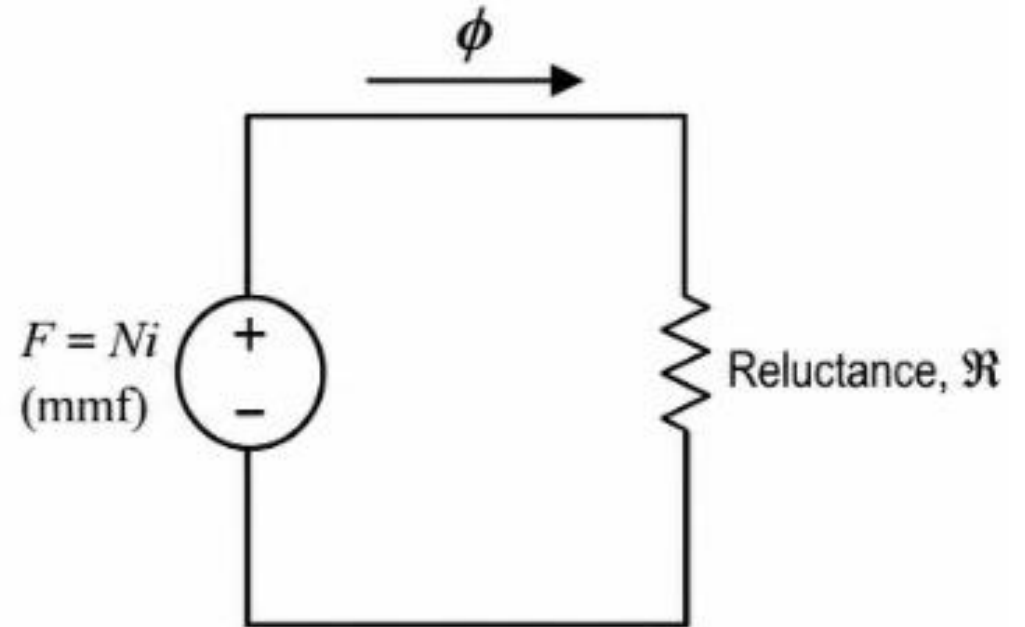
$$\Phi = \frac{\mu N i A}{L_c}$$



Electric Circuit



Magnetic Circuit



$$V = IR \quad \leftrightarrow \quad F = \Phi \mathfrak{R}$$

Main Equations

$$F = N i \quad (\text{A} \cdot \text{turns})$$

$$\mathfrak{R} = \frac{L_c}{\mu A}$$

$$\Phi = \frac{F}{\mathfrak{R}}$$

$$B = \frac{\Phi}{A}$$

Series Path

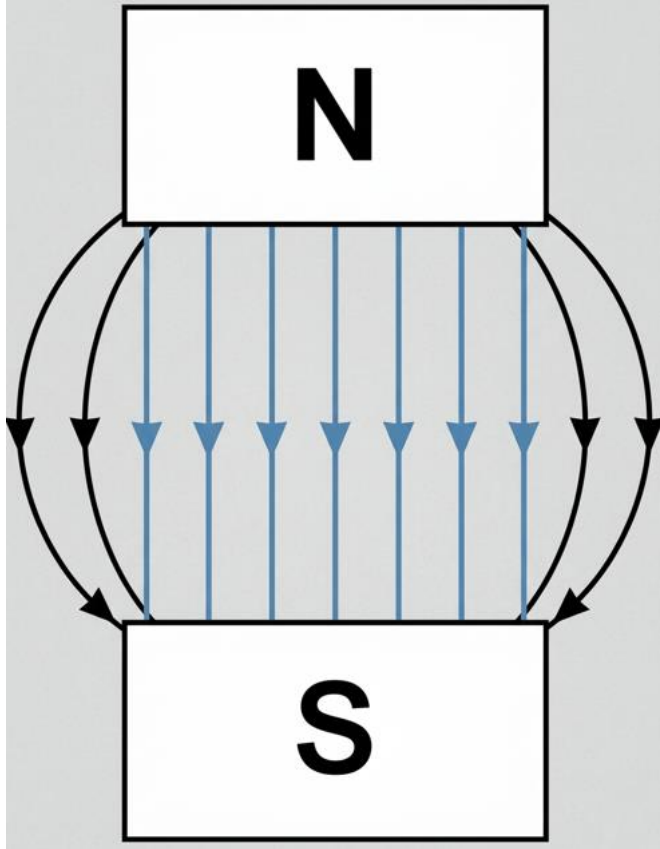
Series reluctances:

$$\mathfrak{R}_{eq} = \mathfrak{R}_1 + \mathfrak{R}_2 + \mathfrak{R}_3 + \dots$$

Parallel Path

Parallel reluctances:

$$1/\mathfrak{R}_{eq} = 1/\mathfrak{R}_1 + 1/\mathfrak{R}_2 + 1/\mathfrak{R}_3 + \dots$$



Air Gap Notes

Flux crosses the air gap from N to S.

Air has high reluctance.

Most mmf drop often occurs in the air gap.

Useful Approximations

$$\mathcal{R}_g = g / (\mu_0 A_g)$$

$$B_g \approx \Phi / A_g$$

$$F_g = \Phi \mathcal{R}_g$$

Given

$$N = 400 \text{ turns}$$

$$i = 1.5 \text{ A}$$

$$l_c = 0.30 \text{ m}$$

$$A = 20 \text{ cm}^2$$

$$\mu_r = 1000$$

This example links current, field intensity, flux density, and flux.

Solution

$$F = Ni = 400(1.5) = 600 \text{ A}\cdot\text{turns}$$

$$H = Ni/l_c = 600/0.30 = 2000 \text{ A/m}$$

$$\mu = \mu_r \mu_0 = 1000(4\pi \times 10^{-7})$$

$$B = \mu H = 2.51 \text{ T}$$

$$A = 20 \text{ cm}^2 = 2 \times 10^{-3} \text{ m}^2$$

$$\Phi = BA = 2.51(2 \times 10^{-3}) = 5.02 \text{ mWb}$$

- Rotational motion uses θ , ω , and α .
- Torque produces rotational acceleration: $\tau = J\alpha$.
- Mechanical output power: $P = \tau\omega$.
- Current produces magnetic field: $\oint \mathbf{H} \cdot d\mathbf{l} = I_{\text{net}}$.
- Magnetic circuits use mmf, flux, and reluctance.
- Air gaps strongly affect machine magnetic behavior.

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Lecture 8

Example 1

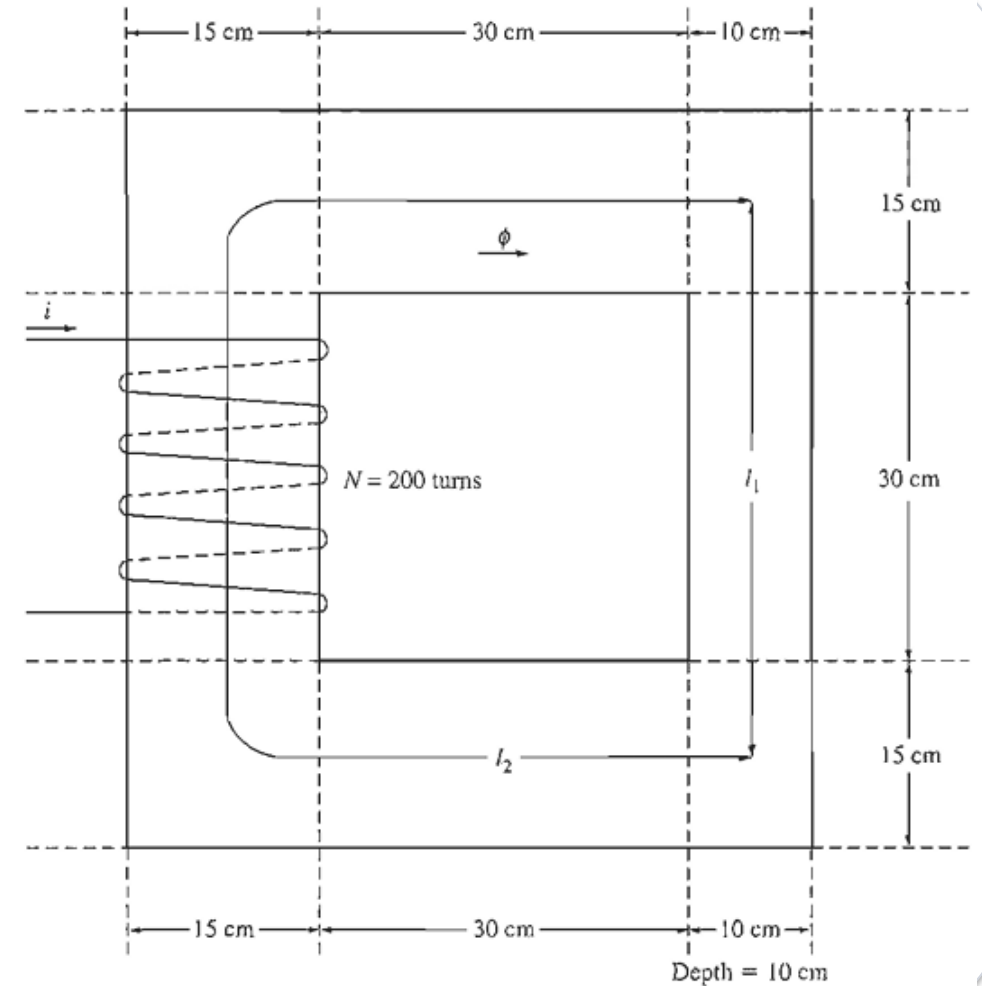
A ferromagnetic core is shown. Three sides of this core are of uniform width, while the fourth side is somewhat thinner. The depth of the core (into the page) is 10 cm, and the other dimensions are shown in the figure. There is a 200-turn coil wrapped around the left side of the core. Assuming a relative permeability μ_r of 2500, how much flux will be produced by a 1 A input current?

