

Design of Reinforced Concrete Structures II

(ECIV 3316)

Chapter 6

Torsion Design of RC Beams

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6.1 Introduction

- ✓ If external loads act far away from the vertical plane of bending, the beam will be subjected to twisting about its longitudinal axis, known as torsion, in addition to the shearing force and bending moment.
- ✓ Torsion on structural elements may be classified into two types; statically determinate, and statically indeterminate.
- ✓ In Figure 6.1 several examples of beams subjected to torsion are shown

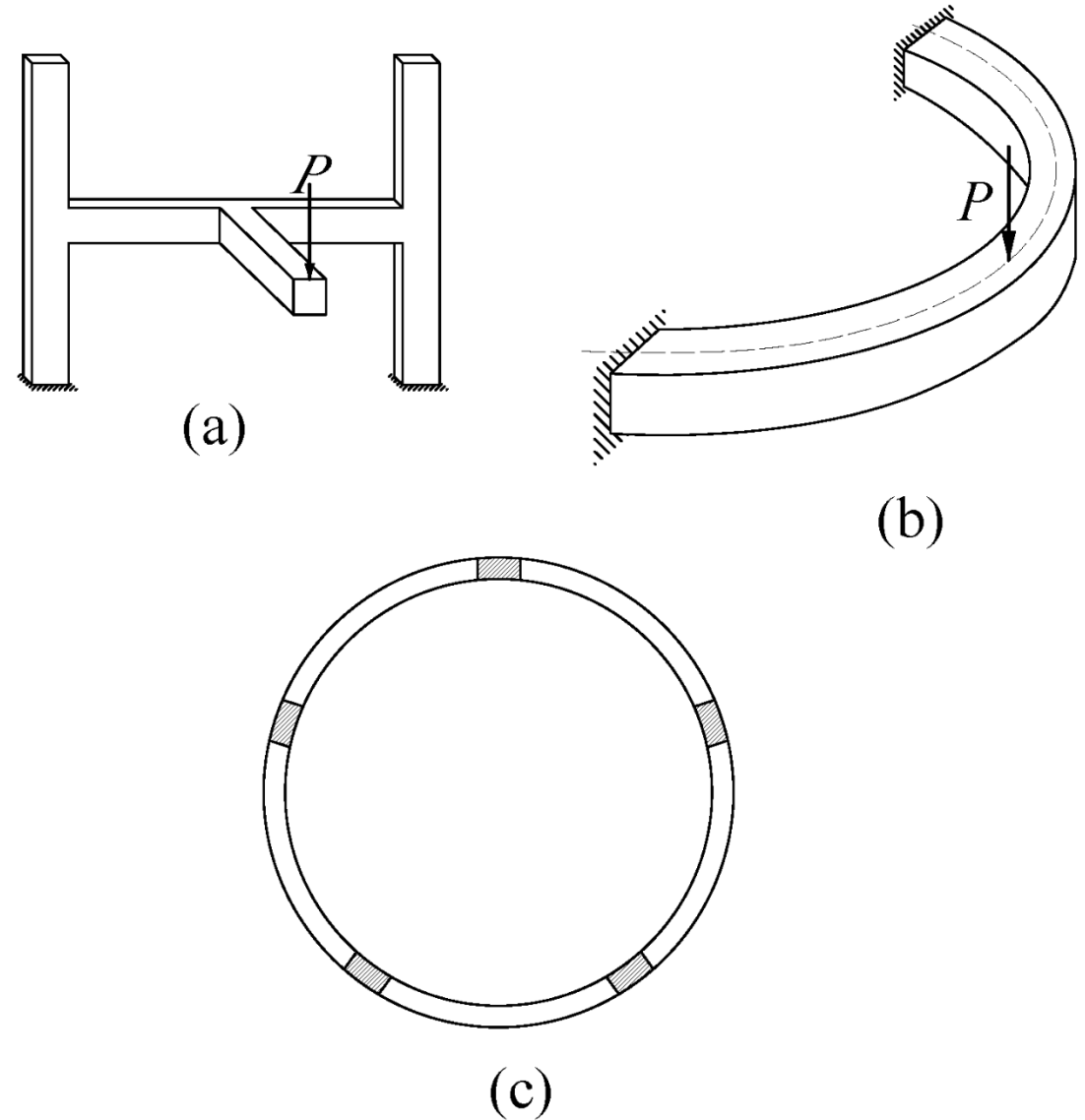


Figure 6.1: Reinforced concrete members subjected to torsion:

6.1 Introduction

- ✓ Shear stresses due to torsion create diagonal tension stresses that produce diagonal cracking. If the member is not adequately reinforced for torsion, a sudden brittle failure can occur.
- ✓ Since shear and moment usually develop simultaneously with torsion, a reasonable design should logically account for the interaction of these forces.
- ✓ However, variable cracking, the inelastic behavior of concrete, and the complex state of stress created by the interaction of shear, moment, and torsion make an exact analysis unfeasible.
- ✓ The current torsion design approach assumes no interaction between flexure, shear and torsion. Reinforcement for each of these forces is designed separately and then combined.

6.2 Shear Stresses Due to Torsion

- ✓ In a rectangular solid section, assuming elastic behavior, the shearing stresses vary in magnitude from zero at the centroid to a maximum at midpoints of the long sides as shown in Figure 6.2.

- ✓ The maximum shear stress τ_{max} is given as

$$\tau_{max} = T/(\alpha x^2 y)$$

where x is the shorter side of the section, y is the longer side of the section, and α is a constant in terms of y/x

- ✓ A close approximation is $\alpha = 1/(3 + 1.8 y/x)$
- ✓ Both solid and hollow members are considered as tubes.

