

Electrical Machines

Chapter 2 TRANSFORMERS

Instructor

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Electrical Machines

Chapter 2 TRANSFORMERS

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Lecture 11

Practical Types of Transformer?



Image A
Oil-Immersed Distribution Transformer



Image B
Pole-Mounted Distribution Transformer



Image C
Pole-Mounted Transformer Bank

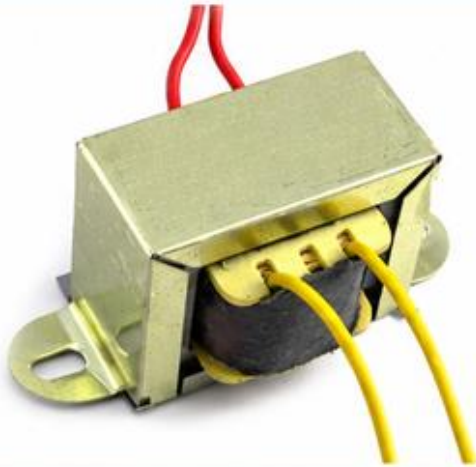


Image D
Small Electronic Transformer



Image E
Power Distribution Transformer

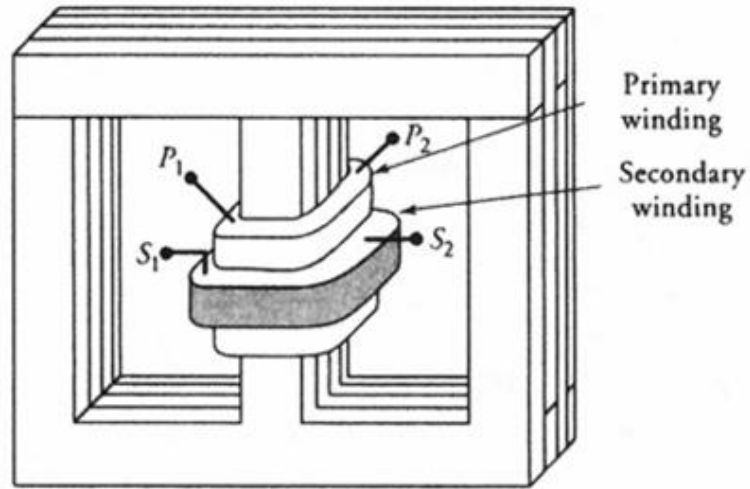


Image F
Substation Power Transformer

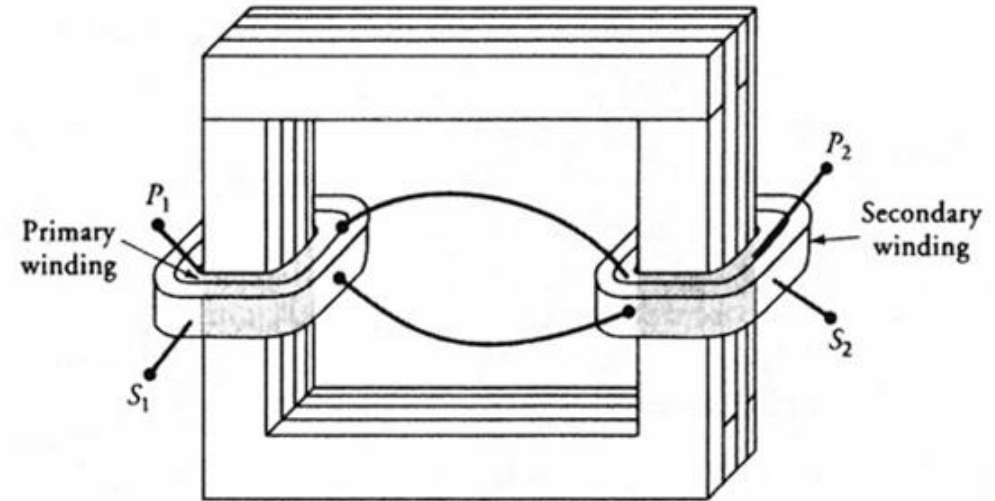
Definition

- A transformer has two windings that are electrically isolated but magnetically coupled.
- **Primary winding: connected to the source.**
- **Secondary winding: connected to the load.**
- Induced emf is proportional to the number of turns in the winding.
- **Step-up transformer: $V_s > V_p$.**
- **Step-down transformer: $V_s < V_p$.**
- **A 1:1 ratio transformer is called an isolation transformer.**

Basic types

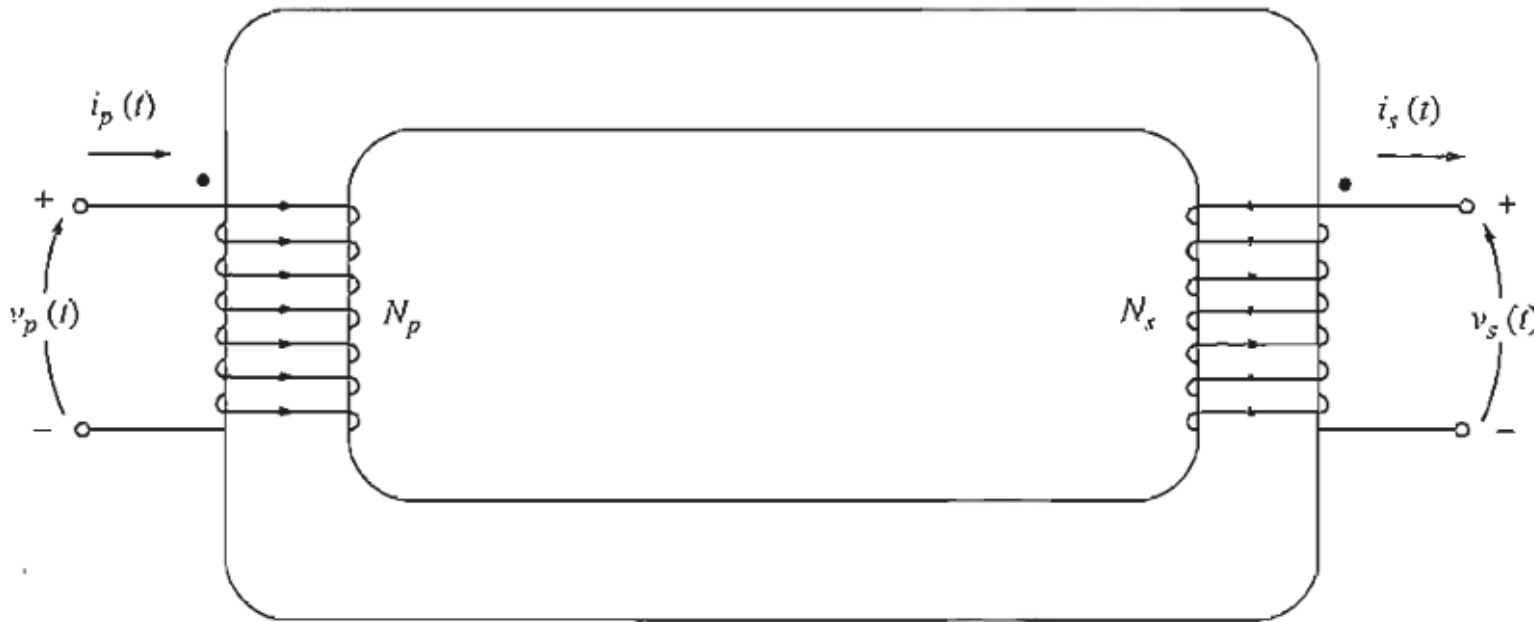


Shell-type



Core-type

- The core is built from thin laminations of highly permeable ferromagnetic material, such as silicon sheet steel.
- Lamination thickness is typically 0.014 to 0.024 inch to reduce core losses.
- A thin varnish coating is applied to provide electrical insulation between laminations.

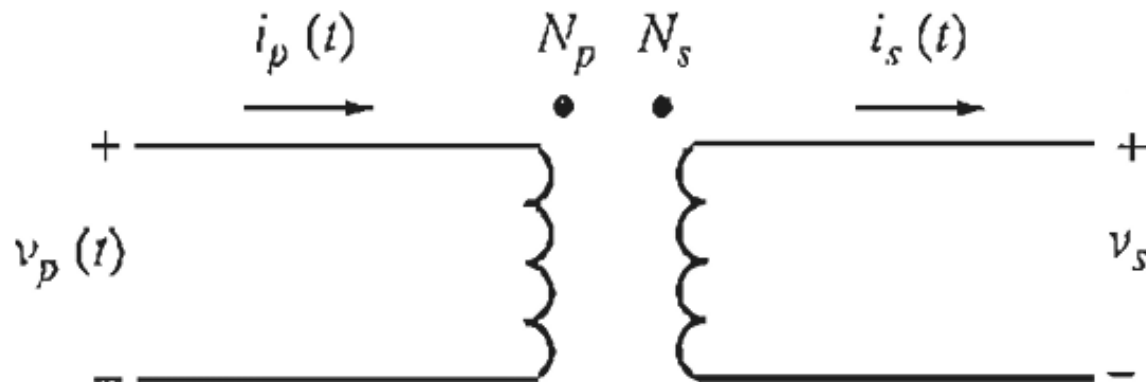


Voltage ratio

$$\frac{V_p}{V_s} = \frac{N_p}{N_s} = a$$

Ampere-turn balance

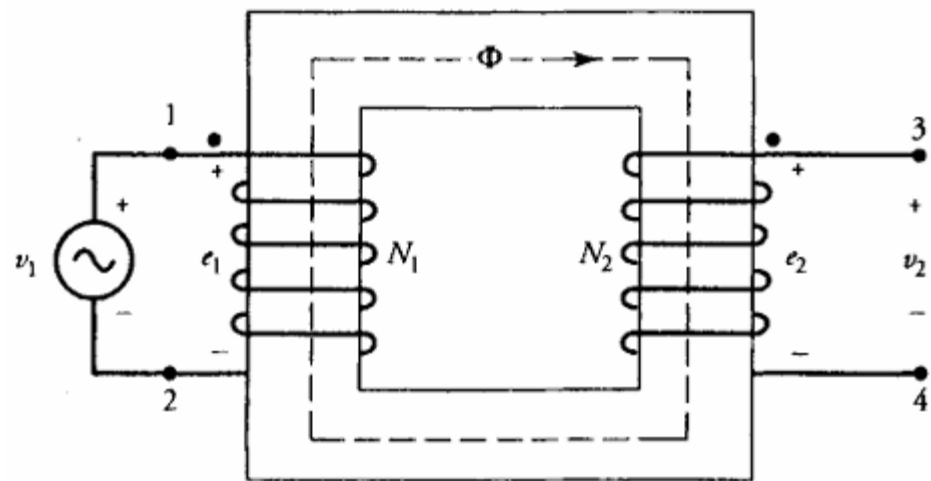
$$N_p i_p = N_s i_s$$



Current ratio

$$\frac{I_p}{I_s} = \frac{N_s}{N_p} = \frac{1}{a}$$

Equivalent concept



$$e_1 = N_1 \frac{d\phi}{dt} = v_1$$

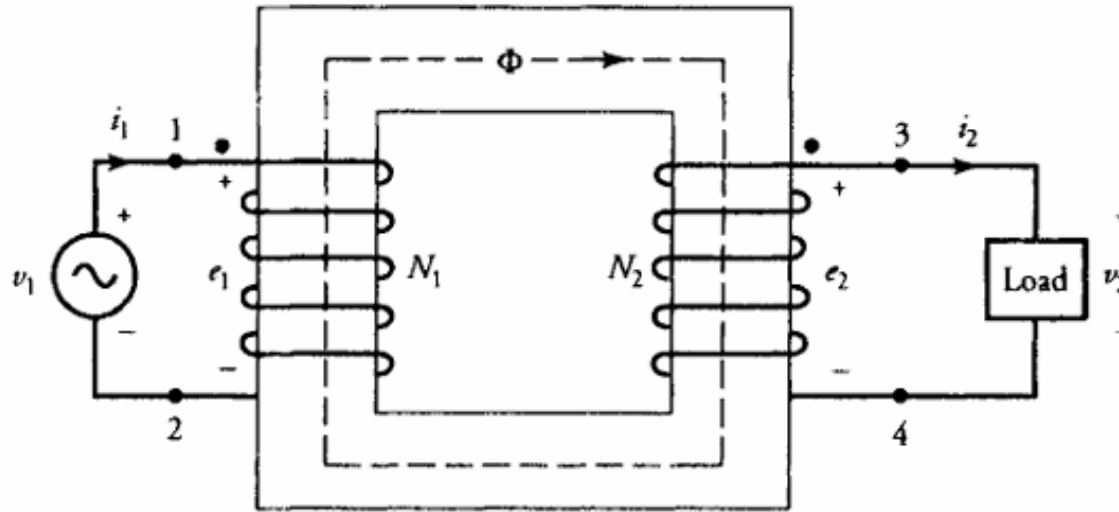
$$e_2 = N_2 \frac{d\phi}{dt} = v_2$$

$$\frac{V_1}{V_2} = \frac{e_1}{e_2} = \frac{N_1}{N_2} = a$$

Ideal transformer properties

- **Highly permeable core: only a very small mmf is needed to establish flux Φ .**
- **No eddy-current loss and no hysteresis loss.**
- **No leakage flux: the flux is fully confined within the core.**
- **Winding resistance is negligible.**

Loaded transformer



$$N_1 i_1 = N_2 i_2$$

$$\frac{i_2}{i_1} = \frac{N_1}{N_2} = a$$

$$v_1 i_1 = v_2 i_2$$

$$\Phi = \Phi_m \sin \omega t$$

$$e_1 = N_1 \omega \Phi_m \cos \omega t$$

$$E_1 = 4.44 f N_1 \Phi_m \angle 0^\circ$$

$$E_2 = 4.44 f N_2 \Phi_m \angle 0^\circ$$

$$V_1 / V_2 = E_1 / E_2 = N_1 / N_2 = a$$

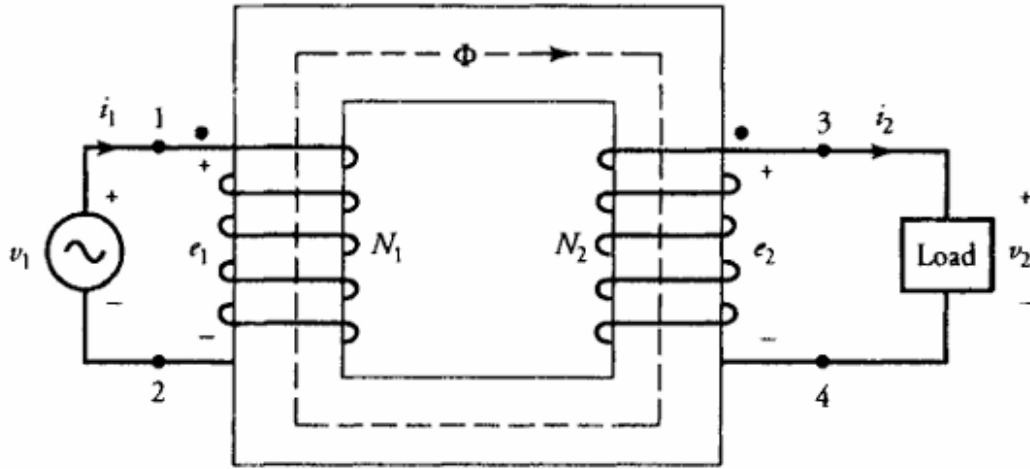
$$I_2 / I_1 = N_1 / N_2 = a$$

$$V_1 I_1^* = V_2 I_2^* \quad \text{or} \quad V_1 I_1 = V_2 I_2$$

Important observations:

- The induced rms voltage is proportional to frequency, turns, and maximum flux.
- Voltage transformation follows the turns ratio.
- Current transformation is inverse to the turns ratio.
- In the ideal model, complex power is conserved across the transformer.

Equivalent concept



$$Z_2 = V_2 / I_2 = (1 / a^2) \cdot (V_1 / I_1)$$

$$Z_2 = (1 / a^2) Z_1$$

$$Z_1 = a^2 Z_2$$

Impedance reflection rule:

- An impedance moved from secondary to primary is multiplied by a^2 .
- An impedance moved from primary to secondary is divided by a^2 .

This relation is widely used to analyze equivalent transformer circuits.

A transformer nameplate specifies the voltage and apparent-power handling capability of each winding.

Example: 5-kVA, 500/250-V step-down transformer

- **Full-load apparent power rating: 5 kVA**
 - Nominal primary voltage: $V_1 = 500 \text{ V}$.
 - Nominal secondary voltage: $V_2 = 250 \text{ V}$.
 - Full-load primary current: $I_1 = 5000 / 500 = 10 \text{ A}$.
 - Full-load secondary current: $I_2 = 5000 / 250 = 20 \text{ A}$.
 - Transformation ratio: $a = V_1 / V_2 = 500 / 250 = 2$.

Quick Summary

$$S = 5 \text{ kVA}$$

$$V_1 = 500 \text{ V}$$

$$V_2 = 250 \text{ V}$$

$$I_1 = 10 \text{ A}$$

$$I_2 = 20 \text{ A}$$

$$a = 2$$

Example 1 — Ideal Transformer

Turns ratio, current, load power, and core flux

An ideal transformer has a 150-turn primary and 750-turn secondary. The primary is connected to a 240-V, 50-Hz source. The secondary winding supplies a load of 4 A at a lagging power factor of 0.8.

Determine:

- (a) the a-ratio,
- (b) the current in the primary,
- (c) the power supplied to the load, and
- (d) the flux in the core.

Solution

$$a = N_1/N_2 = 150/750 = 0.2$$

$$I_1 = I_2/a = 4/0.2 = 20 \text{ A}$$

$$V_2 = V_1/a = 240/0.2 = 1200 \text{ V}$$

$$P_L = V_2 I_2 \cos\theta$$

$$P_L = 1200 \times 4 \times 0.8 = 3840 \text{ W}$$

Flux in the Core

$$\Phi_m = E_1 / (4.44 f N_1)$$

$$\Phi_m = 240 / (4.44 \times 50 \times 150)$$

$$\Phi_m = 7.21 \text{ mWb}$$

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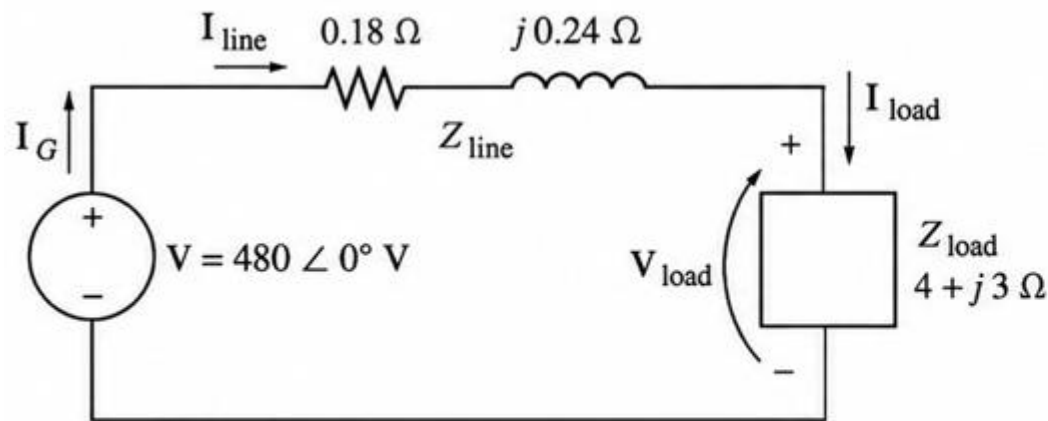
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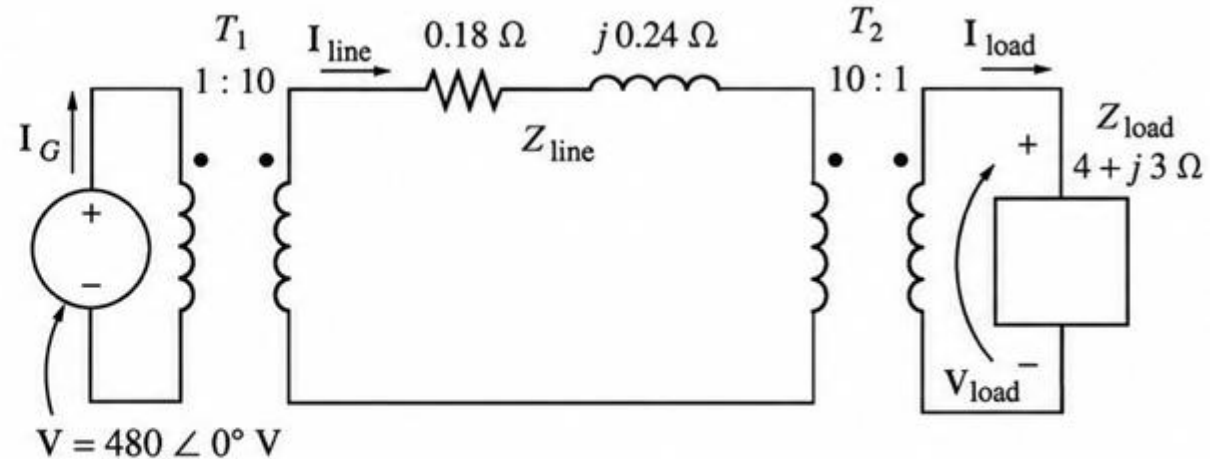
A generator rated at 480-V, 60-Hz is connected to a transmission line with an impedance of $0.18 + j0.24 \Omega$. At the end of the transmission line there is a load of $4 + j3 \Omega$.

(a) If the power system is exactly as described above in Figure (a), what will the voltage at the load be? What will the transmission line losses be?

(b) Suppose a 1:10 step-up transformer is placed at the generator end of the transmission line and a 10:1 step-down transformer is placed at the load end of the line (Figure (b)). What will the load voltage be now? What will the transmission line losses be now?



(a)

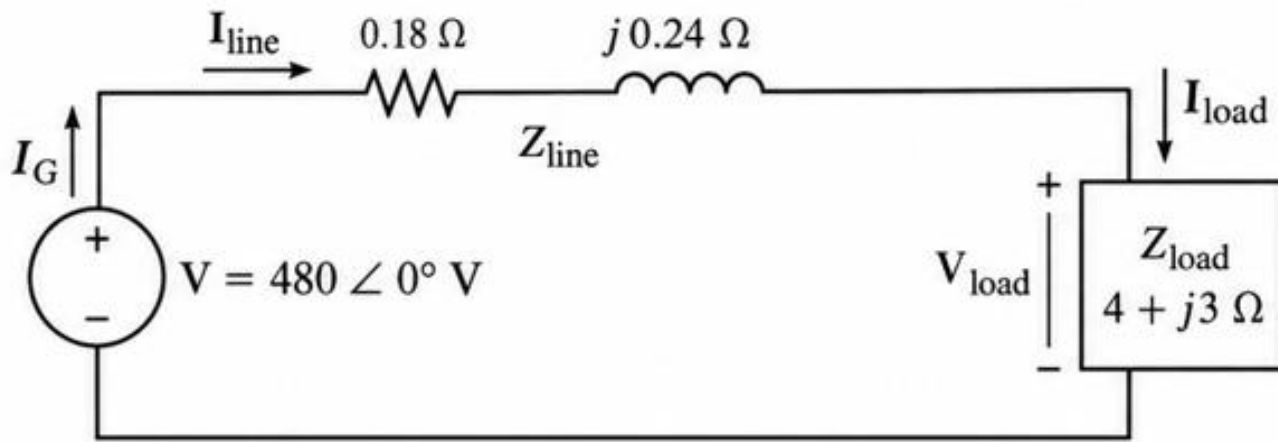


(b)

Solution of Part (a) – System without Transformers

Figure 2–6a shows the power system without transformers. Here $I_G = I_{\text{line}} = I_{\text{load}}$.

Figure (a)



Results for Part (a)

$$I_{\text{line}} = 90.8 \angle -37.8^\circ \text{ A}$$

$$V_{\text{load}} = 454 \angle -0.9^\circ \text{ V}$$

$$P_{\text{loss}} = 1484 \text{ W}$$

1 Line current

The line current in this system is given by

$$\begin{aligned} I_{\text{line}} &= \frac{V}{Z_{\text{line}} + Z_{\text{load}}} \\ &= \frac{480 \angle 0^\circ \text{ V}}{(0.18 \Omega + j0.24 \Omega) + (4 \Omega + j3 \Omega)} \\ &= \frac{480 \angle 0^\circ}{4.18 + j3.24} = \frac{480 \angle 0^\circ}{5.29 \angle 37.8^\circ} \\ &= 90.8 \angle -37.8^\circ \text{ A} \end{aligned}$$

2 Load voltage

Therefore the load voltage is

$$\begin{aligned} V_{\text{load}} &= I_{\text{line}} Z_{\text{load}} \\ &= (90.8 \angle -37.8^\circ \text{ A})(4 \Omega + j3 \Omega) \\ &= (90.8 \angle -37.8^\circ \text{ A})(5 \angle 36.9^\circ \Omega) \\ &= 454 \angle -0.9^\circ \text{ V} \end{aligned}$$

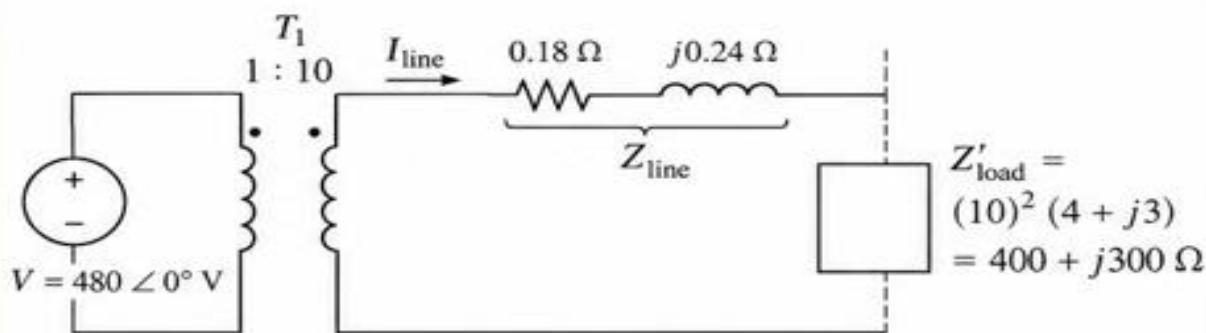
3 Line losses

and the line losses are

$$\begin{aligned} P_{\text{loss}} &= (I_{\text{line}})^2 R_{\text{line}} \\ &= (90.8 \text{ A})^2 (0.18 \Omega) = 1484 \text{ W} \end{aligned}$$

(b) System with Transformers – Part 1

Refer the load to the transmission line's voltage level (eliminate T_2).



(a) T_2 eliminated (load referred to line voltage level)

1 Refer the load to the line voltage level

$$Z'_{\text{load}} = a^2 Z_{\text{load}} = (10)^2 (4 + j3) = 400 + j300 \Omega$$

2 Total impedance at the transmission line level

$$\begin{aligned} Z_{\text{eq}} &= Z_{\text{line}} + Z'_{\text{load}} = (0.18 + j0.24) + (400 + j300) \\ &= 400.18 + j300.24 \Omega = 500.3 \angle 36.88^\circ \Omega \end{aligned}$$

3 Refer to the generator side (eliminate T_1)

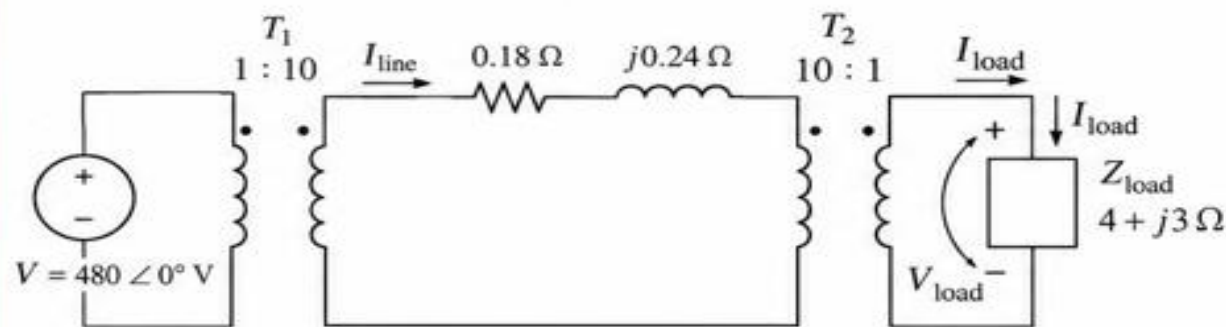
$$\begin{aligned} Z''_{\text{eq}} &= a^2 Z_{\text{eq}} = \left(\frac{1}{10}\right)^2 (400.18 + j300.24) \\ &= 0.0018 + j0.0024 + 4 + j3 = 5.003 \angle 36.88^\circ \Omega \end{aligned}$$

4 Generator current

$$I_G = \frac{480 \angle 0^\circ}{5.003 \angle 36.88^\circ} = 95.94 \angle -36.88^\circ \text{ A}$$

(b) System with Transformers – Part 2

Find the line current, load current, load voltage, and line losses.