Given:

$N = 200$ turns, $i = 1$ A, $\mu_r = 2500$

Core depth = 10 cm

Find the magnetic flux ϕ produced in the core.



Example 1 — Solution

Calculations

$$\mathcal{R}_1 = \frac{\ell_1}{\mu A_1} = \frac{130 \times 10^{-2}}{2500 \times 4\pi \times 10^{-7} \times 15 \times 10 \times 10^{-4}}$$

$$\mathcal{R}_1 = 27586.86 \text{ At/Wb}$$

$$\mathcal{R}_2 = \frac{\ell_2}{\mu A_2} = \frac{45 \times 10^{-2}}{2500 \times 4\pi \times 10^{-7} \times 10 \times 10 \times 10^{-4}}$$

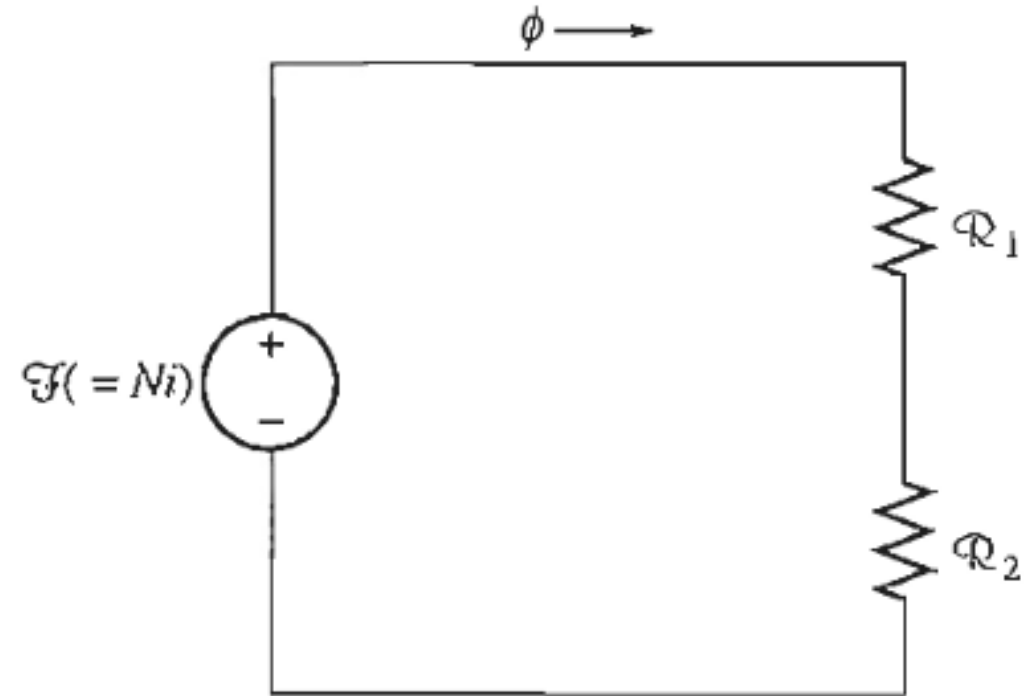
$$\mathcal{R}_2 = 14323.94 \text{ At/Wb}$$

$$\mathcal{R}_{\text{tot}} = \mathcal{R}_1 + \mathcal{R}_2 = 41910.8 \text{ At/Wb}$$

$$\phi = \frac{Ni}{\mathcal{R}_{\text{tot}}} = \frac{200 \times 1}{41910.8} \times 10^3$$

$$\phi = 4.77 \text{ mWb}$$

Equivalent Magnetic Circuit

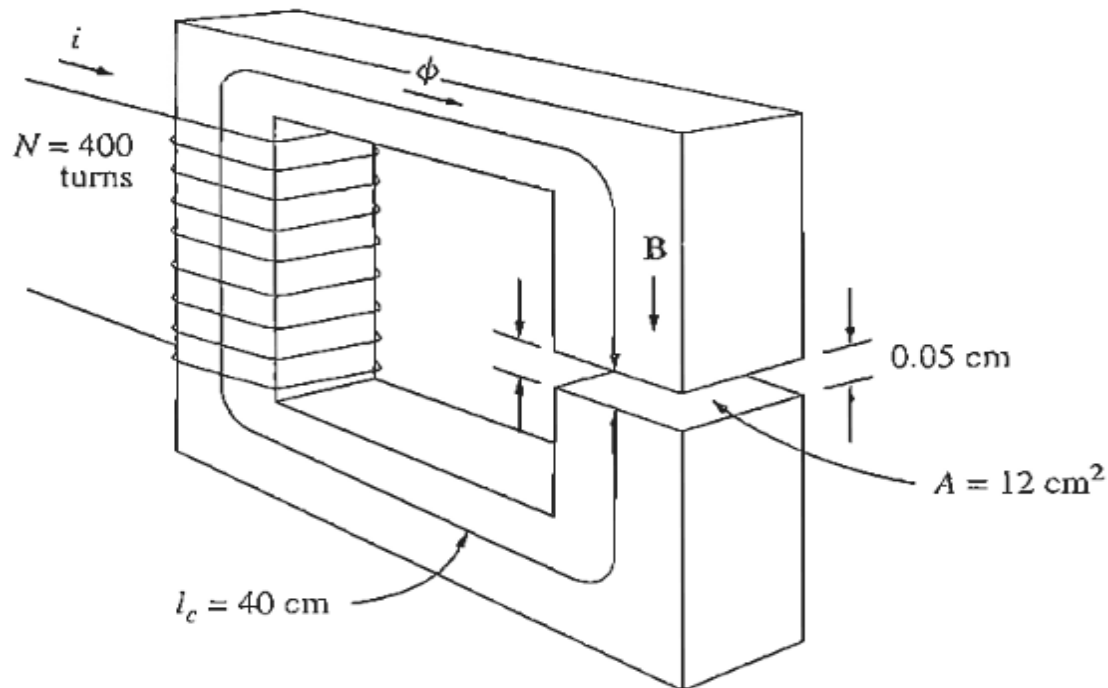


Final Result

$$\phi = 4.77 \text{ mWb}$$

Example 2

Figure shows a ferromagnetic core whose mean path length is 40cm. There is a small gap of 0.05cm in the structure of the otherwise whole core. The csa of the core is 12cm^2 , the relative permeability of the core is 4000, and the coil of wire on the core has 400 turns. Assume that fringing in the air gap increases the effective csa of the gap by 5%. Given this information, find (a) the total reluctance of the flux path (iron plus air gap) (b) the current required to produce a flux density of 0.5T in the air gap.



Given:

$$L_c = 40\text{ cm}$$

$$\text{Air gap} = 0.05\text{ cm}$$

$$A_c = 12\text{ cm}^2$$

$$N = 400\text{ turns}$$

$$\mu_r = 4000$$

Fringing increases A_g by 5%

Find total reluctance and the current for $B = 0.5\text{ T}$.