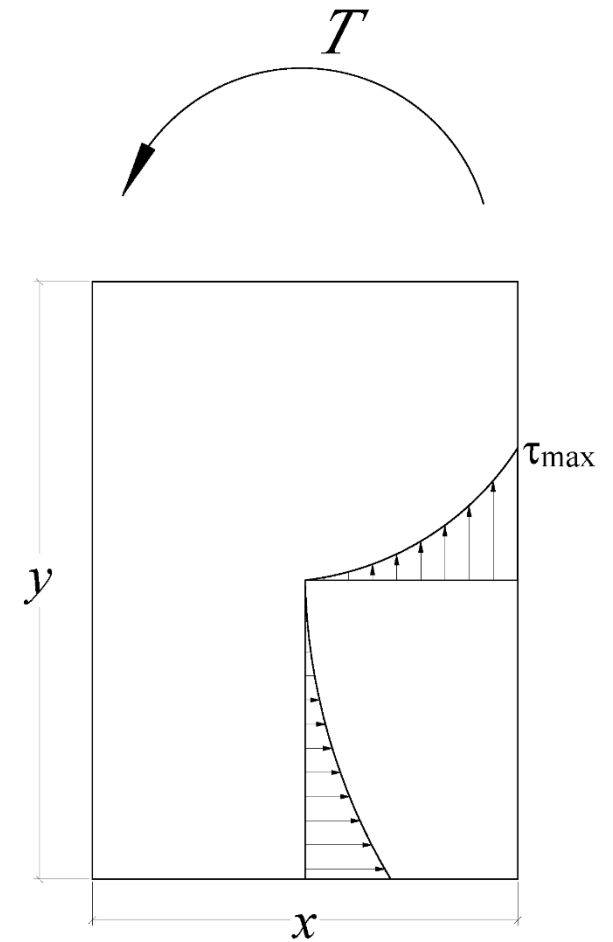


Figure 6.2: Shear stresses in a rectangular section

6.2 Shear Stresses Due to Torsion

- ✓ Experimental test results for solid and hollow beams with the same outside dimensions and identical areas of torsion reinforcement, shown in Figure 6.3, suggest that once torsional cracking has occurred, the concrete in the center of the member has a limited effect on the torsional strength of the cross section and thus can be ignored

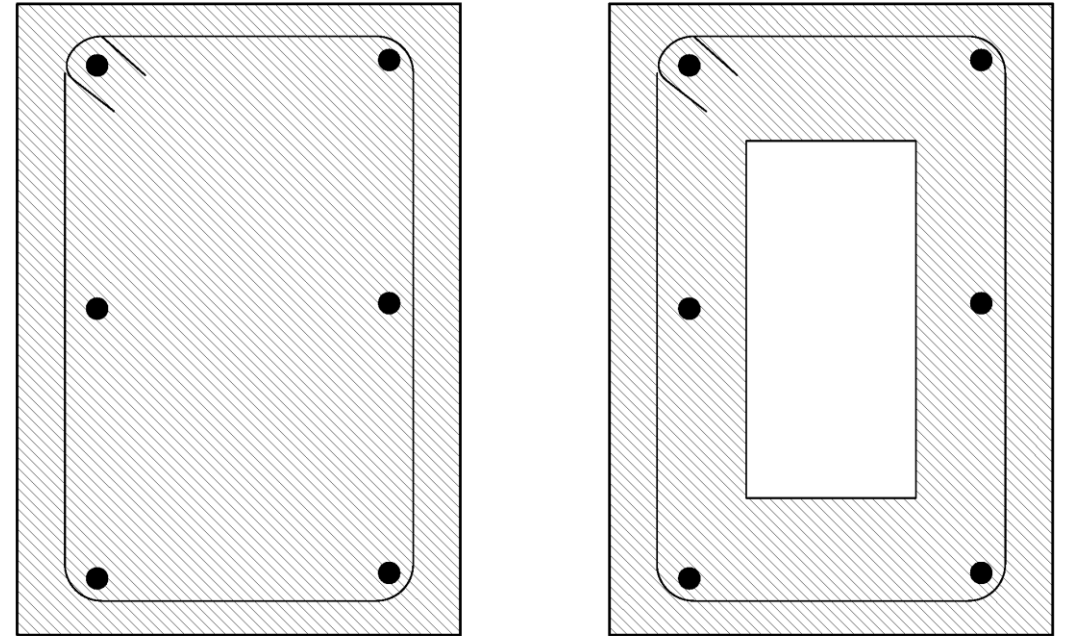


Figure 6.3: Solid and hollow sections of the same size and reinforcement

- ✓ Since 1995, the *ACI Code* has analyzed solid beams as hollow beams for which equations for evaluating shear stresses are easier to develop

6.2 Shear Stresses Due to Torsion

6.2.1 Principal Stresses Due to Pure Tortion

- ✓ When a beam is subjected to pure torsion, shearing stresses develop in the four faces as shown by the elements. When the principal tensile strength exceeds the maximum tensile strength of the beam, cracking will occur spiraling around the outside surface of the beam as shown in Figure 6.4.
- ✓ In a reinforced concrete member, such a crack would cause brittle failure unless torsional reinforcement is provided to limit the growth of this crack. Closed stirrups and longitudinal bars in the corners of the section are usually used as torsional reinforcement.