5 Line current (through T_1)

$$I_{\text{line}} = \frac{N_{P1}}{N_{S1}} I_G = \frac{1}{10} (95.94 \angle -36.88^\circ) = 9.594 \angle -36.88^\circ \text{ A}$$

6 Load current (through T_2)

$$I_{\text{load}} = \frac{N_{P2}}{N_{S2}} I_{\text{line}} = \frac{10}{1} (9.594 \angle -36.88^\circ) = 95.94 \angle -36.88^\circ \text{ A}$$

7 Load voltage

$$\begin{aligned} V_{\text{load}} &= I_{\text{load}} Z_{\text{load}} = (95.94 \angle -36.88^\circ)(4 + j3) \\ &= (95.94 \angle -36.88^\circ)(5 \angle 36.87^\circ) = 479.7 \angle -0.01^\circ \text{ V} \end{aligned}$$

8 Transmission line losses

$$P_{\text{loss}} = (I_{\text{line}})^2 R_{\text{line}} = (9.594)^2 (0.18) = 16.7 \text{ W}$$

Conclusion:

- The load voltage is $479.7 \angle -0.01^\circ \text{ V}$.
- The transmission line losses are 16.7 W.
- Raising the transmission voltage by a factor of 10 reduces the line losses by nearly 90%, and the voltage drop at the load is much smaller compared to the system without transformers.

Real transformers are modeled by adding internal losses and voltage drops to the ideal transformer

Winding Resistances

Copper losses and voltage drops

$$R_1, R_2$$

Leakage Fluxes

Flux that links only one winding

$$X_1, X_2$$

Finite Core Permeability

Magnetizing current is not zero

$$X_m$$

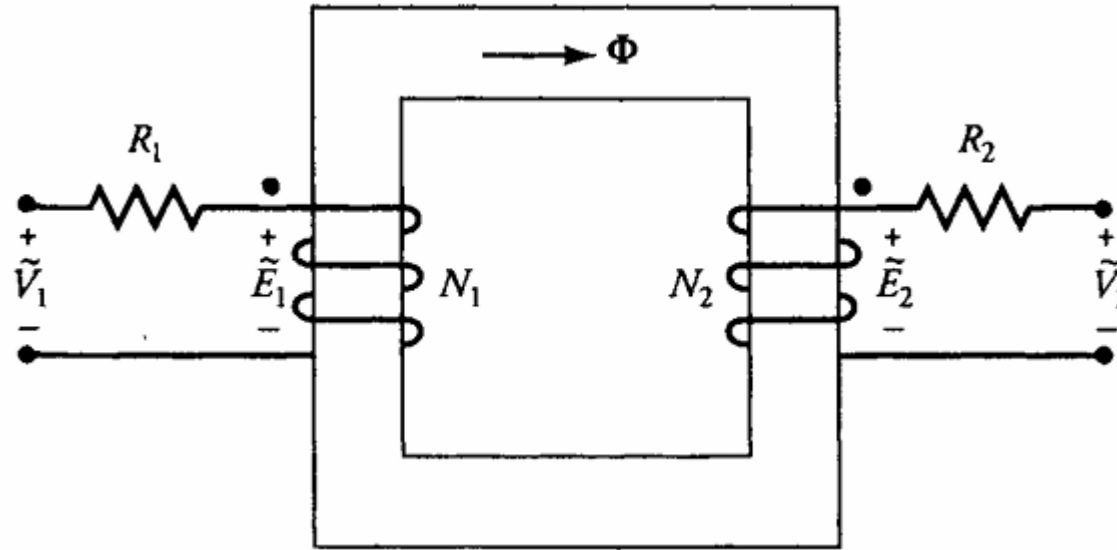
Core Losses

Eddy-current and hysteresis losses

$$R_c$$

These parameters allow the ideal equations to be used with practical voltage drops, losses, and efficiency calculations.

Winding resistance model



Voltage Drops

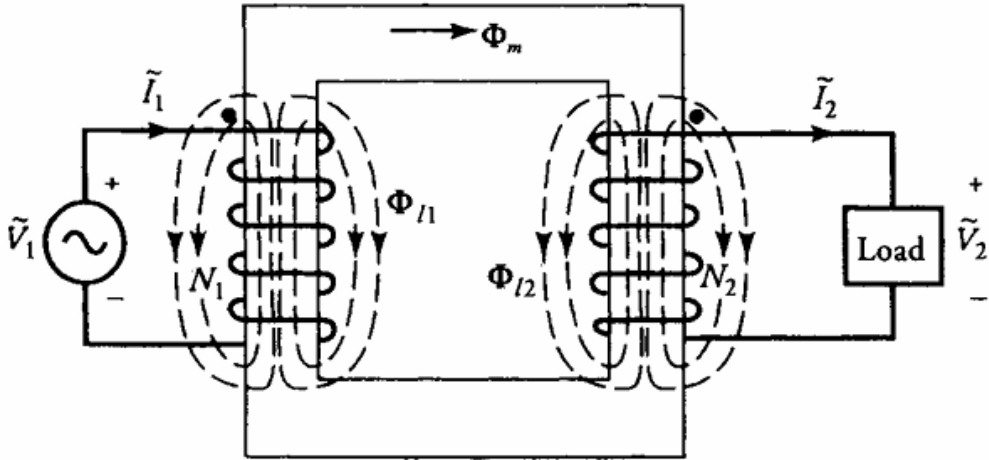
$$V_1 = E_1 + R_1 I_1$$

$$V_2 = E_2 - R_2 I_2$$

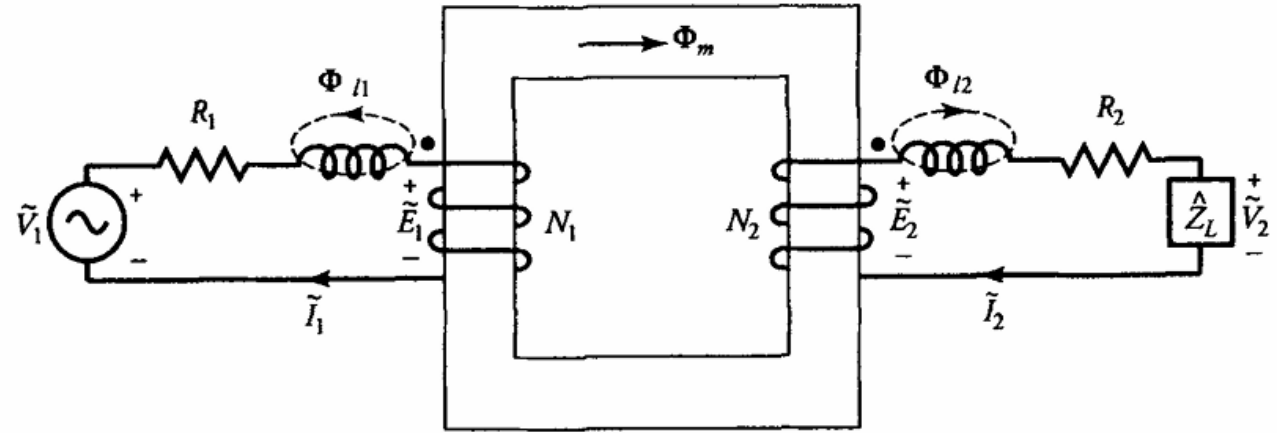
$$P_{cu} = I_1^2 R_1 + I_2^2 R_2$$

- R_1 and R_2 represent copper resistance.
- They reduce terminal voltage under load.
- **They produce copper losses and heat.**

Leakage and mutual flux



Separated leakage paths



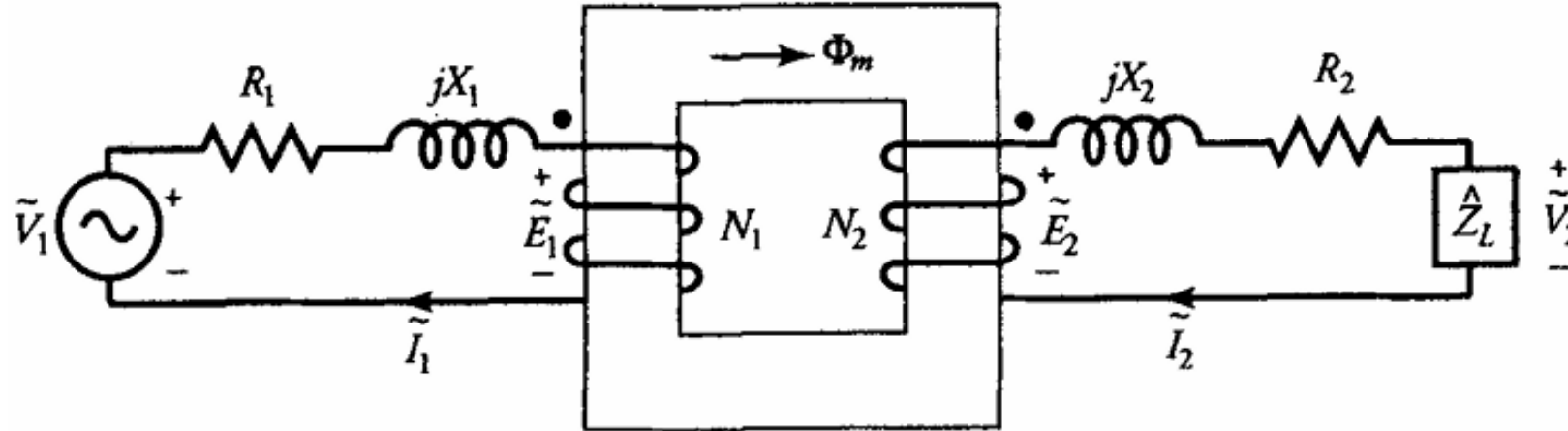
Key distinction

- **Mutual flux Φ_m links both windings.**
- Leakage flux Φ_{l1} or Φ_{l2} links only one winding.

Circuit Representation

Leakage flux $\rightarrow$ leakage reactance
Primary: jX_1 Secondary: jX_2

Equivalent circuit



Primary Side

$$Z_1 = R_1 + jX_1$$

$$V_1 = E_1 + Z_1 I_1$$

Secondary Side

$$Z_2 = R_2 + jX_2$$

$$V_2 = E_2 - Z_2 I_2$$

This model is enough for many voltage-regulation and efficiency calculations when magnetizing current is neglected.

Ideal Core Coupling

$$\frac{E_1}{E_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} = a$$

$$E_1 = a E_2$$

$$I_1 = I_2 / a$$

Use phasors for AC quantities.

Terminal Voltages

$$V_1 = E_1 + (R_1 + jX_1) I_1$$

$$V_2 = E_2 - (R_2 + jX_2) I_2$$

Output power:

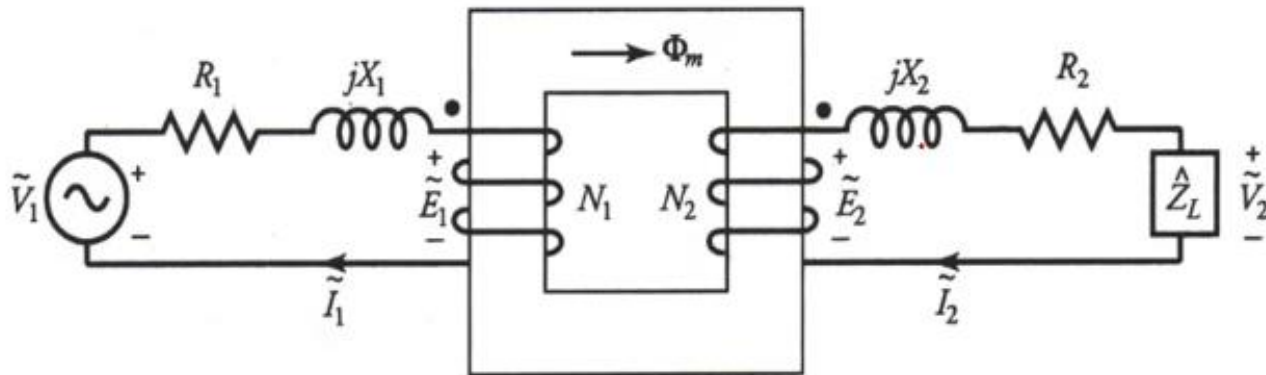
$$P_o = \text{Re}\{V_2 I_2^*\}$$

Input power:

$$P_{in} = \text{Re}\{V_1 I_1^*\}$$

A 23-kVA, 2300/230-V, 60-Hz step-down transformer has $R_1 = 4 \, \Omega$, $R_2 = 0.04 \, \Omega$, $X_1 = 12 \, \Omega$, and $X_2 = 0.12 \, \Omega$. The transformer operates at 75% of rated load. If the load power factor is 0.866 leading, determine the transformer efficiency.

Circuit model



Given data

- $S_{\text{rated}} = 23 \text{ kVA}$
- $V_1/V_2 = 2300/230$
- **$a = 10$**
- Load = 75% rated
- **$\text{pf} = 0.866 \text{ leading} \Rightarrow \theta = 30^\circ$**

Example 3 — Solution

Find I_2 and internal secondary emf E_2

Solution

$$I_2 = (23000 / 230) \times 0.75$$

$$I_2 = 75 \text{ A}$$

$$\text{pf leading} \Rightarrow I_2 = 75 \angle 30^\circ \text{ A}$$

$$Z_2 = R_2 + jX_2$$

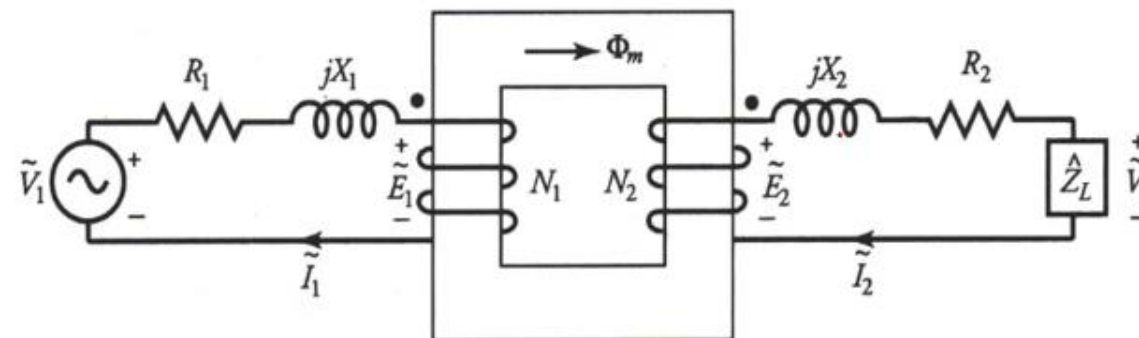
$$Z_2 = 0.04 + j0.12 \ \Omega$$

$$E_2 = V_2 + I_2 Z_2$$

$$E_2 = 230 + (75 \angle 30^\circ)(0.04 + j0.12)$$

$$E_2 = 228.287 \angle 2.33^\circ \text{ V}$$

Circuit reference



Refer to Primary

$$a = 2300 / 230 = 10$$

$$E_1 = a E_2$$

$$E_1 = 10 \times 228.287 \angle 2.33^\circ$$

$$E_1 = 2282.87 \angle 2.33^\circ \text{ V}$$

Example 3 — Solution Continued

Find V_1 , output power, and input power

Primary Calculation

$$I_1 = I_2/a = 75\angle 30^\circ / 10$$

$$I_1 = 7.5\angle 30^\circ \text{ A}$$

$$Z_1 = R_1 + jX_1 = 4 + j12 \Omega$$

$$V_1 = E_1 + I_1 Z_1$$

$$V_1 = 2282.87\angle 2.33^\circ + (7.5\angle 30^\circ)(4 + j12)$$

$$V_1 = 2269.578\angle 4.7^\circ \text{ V}$$

Efficiency compares real output power to real input power.

Power Calculation

$$P_o = \text{Re}\{V_2 I_2^*\}$$

$$P_o = \text{Re}\{230 \times 75\angle -30^\circ\}$$

$$P_o = 14938.94 \text{ W}$$

$$P_{in} = \text{Re}\{V_1 I_1^*\}$$

$$P_{in} = \text{Re}\{2269.578\angle 4.7^\circ \times 7.5\angle -30^\circ\}$$

$$P_{in} = 15389.14 \text{ W}$$

Efficiency Formula

$$\eta = \frac{P_o}{P_{in}} = \frac{14938.94}{15389.14} = 0.971$$

$$\eta = 97.1\%$$

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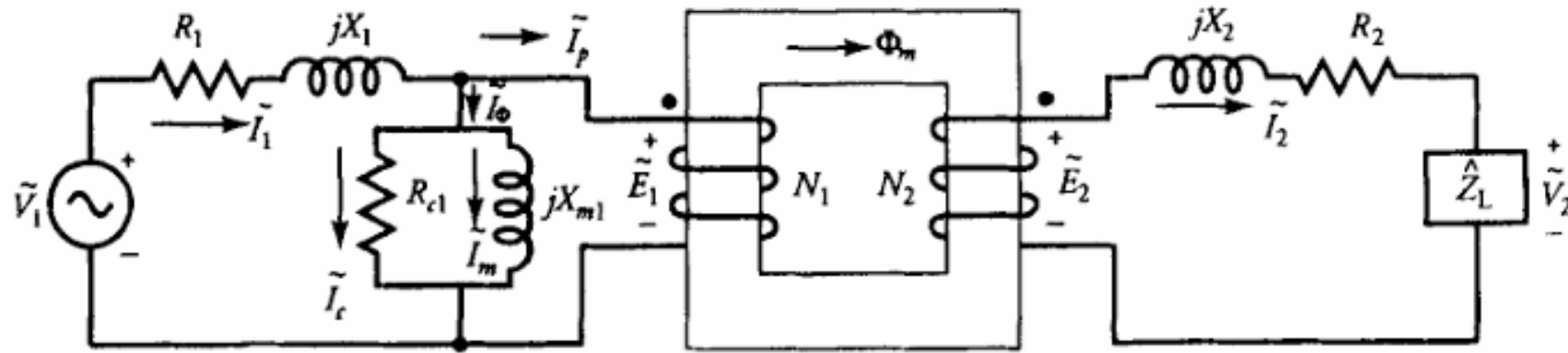
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Lecture 13



Excitation Current

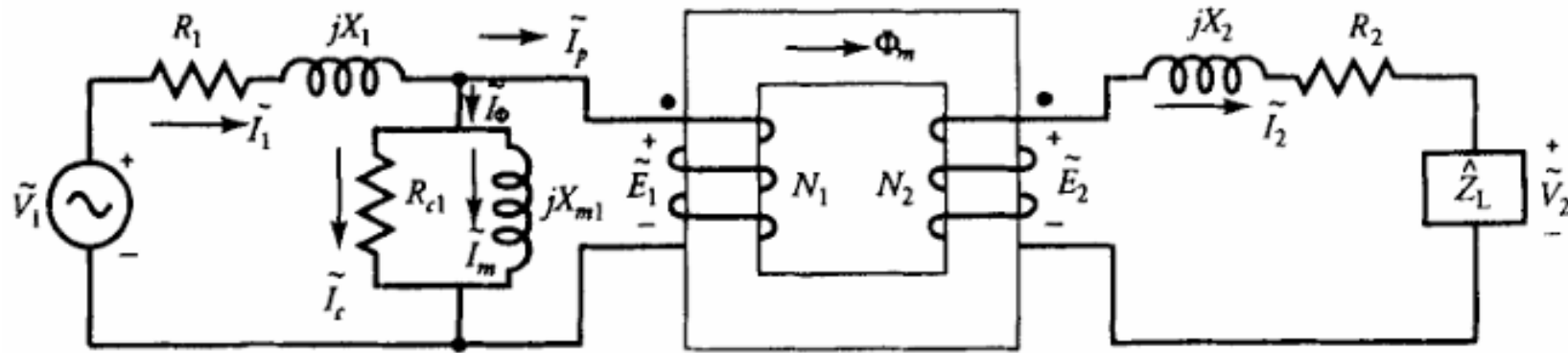
$$I_{\phi} = I_c + I_m$$

$$I_c = \frac{E_1}{R_{c1}} \quad \text{core-loss current}$$

$$I_m = \frac{E_1}{jX_{m1}} \quad \text{magnetizing current}$$

$$X_{m1} = \frac{E_1}{jI_m} \quad \text{magnetizing reactance}$$

- Finite permeability means μ is not infinite.
- A nonzero mmf is needed to establish mutual flux.
- **The unloaded transformer still draws a small current.**
- **Excitation current has core-loss and magnetizing components.**



1

Load current
increases $\tilde{I}_2 \uparrow$

2

Source current
increases $\tilde{I}_1 \uparrow$

3

Primary drop
increases $\tilde{I}_1(R_1 + jX_1) \uparrow$

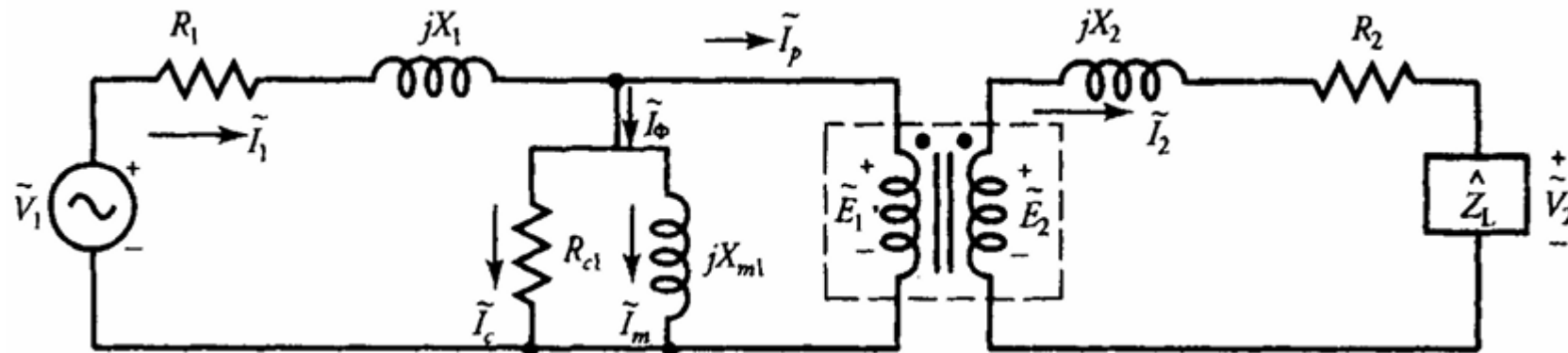
4

Induced emf
drops $\tilde{E}_1 \downarrow$

5

Mutual flux
decreases $\Phi_m \downarrow$

This is why real-transformer terminal voltage changes with load.



Elements

- R_1, R_2 model copper losses.
- jX_1, jX_2 model leakage flux.
- R_{c1} models core losses.
- jX_{m1} models finite permeability.

Useful Relations

$$I_1 = I_p + I_\phi$$

$$I_\phi = I_c + I_m$$

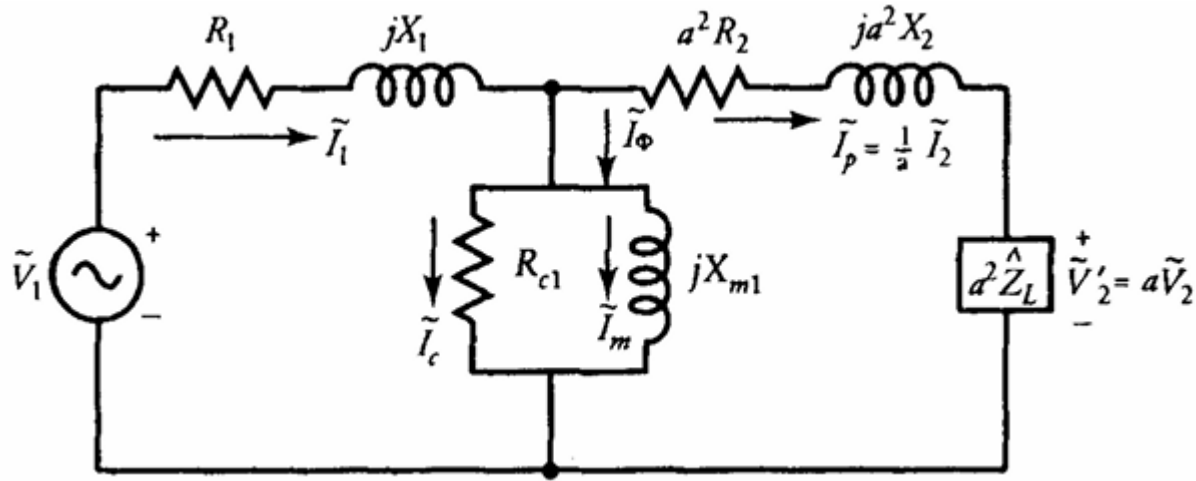
$$V_1 = E_1 + (R_1 + jX_1) I_1$$

$$V_2 = E_2 - (R_2 + jX_2) I_2$$

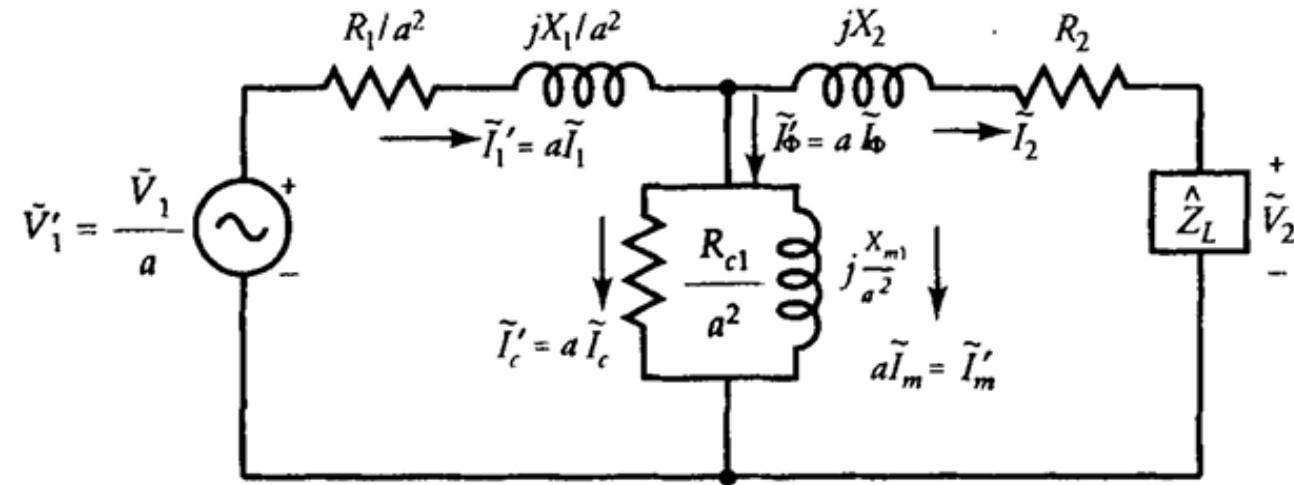
When to Use

- **Most accurate circuit for steady-state calculations.**
- Good for detailed voltage, current, and efficiency analysis.