Example 2 — Solution

$$\mathcal{R}_c = \frac{\ell_c}{\mu A_c} = \frac{40 \times 10^{-2}}{4000 \times 4\pi \times 10^{-7} \times 12 \times 10^{-4}}$$

$$\mathcal{R}_c = 66314.56 \text{ At/Wb}$$

$$\mathcal{R}_a = \frac{\ell_a}{\mu A_a} = \frac{0.05 \times 10^{-2}}{4\pi \times 10^{-7} \times 12.6 \times 10^{-4}}$$

$$\mathcal{R}_a = 315783.62 \text{ At/Wb}$$

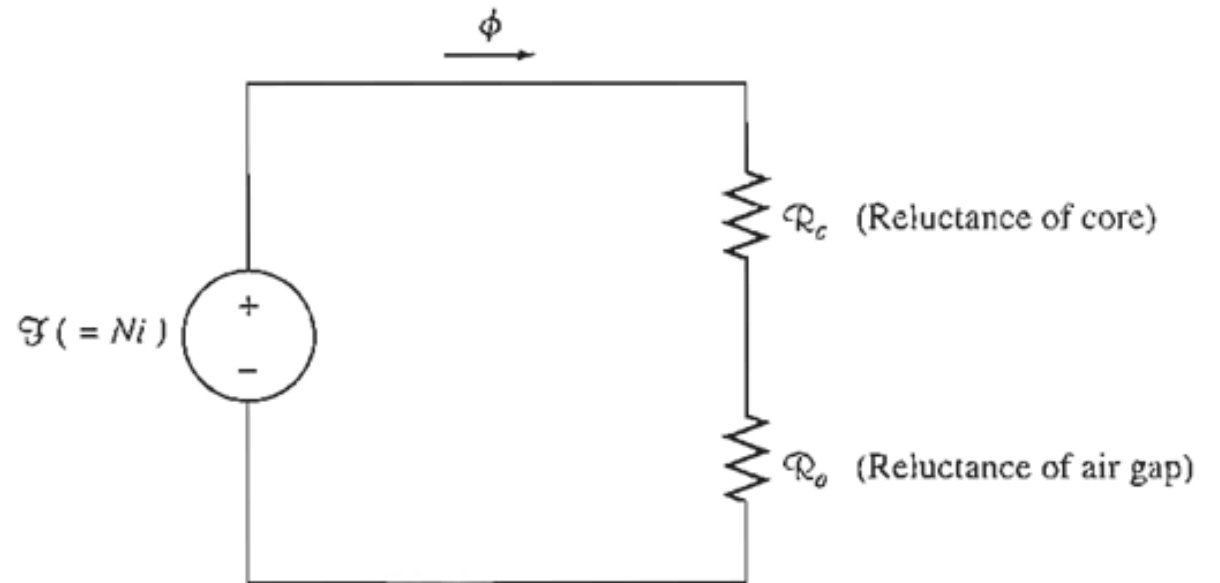
$$\mathcal{R}_{\text{tot}} = \mathcal{R}_c + \mathcal{R}_a = 382098.18 \text{ At/Wb}$$

$$\phi = B \times A = \frac{Ni}{\mathcal{R}_{\text{tot}}} = \frac{400 \times i}{382098.18}$$

$$i = \frac{B \times A \times \mathcal{R}_{\text{tot}}}{400} = \frac{0.5 \times 12.6 \times 10^{-4} \times 382098.18}{400}$$

$$i = 0.602 \times 10^{-3} = 602 \text{ mA}$$

Equivalent Magnetic Circuit

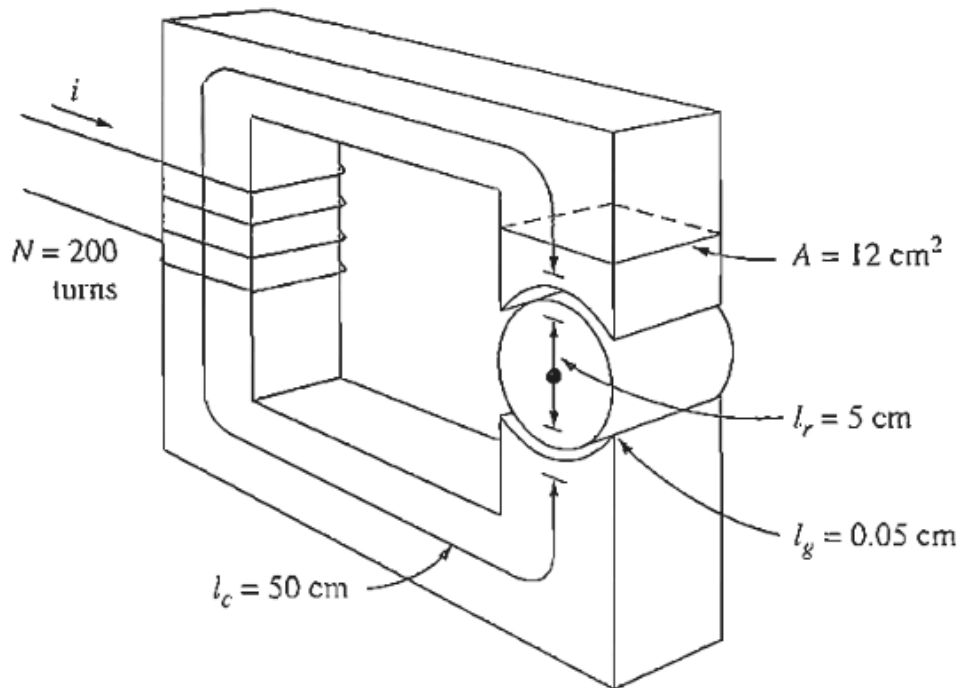


Final Result

$i = 602 \text{ mA}$

Example 3

Figure shows a simplified rotor and stator for a dc motor. The mean path length of the stator is 50cm, and its csa is 12cm^2 . The mean path length of the rotor is 5cm, and its csa also may be assumed to be 12cm^2 . Each air gap between the rotor and the stator is 0.05cm wide, and the csa of each air gap (including fringing) is 14cm^2 . The iron of the core has a relative permeability of 2000, and there are 200 turns of wire on the core. If the current in the wire is adjusted to be 1A, what will the resulting flux density in the air gaps be?



Given:

$$l_s = 50 \text{ cm}, A_s = 12 \text{ cm}^2$$

$$l_r = 5 \text{ cm}, A_r = 12 \text{ cm}^2$$

Two air gaps, each 0.05 cm

$$A_g = 14 \text{ cm}^2 \text{ (with fringing)}$$

$$\mu_r = 2000$$

$$N = 200 \text{ turns}, i = 1 \text{ A}$$

Find the air-gap flux density B.

Example 3 — Solution

$$\phi = \frac{Ni}{\mathcal{R}_{tot}} = \frac{200 \times 1}{750775.55} \times 10^3$$