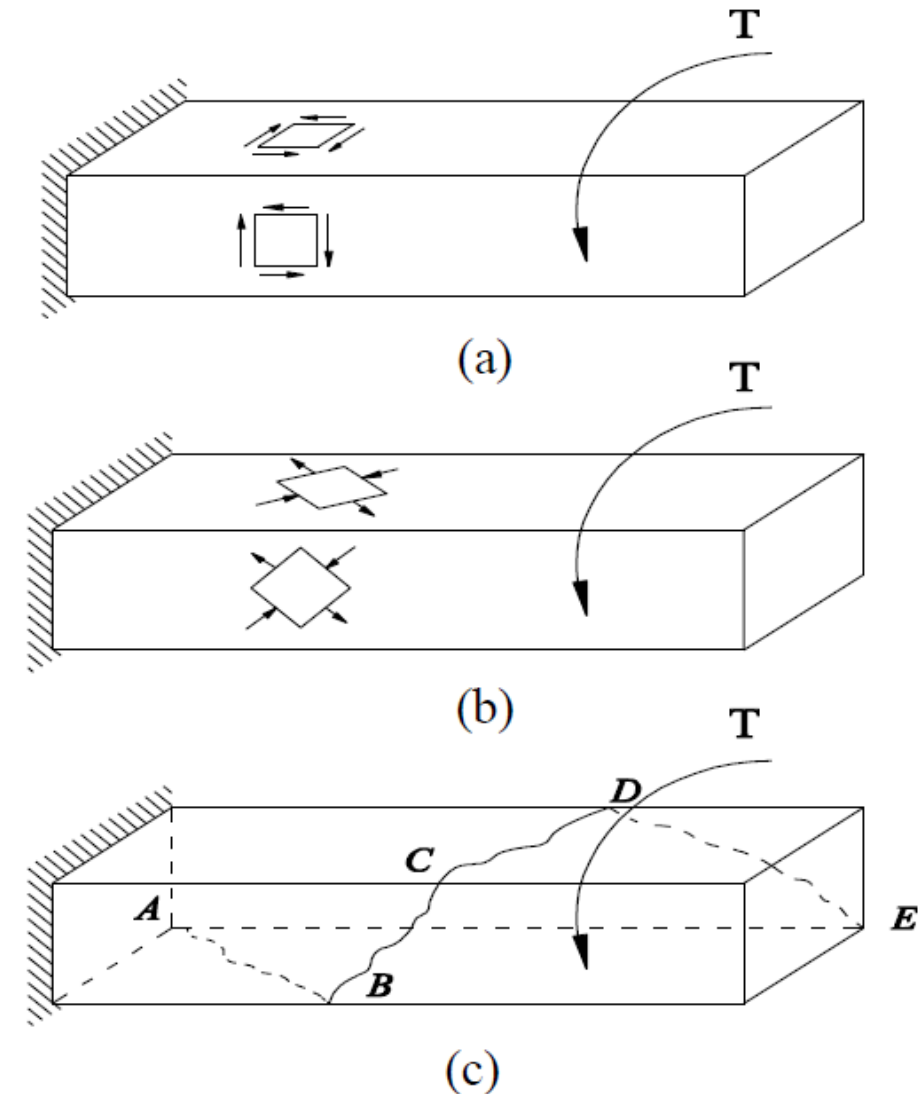


Figure 6.4: Cracking due to pure torsion

6.3 Current ACI Code Design Philosophy

The current design procedure for torsion is based on the following assumptions:

- ✓ Concrete strength in torsion is neglected.
- ✓ Torsion has no effect on shear strength of concrete.
- ✓ No interaction exists between moment, shear, and torsion. Reinforcement for each of the three forces is calculated separately and then combined.
- ✓ Torsion stress determination is based **on thin-walled tube, space truss analogy**. Both **solid and hollow** members are considered **as tubes before and after cracking**, and resistance is assumed to be provided by the outer part of the cross section centered around the stirrups.
- ✓ The basic design equation for torsion is

$$T_u \leq \phi T_n$$

- ✓ Where T_u is the factored torque, T_n is the nominal torsional capacity, and ϕ is the strength reduction factor for torsion, taken as 0.75.

6.4 Limit on Consideration of Torsion

- ✓ In pure torsion, the principal tensile stress σ_1 , shown in Figure 6.6 is equal to the shear stress τ at a given location. $\sigma_1 = \tau = T/(2 A_o t)$

Where:

t is the wall thickness at a point where the shear stress τ is being computed

A_o is the area enclosed by the centerlines of the wall thicknesses.

- ✓ To apply this equation **to solid uncracked sections**, the actual section is **replaced** by an equivalent thin-walled tube with a wall thickness t prior to cracking of $3 A_{cp}/4 p_{cp}$, and an area enclosed by the wall centerline A_o equals $2 A_{cp}/3$