Viewed from the primary side



Viewed from the secondary side

**Referred to Primary**

$$Z_{P-referred} = a^2 Z_s$$

$$R_2 \rightarrow a^2 R_2$$

$$X_2 \rightarrow a^2 X_2$$

$$V_2' = a V_2, \quad I_p = I_2 / a$$

Referred to Secondary

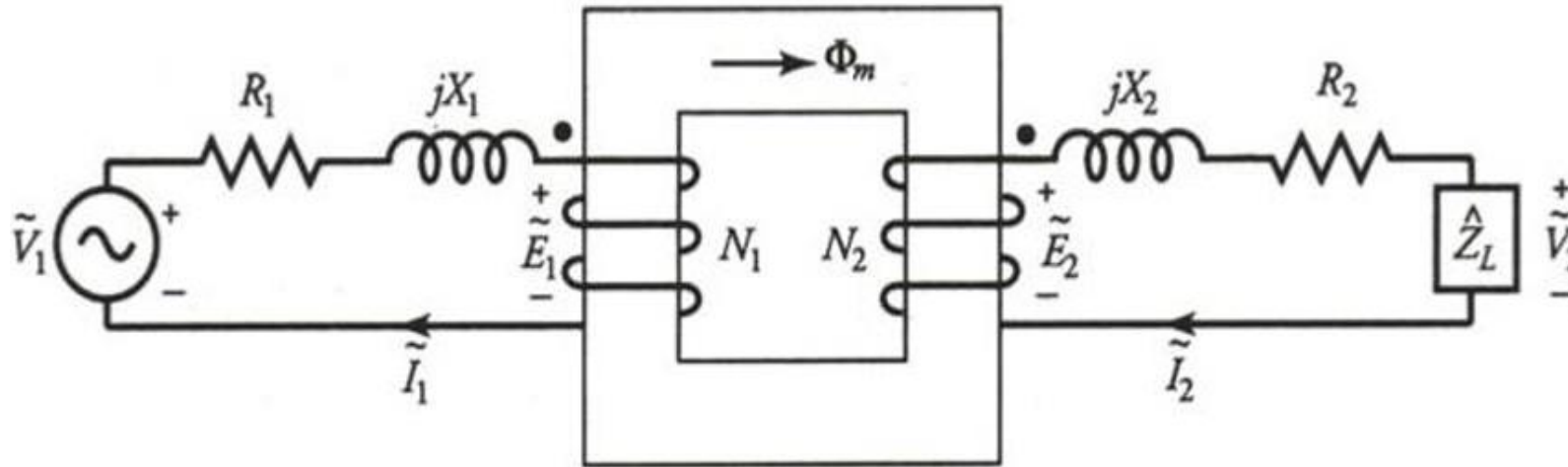
$$Z_{s-referred} = \frac{Z_p}{a^2}$$

$$R_1 \rightarrow R_1 / a^2$$

$$X_1 \rightarrow X_1 / a^2$$

$$V_1' = V_1 / a, \quad I_1' = a I_1$$

- Given $R_{c1} = 20 \text{ k}\Omega$ and $X_{m1} = 15 \text{ k}\Omega$ for the transformer of Example 4.3.
- Find the efficiency for the same load.**



We now include the shunt excitation branch: R_{c1} and $j X_{m1}$

From Example 4.3

$$V_2 = 230 \text{ V}$$

$$I_2 = 75 \angle 30^\circ \text{ A}$$

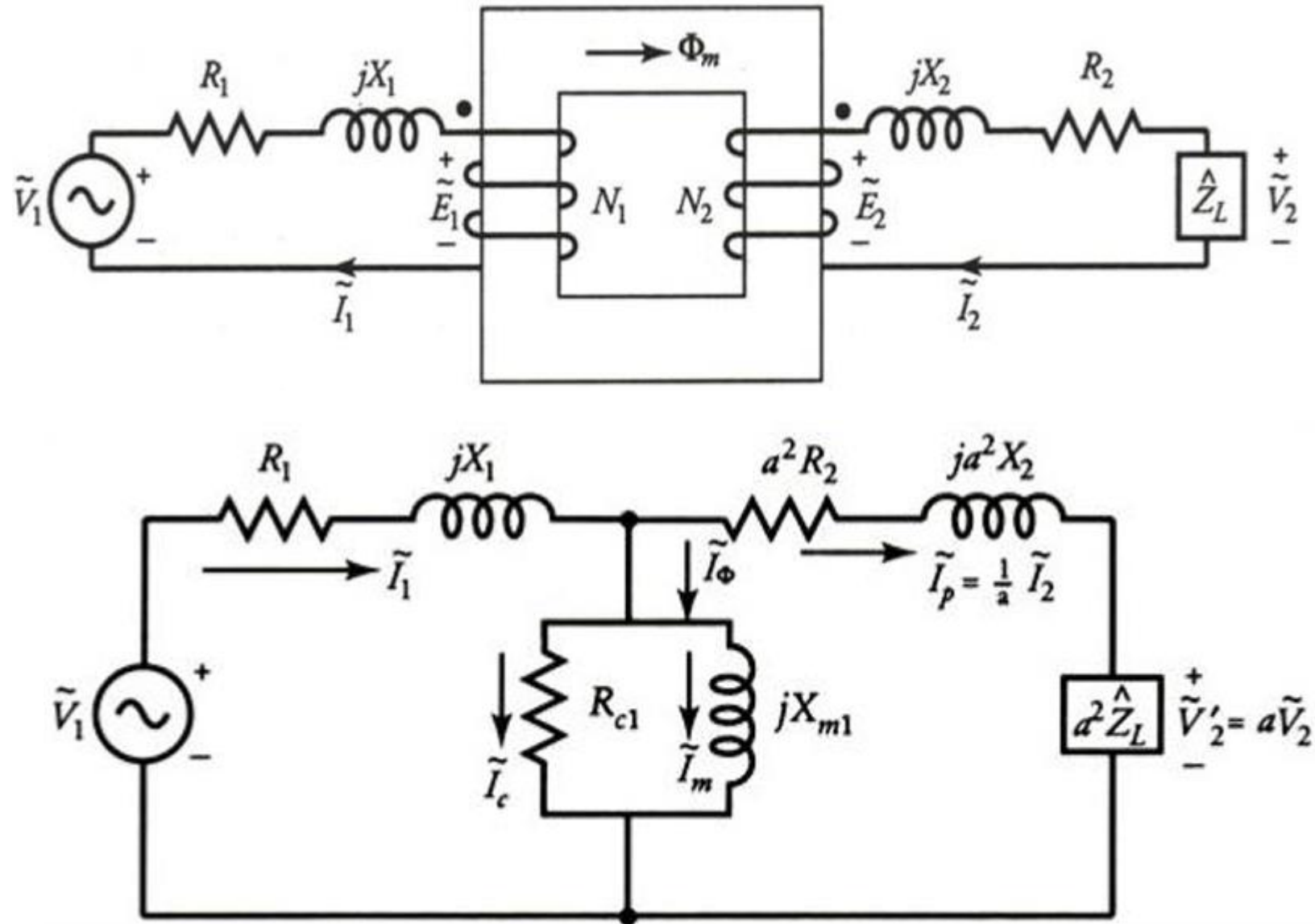
$$a = 10$$

$$E_2 = 228.287 \angle 2.33^\circ \text{ V}$$

$$V_1 = 2282.87 \angle 2.33^\circ \text{ V}$$

$$I_p = 7.5 \angle 30^\circ \text{ A}$$

$$P_o = 14,938.94 \text{ W}$$



Computation

$$I_c = \frac{E_1}{R_{c1}} = 0.114 \angle 2.33^\circ \text{ A}$$

$$I_m = \frac{E_1}{jX_{m1}} = 0.152 \angle (-87.67^\circ) \text{ A}$$

$$I_\phi = I_c + I_m = 0.19 \angle (-50.8^\circ) \text{ A}$$

$$I_1 = I_p + I_\phi = 7.53 \angle 28.57^\circ \text{ A}$$

$$V_1 = E_1 + I_1 Z_1 = 2271.9 \angle 4.71^\circ \text{ V}$$

$$P_{in} = \text{Re}\{V_1 I_1^*\} = 15,645.35 \text{ W}$$

Efficiency

$$\eta = \frac{P_o}{P_{in}} = \frac{14938.94}{15645.35} = 0.955$$

$$\eta = 95.5\%$$

- Including finite permeability adds a small excitation current.
- **The efficiency remains very close to the previous result.**

Final Answer: $\eta = 95.5\%$

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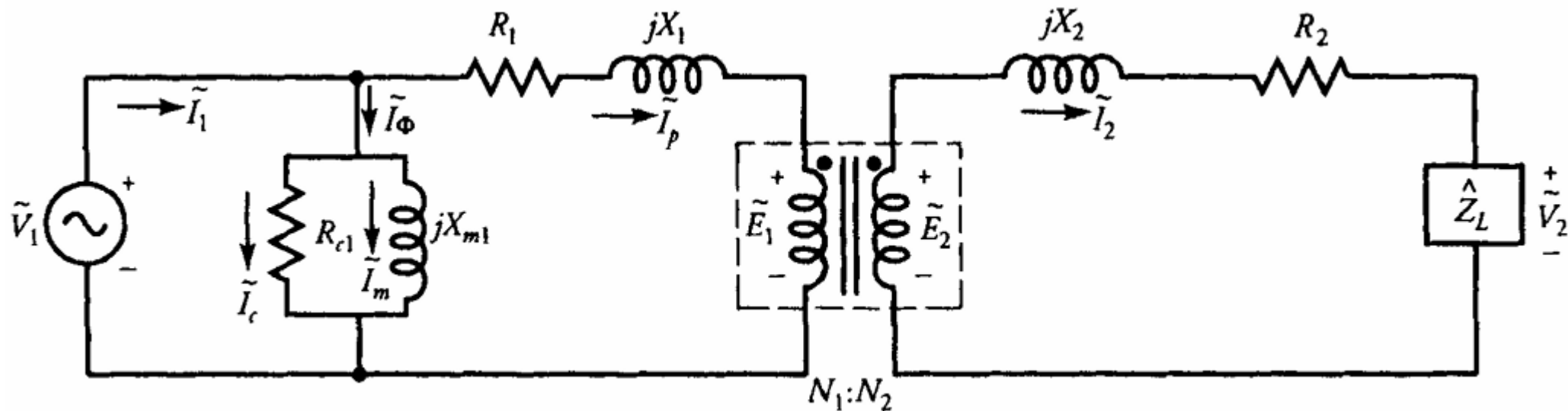
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Lecture 14

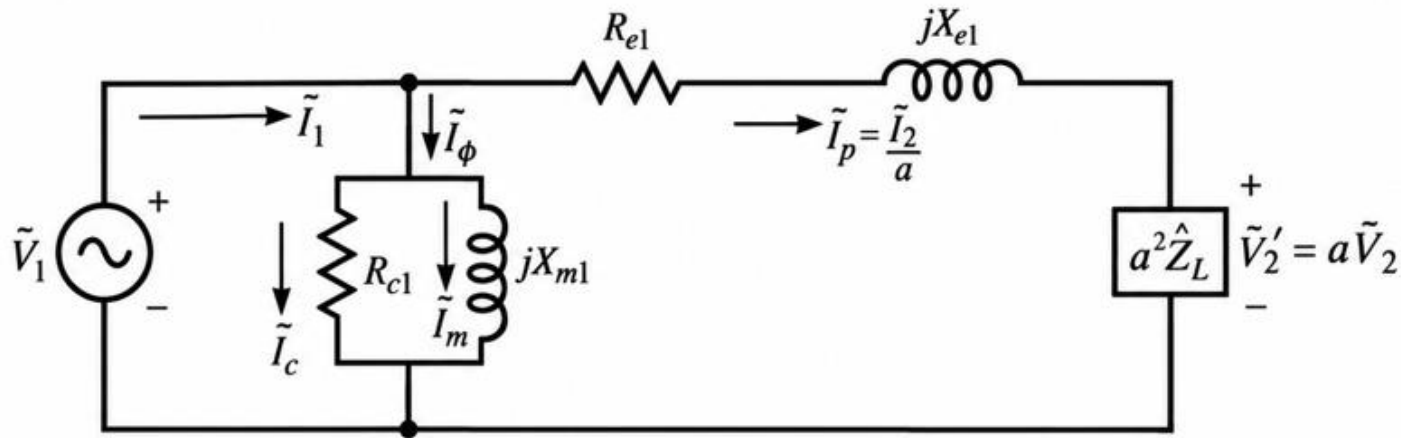


Physical Reason

- **Low core loss** $\rightarrow$ high R_{c1}
- **High permeability** $\rightarrow$ high X_{m1}
- Parallel branch impedance is very high.
- **Excitation current is small.**

Approximation

- The voltage drop across series impedance is small compared with applied voltage.
- **Therefore, the shunt branch can be moved to the input terminals.**
- **This greatly simplifies circuit calculations.**



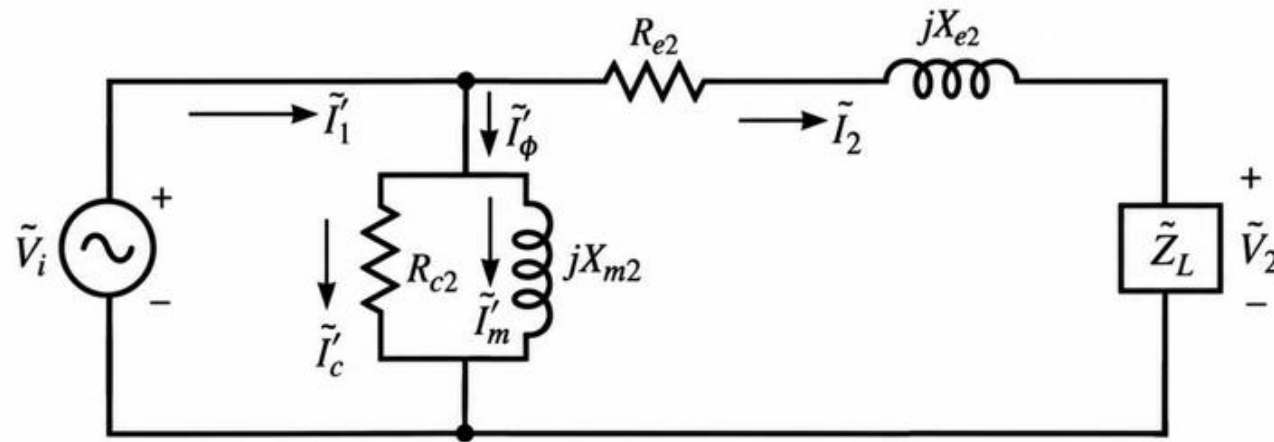
Approximate equivalent circuit of a transformer as viewed from the primary side.

Referred to Primary Side

$$Z_{e1} = R_{e1} + jX_{e1}$$

$$R_{e1} = R_1 + a^2 R_2$$

$$X_{e1} = X_1 + a^2 X_2$$



An approximate equivalent circuit of a transformer as viewed from the secondary side.

Referred to Secondary Side

$$Z_{e2} = R_{e2} + jX_{e2}$$

$$R_{e2} = R_2 + R_1 / a^2$$

$$X_{e2} = X_2 + X_1 / a^2$$

$$R_{c2} = R_{c1} / a^2, \quad X_{m2} = X_{m1} / a^2$$

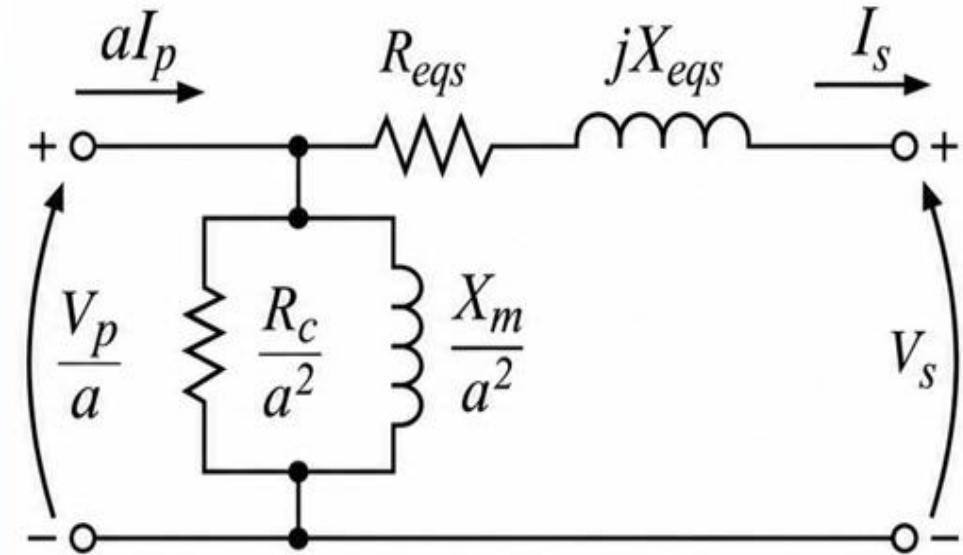
Phasor relation

$$\frac{V_p}{a} = V_s + R_{eqs} I_s + jX_{eqs} I_s$$

Meaning

- V_s is used as the reference phasor.
- $R_{eqs} I_s$ is in phase with load current.
- $jX_{eqs} I_s$ leads the load current by 90° .

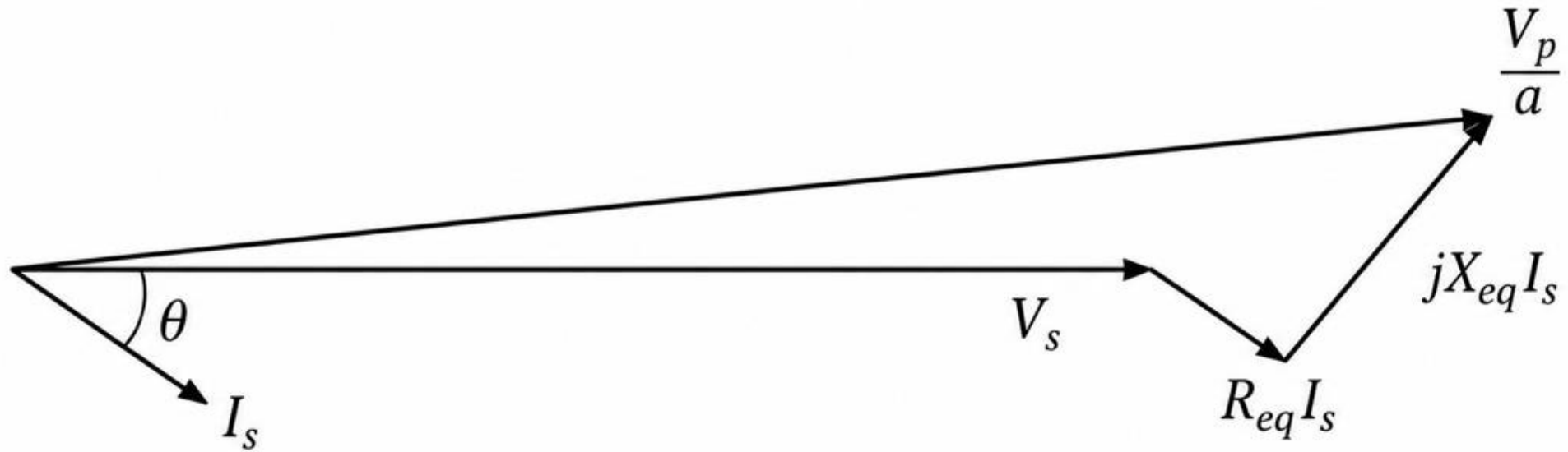
Secondary-referred circuit



$$X_{eqs} = \frac{X_p}{a^2} + X_s$$

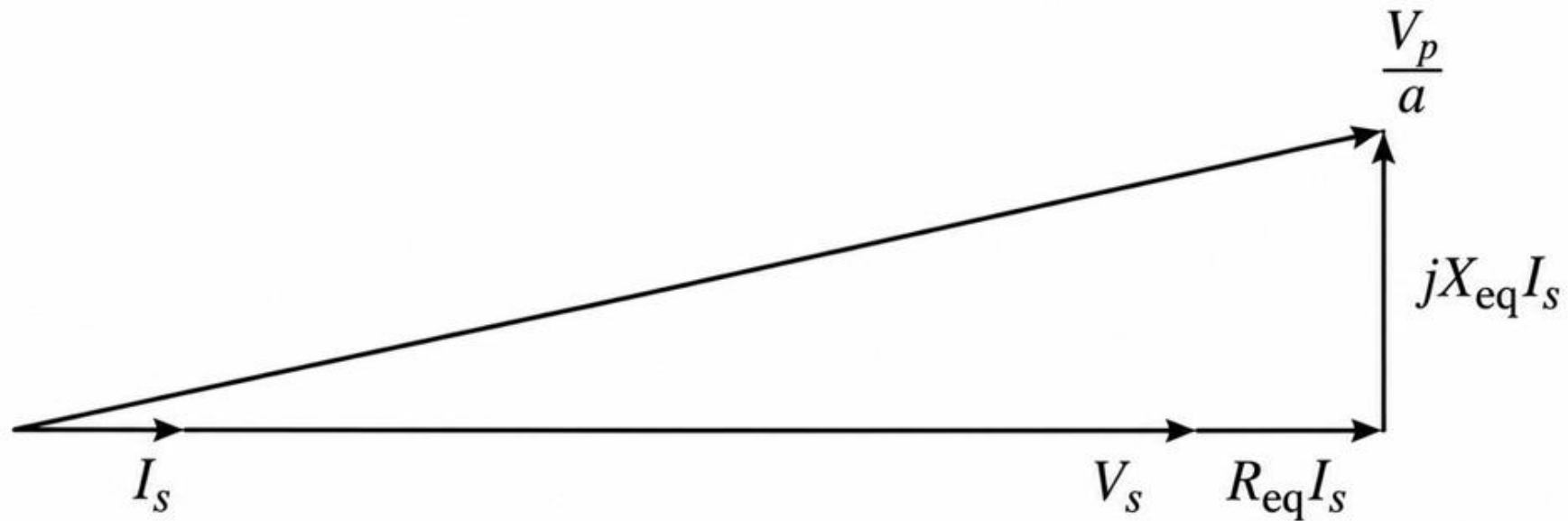
$$R_{eqs} = \frac{R_p}{a^2} + R_s$$

Phasor Diagram – Lagging Power Factor



$$\frac{V_p}{a} = V_s + R_{eq} I_s + jX_{eq} I_s$$

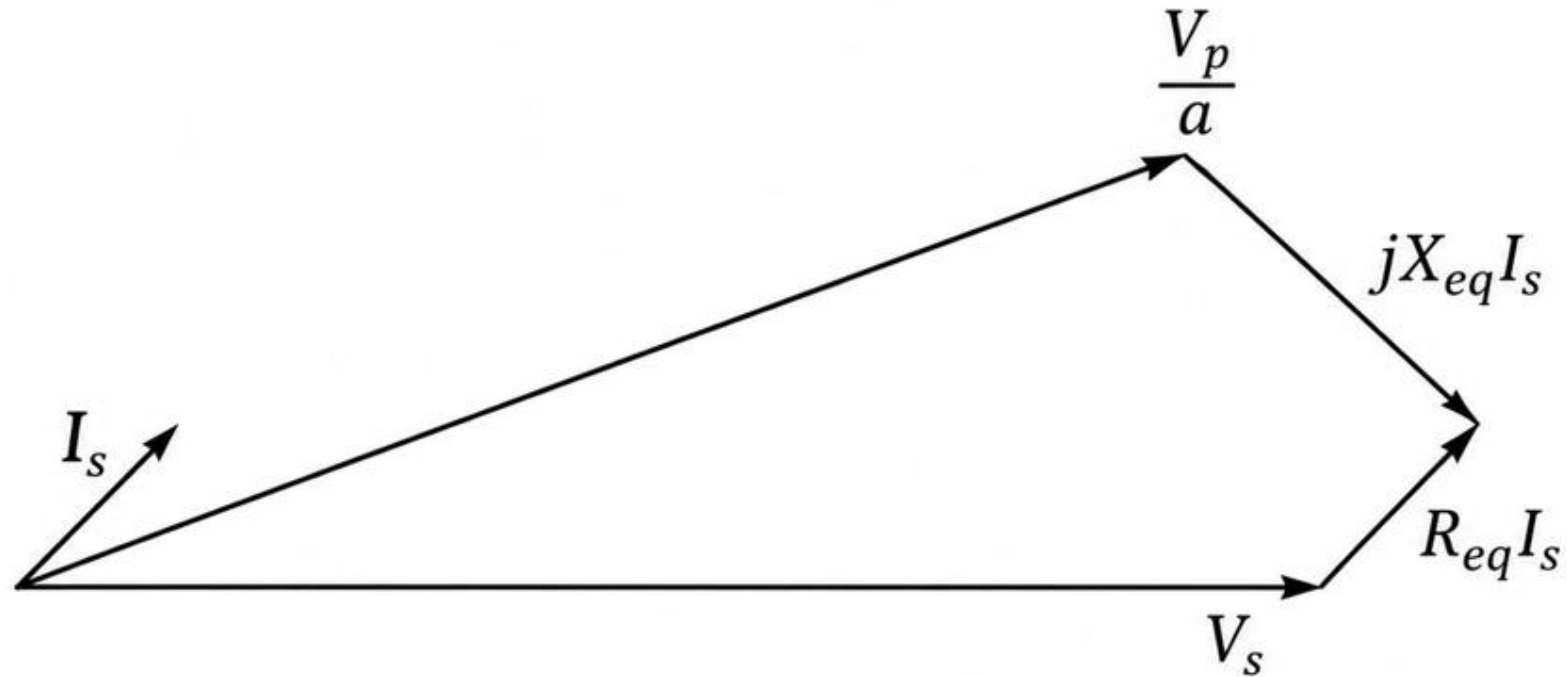
Phasor Diagram – Unity Power Factor



I_s in phase with V_s

$$\frac{V_p}{a} = V_s + R_{eq}I_s + jX_{eq}I_s$$

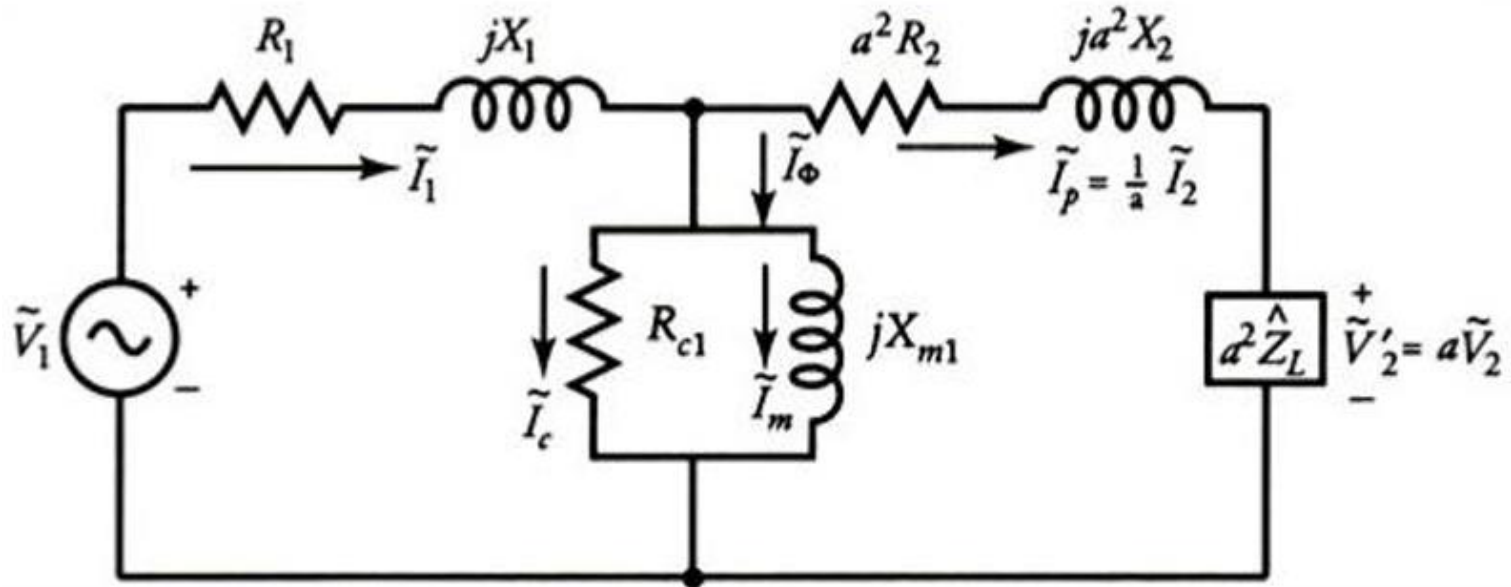
Phasor Diagram – Leading Power Factor



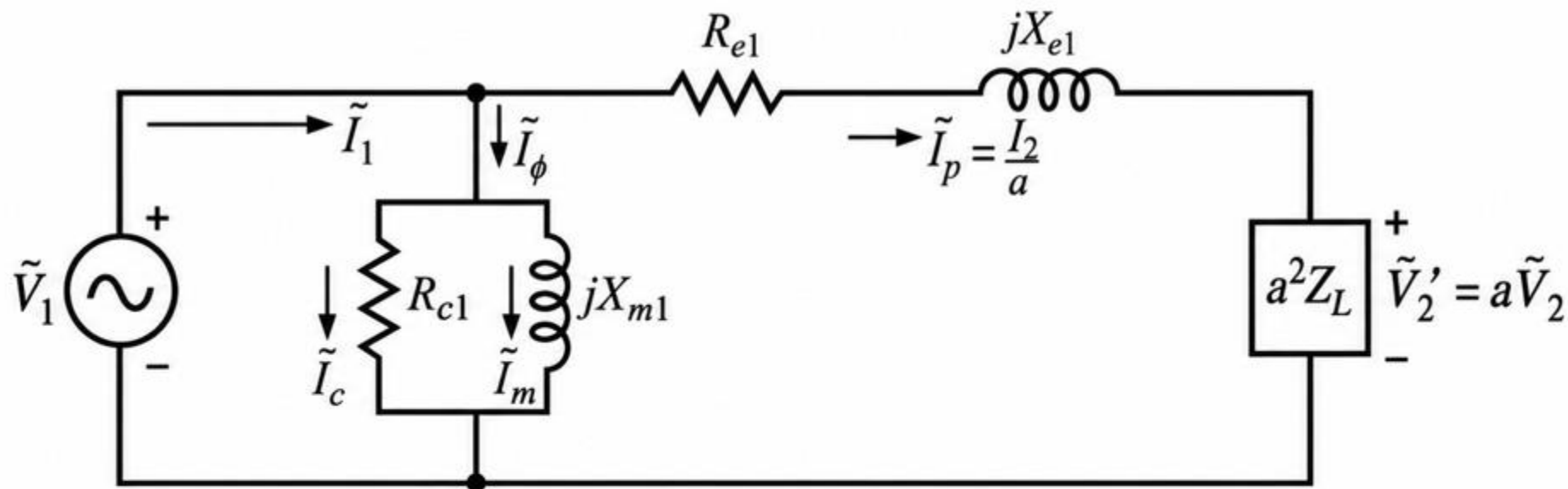
$$\frac{V_p}{a} = V_s + R_{eq}I_s + jX_{eq}I_s$$

I_s leads V_s

- Analyze the transformer of Examples 3 and 4 using the approximate equivalent circuit as viewed from the primary side. • **Also sketch its phasor diagram.**



- Use the primary-referred approximate model: R_{e1} , X_{e1} , and $V_2' = aV_2$.