$$\phi = 0.2664 \text{ mWb}$$

$$B = \frac{\phi}{A} = \frac{0.2664 \times 10^{-3}}{14 \times 10^{-4}}$$

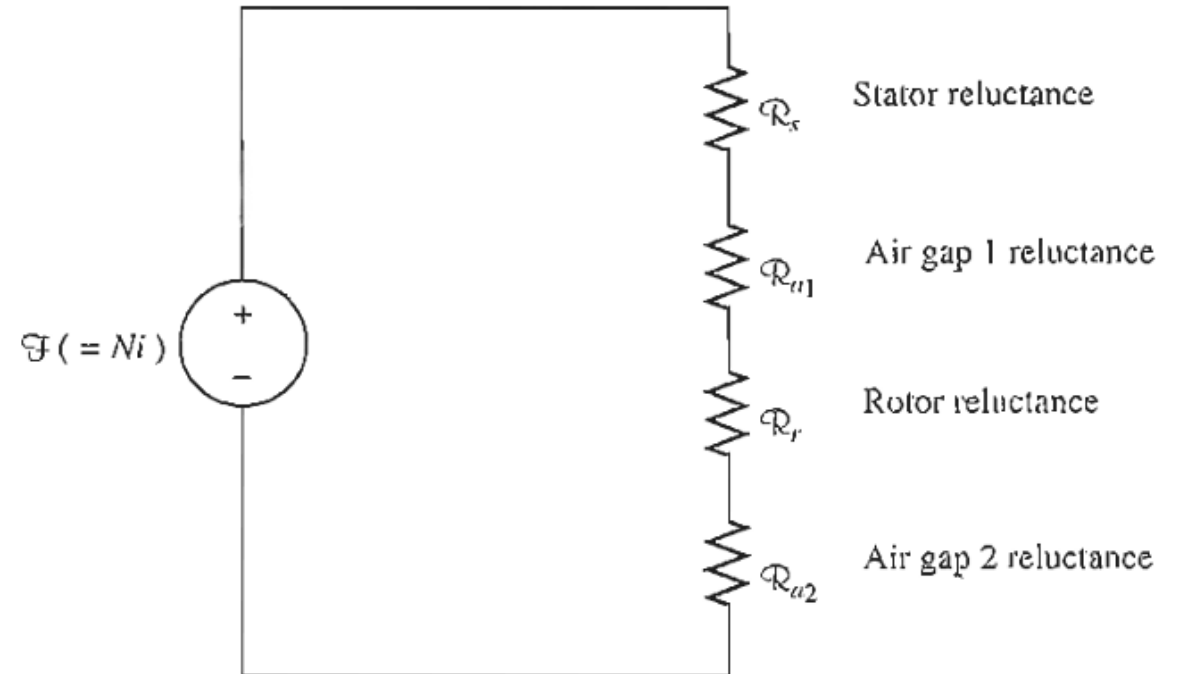
$$B = 0.1903 \text{ T}$$

$$\mathcal{R}_{a1} = \mathcal{R}_{a2} = 284205.26 \text{ At/Wb}$$

$$\mathcal{R}_{tot} = \mathcal{R}_s + \mathcal{R}_r + 2\mathcal{R}_a$$

$$\mathcal{R}_{tot} = 750775.55 \text{ At/Wb}$$

Equivalent Magnetic Circuit



Final Result

$$B = 0.1903 \text{ T}$$

Key Ideas

Angular motion uses θ , ω , and α .

Mechanical power in rotation: $P = \tau \omega$.

Magnetic-field intensity: $H = Ni / l$.

Flux density: $B = \mu H$.

Magnetic flux: $\phi = BA$.

Reluctance: $\mathfrak{R} = l / (\mu A)$.

Magnetic-circuit law: $F = \phi \mathfrak{R}$, where $F = Ni$.

Worked Examples

Example 1.1: $\phi = 4.77 \text{ mWb}$

Example 1.2: $i = 602 \text{ mA}$

Example 1.3: $B = 0.1903 \text{ T}$

These examples support the magnetic-circuit foundations used in electrical machines.

$$\theta, \omega, \alpha \quad \bullet \quad \tau = F r \sin\theta \quad \bullet \quad P = \tau\omega \quad \bullet \quad H = Ni/l \quad \bullet \quad B = \mu H \quad \bullet \quad \phi = BA \quad \bullet \quad \mathfrak{R} = l/(\mu A)$$

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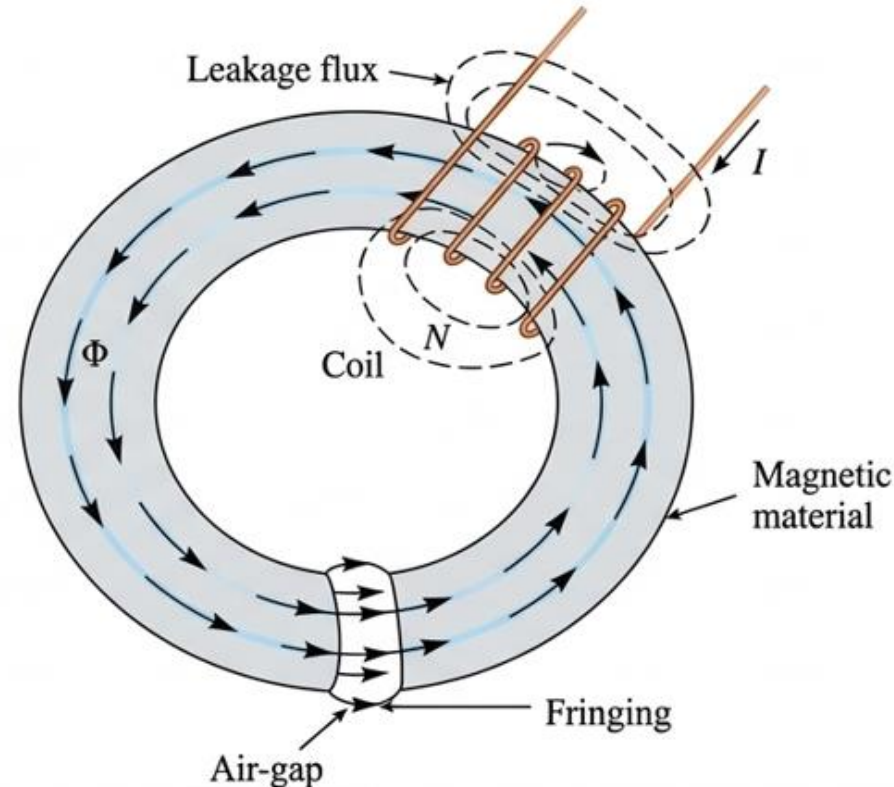
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MSc – Electrical Engineering

Lecture 9

Example 4

An electromagnet of square cross-section has a tightly wound coil with 1500 turns. The inner and outer radii of the magnetic core are 10 cm and 12 cm, respectively. The length of the air-gap is 1 cm. If the current in the coil is 4 A and the relative permeability of the magnetic material is 1200, determine the flux density in the magnetic circuit.



Example 4 — Solution

$$l_m = 2\pi \times 11 - 1 = 68.12 \text{ cm}$$

$$A_m = A_g = 2 \times 2 = 4 \text{ cm}^2$$

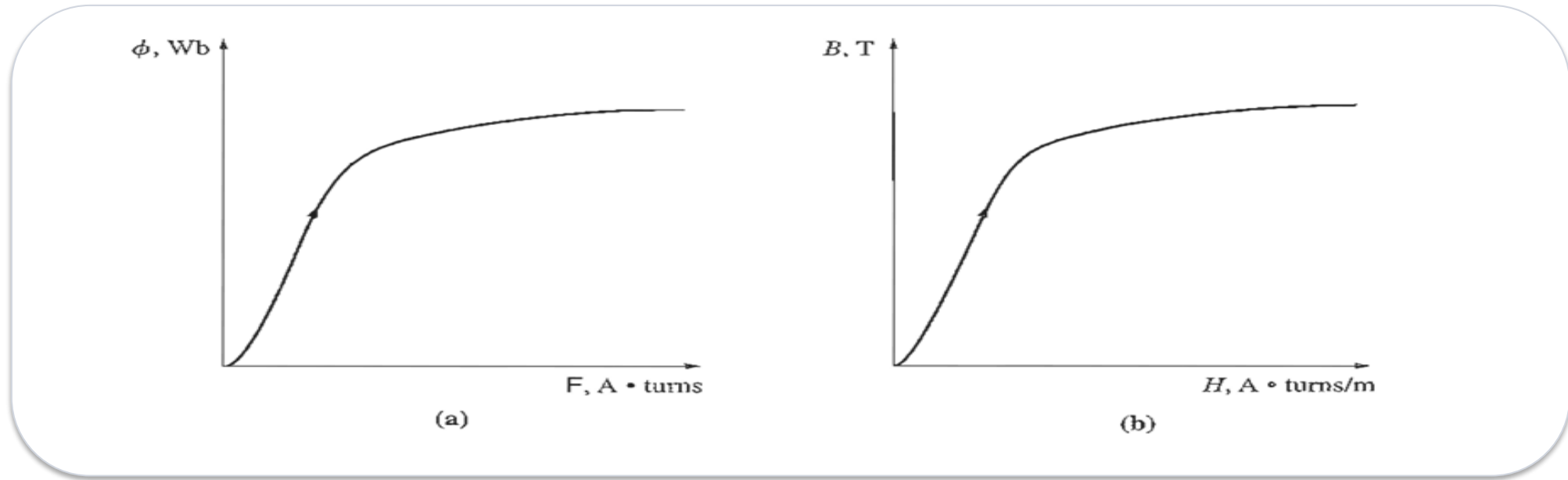
$$R_m = \frac{68.12 \times 10^{-2}}{1200 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 1.129 \times 10^6 \text{ A.t/Wb}$$

$$R_g = \frac{1 \times 10^{-2}}{4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 19.894 \times 10^6 \text{ A.t/Wb}$$

$$R = R_m + R_g = 21.023 \times 10^6 \text{ A.t/Wb}$$

$$\Phi = \frac{1500 \times 4}{21.023 \times 10^6} = 285.402 \times 10^{-6} \text{ Wb}$$

$$B_m = B_g = \frac{285.402 \times 10^{-6}}{4 \times 10^{-4}} = 0.714 \text{ T}$$