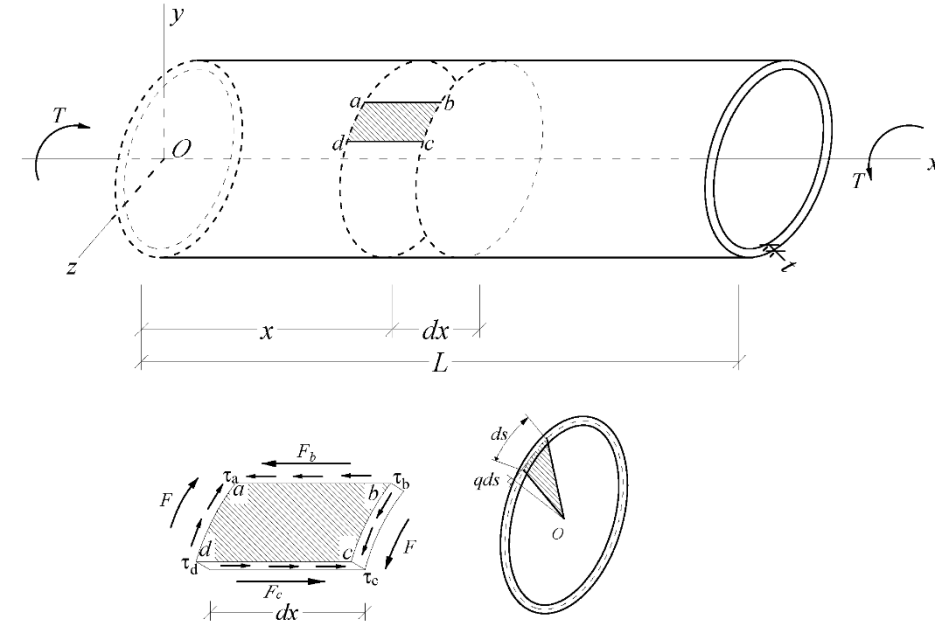


Figure 6.5: Shear stresses in a thin-walled tube

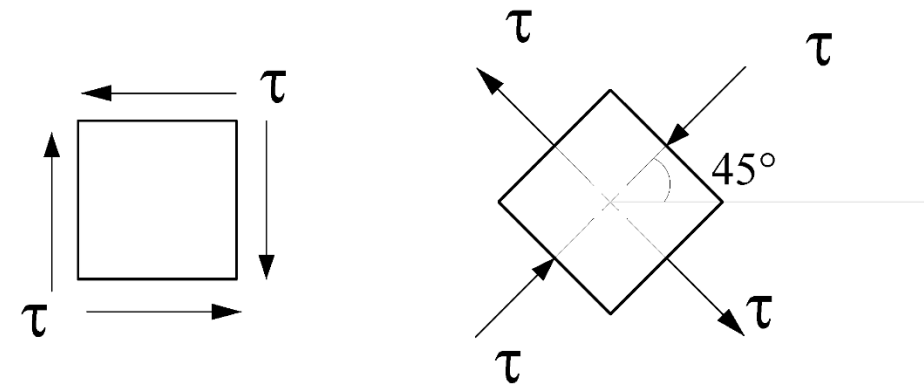


Figure 6.6: Principal stresses due to pure torsion

6.4 Limit on Consideration of Torsion

- ✓ where p_{cp} is the perimeter of the concrete section and A_{cp} is the area enclosed by this perimeter

$$\sigma_1 = \tau = \frac{T}{2 \left(\frac{2 A_{cp}}{3} \right) \left(\frac{3 A_{cp}}{4 p_{cp}} \right)} \text{ and So } \sigma_1 = \tau = T \frac{p_{cp}}{(A_{cp})^2}$$

- ✓ **Torsional cracking** is assumed to occur when the principal tensile stress σ_1 reaches the tensile strength of concrete in biaxial tension-compression, **taken as $0.33 \sqrt{f'_c}$**
- ✓ Substituting this in previous *Eqn.* gives the cracking torque T_{cr} for solid and hollow cross sections as, $T_{cr} = 0.33 \sqrt{f'_c} (A_{cp}^2 / p_{cp})$
- ✓ The *ACI* requires that torsion be considered in design if **T_u exceeds T_{th}**

For solid sections: $T_u \geq \phi T_{th}$ where $T_u \geq \phi 0.083 \lambda \sqrt{f'_c} (A_{cp}^2 / p_{cp})$

For hollow sections: $T_u \geq \phi T_{th}$ where $T_u \geq \phi 0.083 \lambda \sqrt{f'_c} (A_g^2 / p_{cp})$

- ✓ $T_u < \phi T_{th}$ will not cause a structurally significant reduction in either flexural or shear strength **and can be ignored.**

6.4 Limit on Consideration of Torsion

- ✓ For an isolated member with or without flanges, A_{cp} is the area of the entire cross section including the area of voids in hollow cross sections, and p_{cp} is the perimeter of the entire cross section as shown Figure 6.7