Equivalent Parameters

$$V_2' = aV_2 = 10 \times 230 \angle 0^\circ = 2300 \angle 0^\circ \text{ V}$$

$$I_p = 7.5 \angle 30^\circ \text{ A}$$

$$R_{e1} = R_1 + a^2 R_2 = 4 + 10^2 \times 0.04 = 8 \, \Omega$$

$$X_{e1} = X_1 + a^2 X_2 = 12 + 10^2 \times 0.12 = 24 \, \Omega$$

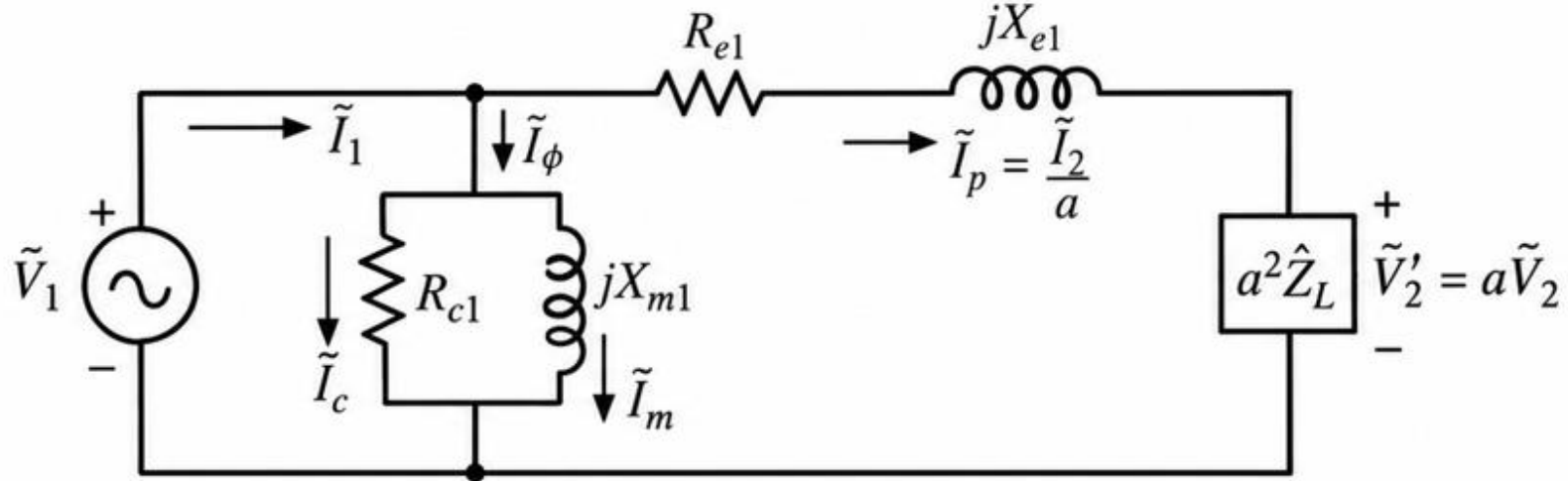
$$Z_{e1} = 8 + j24 \, \Omega$$

Supply Voltage

$$V_1 = V_2' + I_p Z_{e1}$$

$$V_1 = 2300 \angle 0^\circ + 7.5 \angle 30^\circ (8 + j24)$$

$$V_1 = 2269.59 \angle 4.7^\circ \text{ V}$$



Excitation and Total Current

$$I_c = 2269.59 \angle 4.7^\circ / 20000 = 0.113 \angle 4.7^\circ \text{ A}$$

$$I_m = 2269.59 \angle 4.7^\circ / j15000 = 0.151 \angle -85.3^\circ \text{ A}$$

$$I_1 = I_p + I_c + I_m = 7.54 \angle 28.6^\circ \text{ A}$$

Power

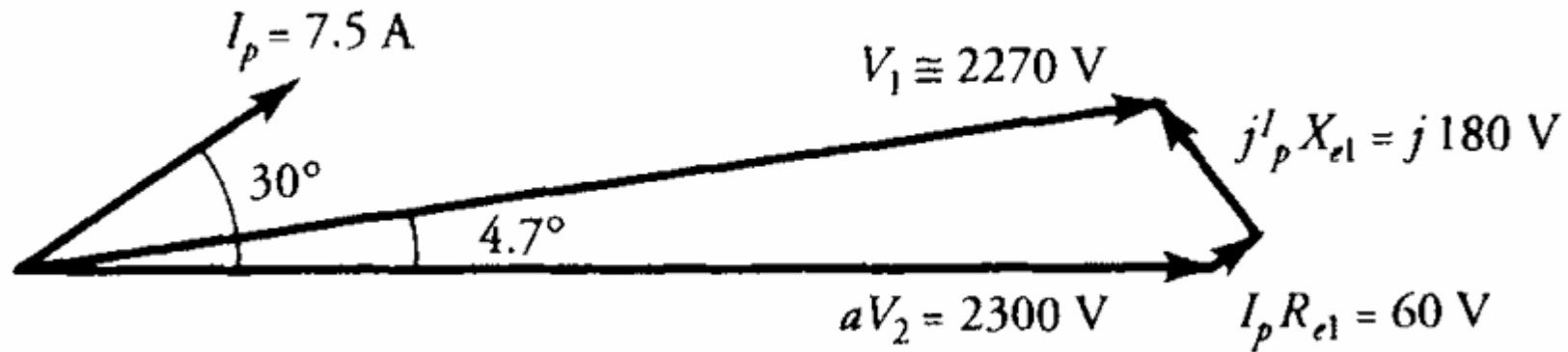
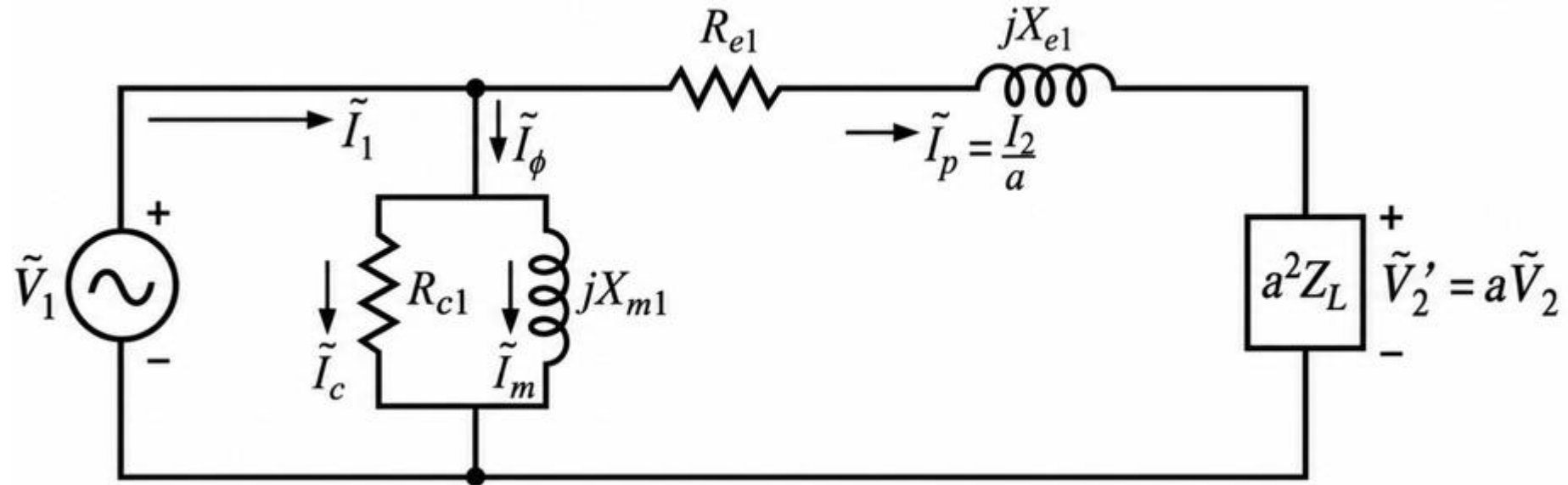
$$P_o = \text{Re}\{2300 \angle 0^\circ \times 7.5 \angle -30^\circ\}$$

$$P_o = 14,938.94 \text{ W}$$

$$P_{in} = \text{Re}\{2269.59 \angle 4.7^\circ \times 7.54 \angle -28.6^\circ\}$$

$$P_{in} = 15,645.36 \text{ W}$$

$$\eta = P_o / P_{in} = 14,938.94 / 15,645.36 = 0.955 = 95.5\%$$



Electrical Machines

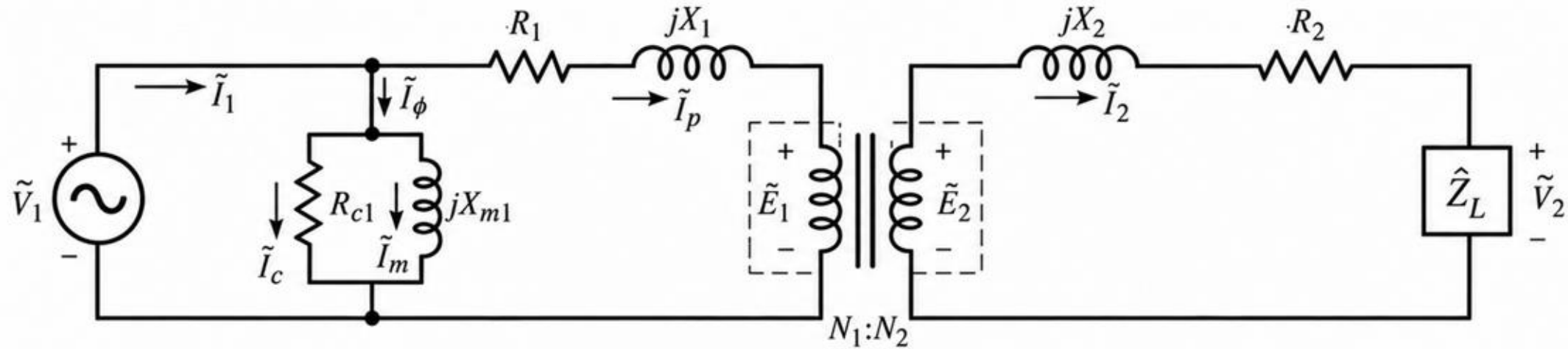
Chapter 2 TRANSFORMERS

Instructor

Eng. Nael S. Dokhan

MSc – Electrical Engineering

Lecture 15



Common Forms

$$\text{VR}\% = \frac{V_{2NL} - V_{2FL}}{V_{2FL}} \times 100$$

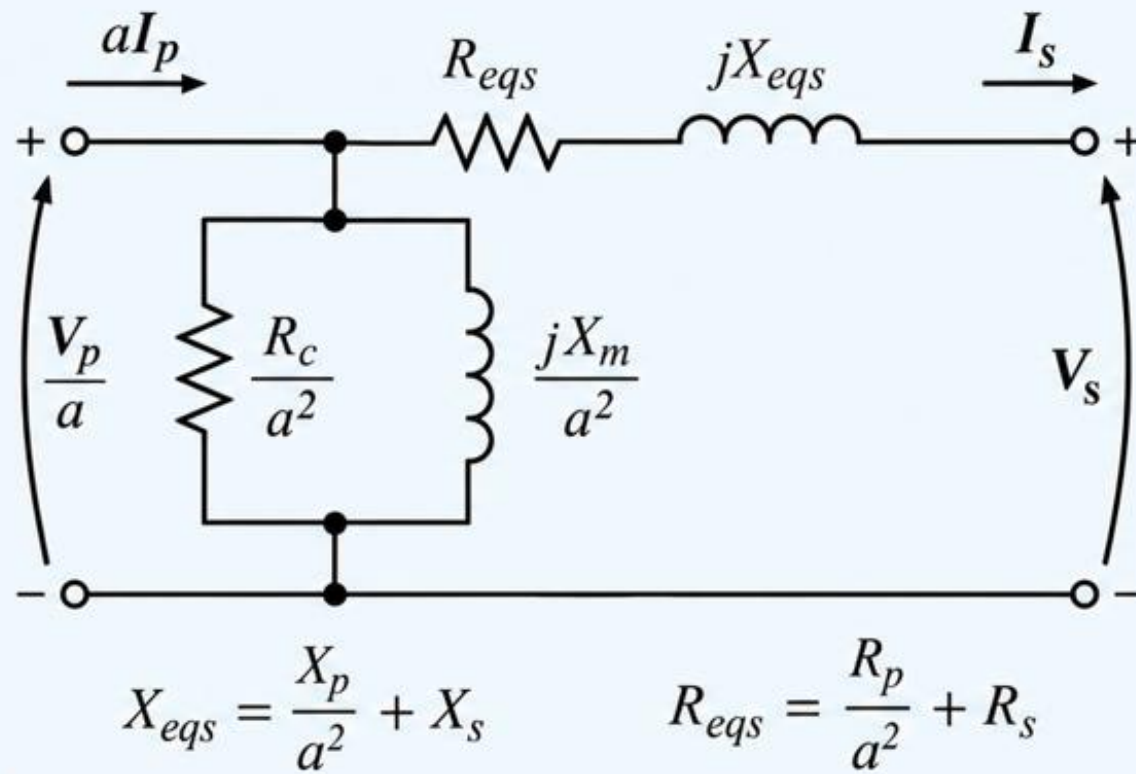
$$\text{VR}\% = \frac{V_1 - aV_2}{aV_2} \times 100$$

$$\text{VR}\% = \frac{\frac{V_1}{a} - V_2}{V_2} \times 100$$

Meaning

- V_{2NL} : no-load secondary voltage
- V_{2FL} : full-load secondary voltage
- **Positive regulation means terminal voltage drops under load.**
- **Good regulation means smaller voltage change.**

Transformer Equivalent Circuit (Referred to Secondary)



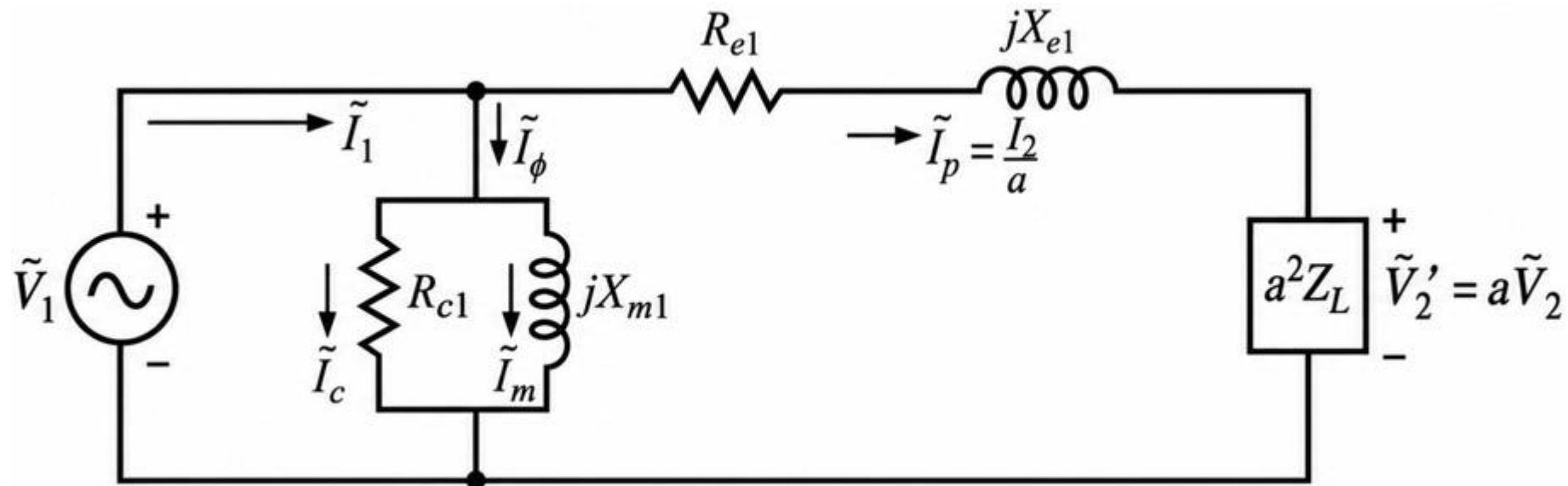
Transformer Efficiency

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

$$\eta = \frac{V_s I_s \cos \theta}{P_{Cu} + P_{core} + V_s I_s \cos \theta} \times 100\%$$

A 23-kVA, 2300/230-V, 60-Hz step-down transformer has $R_1=4\ \Omega$, $R_2=0.04\ \Omega$, $X_1=12\ \Omega$, $X_2=0.12\ \Omega$, $R_{c1}=20\ \text{k}\Omega$, and $X_{m1}=15\ \text{k}\Omega$. It operates at rated voltage and rated load with $\text{pf} = 0.866$ lagging. Determine efficiency and plot the phasor diagram.



Equivalent Parameters and Input Voltage

$$V'_2 = aV_2 = 10 \times 230 \angle 0^\circ = 2300 \angle 0^\circ \text{ V}$$

$$I'_2 = 10 \angle -30^\circ \text{ A}$$

$$R_{e1} = R_1 + a^2 R_2 = 4 + 10^2 \times 0.04 = 8 \ \Omega$$

$$X_{e1} = X_1 + a^2 X_2 = 12 + 10^2 \times 0.12 = 24 \ \Omega$$

$$Z_{e1} = R_{e1} + jX_{e1} = 8 + j24 \ \Omega$$

$$V_1 = V'_2 + I'_2 Z_{e1} = 2300 \angle 0^\circ + 10 \angle -30^\circ (8 + j24)$$

$$V_1 = 2494.93 \angle 3.8575^\circ \text{ V}$$

Power and Efficiency

$$P_c = \frac{V_1^2}{R_c} = \frac{2494.93^2}{20000}$$

$$P_c = 311.2349 \text{ W}$$

$$P_{cu} = I_2'^2 \times R_{e1}$$

$$P_{cu} = 10^2 \times 8 = 800 \text{ W}$$

$$P_{out} = 23000 \times 0.866$$

$$P_{out} = 19918 \text{ W}$$

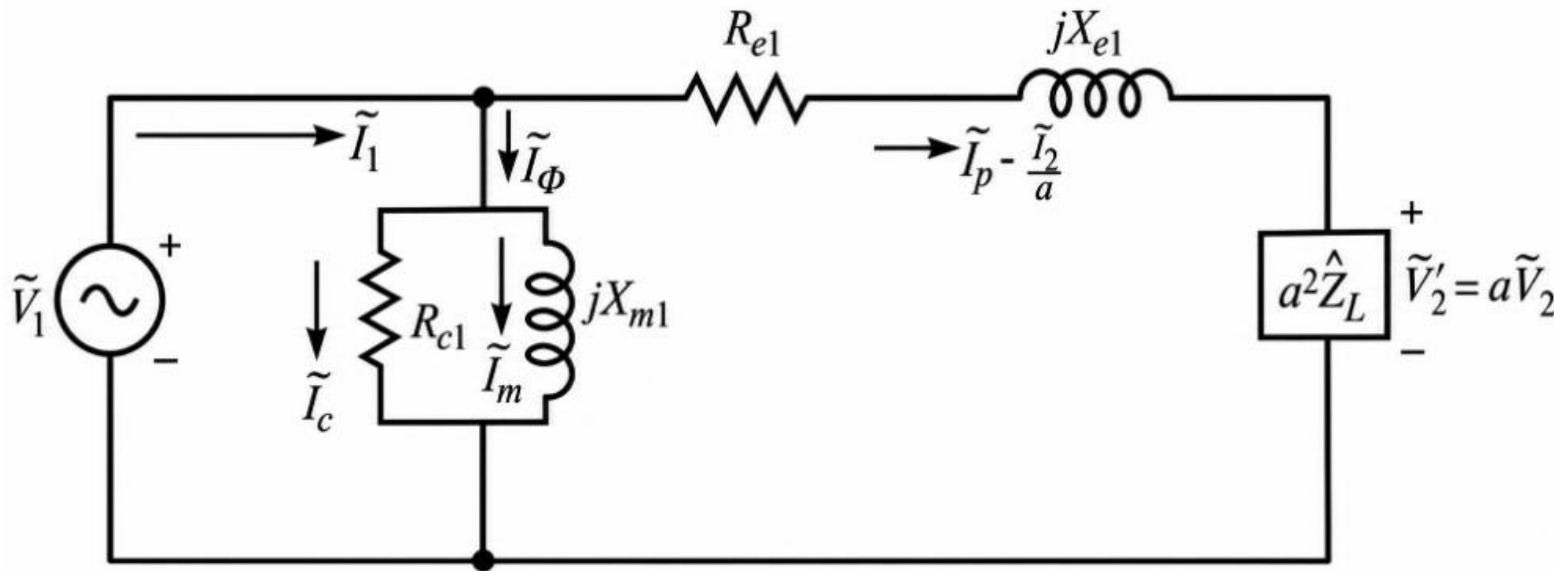
$$\eta = \frac{P_{out}}{P_{out} + P_{loss}}$$

$$\eta = \frac{19918}{19918 + 311.2349 + 800} \times 100$$

$$\eta = 94.72\%$$

A 2.2-kVA, 440/220-V, 50-Hz step-down transformer has parameters referred to the primary side: $R_{e1} = 3 \Omega$, $X_{e1} = 4 \Omega$, $R_{c1} = 2.5 \text{ k}\Omega$, and $X_{m1} = 2 \text{ k}\Omega$. The transformer delivers full load at rated voltage with $\text{pf} = 0.707$ lagging. Determine efficiency and voltage regulation.

Approximate circuit



Given / Required

$$S = 2200 \text{ VA}$$

$$V_1/V_2 = 440/220$$

$$\text{pf} = 0.707 \text{ lagging}$$

Find: η and $\text{VR}\%$

1 From the given data,

$$a = \frac{440}{220} = 2 \quad \tilde{V}_2 = 220 \text{ V} \quad S = 2200 \text{ VA} \quad I_2 = \frac{2200}{220} = 10 \text{ A}$$

2 For a lagging power factor of 0.707, $\theta = -45^\circ$
 Using load voltage as a reference, $\tilde{I}_2 = 10\angle -45^\circ \text{ A}$.
 Referring to the equivalent circuit,

$$\tilde{I}_p = \frac{\tilde{I}_2}{a} = 5\angle -45^\circ \text{ A}$$

$$\tilde{V}'_2 = a\tilde{V}_2 = 440\angle 0^\circ \text{ V}$$

3 Thus,

$$\begin{aligned} \tilde{V}_1 &= \tilde{V}'_2 + \tilde{I}_p(R_{e1} + jX_{e1}) \\ &= 440 + (5\angle -45^\circ)(3 + j4) \\ &= 464.762\angle 0.44^\circ \text{ V} \end{aligned}$$

The core-loss and magnetizing currents are

$$\tilde{I}_c = \frac{464.762 \angle 0.44^\circ}{2500} = 0.186 \angle 0.44^\circ \text{ A}$$

$$\tilde{I}_m = \frac{464.762 \angle 0.44^\circ}{j2000} = 0.232 \angle -89.56^\circ \text{ A}$$

The current supplied by the source is

$$\begin{aligned} \tilde{I}_1 &= \tilde{I}_p + \tilde{I}_c + \tilde{I}_m = 5 \angle -45^\circ + 0.186 \angle 0.44^\circ + 0.232 \angle -89.56^\circ \\ &= 5.296 \angle -45.33^\circ \text{ A} \end{aligned}$$

The power output, the power input, and the efficiency are

$$P_o = \text{Re}[(440)(5 \angle 45^\circ)] = 1555.63 \text{ W}$$

$$P_{in} = \text{Re}[(464.762 \angle 0.44^\circ)(5.296 \angle 45.33^\circ)] = 1716.91 \text{ W}$$

$$\eta = \frac{1555.63}{1716.91} = 0.906 \quad \text{or} \quad 90.6\%$$

The voltage regulation is

$$VR\% = \frac{464.762 - 440}{440} \times 100 = 5.63\%$$

$$\eta = 90.6\%$$

$$VR = 5.63\%$$

Electrical Machines

Chapter 2 TRANSFORMERS

Instructor

Eng. Nael S. Dokhan

MSc – Electrical Engineering

Lecture 16

Open-circuit test idea

One winding is left open while the other is excited by rated voltage and rated frequency.

For safety, this test is normally performed on the low-voltage side.

Measured quantities

V_{oc} : open-circuit voltage

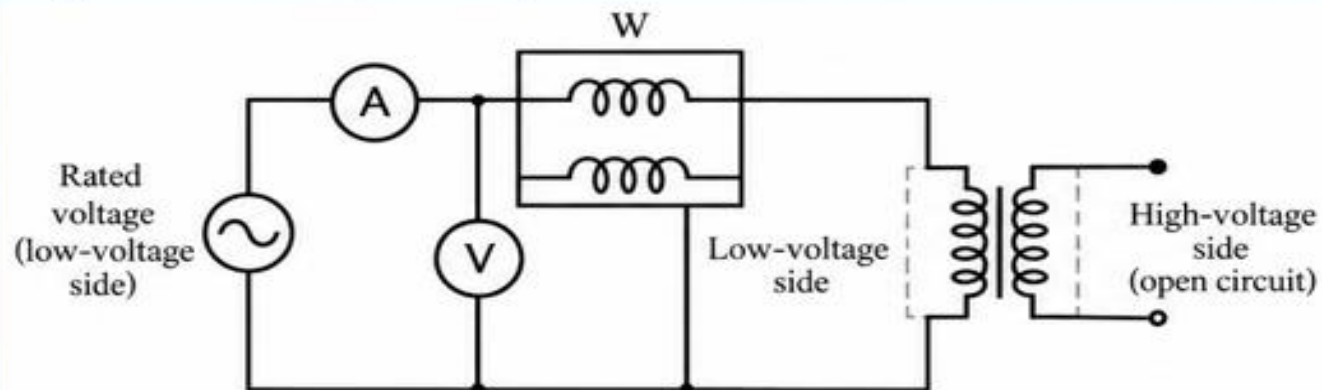
I_{oc} : no-load current

P_{oc} : core loss at rated voltage

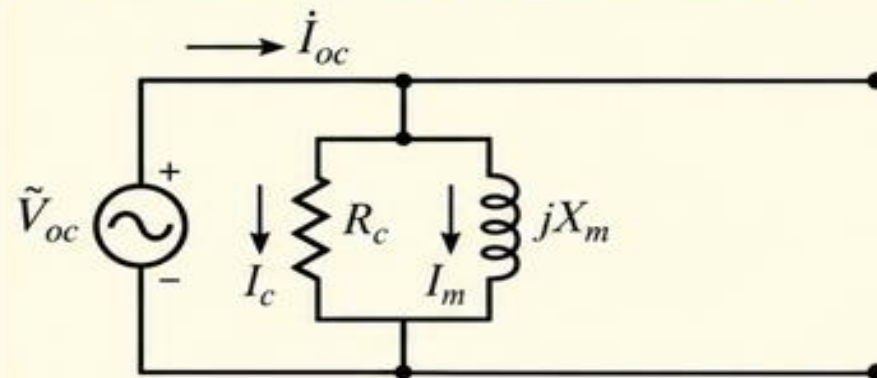
Used to determine R_c and X_m .



Open-Circuit Test Setup



Equivalent Circuit (Open-Circuit Test)



Formulas (Open-Circuit Test)

$$S_{oc} = V_{oc} I_{oc} \quad \phi_{oc} = \cos^{-1} \left(\frac{P_{oc}}{S_{oc}} \right) \quad \text{①}$$

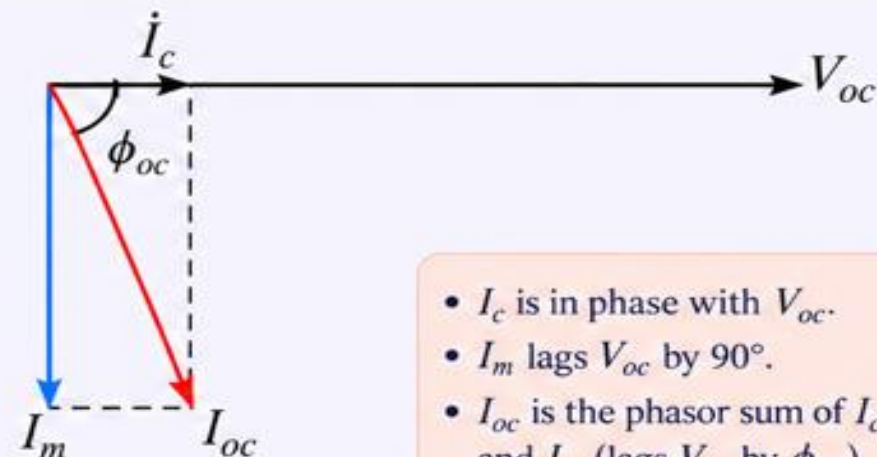
$$I_c = I_{oc} \cos \phi_{oc} \quad I_m = I_{oc} \sin \phi_{oc} \quad \text{②}$$

$$R_c = \frac{V_{oc}}{I_c} = \frac{V_{oc}^2}{P_{oc}} \quad X_m = \frac{V_{oc}}{I_m} = \frac{V_{oc}^2}{Q_{oc}} \quad \text{③}$$

$$Q_{oc} = \sqrt{S_{oc}^2 - P_{oc}^2} \quad \text{④}$$

V_{oc} : applied (rated) voltage at no-load
 I_{oc} : no-load current
 P_{oc} : core (iron) loss
 Q_{oc} : reactive power at no-load
 R_c : core-loss resistance
 X_m : magnetizing reactance
 ϕ_{oc} : no-load power factor angle

Phasor Diagram (Open-Circuit Test)



- I_c is in phase with V_{oc} .
- I_m lags V_{oc} by 90° .
- I_{oc} is the phasor sum of I_c and I_m (lags V_{oc} by ϕ_{oc}).