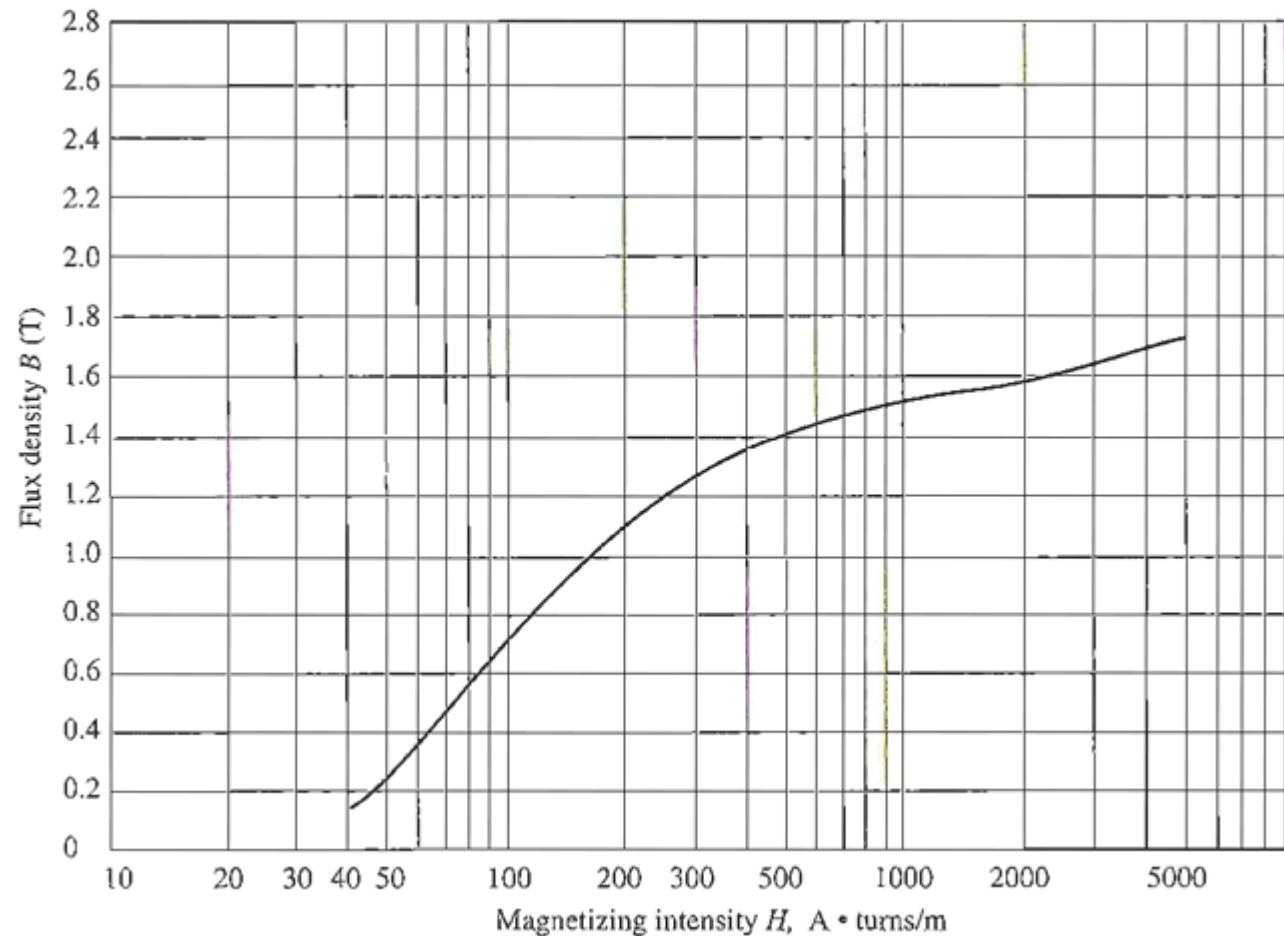
$$H = \frac{Ni}{\ell_c} = \frac{\mathcal{F}}{\ell_c}$$

$$B = \mu \times H$$

$$\phi = B \times A = \mu \times H \times A$$

Example 5

Find the relative permeability of the typical ferromagnetic material whose magnetization curve is shown at (a) $H = 50$, (b) $H = 100$, (c) $H = 500$, and (d) $H = 1000 \text{ A-turn/m}$.



Example 5 — Solution

a

$$\mu = \frac{B}{H}$$

$$\text{at } H = 50 \text{ A-turn/m} \quad B = 0.25 \text{ T}$$

$$\mu = \frac{0.25}{50} = 0.005 \text{ H/m}$$

$$\mu_r = \frac{\mu}{\mu_o} = \frac{0.005}{4\pi \times 10^{-7}} = 3980$$

b

$$\mu = \frac{B}{H}$$

$$\text{at } H = 100 \text{ A-turn/m} \quad B = 0.72 \text{ T}$$

$$\mu = \frac{0.72}{100} = 0.0072 \text{ H/m}$$

$$\mu_r = \frac{\mu}{\mu_o} = \frac{0.0072}{4\pi \times 10^{-7}} = 5730$$

c

$$\mu = \frac{B}{H}$$

$$\text{at } H = 500 \text{ A-turn/m} \quad B = 1.4 \text{ T}$$

$$\mu = \frac{1.4}{500} = 0.0028 \text{ H/m}$$

$$\mu_r = \frac{\mu}{\mu_o} = \frac{0.0028}{4\pi \times 10^{-7}} = 2230$$

d

$$\mu = \frac{B}{H}$$

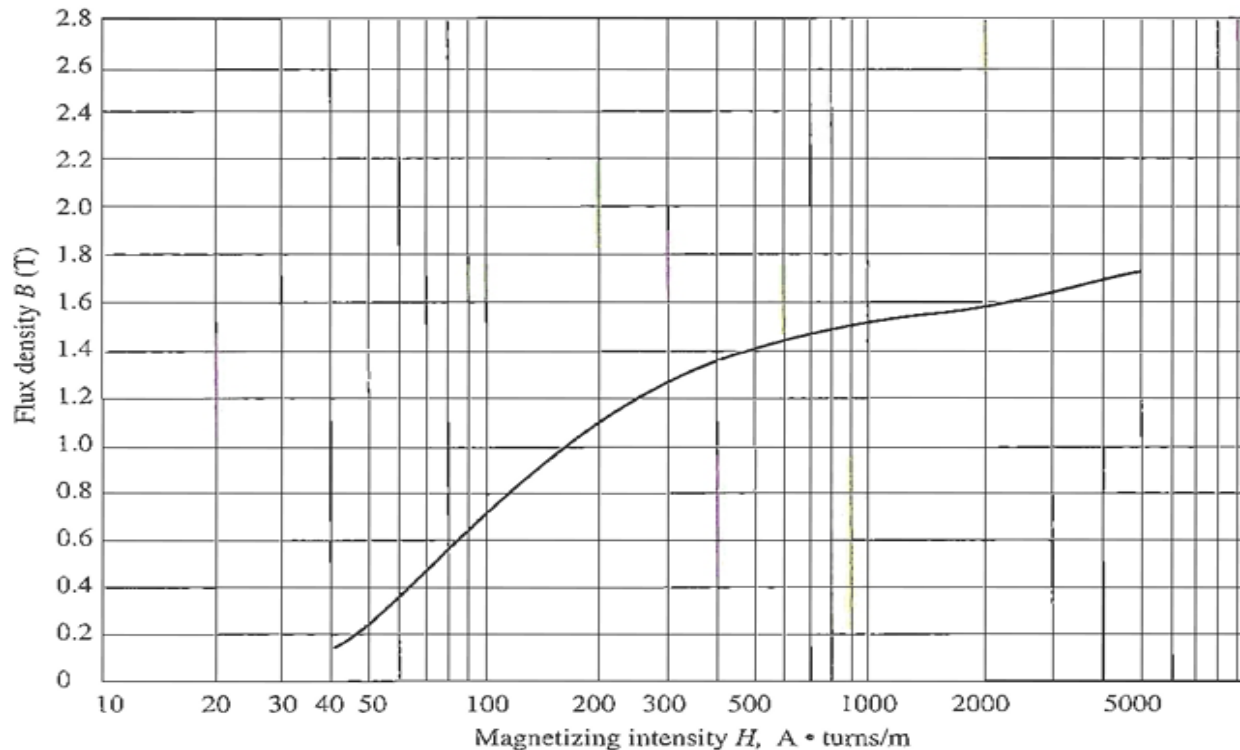
$$\text{at } H = 1000 \text{ A-turn/m} \quad B = 1.51 \text{ T}$$

$$\mu = \frac{1.51}{1000} = 0.00151 \text{ H/m}$$

$$\mu_r = \frac{\mu}{\mu_o} = \frac{0.00151}{4\pi \times 10^{-7}} = 1200$$

Example 6

A square magnetic core has a mean path length of 55cm and a csa of 150cm^2 . A 200-turn coil of wire is wrapped around one leg of the core. The core is made of a material having the magnetization curve shown below. Find: (a) How much current is required to produce 0.012 Wb of flux in the core?, (b) What is the core's relative permeability at that current level?, and (c) What is its reluctance?