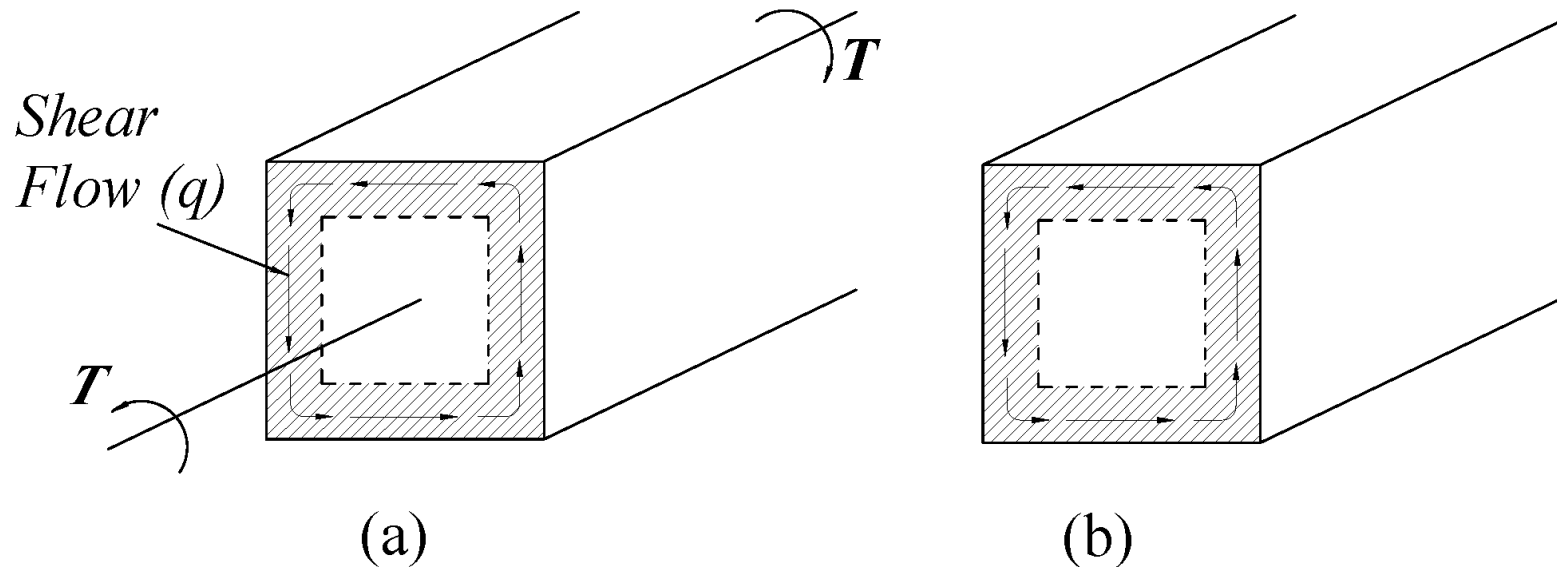


Figure 6.7: Definition of A_{cp} : (a) thin-walled tube; (b) area enclosed by shear flow path

6.4 Limit on Consideration of Torsion

- ✓ For a T-beam cast monolithically with a slab, A_{cp} and p_{cp} can include portions of the adjacent slab conforming to the following:
 - For monolithic construction, a beam includes that portion of slab on each side of the beam extending a distance equal to the projection of the beam above or below the slab, whichever is greater, but not greater than four times the slab thickness.

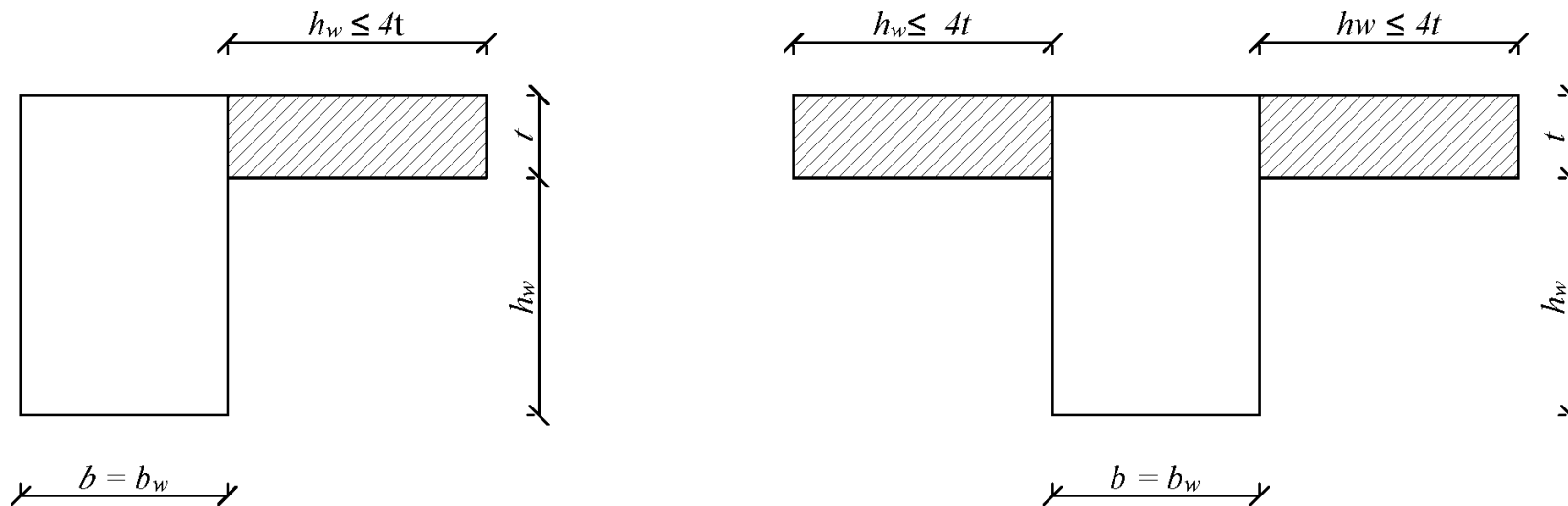


Figure 6.8: L and T beams in monolithic construction

6.5 Ensuring Ductile Mode of Failure

For solid sections, ACI Code requires that the following equation be satisfied

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} \leq \phi \left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right) \quad (6.18)$$

For hollow sections, *ACI Code 22.7.7.1* requires that

$$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right) \leq \phi \left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right) \quad (6.19)$$

T_u = factored torsional moment at section

V_u = factored shear force at section

b_w = web width

d = effective depth

p_h = perimeter of centerline of outermost closed transverse torsional reinforcement

A_{oh} = area enclosed by centerline of outermost closed transverse torsional reinforcement

6.5 Ensuring Ductile Mode of Failure

- ✓ *ACI* requires that if the wall thickness varies around the perimeter of a hollow section, last *Eqn.* be evaluated at the location where the left-hand side of this equation is a maximum.
- ✓ Furthermore, if the wall thickness is less than A_{oh}/p_h , *ACI* requires that the second term in *Eqn.* (6.19) be taken as $\left(\frac{T_u}{1.70 A_{oh} t}\right)$, where t is the thickness of the wall of the hollow section at the location where the stresses are being checked.