Short-circuit test idea

- The test is designed to determine the equivalent winding resistance and leakage reactance.
- A short circuit is placed across one winding, and the other winding is excited at rated frequency.
- The applied voltage is increased carefully until rated current flows.
- For safety, the short-circuit test is commonly conducted on the high-voltage side.

Measured quantities

V_{sc} : open-circuit voltage

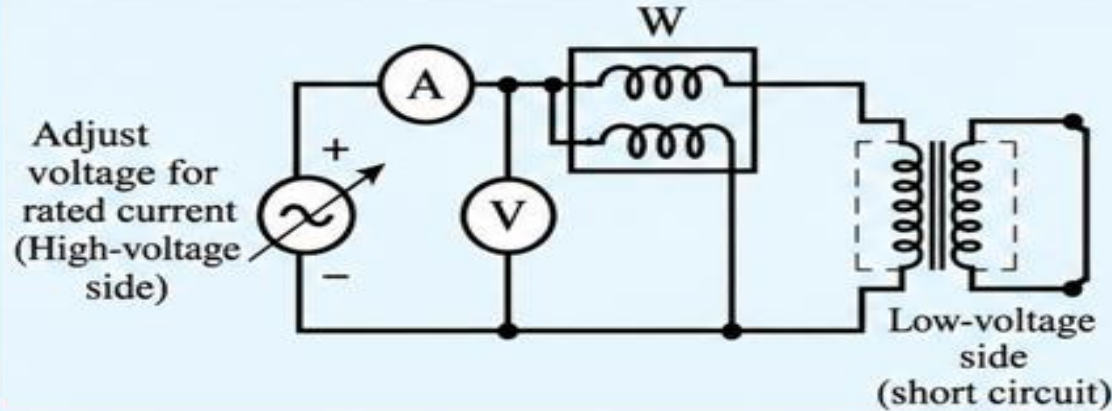
I_{sc} : rated current

P_{sc} : copper loss at rated current

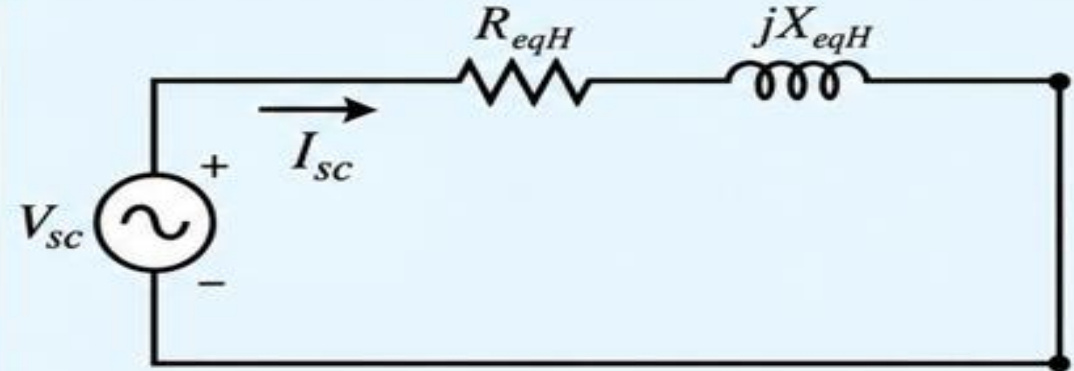
Used to determine R_{eq} and X_{eq} .



Short-Circuit Test Setup



Equivalent Circuit for the Short-Circuit Test



Formulas for Short-Circuit Test

Basic Formulas (High-Voltage Side)

$$S_{sc} = V_{sc} \times I_{sc}$$

$$\phi_{sc} = \cos^{-1} \left[\frac{P_{sc}}{S_{sc}} \right]$$

$$R_{eqH} = \frac{P_{sc}}{I_{sc}^2}$$

$$Z_{eqH} = \frac{V_{sc}}{I_{sc}}$$

$$X_{eqH} = \sqrt{Z_{eqH}^2 - R_{eqH}^2}$$

$$a = \frac{N_H}{N_L}$$

Referred High-Side Relations

$$R_{eqH} = R_H + a^2 R_L$$

$$X_{eqH} = X_H + a^2 X_L$$

$$I_H^2 R_H = I_L^2 R_L$$

$$R_H = a^2 R_L = 0.5 R_{eqH}$$

$$X_H = a^2 X_L = 0.5 X_{eqH}$$

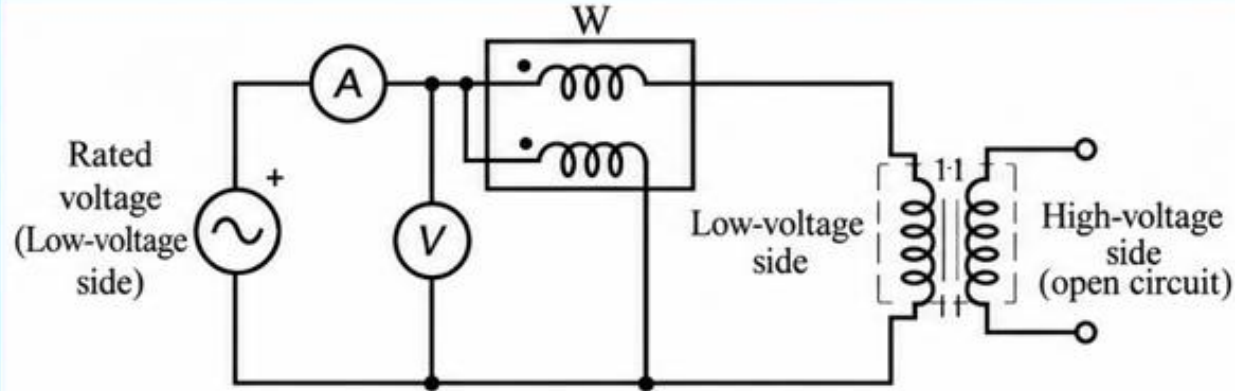
The following data were obtained from testing a **48-kVA, 4800/240-V**, step-down transformer:

Test	Voltage (V)	Current (A)	Power (W)
Open-circuit test	240	2	120
Short-circuit test	150	10	600

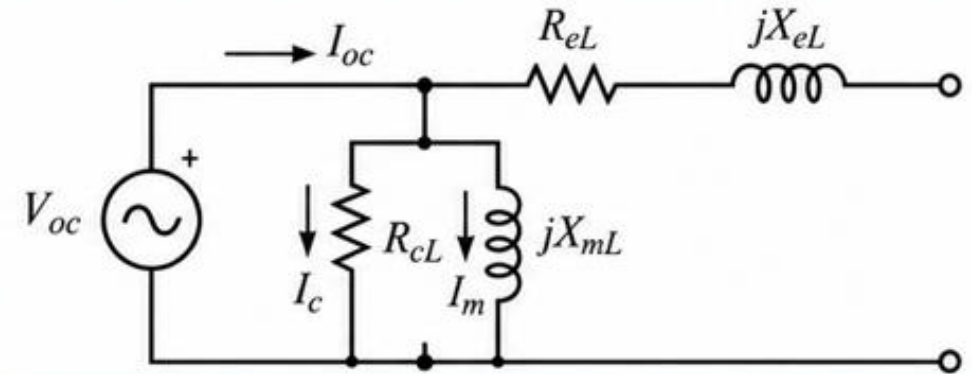
Determine the equivalent circuit of the transformer as viewed from:

- (a) the high-voltage side**
- (b) the low-voltage side**

OC-Test Setup



Equivalent Circuit



Calculations (Low-Voltage Side)

$$S_{oc} = V_{oc} \times I_{oc} = 240 \times 2 = 480 \text{ VA}$$

$$R_{cL} = \frac{V_{oc}^2}{P_{oc}} = \frac{240^2}{120} = 480 \text{ } \Omega$$

$$Q_{oc} = \sqrt{S_{oc}^2 - P_{oc}^2} = \sqrt{480^2 - 120^2} = 464.76 \text{ VAR}$$

$$X_{mL} = \frac{V_{oc}^2}{Q_{oc}} = \frac{240^2}{464.76} = 123.94 \text{ } \Omega$$

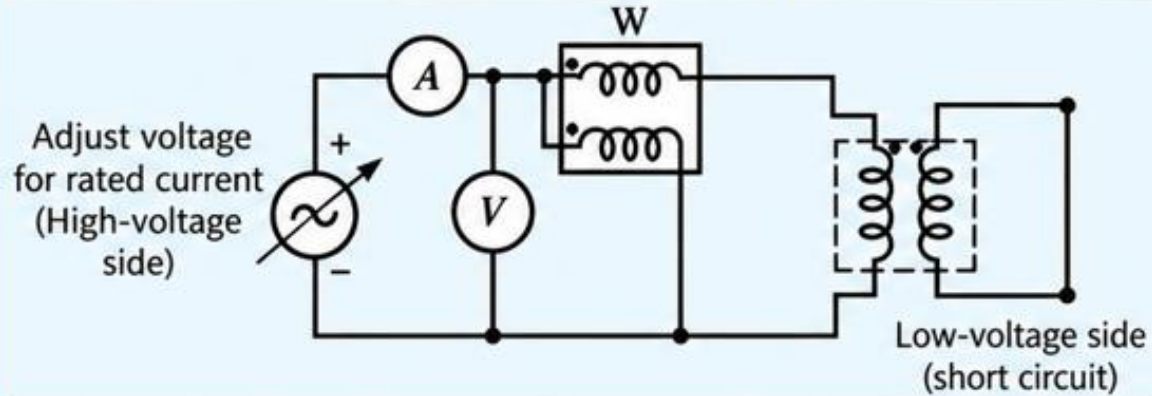
Referred to High-Voltage Side

$$a = \frac{4800}{240} = 20$$

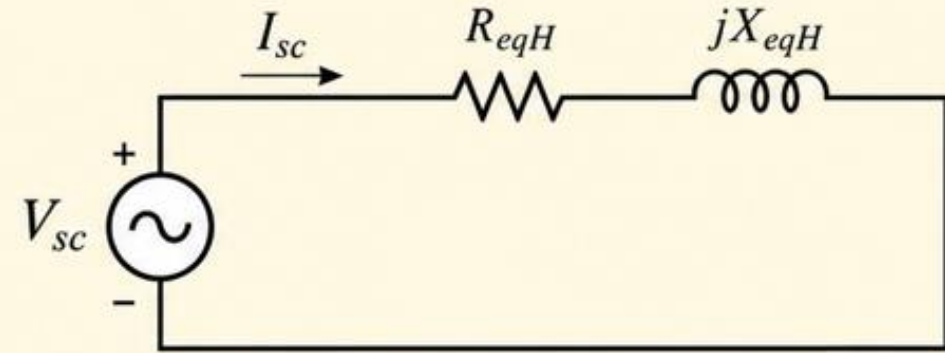
$$R_{cH} = a^2 R_{cL} = 20^2 (480) = 192 \text{ k}\Omega$$

$$X_{mH} = a^2 X_{mL} = 20^2 (123.94) = 49.58 \text{ k}\Omega$$

SC-Test Setup



Equivalent Circuit



Equivalent Parameters

$$R_{eqH} = \frac{P_{sc}}{I_{sc}^2} = \frac{600}{10^2} = 6 \, \Omega$$

$$Z_{eqH} = \frac{V_{sc}}{I_{sc}} = \frac{150}{10} = 15 \, \Omega$$

$$X_{eqH} = \sqrt{Z_{eqH}^2 - R_{eqH}^2} = \sqrt{15^2 - 6^2} = 13.75 \, \Omega$$

$$R_{eqL} = \frac{R_{eqH}}{a^2} = \frac{6}{20^2} = 0.015 \, \Omega \text{ or } 15 \, \text{m}\Omega$$

$$X_{eqL} = \frac{X_{eqH}}{a^2} = \frac{13.75}{20^2} = 0.034 \, \Omega \text{ or } 34 \, \text{m}\Omega$$

Separated Winding Parameters

Turns ratio: $a = 20$

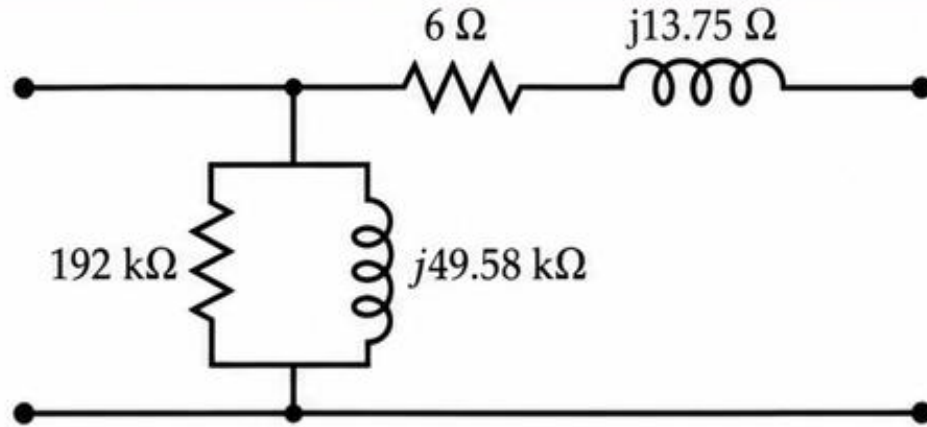
$$R_H = 0.5 R_{eqH} = 0.5 \times 6 = 3 \, \Omega$$

$$X_H = 0.5 X_{eqH} = 0.5 \times 13.75 = 6.88 \, \Omega$$

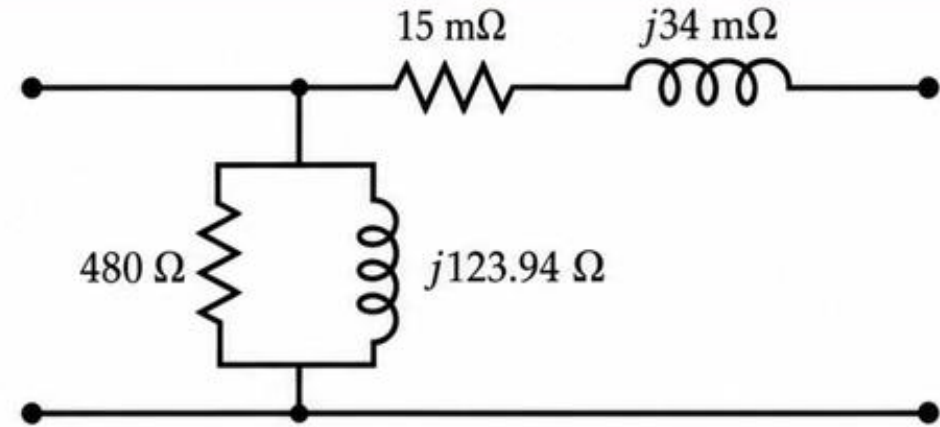
$$R_L = \frac{0.5 R_{eqH}}{a^2} = \frac{0.5 \times 6}{20^2} = 0.0075 \, \Omega \text{ or } 7.5 \, \text{m}\Omega$$

$$X_L = \frac{0.5 X_{eqH}}{a^2} = \frac{0.5 \times 13.75}{20^2} = 0.017 \, \Omega \text{ or } 17 \, \text{m}\Omega$$

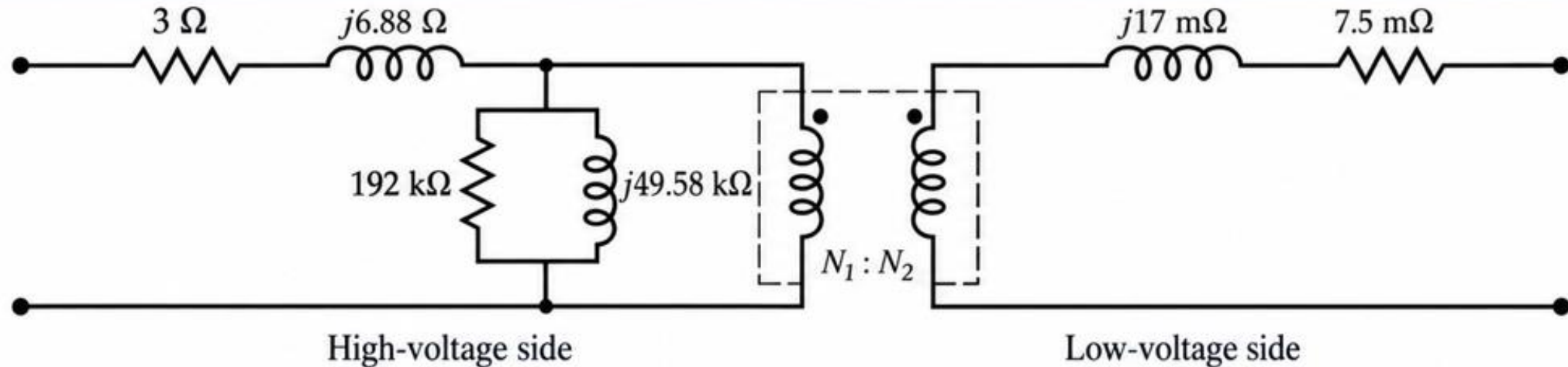
High-Voltage Side Equivalent Circuit



Low-Voltage Side Equivalent Circuit



Complete Equivalent Circuit



Electrical Machines

Chapter 2 TRANSFORMERS

Instructor

Eng. Nael S. Dokhan

MSc – Electrical Engineering

Lecture 17

The equivalent circuit impedances of a 20-kVA, 8000/240 V, 60-Hz transformer are to be determined. The open-circuit test was performed on the secondary side of the transformer (to reduce the maximum voltage to be measured) and the short-circuit test was performed on the primary side of the transformer (to reduce the maximum current to be measured). The following data were taken:

Open-circuit test (on secondary)	Short-circuit test (on primary)
$V_{OC} = 240 \text{ V}$	$V_{SC} = 489 \text{ V}$
$I_{OC} = 7.133 \text{ A}$	$I_{SC} = 2.5 \text{ A}$
$P_{OC} = 400 \text{ W}$	$P_{SC} = 240 \text{ W}$

Find the impedances of the approximate equivalent circuit referred to the primary side, and sketch that circuit.

- 1 The turns ratio of this transformer is $a = 8000/240 = 33.3333$. The power factor during the *open-circuit* test is

$$\text{PF} = \cos \theta = \frac{P_{OC}}{V_{OC} I_{OC}}$$

$$\text{PF} = \cos \theta = \frac{400 \text{ W}}{(240 \text{ V})(7.133 \text{ A})}$$

$$\text{PF} = 0.234 \text{ lagging}$$

- 2 The excitation admittance is given by

$$Y_E = \frac{I_{OC}}{V_{OC}} \angle -\cos^{-1} \text{PF}$$

$$Y_E = \frac{7.133 \text{ A}}{240 \text{ V}} \angle -\cos^{-1} 0.234$$

$$Y_E = 0.0297 \angle -76.5^\circ \text{ S}$$

$$Y_E = 0.00693 - j0.02888 = \frac{1}{R_C} - j \frac{1}{X_M}$$

- 3 Therefore, the values of the excitation branch *referred to the low-voltage (secondary) side* are

$$R_C = \frac{1}{0.00693} = 144 \Omega$$

$$X_M = \frac{1}{0.02888} = 34.63 \Omega$$

1. Short-Circuit Test Calculations

The power factor during the *short-circuit* test is

$$\text{PF} = \cos \theta = \frac{P_{\text{SC}}}{V_{\text{SC}} I_{\text{SC}}}$$

$$\text{PF} = \cos \theta = \frac{240 \text{ W}}{(489 \text{ V})(2.5 \text{ A})} = 0.196 \text{ lagging}$$

The series impedance is given by

$$Z_{\text{SE}} = \frac{V_{\text{SC}}}{I_{\text{SC}}} \angle \cos^{-1} \text{PF}$$

$$Z_{\text{SE}} = \frac{489 \text{ V}}{2.5 \text{ A}} \angle 78.7^\circ$$

$$Z_{\text{SE}} = 195.6 \angle 78.7^\circ = 38.4 + j192 \Omega$$

2. Equivalent Resistance and Reactance (Referred to High-Voltage Side)

Therefore, the equivalent resistance and reactance *referred to the high-voltage (primary) side* are

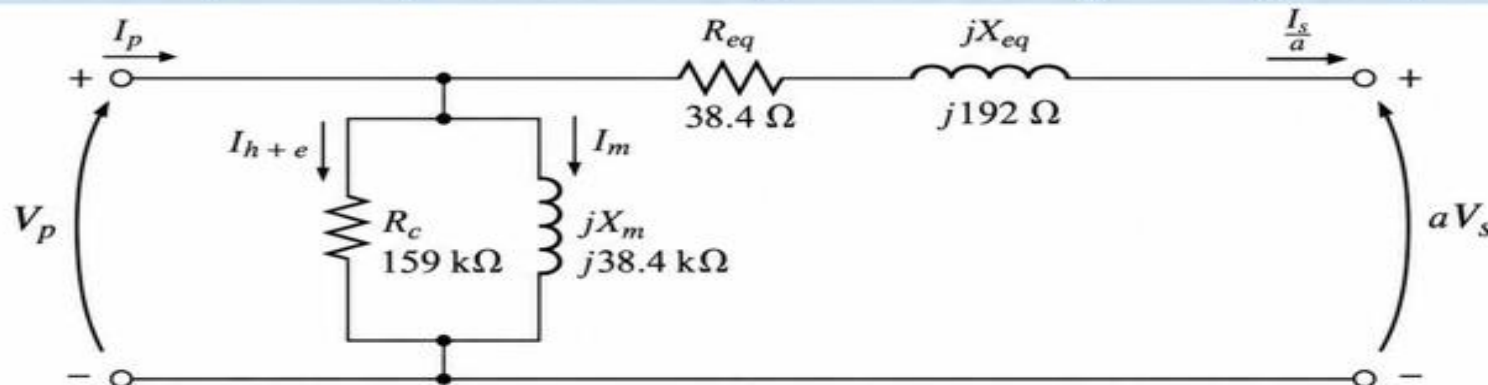
$$R_{\text{eq}} = 38.4 \Omega \quad X_{\text{eq}} = 192 \Omega$$

The resulting simplified equivalent circuit referred to the high-voltage (*primary*) side can be found by converting the excitation branch values to the *high-voltage* side.

$$R_{C,P} = a^2 R_{C,S} = (33.333)^2 (144 \Omega) = 159 \text{ k}\Omega$$

$$X_{M,P} = a^2 X_{M,S} = (33.333)^2 (34.63 \Omega) = 38.4 \text{ k}\Omega$$

3. Simplified Equivalent Circuit (Referred to High-Voltage Side)



A 15-kVA, 2300/230-V transformer is to be tested to determine its excitation branch components, its series impedances, and its voltage regulation. The following test data have been taken from the transformer.

Open-circuit test (low voltage side)	Short-circuit test (high voltage side)
$V_{OC} = 230 \text{ V}$	$V_{SC} = 47 \text{ V}$
$I_{OC} = 2.1 \text{ A}$	$I_{SC} = 6.0 \text{ A}$
$P_{OC} = 50 \text{ W}$	$P_{SC} = 160 \text{ W}$

- (a) Find the equivalent circuit of this transformer referred to the high-voltage side.
- (b) Find the equivalent circuit of this transformer referred to the low-voltage side.
- (c) Calculate the full-load voltage regulation at 0.8 lagging power factor, 1.0 power factor, and at 0.8 leading power factor using the exact equation for V_p .
- (d) What is the efficiency of the transformer at full load with a power factor of 0.8 lagging?

Equivalent Circuit Referred to the Primary Side

1 Turns Ratio

$$a = \frac{2300}{230} = 10$$

$$a = 10$$

2 Open-Circuit Test (Excitation Branch)

Open-circuit impedance angle:

$$\theta_{OC} = \cos^{-1} \left(\frac{P_{OC}}{V_{OC} I_{OC}} \right) = \cos^{-1} \left(\frac{50}{230 \times 2.1} \right) = 84^\circ$$

Excitation admittance:

$$Y_E = \frac{I_{OC}}{V_{OC}} \angle (-84^\circ) = \frac{2.1}{230} \angle -84^\circ \text{ S}$$

$$Y_E = 0.00913 \angle -84^\circ \text{ S} = 0.000954 - j0.00908 \text{ S}$$

Excitation branch referred to the secondary:

$$R_{C,S} = \frac{1}{0.000954} = 1050 \Omega$$

$$X_{M,S} = \frac{1}{0.00908} = 110 \Omega$$

3 Short-Circuit Test (Series Branch)

Short-circuit impedance angle:

$$\theta_{SC} = \cos^{-1} \left(\frac{P_{SC}}{V_{SC} I_{SC}} \right) = \cos^{-1} \left(\frac{160}{47 \times 6} \right) = 55.4^\circ$$

Equivalent series impedance (referred to secondary):

$$Z_{SE} = \frac{V_{SC}}{I_{SC}} \angle \theta_{SC} = \frac{47}{6} \angle 55.4^\circ \Omega$$

$$Z_{SE} = 7.833 \angle 55.4^\circ \Omega = 4.45 + j6.45 \Omega$$

Series elements referred to the primary side:

$$R_{eq,P} = 4.45 \Omega$$

$$X_{eq,P} = 6.45 \Omega$$

4 Convert Excitation Branch to Primary Side

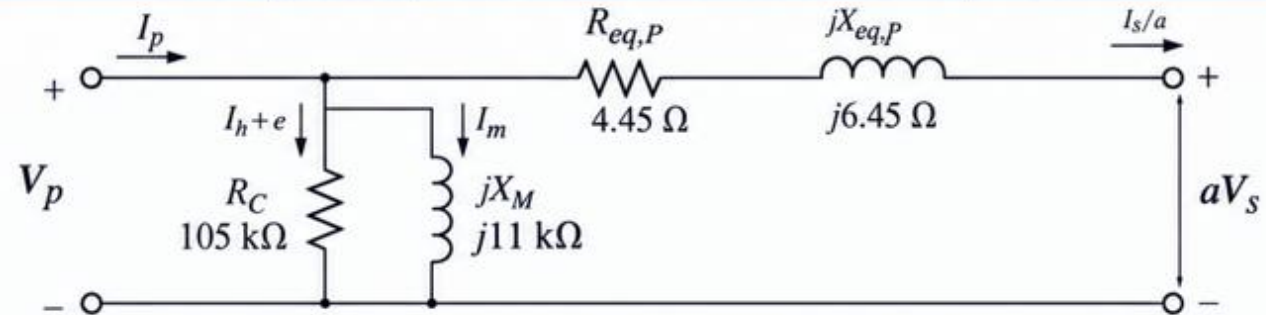
$$R_{C,P} = a^2 R_{C,S} = (10)^2 (1050 \Omega) = 105 \text{ k}\Omega$$

$$X_{M,P} = a^2 X_{M,S} = (10)^2 (110 \Omega) = 11 \text{ k}\Omega$$

$$R_{C,P} = 105 \text{ k}\Omega$$

$$X_{M,P} = 11 \text{ k}\Omega$$

Simplified Equivalent Circuit Referred to the Primary Side



Equivalent Circuit Referred to the Low-Voltage Side

Method

To find the equivalent circuit referred to the low-voltage side, it is simply necessary to divide the impedances by a^2 , where

$$a = \frac{N_p}{N_s} = 10$$

Thus, each impedance on the high-voltage side is divided by a^2 to obtain the low-voltage referred values.

Equivalent Impedances (Referred to Low-Voltage Side)

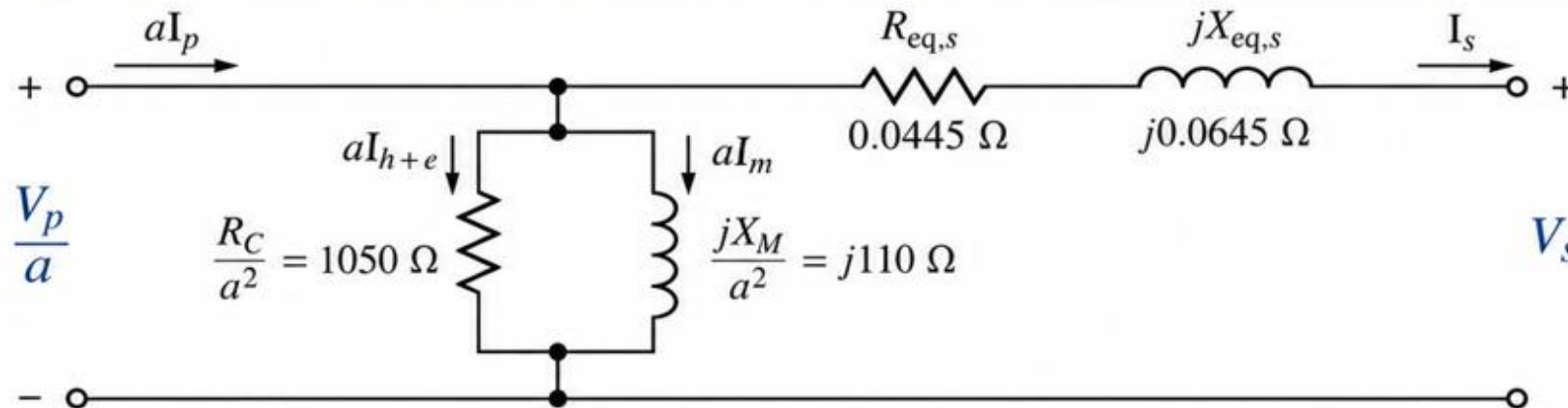
$$R_C = 1050 \, \Omega$$

$$R_{eq} = 0.0445 \, \Omega$$

$$X_M = 110 \, \Omega$$

$$X_{eq} = 0.0645 \, \Omega$$

Simplified Equivalent Circuit (Referred to Low-Voltage Side)



(c) Transformer Secondary Voltage and Voltage Regulation

1 Rated Secondary Current

$$I_{s,\text{rated}} = \frac{S_{\text{rated}}}{V_{s,\text{rated}}} = \frac{15,000 \text{ VA}}{230 \text{ V}} = 65.2 \text{ A}$$

2 Phasor Voltage Calculation (0.8 Lagging Power Factor)

At PF = 0.8 lagging, current $I_s = 65.2 \angle -36.9^\circ \text{ A}$.