a

$$B = \frac{\phi}{A} = \frac{0.012}{0.015} = 0.8 \text{ T}$$

$$H = 115 \text{ A-turn/m (from curve)}$$

$$\mathcal{F} = Ni = H\ell = 115 \times 0.55$$

$$\mathcal{F} = 63.25 \text{ A-turn}$$

$$i = \frac{\mathcal{F}}{N} = \frac{63.25}{200} = 0.316 \text{ A}$$

b

$$\mu = \frac{B}{H} = \frac{0.8}{115} = 0.00696 \text{ H/m}$$

$$\mu_r = \frac{\mu}{\mu_o} = \frac{0.00696}{4\pi \times 10^{-7}} = 5540$$

c

$$\mathcal{R} = \frac{\mathcal{F}}{\phi} = \frac{63.25}{0.012}$$

$$\mathcal{R} = 5270 \text{ A-turn/Wb}$$

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Lecture 10

If a flux passes through a turn of a coil of wire, voltage will be induced in the turn of the wire that is directly proportional to the rate of change in the flux with respect of time (Faraday's Law).

$$e_{ind} = -\frac{d\phi}{dt}$$

where

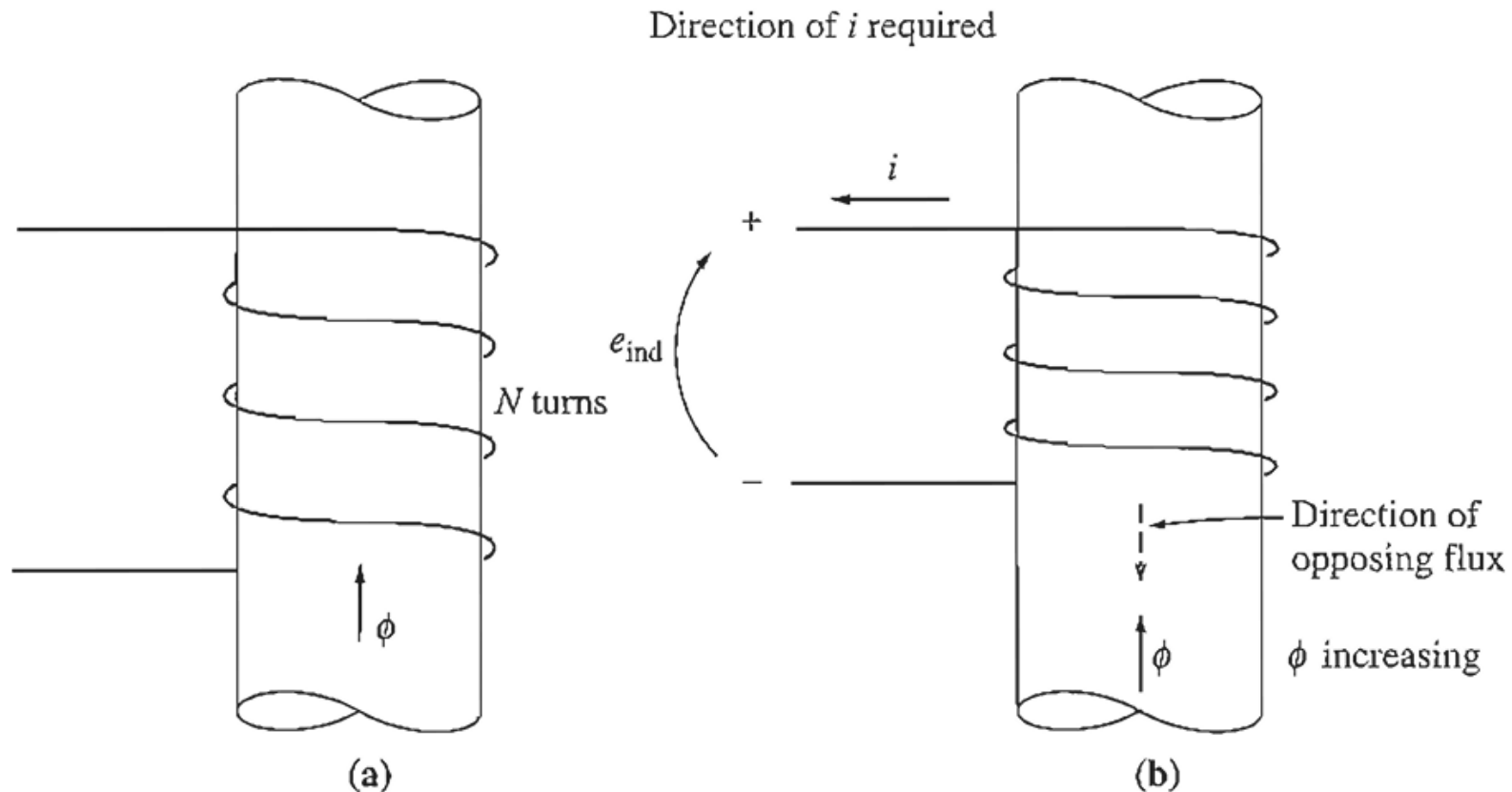
e_{ind} = voltage induced in the coil

N = number of turns of wire in coil

ϕ = flux passing through coil

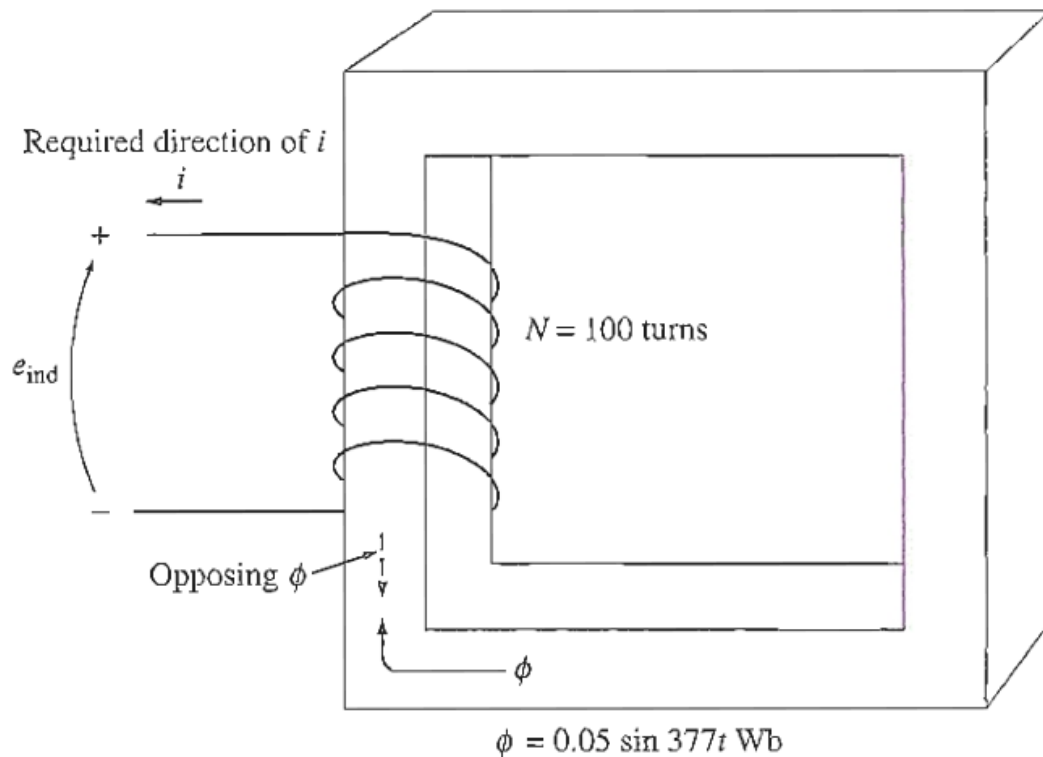
$$e_{ind} = -N \frac{d\phi}{dt}$$

The direction of the build-up voltage in the coil is as such that if the coils were short circuited, it would produce current that would cause a flux opposing the original flux change (Lenz's Law).