6.6 Critical Section for Torsion

- ✓ According to *ACI Code 9.4.4.3*, sections located less than a distance d from the face of a support are designed for not less than the factored torque computed at a distance d .
- ✓ If a concentrated torque occurs within this distance, the critical section for design must be taken at the face of the support.

6.7 Torsional Reinforcement

- ✓ Both longitudinal and transverse reinforcement are required to resist diagonal tension stresses due to torsion.
- ✓ The stirrups must be closed, since torsion cracks can spiral around the beam.

6.7.1 Transverse Reinforcement

A beam subjected to pure torsion can be modeled as a hollow-tube space truss consisting of closed stirrups, longitudinal bars in the corners, and diagonal concrete compression members which spiral around the beam between cracks. The height and width of the truss are y_o and x_o , measured between the centers of the corner bars. The angle of the crack is θ , generally taken as 45° for reinforced concrete.

The shear flow q , is given by

$$q = T / (2 A_o) = \tau t$$

6.7 Torsional Reinforcement

$$V_2 = V_4 = q y_o$$

Substituting *Eqn. (6.11)* into *Eqn. (6.20)* gives

$$V_2 = V_4 = T y_o / (2 A_o) \quad (6.21)$$

Similarly

$$V_1 = V_3 = T x_o / (2 A_o) \quad (6.22)$$

he top crack in Figure 6.9(b) intersects a number of stirrups n , where

$$n = \frac{y_o \cot \theta}{S} \quad (6.23)$$

where S is the spacing of the stirrups.

The force in each stirrup at ultimate torque, assuming yielding of all stirrups is equal to $A_t f_{yt}$, where A_t is the area of one leg of a closed stirrup resisting torsion within a distance S , and f_{yt} is the yield stress of the transverse reinforcement.

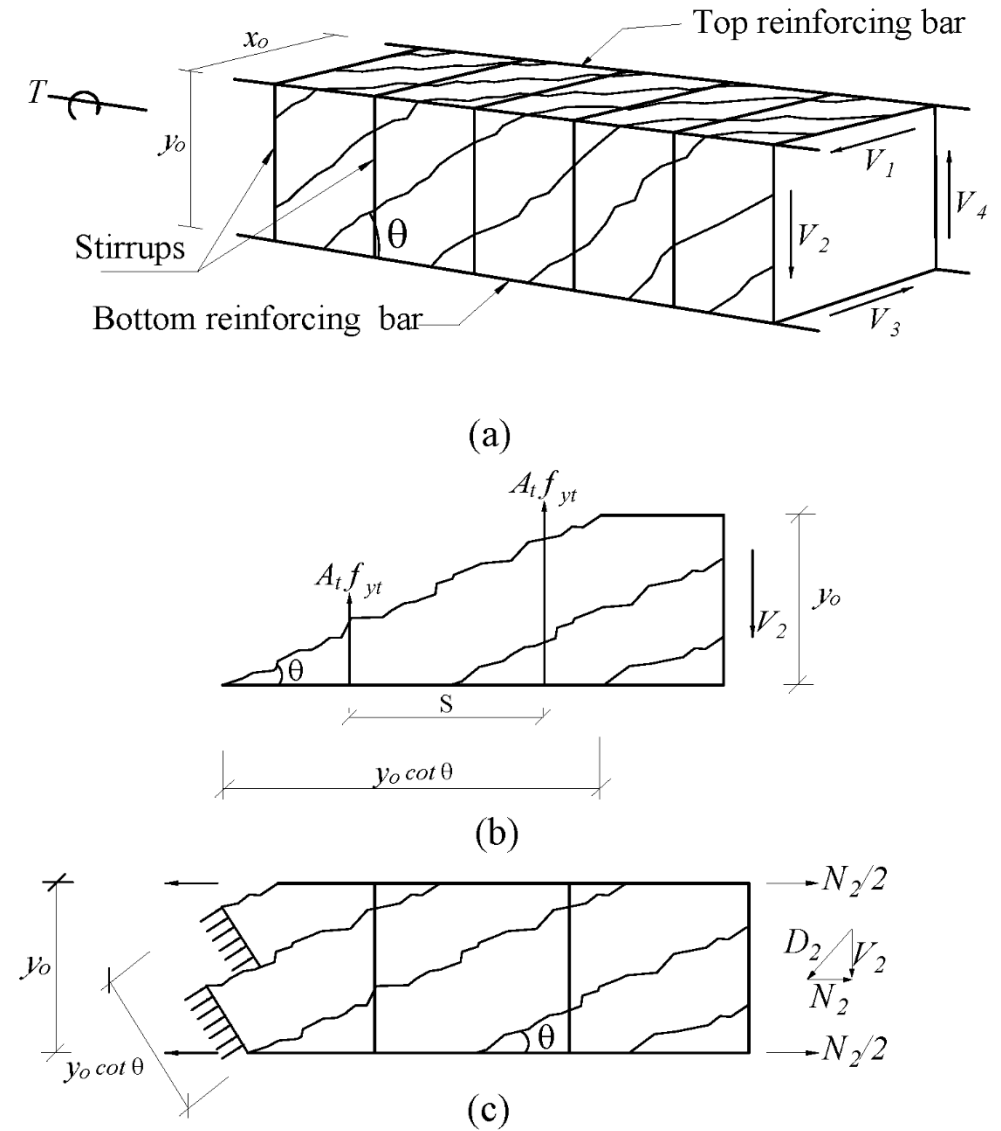


Figure 6.9 Torsional reinforcement:
(a) Space truss analogy (b) forces in stirrups;
(c) resolution of shear force V_2