Use

$$\frac{V_p}{a} = V_s + R_{\text{eq}} I_s + jX_{\text{eq}} I_s$$

Substituting the values:

$$\begin{aligned} \frac{V_p}{a} &= 230 \angle 0^\circ \text{ V} + (0.0445 \Omega)(65.2 \angle -36.9^\circ \text{ A}) + j(0.0645 \Omega)(65.2 \angle -36.9^\circ \text{ A}) \\ &= 230 \angle 0^\circ \text{ V} + 2.90 \angle -36.9^\circ \text{ V} + 4.21 \angle 53.1^\circ \text{ V} \\ &= 230 + 2.32 - j1.74 + 2.52 + j3.36 \\ &= 234.84 + j1.62 = 234.85 \angle 0.40^\circ \text{ V} \end{aligned}$$

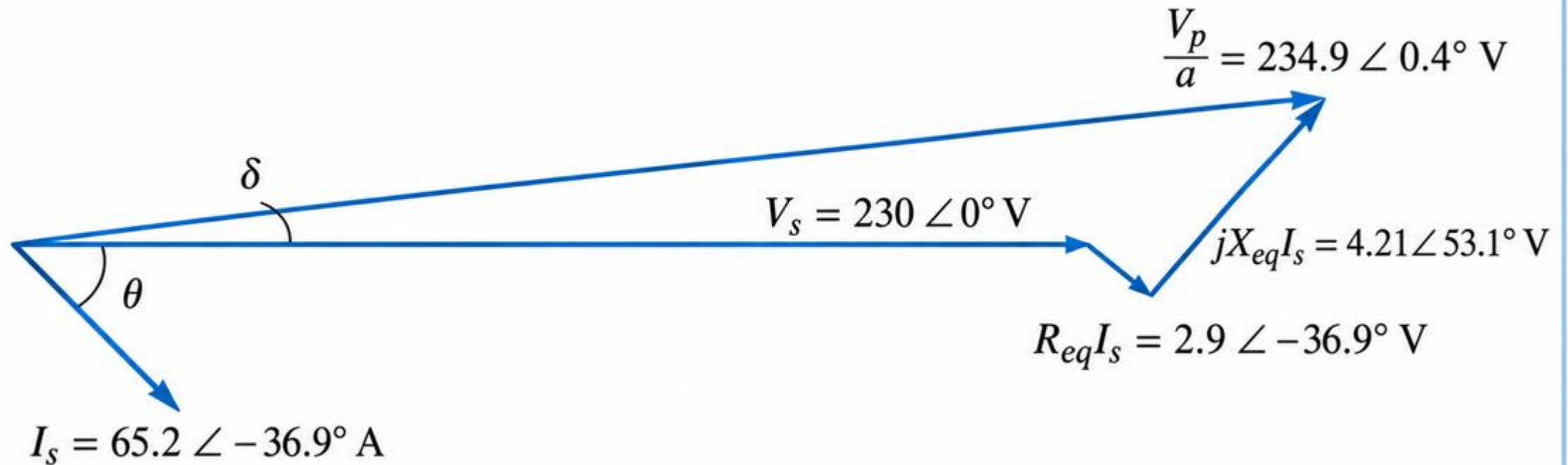
3 Voltage Regulation

The resulting voltage regulation is

$$\begin{aligned} \text{VR} &= \frac{V_p/a - V_{s,\text{fl}}}{V_{s,\text{fl}}} \times 100\% \\ &= \frac{234.85 \text{ V} - 230 \text{ V}}{230 \text{ V}} \times 100\% = 2.1\% \end{aligned}$$

**Voltage Regulation = 2.1 %
at 0.8 lagging PF**

Phasor Diagram at 0.8 Lagging Power Factor



Voltage Regulation at Full Load (Unity Power Factor, PF = 1.0)

At PF = 1.0, the load current is in phase with the voltage:

$$I_s = 65.2 \angle 0^\circ \text{ A}$$

Using $\frac{V_s}{\hat{a}} = V_s + R_{eq}I_s + jX_{eq}I_s$:

$$\frac{V_p}{\hat{a}} = 230 \angle 0^\circ + (0.0445 \Omega)(65.2 \angle 0^\circ) + j(0.0645 \Omega)(65.2 \angle 0^\circ)$$

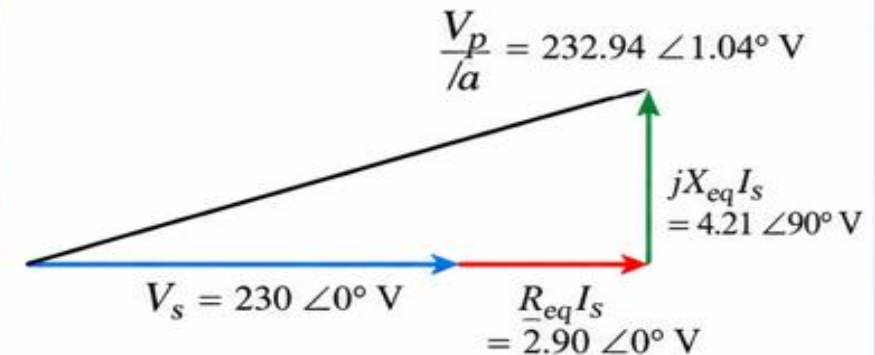
$$= 230 \angle 0^\circ + 2.90 \angle 0^\circ + 4.21 \angle 90^\circ$$

$$= 230 + 2.90 + j4.21$$

$$= 232.94 + j4.21$$

$$= 232.94 \angle 1.04^\circ \text{ V}$$

Phasor Diagram (PF = 1.0)



Voltage Regulation

$$\text{VR} = \left(\frac{\frac{V_p}{\hat{a}} - V_{s,\text{fl}}}{V_{s,\text{fl}}} \right) \times 100\% = \frac{232.94 \text{ V} - 230 \text{ V}}{230 \text{ V}} \times 100\% = 1.28\%$$

Voltage Regulation at 0.8 Leading Power Factor

1 Calculate the sending-end voltage

At PF = 0.8 leading, current $I_s = 65.2 \angle 36.9^\circ$ A. Therefore,

$$\begin{aligned}\frac{V_p}{a} &= 230 \angle 0^\circ \text{ V} + (0.0445 \, \Omega)(65.2 \angle 36.9^\circ \text{ A}) + j(0.0645 \, \Omega)(65.2 \angle 36.9^\circ \text{ A}) \\ &= 230 \angle 0^\circ \text{ V} + 2.90 \angle 36.9^\circ \text{ V} + 4.21 \angle 126.9^\circ \text{ V} \\ &= 230 + 2.32 + j1.74 - 2.52 + j3.36 \\ &= 229.80 + j5.10 = 229.85 \angle 1.27^\circ \text{ V}\end{aligned}$$

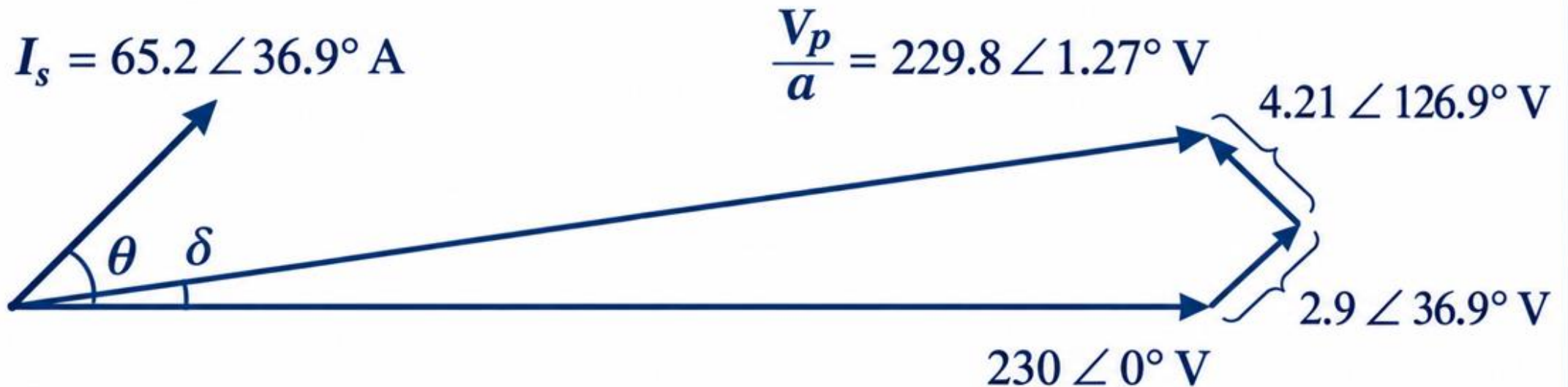
2 Calculate the voltage regulation

The resulting voltage regulation is

$$\text{VR} = \frac{229.85 \text{ V} - 230 \text{ V}}{230 \text{ V}} \times 100\% = -0.062\%$$

$$\text{Voltage Regulation} = -0.062\%$$

Phasor Diagram at 0.8 Leading Power Factor



Transformer Efficiency at Full Load (0.8 Lagging PF)**1 Copper losses**

$$P_{Cu} = (I_S)^2 R_{eq} = (65.2 \text{ A})^2 (0.0445 \Omega) = 189 \text{ W}$$

2 Core losses

$$P_{core} = \frac{(V_p/a)^2}{R_C} = \frac{(234.85 \text{ V})^2}{1050 \Omega} = 52.5 \text{ W}$$

3 Output power at this power factor

$$\begin{aligned} P_{out} &= V_S I_S \cos \theta \\ &= (230 \text{ V})(65.2 \text{ A}) \cos 36.9^\circ = 12,000 \text{ W} \end{aligned}$$

4 Efficiency

$$\begin{aligned} \eta &= \frac{V_S I_S \cos \theta}{P_{Cu} + P_{core} + V_S I_S \cos \theta} \times 100\% \\ \eta &= \frac{12,000 \text{ W}}{189 \text{ W} + 52.5 \text{ W} + 12,000 \text{ W}} \times 100\% \end{aligned}$$

$$\eta = 98.03\%$$

Electrical Machines

Chapter 2 TRANSFORMERS

Instructor

Eng. Nael S. Dokhan

MSc – Electrical Engineering

Lecture 18

Key points

Two transformer windings are connected electrically.

Energy transfer occurs by conduction and magnetic induction.

Lower first cost than a two-winding transformer of similar rating.

Can deliver more kVA for the same physical size.

Often more efficient and needs lower excitation current.

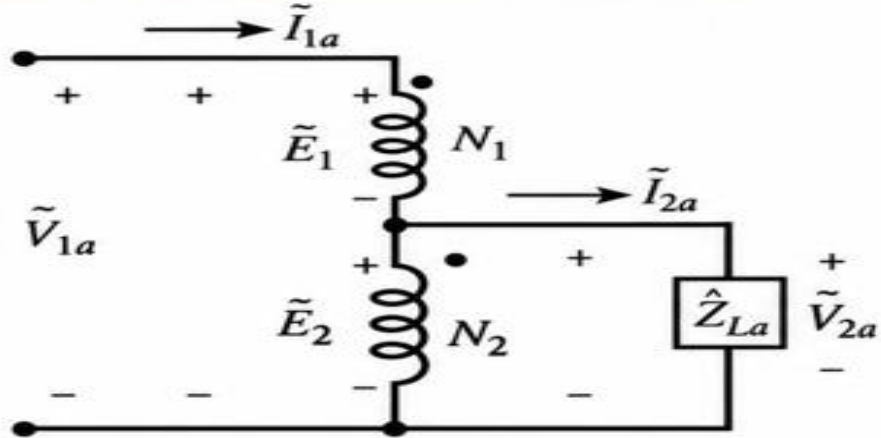
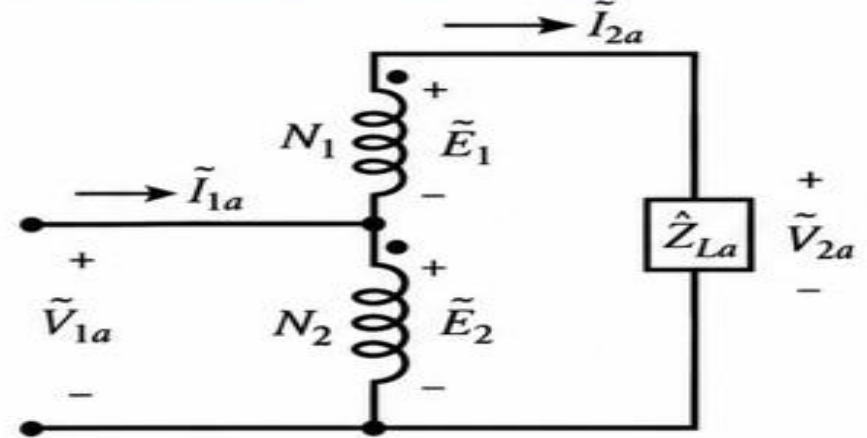
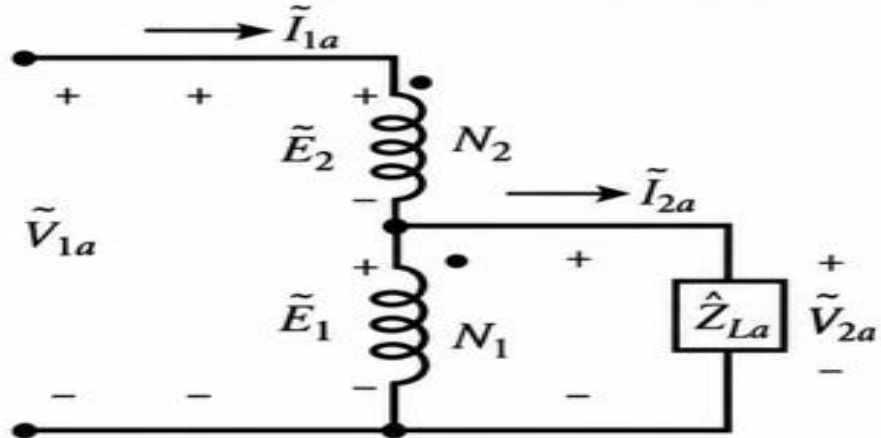
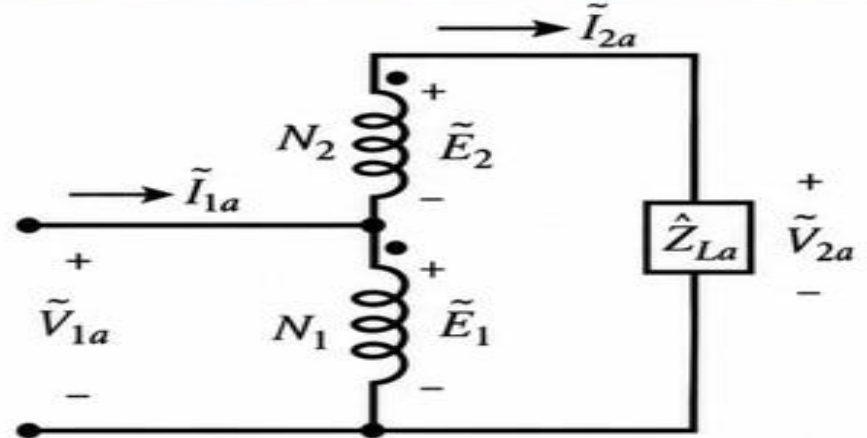
Advantage

Higher kVA rating for the same core and copper.

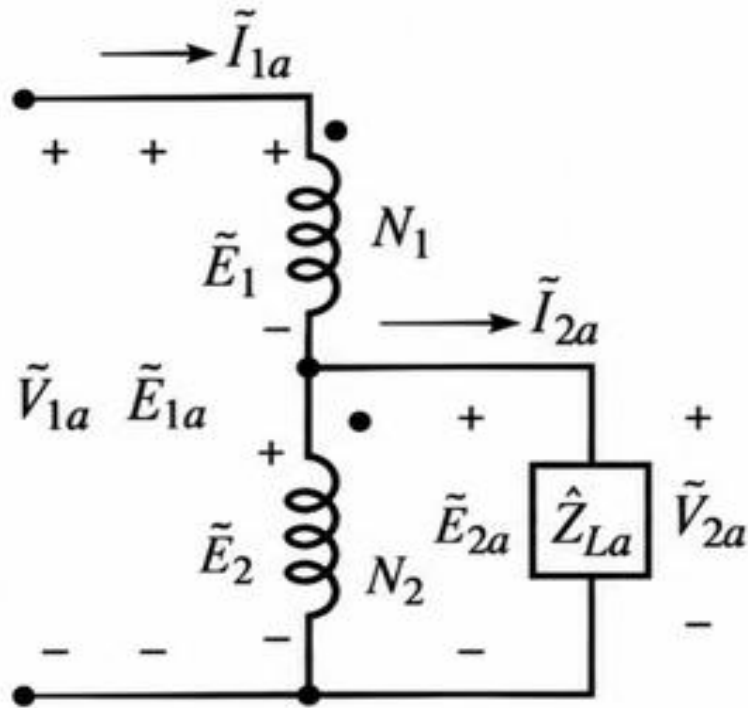
Limitation

No electrical isolation between source and load.

Use autotransformers when isolation is not required and voltage change is moderate.

(a) $(V_1 + V_2) / V_2$ step-down connection.**(b)** $V_2 / (V_1 + V_2)$ step-up connection.**(c)** $(V_1 + V_2) / V_1$ step-down connection.**(d)** $V_1 / (V_1 + V_2)$ step-up connection.

1 Autotransformer Connection



(a) $(V_1 + V_2) / V_2$ step-down connection.

2 Voltage Relations

$$V_{1a} = E_{1a} = E_1 + E_2$$

$$V_{2a} = E_{2a} = E_2$$

$$\frac{V_{1a}}{V_{2a}} = \frac{E_{1a}}{E_{2a}} = \frac{E_1 + E_2}{E_2}$$

$$\frac{V_{1a}}{V_{2a}} = \frac{N_1 + N_2}{N_2} = 1 + a = a_r$$

$$a = \frac{N_1}{N_2} \quad (\text{turns ratio})$$

3 Current Relations

$$(N_1 + N_2) I_{1a} = N_2 I_{2a}$$

$$\frac{I_{2a}}{I_{1a}} = \frac{N_1 + N_2}{N_2} = 1 + a = a_r$$

$$S_{oa} = V_{2a} I_{2a}$$

$$S_{oa} = \left[\frac{V_{1a}}{a_r} \right] [a_r I_{1a}] = V_{1a} I_{1a}$$

$$S_{oa} = S_{ia}$$

4 Apparent Power Relation

$$V_{2a} = V_2$$

$$I_{2a} = a_r I_{1a} = (a + 1) I_{1a}$$

$$I_{1a} = I_1$$

$$S_{oa} = V_2 I_1 (a + 1)$$

$$S_{oa} = V_1 I_2 \frac{(a + 1)}{a} = S_o \left[1 + \frac{1}{a} \right]$$

5 Power Transfer Interpretation

$$S_{oa} = V_2 I_2 \frac{(a + 1)}{a} = S_o \left[1 + \frac{1}{a} \right]$$

S_o = power transfer through induction

$\frac{S_o}{a}$ = power transfer through conduction

Note:

- Part of the power is transferred inductively (S_o).
- A larger portion $\left(\frac{S_o}{a} \right)$ is transferred conductively through the common winding.
- Therefore, an autotransformer is more economical and smaller than a two-winding transformer of the same rating.

A 24-kVA, 2400/240-V distribution transformer is to be connected as an autotransformer. For each possible combination, determine (a) The primary winding voltage (b) The secondary winding voltage (c) The transformation ratio (d) The nominal rating of the autotransformer

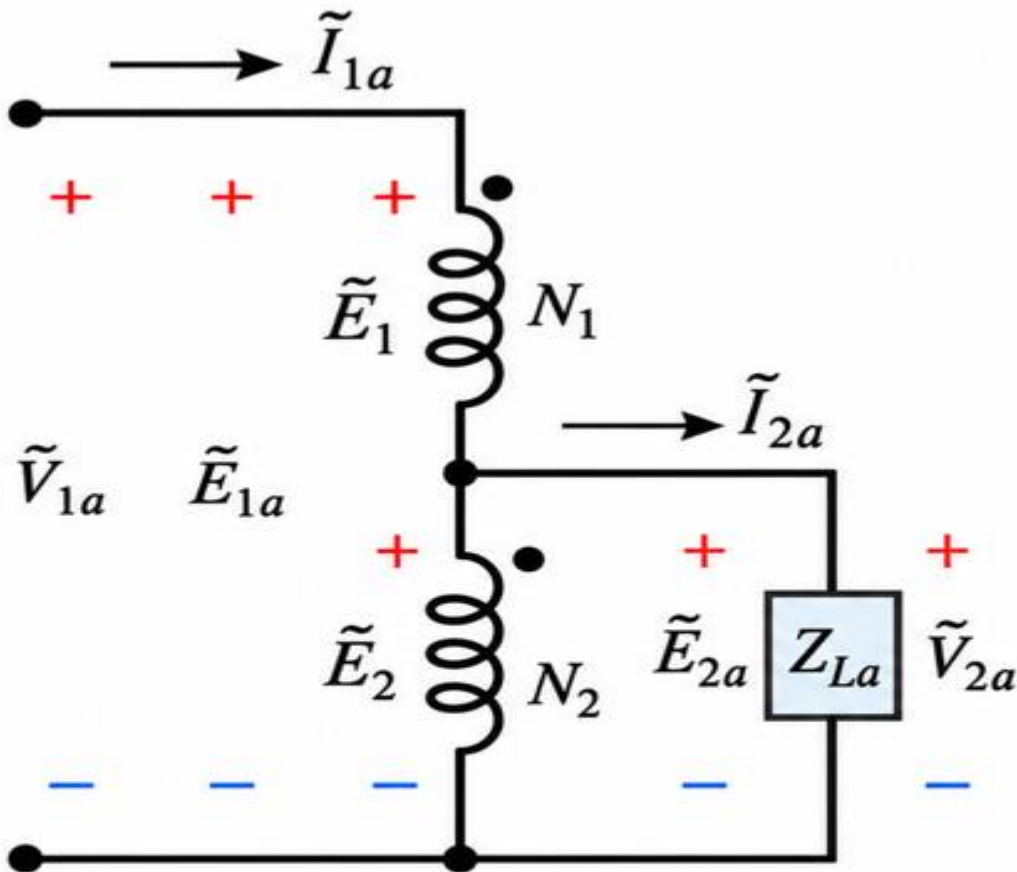
Given

For the original two-winding transformer:

$$V_1 = 2400 \text{ V}, \quad V_2 = 240 \text{ V}, \quad S_0 = 24 \text{ kVA}, \quad I_1 = 10 \text{ A}, \quad I_2 = 100 \text{ A}$$

Example 11 — Solution

24-kVA distribution transformer as an autotransformer



(a) $(V_1 + V_2) / V_2$ step-down connection.

Calculations

$$V_{1a} = E_1 + E_2 = 2400 + 240 = 2640 \text{ V}$$

$$V_{2a} = 240 \text{ V}$$

$$a_T = \frac{V_{1a}}{V_{2a}} = \frac{2640}{240} = 11$$

$$S_{oa} = V_{2a} I_{2a} = V_{1a} I_{1a}$$

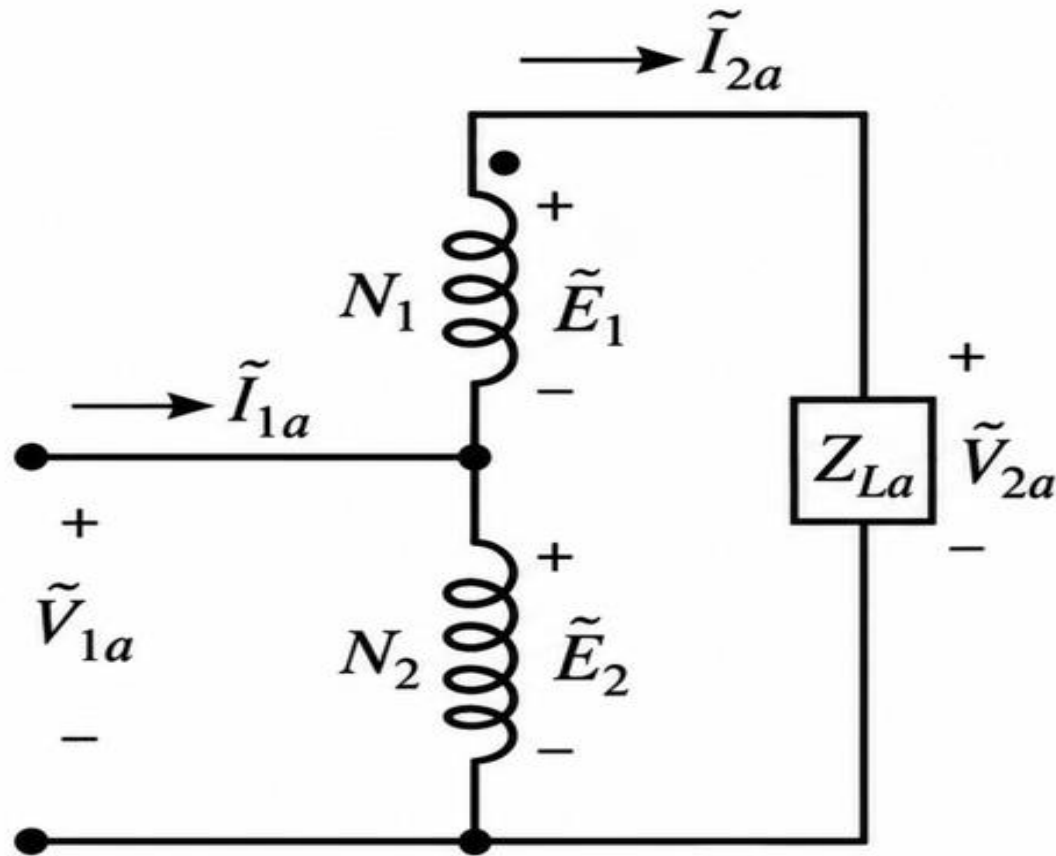
$$S_{oa} = 2640 * 10 = 26400 \text{ VA} = 26.4 \text{ kVA}$$

Result:

26.4 kVA, 2640/240-V

Example 11 — Solution Continued

24-kVA distribution transformer as an autotransformer



(b) $V_2/(V_1 + V_2)$ step-up connection.

Calculations

$$V_{1a} = E_2 = 240 \text{ V}$$

$$V_{2a} = E_1 + E_2 = 2400 + 240 = 2640 \text{ V}$$

$$a_T = \frac{V_{1a}}{V_{2a}} = \frac{240}{2640} = 0.091$$

$$S_{oa} = V_{2a} I_{2a} = V_{2a} I_1$$

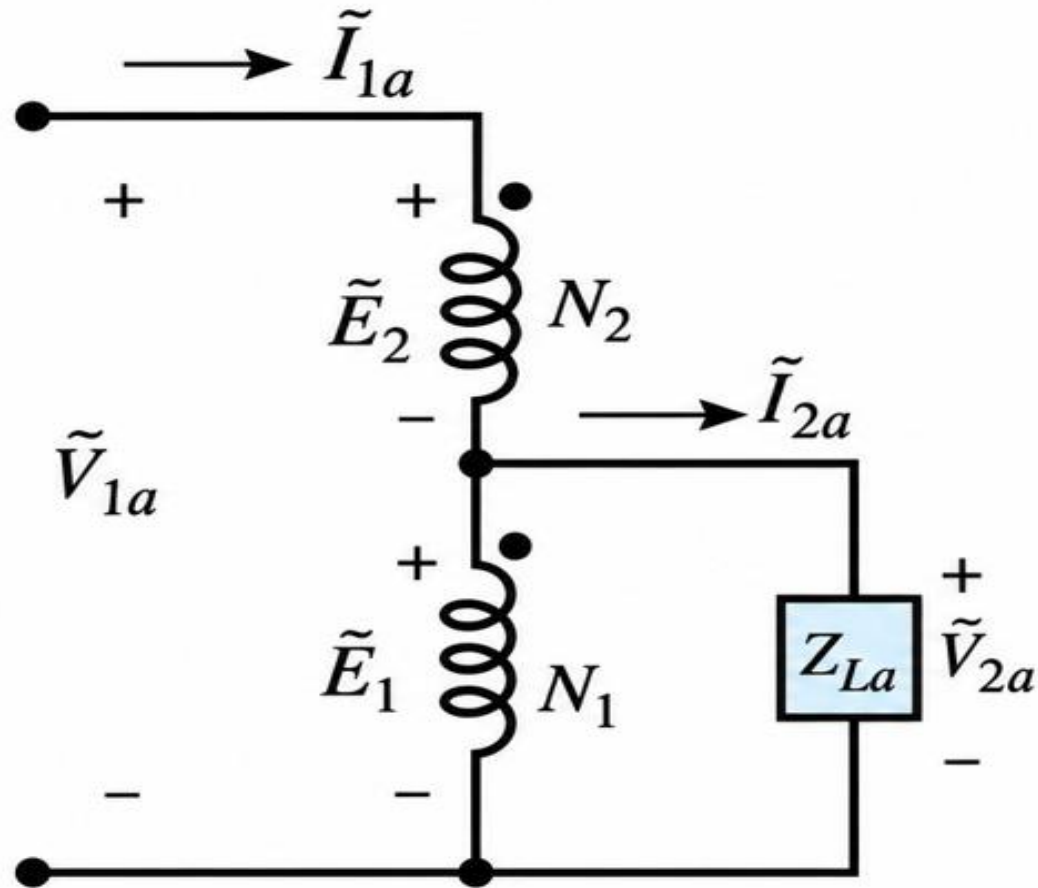
$$S_{oa} = 2640 * 10 = 26400 \text{ VA} = 26.4 \text{ kVA}$$

Result:

26.4 kVA, 240/2640-V

Example 11 — Solution Continued

24-kVA distribution transformer as an autotransformer



(c) $(V_1 + V_2) / V_1$ step-down connection.