Example 7

The coil of wire wrapped around the iron core in the figure shown has 100 turns. If the flux in the core is given by $\Phi = 0.05 \sin 377 t$ Wb Determine the voltage induced at the terminals of the coil and its polarity when flux is increasing in the reference direction shown.



Solution

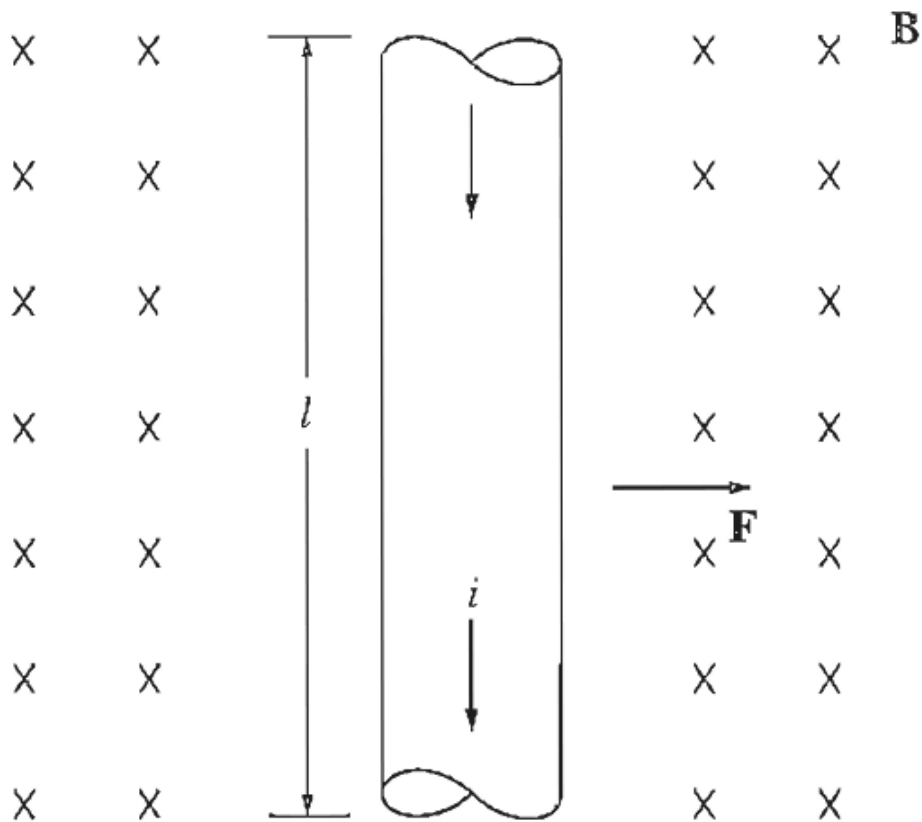
$$e_{ind} = N \frac{d\phi}{dt}$$

$$e_{ind} = 100 \frac{d}{dt} (0.05 \sin 377t)$$

$$e_{ind} = 1885 \cos 377t \text{ V}$$

$$e_{ind} = 1885 \sin(377t + 90^\circ) \text{ V}$$

A magnetic field produces a force on a current-carrying wire placed inside it. The force direction is determined by the **right-hand rule**.



Fleming's Right-Hand Rule

Three perpendicular directions for a force on a current-carrying conductor

Index Finger
(current direction)

Middle Finger
(magnetic flux density direction)

Thumb
(resultant force)

Use your right hand.
Index, middle and thumb
are mutually perpendicular.

Vector form:

$$\mathbf{F} = i(\boldsymbol{\ell} \times \mathbf{B})$$

Magnitude:

$$F = i\ell B \sin \theta$$

Symbols:

F : induced force on the wire (N)

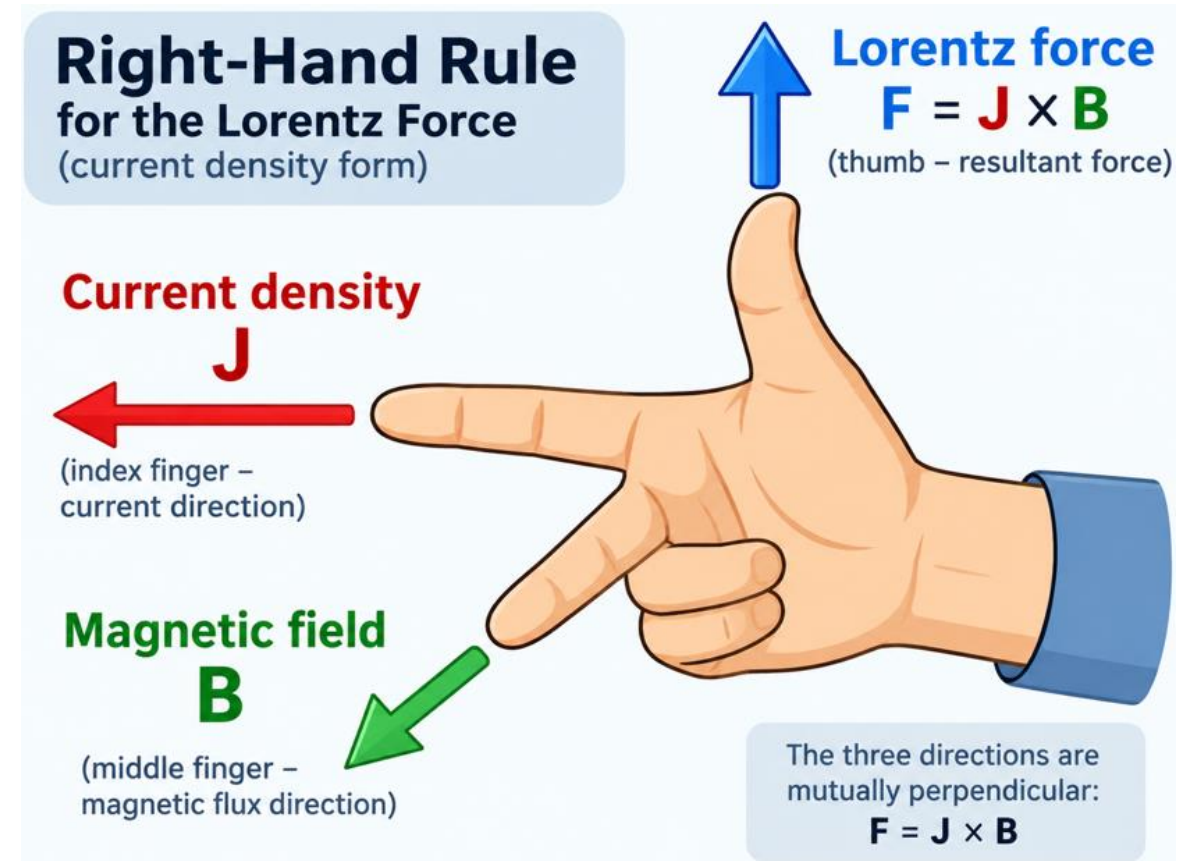
i : current in the wire (A)

ℓ : wire length vector, direction of current (m)

B : magnetic flux density vector (T)