6.7 Torsional Reinforcement

From equilibrium of forces in the vertical direction

$$V_2 = n A_t f_{yt} \quad (6.24)$$

Substituting *Eqn. (6.23)* into *Eqn. (6.24)* gives

$$V_2 = \frac{y_o \cot \theta A_t f_{yt}}{S} \quad (6.25)$$

Equating *Eqn. (6.25)* and (6.21) results in

$$\frac{T_n y_o}{2 A_o} = \frac{y_o \cot \theta A_t f_{yt}}{S}$$

where T_n is the nominal torsion capacity, or

$$\frac{A_t}{S} = \frac{T_n}{2 A_o f_{yt} \cot \theta} = \frac{T_u}{2 A_o \phi f_{yt} \cot \theta} \quad (6.26)$$

Where θ may be taken any angle between 30° and 60° . *ACI Code 22.7.6.1* permits θ to be taken as 45° and the area A_o to be taken as $0.85 A_{oh}$, where A_{oh} is the area enclosed by the centerline of outermost closed stirrups.

6.7 Torsional Reinforcement

6.7.2 Longitudinal Reinforcement

The force V_2 in Figure 6.9 (c) can be resolved into a diagonal compression force, D_2 , parallel to the concrete struts and an axial tension force, N_2 , where D_2 and N_2 are given by

$$N_2 = V_2 \cot \theta \quad (6.27)$$

Similarly, on the top and bottom faces

$$N_1 = V_1 \cot \theta \quad (6.28)$$

The total longitudinal force is

$$N = 2 (N_1 + N_2) \quad (6.29)$$

Substituting *Eqn.* (6.27) and (6.28) into *Eqn.* (6.29) gives

$$N = 2 (V_2 \cot \theta + V_1 \cot \theta) \quad (6.30)$$

6.7 Torsional Reinforcement

$$N = \frac{T_n}{2 A_o} [2(x_o + y_o)] \cot \theta \quad (6.31)$$

The force in the corner bars at ultimate strength is given as

$$N = A_l f_y \quad (6.32)$$

where A_l is total area of longitudinal torsion reinforcement.

But $2(x_o + y_o) = p_h \cong$ perimeter of the closed stirrup.

$$A_l = \frac{T_n p_h \cot \theta}{2 A_o f_y} \quad (6.33)$$

Substituting *Eqn.* (6.26) into *Eqn.* (6.33) gives

$$A_l = \left(\frac{A_t}{S} \right) p_h \left(\frac{f_{yt}}{f_y} \right) \cot^2 \theta \quad (6.34)$$

6.7 Torsional Reinforcement

6.7.3 Minimum Amount of Torsion Reinforcement

6.7.3.1 Longitudinal Reinforcement

To ensure that concrete beams will fail in a ductile manner, *ACI Code 9.6.4.3* requires that the minimum area of longitudinal steel is the lesser of

$$A_{l,min} = \frac{0.42\sqrt{f'_c}A_{cp}}{f_y} - \left(\frac{A_t}{S}\right) p_h \frac{f_{yt}}{f_y} \quad (6.35)$$

and,

$$A_{l,min} = \frac{0.42\sqrt{f'_c}A_{cp}}{f_y} - \left(\frac{0.175 b_w}{f_{yt}}\right) p_h \frac{f_{yt}}{f_y} \quad (6.36)$$

6.7 Torsional Reinforcement

6.7.3.2 Transverse Reinforcement

ACI Code specifies that where torsion reinforcement is required, the minimum area of transverse closed stirrups for combined action of shear and torsion is computed by:

$$(A_v + 2A_t) = \frac{0.062\sqrt{f_c'} b_w S}{f_{yt}} \geq \frac{0.35b_w S}{f_{yt}} \quad (6.37)$$

Where, A_v is the area of two legs of a closed stirrup while A_t is the area of one leg of a closed stirrup.

The torsional and shear reinforcement are added up.

$$Total \left(\frac{A_{v+t}}{s} \right) = \frac{A_v}{s} + 2 \frac{A_t}{s}$$

If a stirrup group has more **than two legs** for shear, only the legs **adjacent to the sides** of the beam **are included** in this summation because the inner legs would be ineffective for resisting torsion.

6.7 Torsional Reinforcement

6.7.4 Details of Torsion Reinforcement

6.7.4.1 Transverse Reinforcement

- ✓ When a rectangular beam fails in torsion, the corners of the beam tend to spall off due to the compressive stresses in the concrete diagonals of the space truss.
- ✓ In tests, closed stirrups anchored by 90-degree hooks failed when this occurred. For this reason, 135-degree hooks are preferable for torsional stirrups in all cases. In regions where this spalling is prevented by an adjacent slab or flange, *ACI Code 25.7.1.6.1* relaxes this and allows 90 degree hooks, as shown in Figure 6.10