Calculations

$$V_{1a} = E_1 + E_2 = 2400 + 240 = 2640 \text{ V}$$

$$V_{2a} = E_1 = 2400 \text{ V}$$

$$a_T = \frac{V_{1a}}{V_{2a}} = \frac{2640}{2400} = 1.1$$

$$S_{oa} = V_{2a} I_{2a} = V_{1a} I_2$$

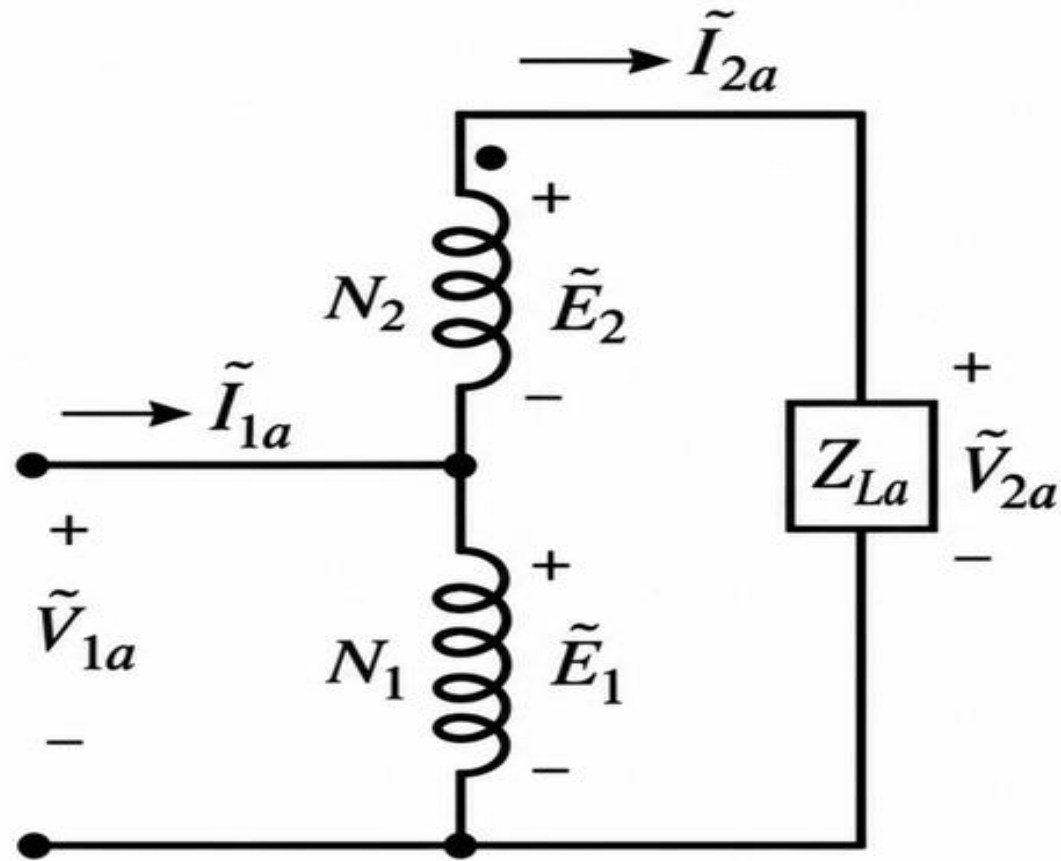
$$S_{oa} = 2640 * 100 = 264000 \text{ VA} = 264 \text{ kVA}$$

Result:

264 kVA, 2640/2400-V

Example 11 — Solution Continued

24-kVA distribution transformer as an autotransformer



(d) $V_1/(V_1 + V_2)$ step-up connection.

Calculations

$$V_{1a} = E_1 = 2400 \text{ V}$$

$$V_{2a} = E_1 + E_2 = 2400 + 240 = 2640 \text{ V}$$

$$a_T = \frac{V_{1a}}{V_{2a}} = \frac{2400}{2640} = 0.91$$

$$S_{oa} = V_{2a} I_{2a} = V_{1a} I_2$$

$$S_{oa} = 2640 * 100 = 264000 \text{ VA} = 264 \text{ kVA}$$

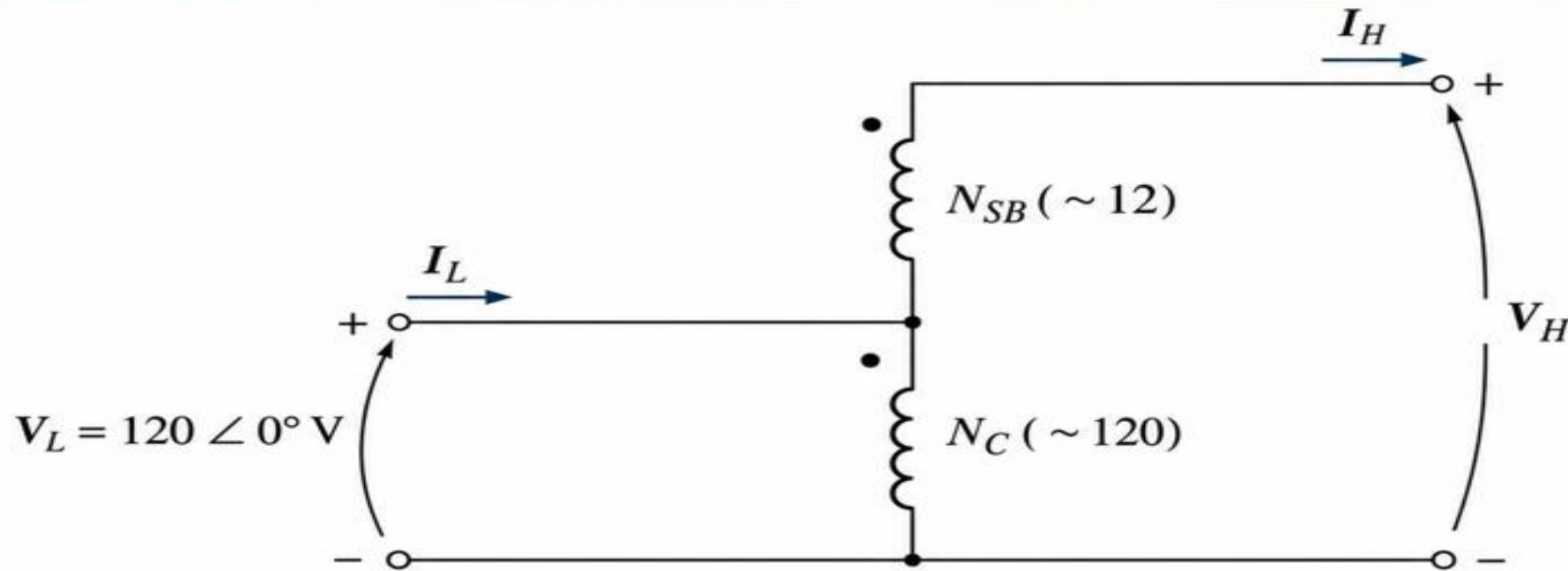
Result:

264 kVA, 2400/2640-V

Example 12

A 100-VA, 120/12-V transformer is to be connected so as to form a step-up autotransformer (see figure). A primary voltage of 120 V is applied to the transformer.

- (a) What is the secondary voltage of the transformer?
- (b) What is the maximum voltampere rating in this mode of operation?
- (c) Calculate the rating advantage of this autotransformer connection over the transformer's rating in conventional 120/12-V operation.



(a) Secondary Voltage

For a step-up autotransformer,

$$V_H = \frac{N_{SE} + N_C}{N_C} V_L$$

$$V_H = \frac{12 + 120}{120} (120 \text{ V})$$

$$V_H = 132 \text{ V}$$

(b) Maximum Voltampere Rating

The series winding voltage is 12 V and its rating is 100 VA, so the maximum series winding current is:

$$I_{SE, \max} = \frac{S_{\max}}{V_{SE}} = \frac{100 \text{ VA}}{12 \text{ V}} = 8.33 \text{ A}$$

Since $I_{SE} = I_S = I_H$ and $V_S = V_H = 132 \text{ V}$,

$$S_{\text{out}} = V_S I_S = V_H I_H = (132 \text{ V})(8.33 \text{ A})$$

$$S_{\text{out}} = 1100 \text{ VA}$$

(c) Rating Advantage

The rating advantage is:

$$\frac{S_{IO}}{S_W} = \frac{1100 \text{ VA}}{100 \text{ VA}} = 11$$

Also,

$$\begin{aligned} \frac{S_{IO}}{S_W} &= \frac{N_{SE} + N_C}{N_{SE}} \\ &= \frac{132}{12} = 11 \end{aligned}$$

$$11$$

The apparent power rating is increased by a factor of 11.

Electrical Machines

Chapter 2 TRANSFORMERS

Instructor

Eng. Nael S. Dokhan

MSc – Electrical Engineering

Lecture 19

Main concepts

Power is generated, transmitted, and consumed mainly in three-phase form.

A three-phase transformer may be built from three identical single-phase transformers.

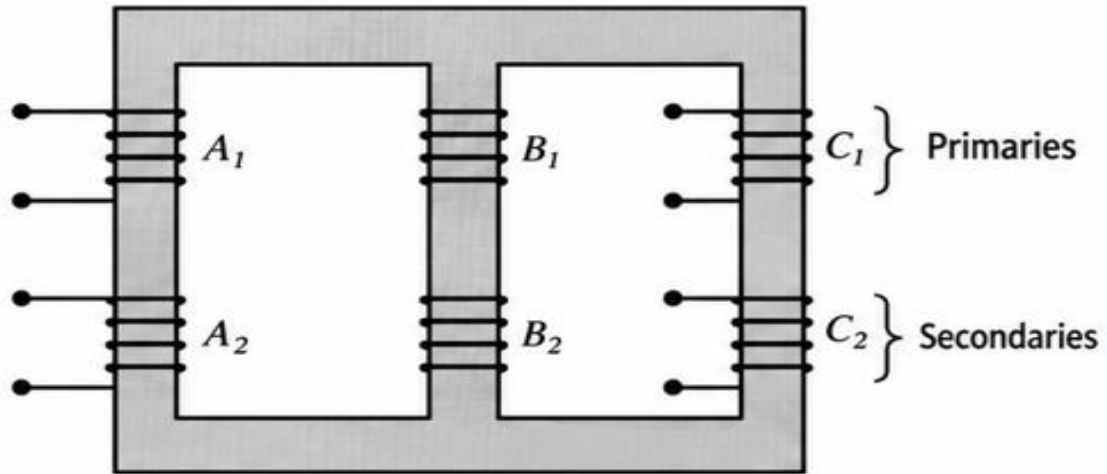
For economy, all six windings are usually placed on a common magnetic core.

The common core can be core-type or shell-type.

Shell-type construction usually has less waveform distortion than core-type construction.

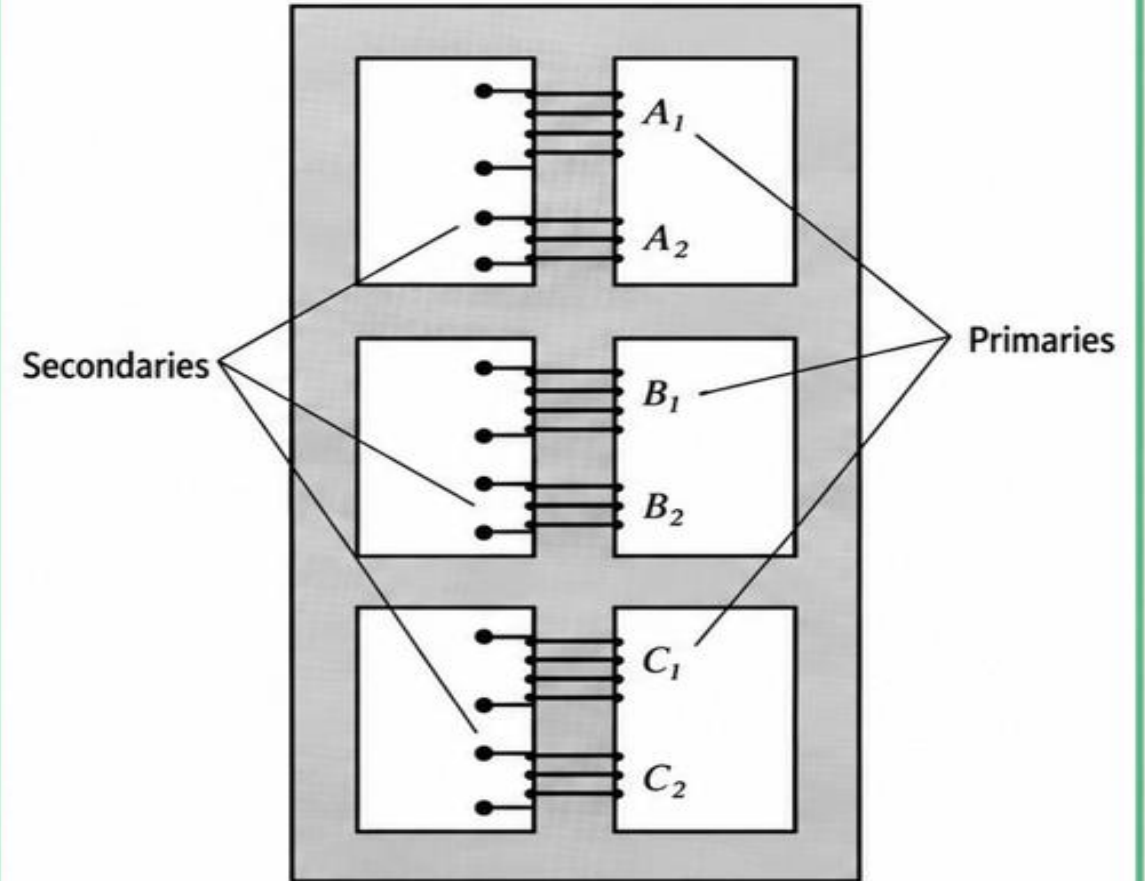
Either side may be connected in wye (Y) or delta (Δ).

Core Type



Windings on separate limbs

Shell-Type



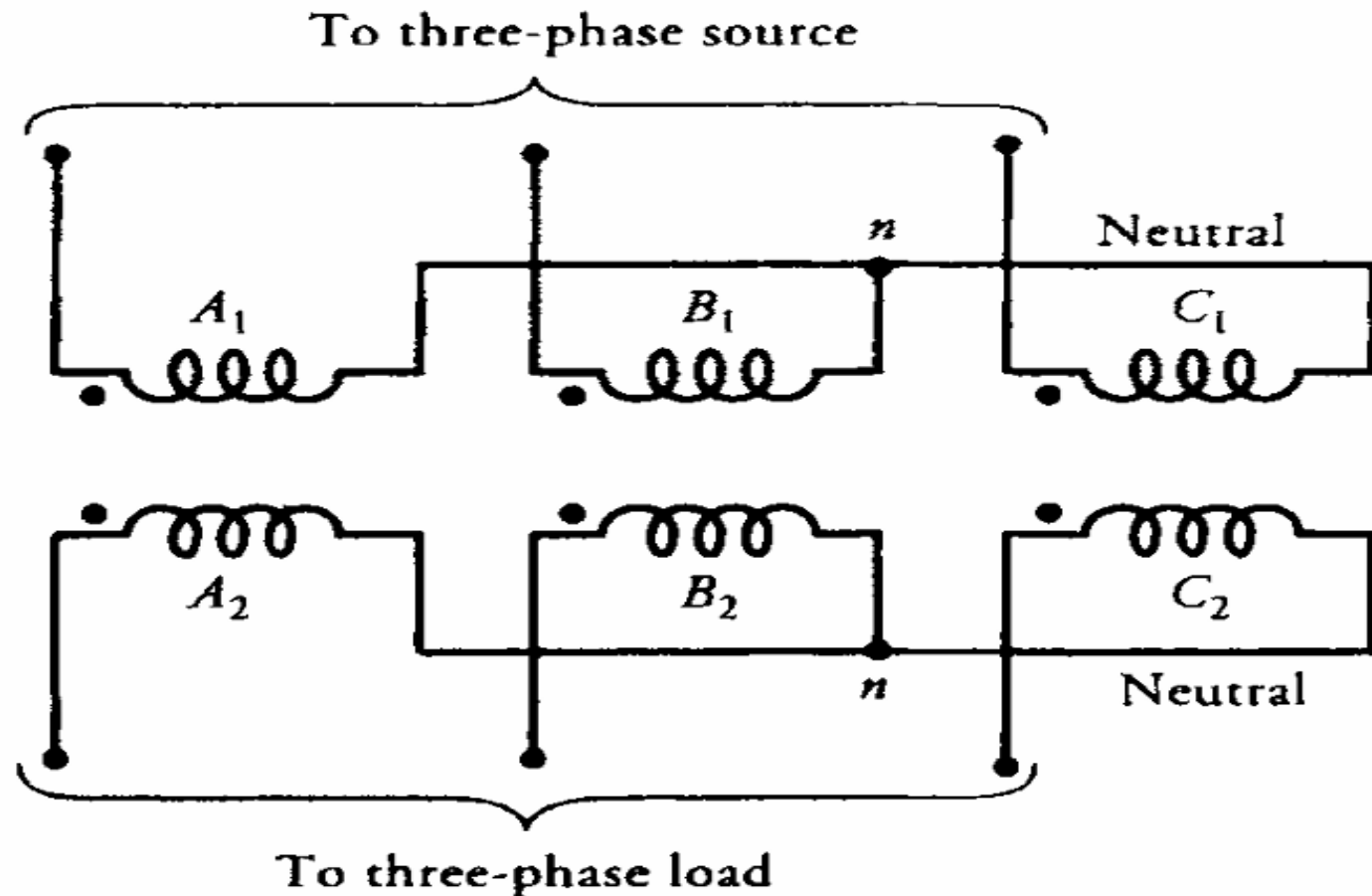
Windings on central limb

Three-phase transformer with both primary and secondary windings in star (Y)

Y-Y Connection

Both primary and secondary windings are connected in star (Y).

Three-Phase Transformer Connection (Y-Y)

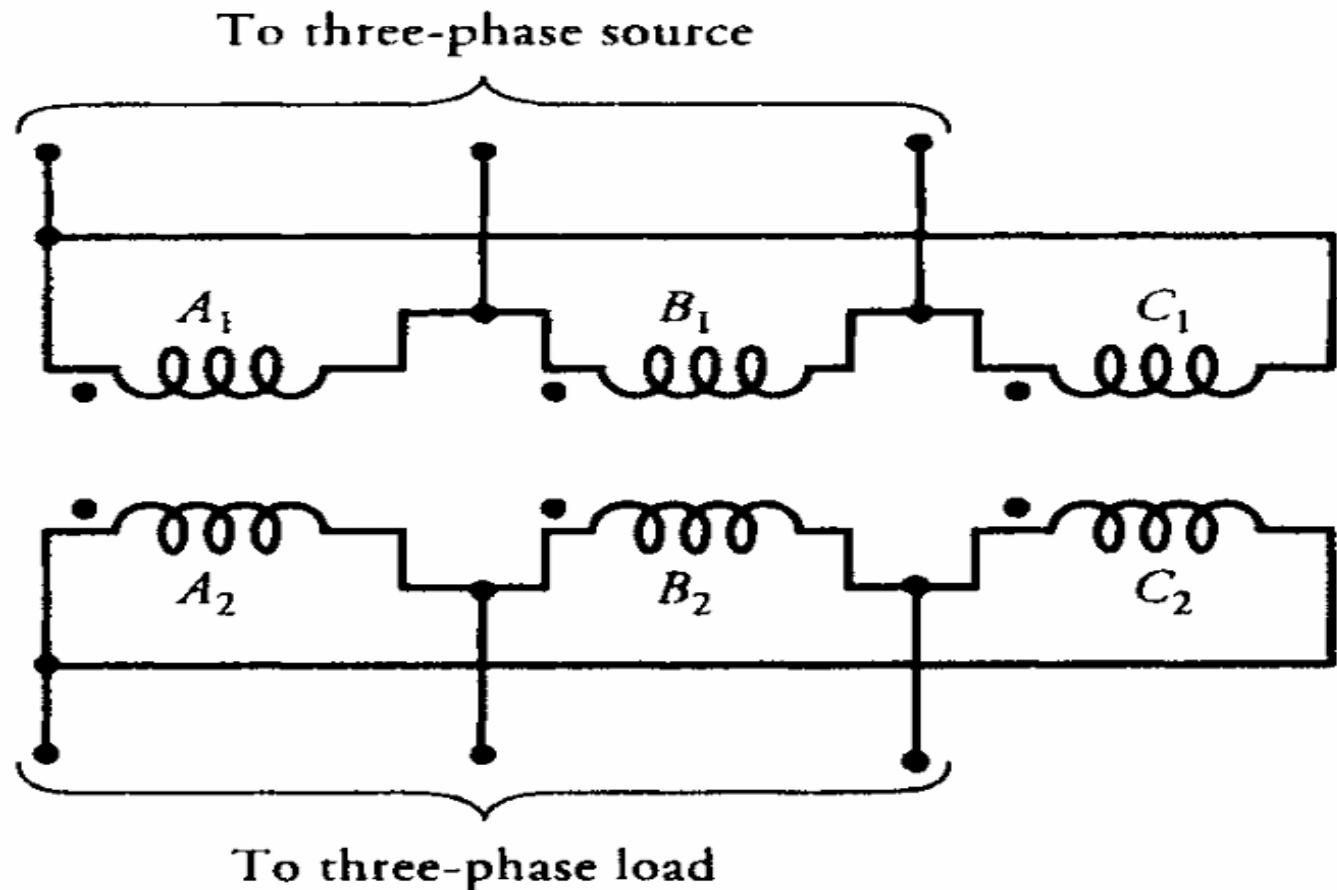


Three-phase transformer with both primary and secondary windings in delta (Δ)

Δ - Δ Connection

Both primary and secondary windings are connected in delta (Δ).

Three-Phase Transformer Connection (Δ - Δ)

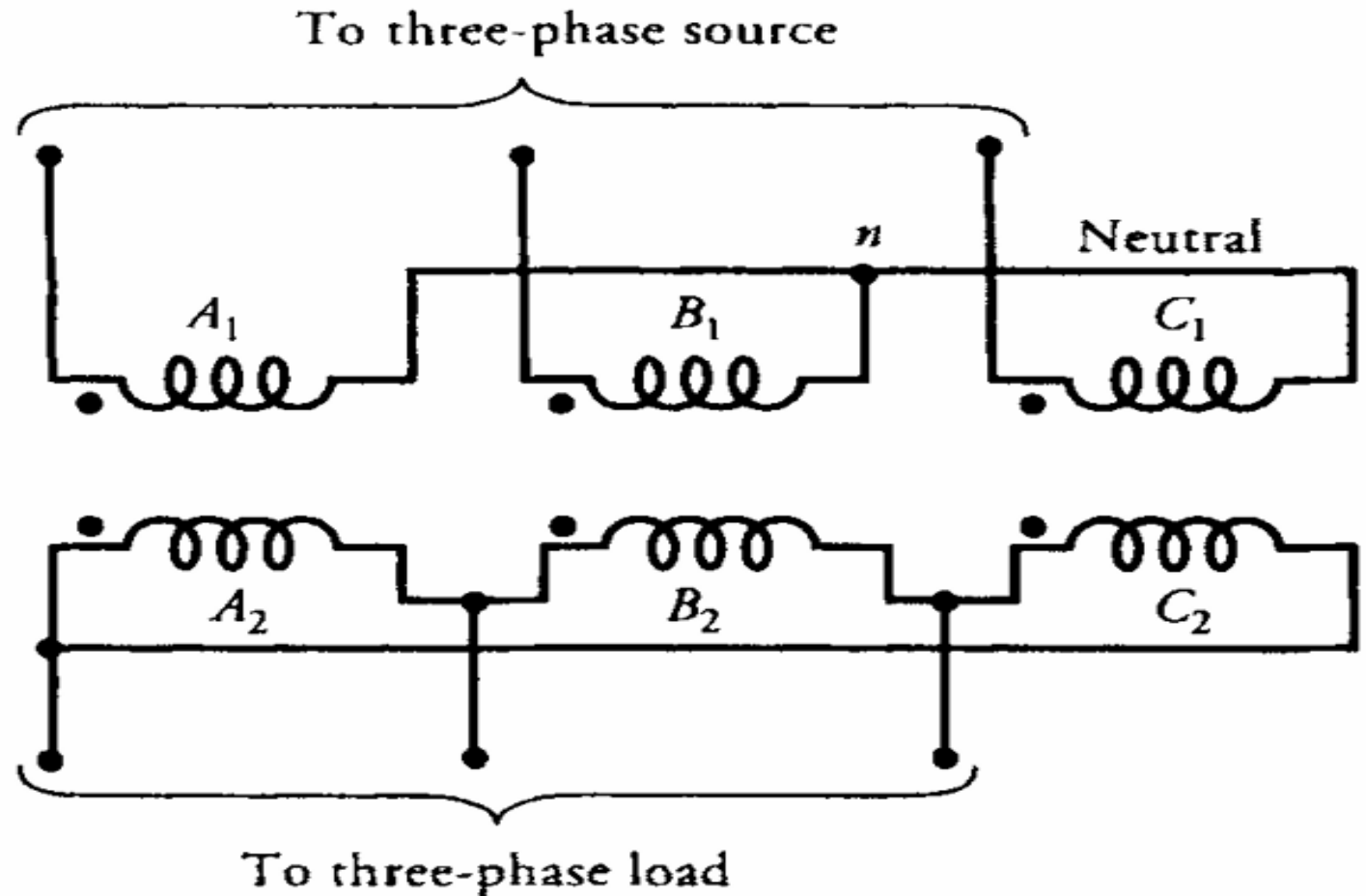


Three-phase transformer with primary windings in star (Y) and secondary windings in delta (Δ)

Y- Δ Connection

Primary windings are connected in star (Y), while secondary windings are connected in delta (Δ).

Three-Phase Transformer Connection (Y- Δ)

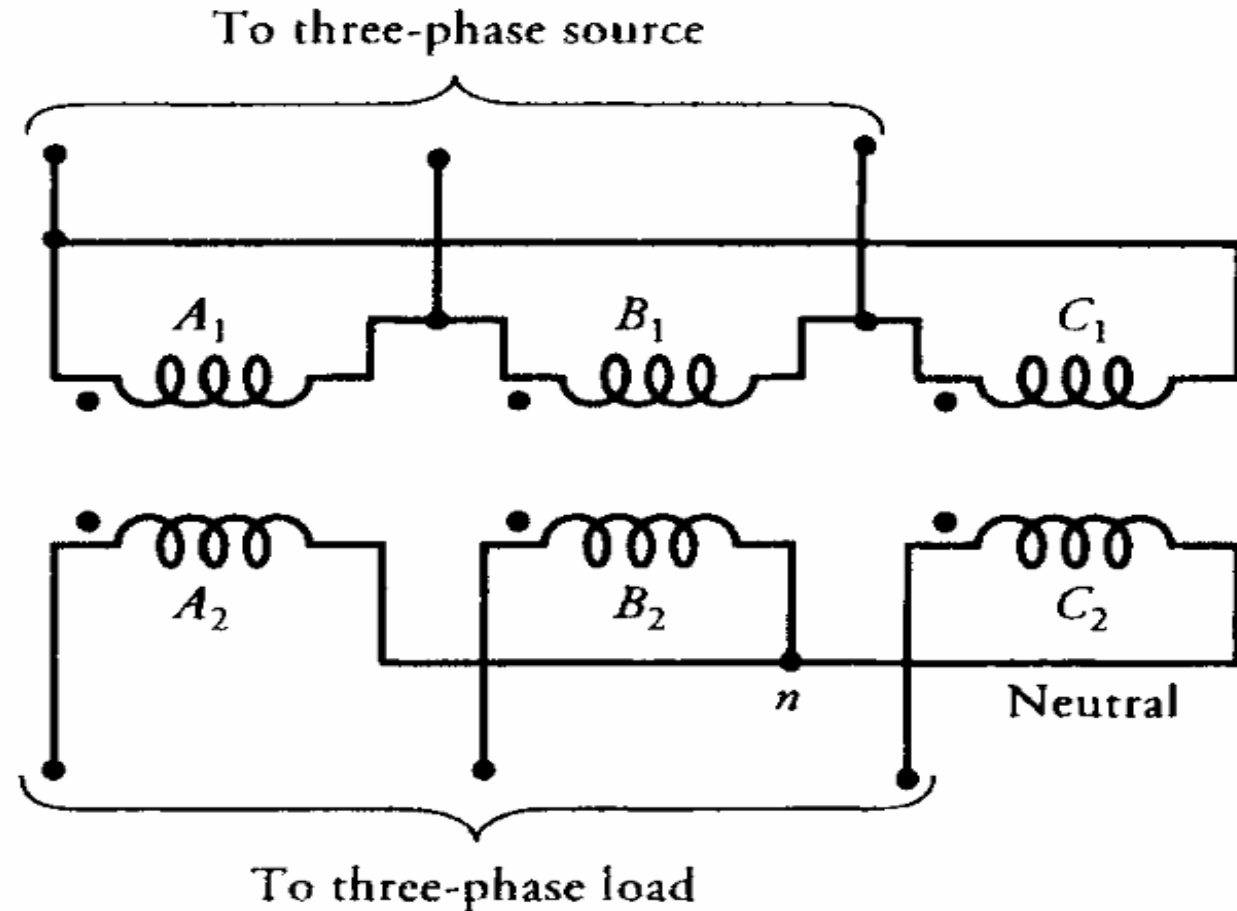


Three-phase transformer with primary windings in delta (Δ) and secondary windings in star (Y)

Δ -Y Connection

Primary windings are connected in delta (Δ), while secondary windings are connected in star (Y).