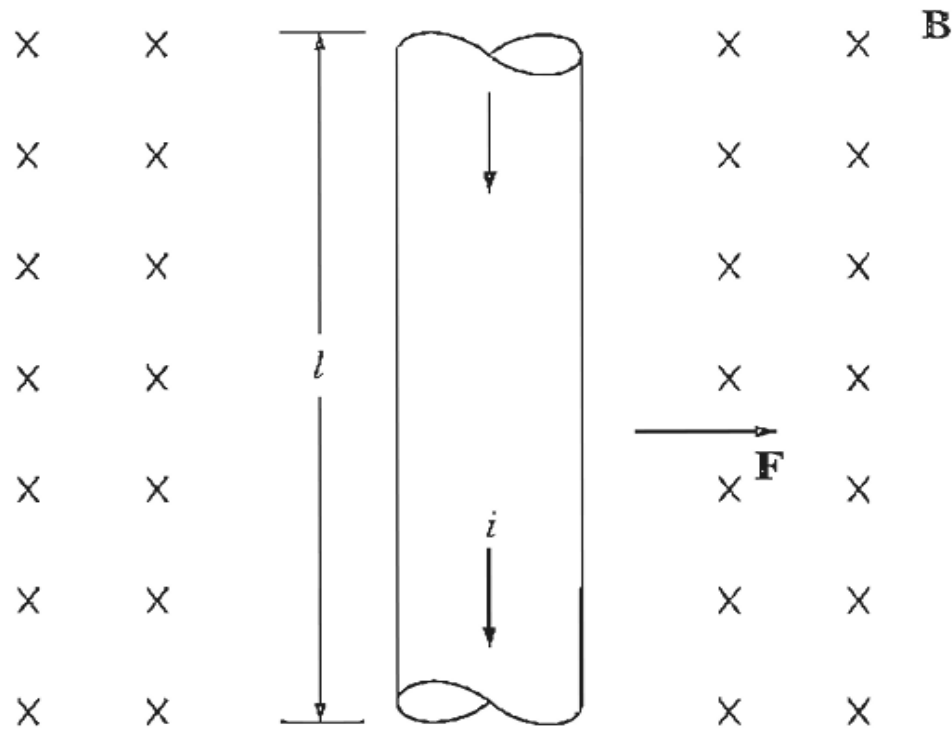
θ : angle between ℓ and B

Direction: right-hand rule



Example 8

The figure shows a wire carrying a current in the presence of a magnetic field. The magnetic flux density is 0.25 T, directed into the page. If the wire is 1 m long and carries 0.5 A of current in the direction from the top of the page to the bottom, what are the magnitude and direction of the force induced on the wire?



Solution

$$F = Bli \sin \theta$$

$$F = 0.25 \times 1 \times 0.5 \sin 90^\circ$$

$$F = 0.125 \text{ N (directed to the right)}$$

When a conductor moves through a magnetic field, a voltage is induced in it.

$$e_{\text{ind}} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l}$$

Symbols:

e_{ind} : induced voltage

$\mathbf{v}$: velocity of the wire

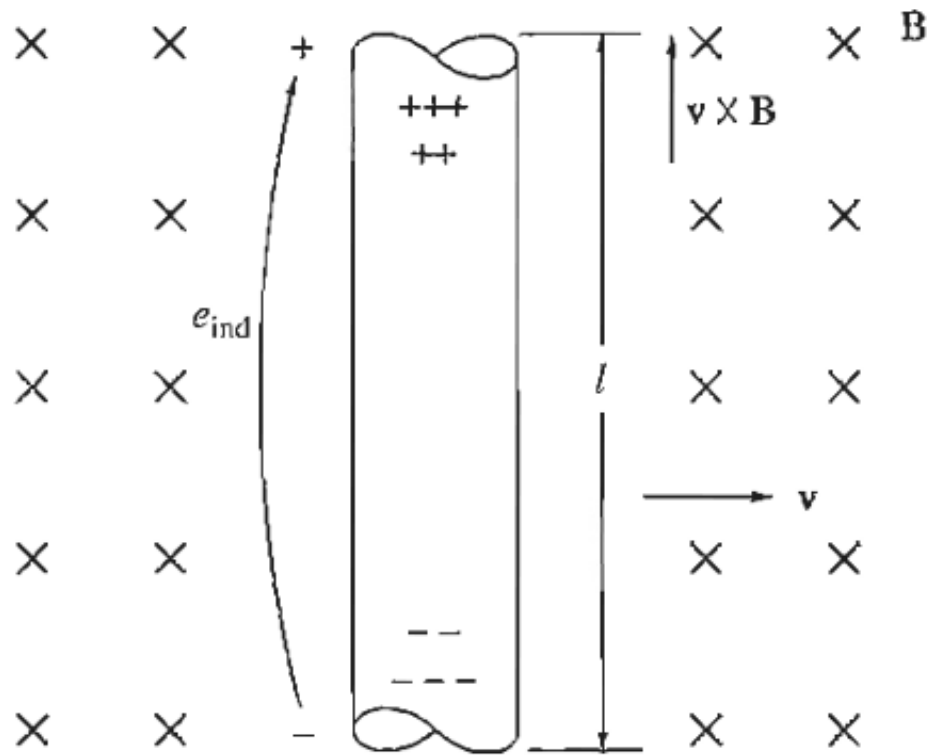
$\mathbf{B}$: magnetic flux density vector

$\mathbf{l}$: length vector of the conductor in the magnetic field

The positive end is in the direction of $\mathbf{v} \times \mathbf{B}$.

Example 9

The figure shows a conductor moving with a velocity of 5 m/s to the right in the presence of a magnetic field. The flux density is 0.5 T into the page, and the wire is 1 m length, oriented as shown. What are the magnitude and polarity of the resulting induced voltage?

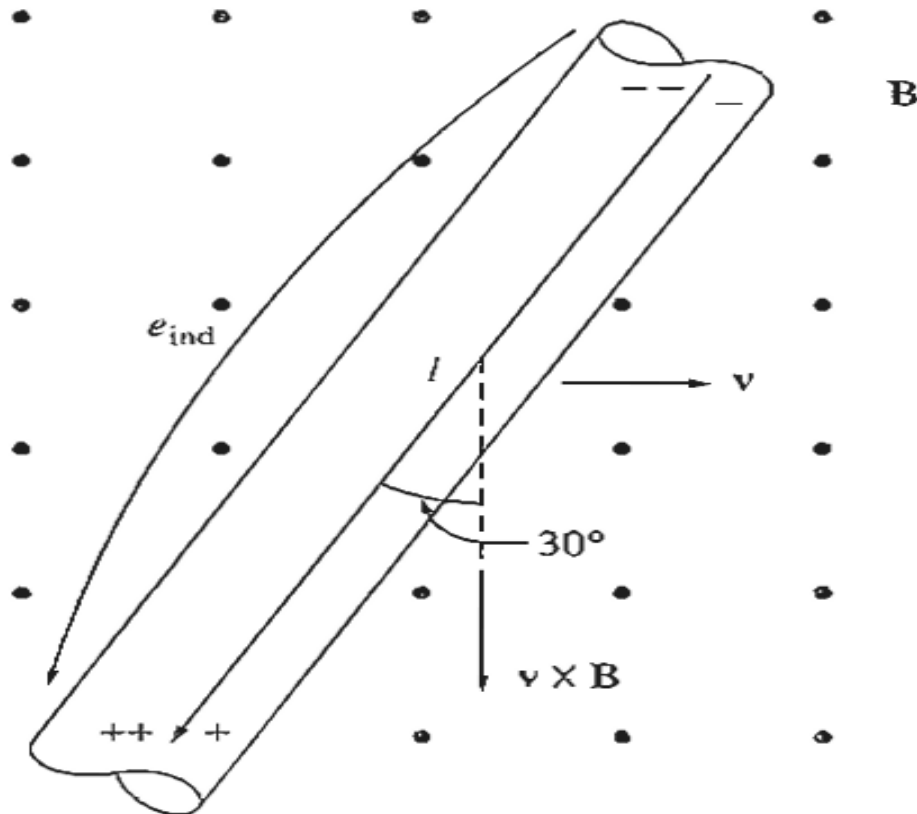


Solution

$$\begin{aligned}
 e_{ind} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \\
 &= (vB \sin 90^\circ) l \cos 0^\circ \\
 &= vBl \\
 &= (5.0 \text{ m/s})(0.5 \text{ T})(1.0 \text{ m}) \\
 &= 2.5 \text{ V}
 \end{aligned}$$

Example 10

Figure shows a conductor moving with a velocity of 10 m/s to the right in a magnetic field. The flux density is 0.5 T, out of the page, and the wire is 1 m in length. What are the magnitude and polarity of the resulting induced voltage?



Solution

$$\begin{aligned}
 e_{\text{ind}} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \\
 &= (vB \sin 90^\circ) l \cos 30^\circ \\
 &= (10.0 \text{ m/s})(0.5 \text{ T})(1.0 \text{ m}) \cos 30^\circ \\
 &= 4.33 \text{ V}
 \end{aligned}$$

Thank you