Three-Phase Transformer Connection (Δ -Y)



Under steady state, a 3-phase transformer behaves like three 1-phase transformers.

Assume identical phases, balanced load, and sinusoidal waveforms.

Analyze one phase, then infer the other phases by symmetry.

Use the per-phase equivalent circuit for calculations.

Delta-connected windings can be converted to equivalent wye windings.

Useful relations

$$Z_Y = \frac{Z_{\Delta}}{3}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = I_L \quad (Y \text{ Connection})$$

$$V_{ph} = V_L \quad (\Delta \text{ Connection})$$

Phase shift note

There may be a $\pm 30^\circ$ phase shift depending on the Y/ Δ connection.

Example 13

Three single-phase units connected as a three-phase transformer

A three-phase transformer is assembled by connecting three 720-VA, 360/120-V, single-phase transformers. The constants for each transformer are $R_H = 18.9 \Omega$, $X_H = 21.6 \Omega$, $R_L = 2.1 \Omega$, $X_L = 2.4 \Omega$, $R_{cH} = 86.4 \text{ k}\Omega$, and $X_{mH} = 6.84 \text{ k}\Omega$. For each of the four configurations, determine the nominal voltage and power ratings of the three-phase transformer. Draw the winding arrangements and the per-phase equivalent circuit for each configuration.

Total Rating

$$S_{3\phi} = 3 \times 720 = 2160 \text{ VA}$$

$$S_{3\phi} = 2.16 \text{ kVA}$$

Given per single-phase transformer

$$720 \text{ VA, } 360/120 \text{ V}$$

$$R_H = 18.9 \Omega, \quad X_H = 21.6 \Omega$$

$$R_L = 2.1 \Omega, \quad X_L = 2.4 \Omega$$

$$R_{cH} = 86.4 \text{ k}\Omega, \quad X_{mH} = 6.84 \text{ k}\Omega$$

Configurations: Y-Y, Δ - Δ , Y- Δ , Δ -Y

Example 13 — Solution

Three single-phase units connected as a three-phase transformer

For Y-Y connection

$$V_{1L} = \sqrt{3} \times 360 = 623.54 \text{ V}$$

$$V_{2L} = \sqrt{3} \times 120 = 207.85 \text{ V}$$

The nominal ratings are

2.16 kVA, 624/208 V, Y-Y connection

For Δ-Δ connection

$$V_{1L} = 360 \text{ V}$$

$$V_{2L} = 120 \text{ V}$$

The nominal ratings are

2.16 kVA, 360/120 V, Δ-Δ connection

For Y-Δ connection

$$V_{1L} = 360 \times \sqrt{3} = 624 \text{ V}$$

$$V_{2L} = 120 \text{ V}$$

The nominal ratings are

2.16 kVA, 624/120 V, Y-Δ connection

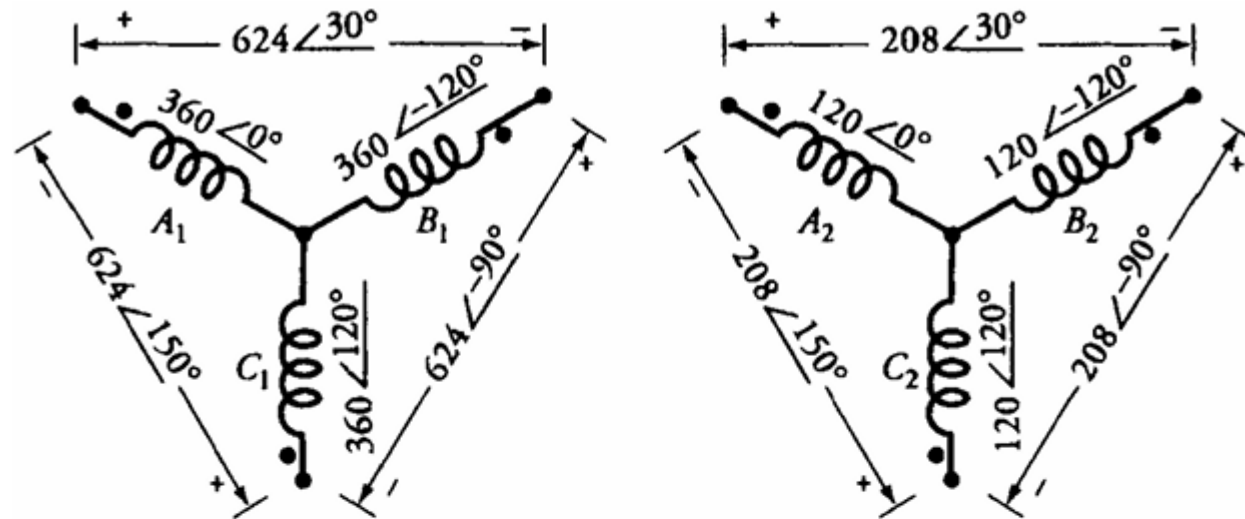
For Δ-Y connection

$$V_{1L} = 360 \text{ V}$$

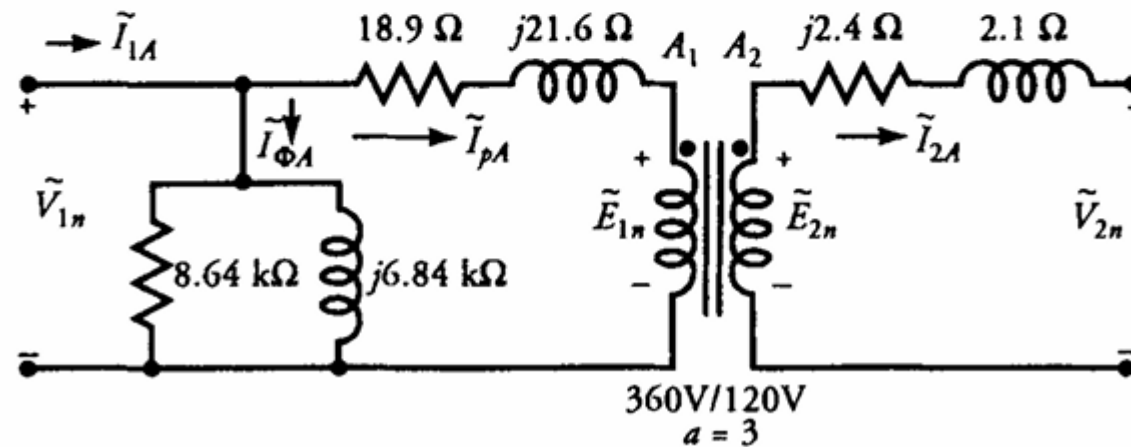
$$V_{2L} = 120 \times \sqrt{3} = 208 \text{ V}$$

The nominal ratings are

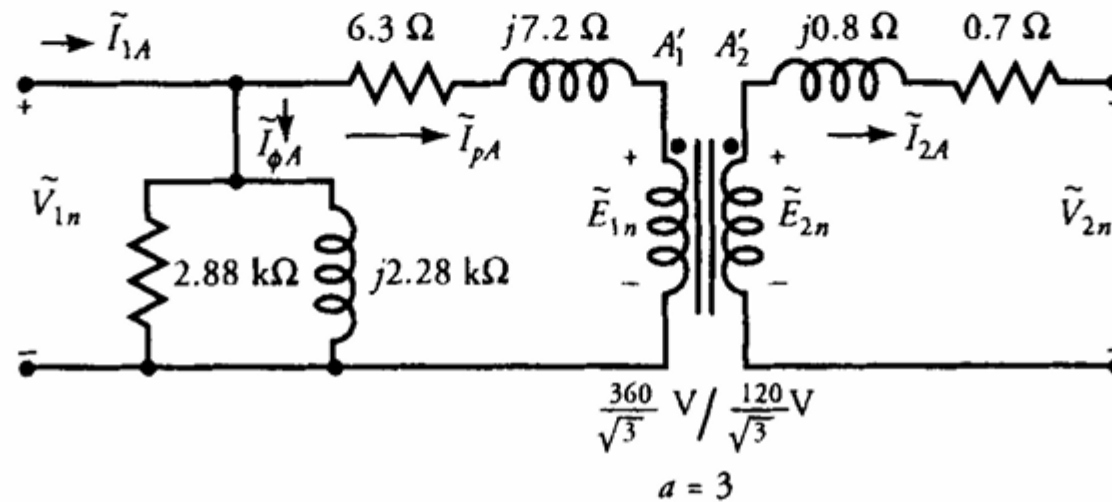
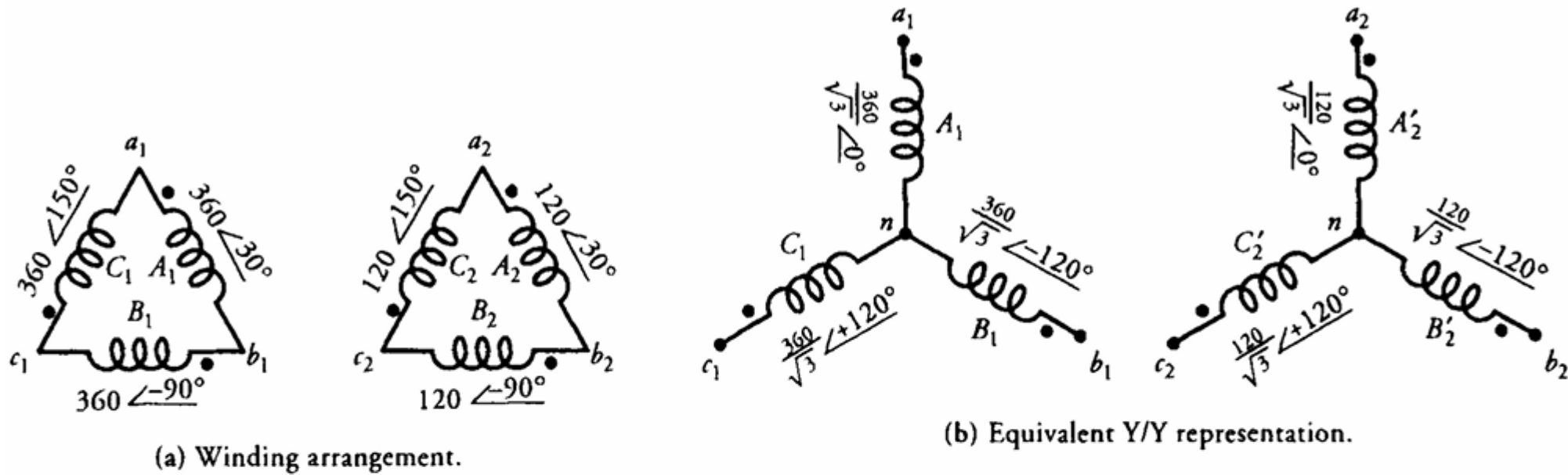
2.16 kVA, 360/208 V, Δ-Y connection

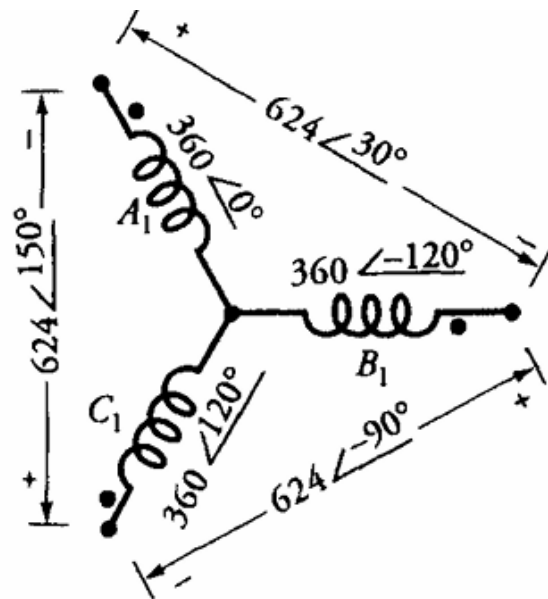


(a) Winding arrangement, phase and line voltages.

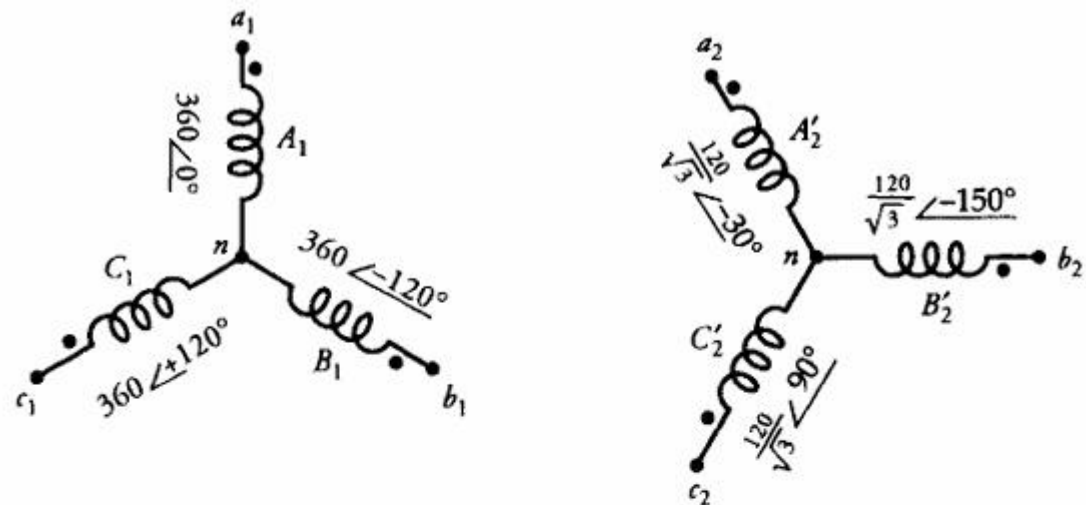
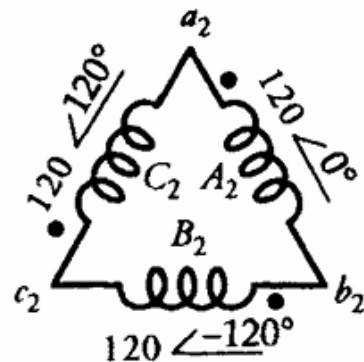


(b) Per-phase equivalent circuit.

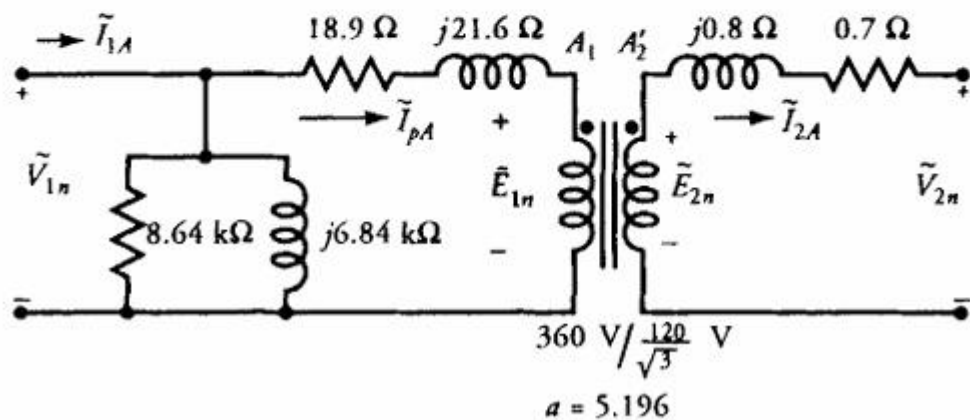




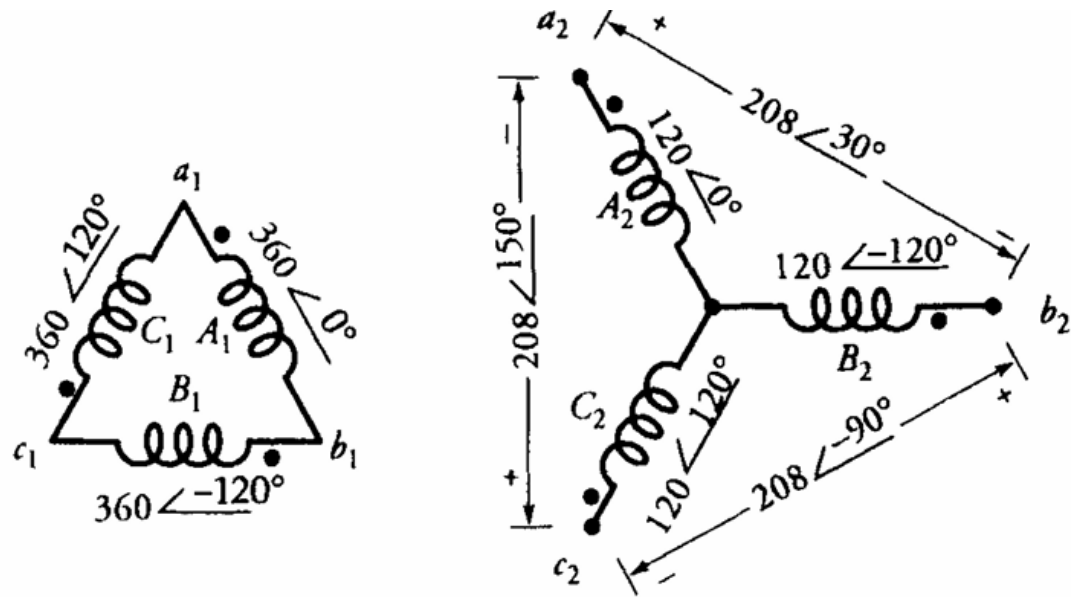
(a) Winding arrangement.



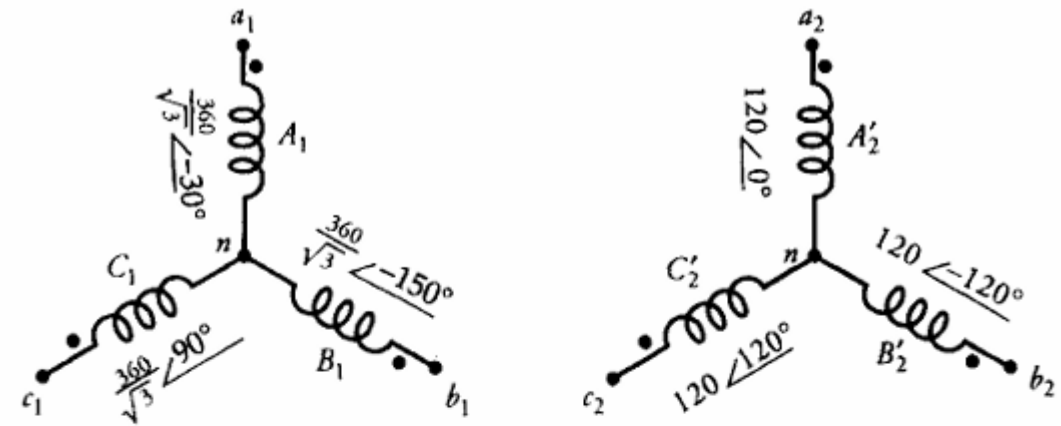
(b) Equivalent Y/Y representation



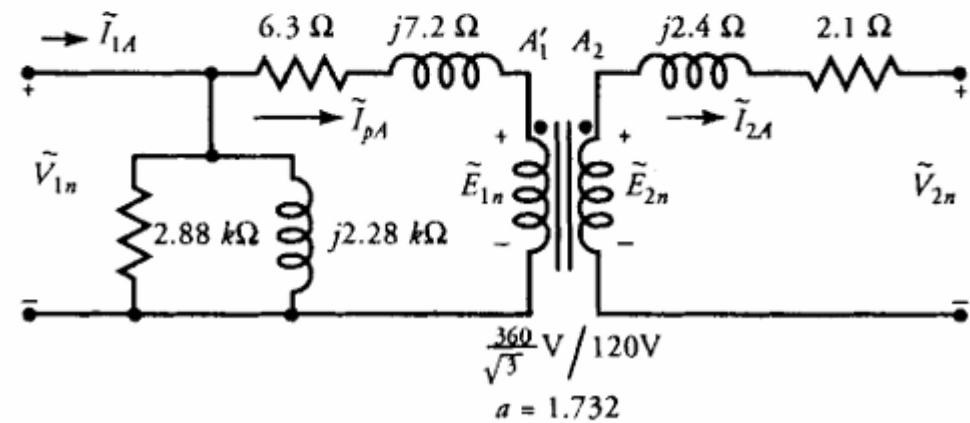
(c) Per-phase equivalent circuit



(a) Winding arrangement.



(b) Equivalent Y/Y representation.



(c) Per-phase equivalent circuit.

Electrical Machines

Chapter 2 TRANSFORMERS

Instructor

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MSc – Electrical Engineering

Lecture 20

Example 14Three single-phase transformers forming a Y- Δ step-up transformer

Three single-phase transformers, each rated at 12 kVA, 120/240 V, 60 Hz, are connected to form a three-phase, step-up, Y/ Δ connection. The parameters of each transformer are $R_H = 133.5 \text{ m}\Omega$, $X_H = 201 \text{ m}\Omega$, $R_L = 39.5 \text{ m}\Omega$, $X_L = 61.5 \text{ m}\Omega$, $R_{cL} = 240 \text{ }\Omega$, and $X_{mL} = 290 \text{ }\Omega$. What are the nominal voltage, current, and power ratings of the three-phase transformer. When it delivers the rated load at the rated voltage and 0.8 pf lagging, determine the line voltages, the line currents, and the efficiency of the transformer.

Example 14 — Solution

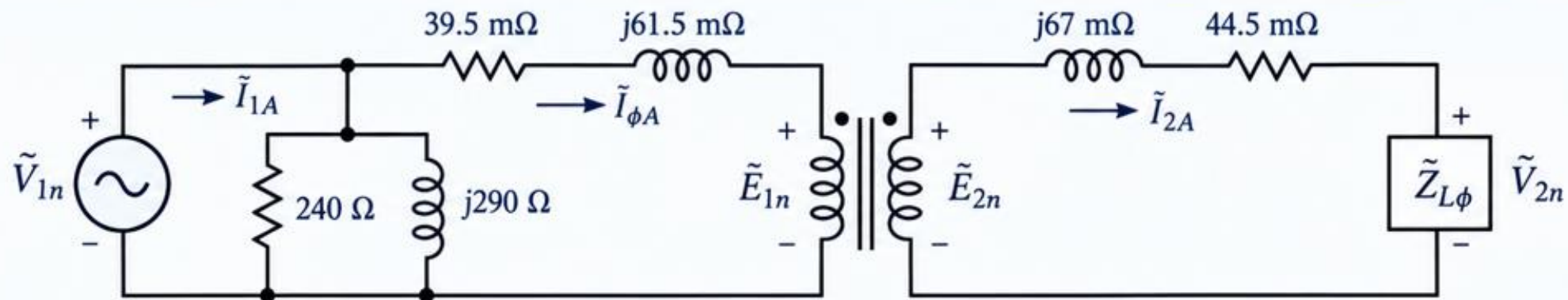
For each single-phase transformer with a power rating of 12 kVA, the primary (120-V side) and the secondary (240-V side) currents are 100 A and 50 A, respectively. Nominal power rating when the three transformers are connected to form a three-phase transformer is 36 kVA. The other nominal ratings of the three-phase transformer are tabulated below.

For the actual Y/ Δ connection

	Primary	Secondary
Phase voltage	120 V	240 V
Line voltage	208 V	240 V
Phase current	100 A	50 A
Line current	100 A	86.6 A

For the equivalent Y/Y connection

	Primary	Secondary
Phase voltage	120 V	138.564 V
Line voltage	208 V	240 V
Phase current	100 A	86.6 A
Line current	100 A	86.6 A



Per-phase equivalent circuit for the Y/Y connection

1 Load Voltage, Load Current, Turns Ratio, and Primary Current

- Assuming the rated load voltage on a per-phase basis for the equivalent Y/Y connection as the reference, then

$$\tilde{V}_{2n} = 138.564 \angle 0^\circ \text{ V}$$
- For a 0.8 lagging power factor, the load current is

$$\tilde{I}_{2A} = 86.6 \angle -36.87^\circ \text{ A}$$
- The a -ratio of the equivalent Y/Y transformer is

$$a = \frac{120}{138.564} = 0.866$$
- Thus, from Eq. (4.45), the per-phase current in the primary winding is

$$\tilde{I}_{\phi A} = \frac{86.6 \angle -36.87^\circ}{0.866} \angle 30^\circ = 100 \angle -6.87^\circ \text{ A}$$

2 Per-Phase Voltage Induced in the Equivalent Y-Connected Secondary Winding

- The per-phase voltage induced in the equivalent Y-connected secondary winding is

$$\begin{aligned} \tilde{E}_{2n} &= \tilde{V}_{2n} + \tilde{I}_{2A} (0.0445 + j0.067) \\ &= 138.564 + (86.6 \angle -36.87^\circ)(0.0445 + j0.067) \\ &= 145.147 \angle 0.92^\circ \text{ V} \end{aligned}$$

3 Induced Emfs in the Actual Δ -Connected Secondary and Y-Connected Primary Windings

- The induced emf in the actual Δ -connected secondary winding is

$$\tilde{E}_{2L} = \sqrt{3} \tilde{E}_{2n} \angle 30^\circ = 251.4 \angle 30.92^\circ \text{ V}$$

- The induced emf in the Y-connected primary winding, from Eq. (4.44), is

$$\tilde{E}_{1n} = a \tilde{E}_{2n} \angle 30^\circ = 125.7 \angle 30.92^\circ \text{ V}$$

4 Primary-side per-phase current and voltage

We can now compute the per-phase current and voltage on the primary side of the Y-connected transformer as

$$\begin{aligned}\tilde{I}_{1A} &= \tilde{I}_{pA} + \tilde{E}_{1n} \left[\frac{1}{240} + \frac{1}{j290} \right] \\ &= 100 \angle -6.87^\circ + (125.7 \angle 30.92^\circ) \left[\frac{1}{240} + \frac{1}{j290} \right]\end{aligned}$$

$$\tilde{I}_{1A} = 100.68 \angle -6.88^\circ \text{ A}$$

$$\begin{aligned}\tilde{V}_{1n} &= \tilde{E}_{1n} + (0.0395 + j0.0615) \tilde{I}_{1A} \\ &= 125.7 \angle 30.92^\circ + (0.0395 + j0.0615)(100.68 \angle -6.88^\circ)\end{aligned}$$

$$\tilde{V}_{1n} = 132.66 \angle 31.98^\circ \text{ V}$$

5 Line voltage on the primary side

Hence, the line voltage on the primary side is

$$\begin{aligned}\tilde{V}_{1L} &= \sqrt{3} \tilde{V}_{1n} \angle 30^\circ \\ &= 229.77 \angle 61.98^\circ \text{ V}\end{aligned}$$

6 Total power and efficiency

Total power supplied by the three-phase transformer is $P_o = 3 \operatorname{Re} [138.564 \times 86.6 \angle 36.87^\circ]$
 $\approx 28,800 \text{ W}$ or **28.8 kW**

Total power supplied by the three-phase source to the transformer is $P_{in} = 3 \operatorname{Re} [(132.66 \angle 31.98^\circ)(100.68 \angle -6.88^\circ)]$
 $\approx 31,200 \text{ W}$ or **31.2 kW**

Hence, the efficiency of the three-phase transformer is $\eta = \frac{28.8}{31.2} = 0.923$ or **92.3%**

Core concepts

Ideal transformer: V ratio follows turns ratio.

Nonideal transformer: winding resistance, leakage reactance, finite permeability, and core loss.

Exact and approximate equivalent circuits simplify analysis.

OC and SC tests determine equivalent-circuit parameters.

Voltage regulation measures terminal-voltage change under load.

Applications covered

Autotransformer connections and rating gain.

Three-phase transformer connection choices: $Y-Y$, $\Delta-\Delta$, $Y-\Delta$, and $\Delta-Y$.

Per-phase equivalent circuit approach for balanced three-phase transformer systems.

Thank you