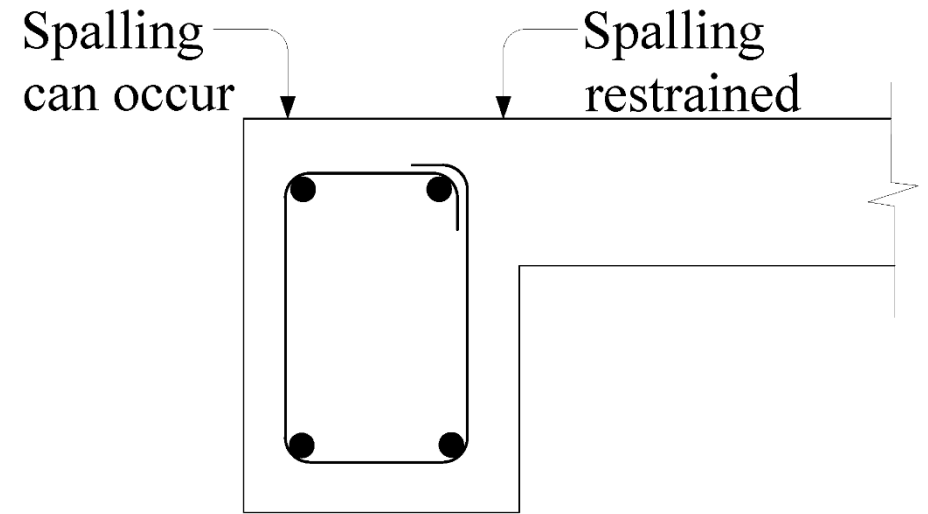


Figure 6.10: Spalling of corners of beams loaded in torsion

6.7 Torsional Reinforcement

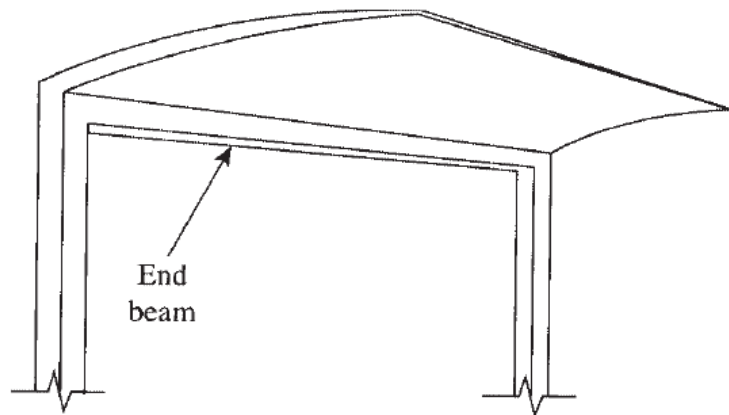
6.7.4.2 Longitudinal Reinforcement

- ✓ *ACI Code* requires that longitudinal torsion reinforcement be developed at both ends.
- ✓ If high torsion acts near the end of a beam, the longitudinal torsion reinforcement should be adequately anchored.
- ✓ Sufficient development length should be provided outside the inner face of the support to develop the needed tension force in the bars. This may require hooks or horizontal U-shaped bars lapped with the longitudinal torsion reinforcement.
- ✓ *ACI Code* require that torsion reinforcement be provided for a distance of at least $(b_t + d)$ beyond the point theoretically required for torsional reinforcement, where b_t is width of that part of cross section containing the closed stirrups, and d is the effective depth of section.

6.8 Equilibrium and Compatibility Torsion

6.8.1 Equilibrium Torsion

- ✓ The torsional moment cannot be reduced by redistribution of internal forces. This is referred to as equilibrium torsion since the torsional moment is required to keep the structure in equilibrium.
- ✓ According to ACI Code, if the factored torsional moment T_u in a member is required to maintain equilibrium and exceeds the minimum given value, the member is to be designed to carry that torsional moment. An example for equilibrium torsion is shown in Figure 6.12(a).



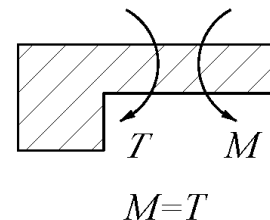
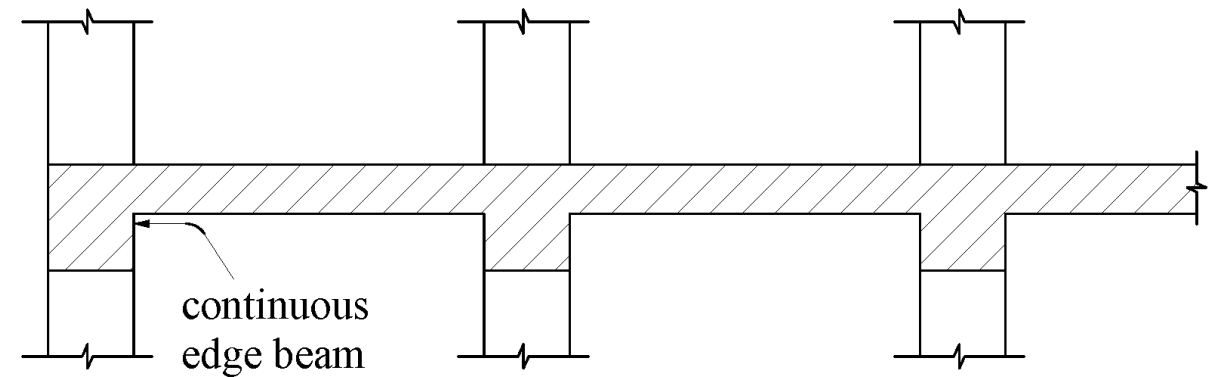
Equilibrium torsion: (a) design torque cannot be reduced

6.8 Equilibrium and Compatibility Torsion

6.8.2 Compatibility Torsion

- ✓ The torsional moment can be reduced by redistribution of internal forces after cracking. This type of torsion is referred to as compatibility torsion, an example of which is shown Figure 6.12(b).
- ✓ For statically indeterminate structures where reduction of the torsional moment in a member can occur due to redistribution of internal forces upon cracking, the maximum factored torsional moment T_u is permitted to be reduced to

$$T_u = \frac{\phi 0.33 \lambda \sqrt{f'_c} (A_{cp}^2)}{p_{cp}}$$



Compatibility Torsion:
(b) design torque may be reduced

6.7 Torsional Reinforcement

6.7.5 Spacing of Torsion Reinforcement

6.7.5.1 Transverse Reinforcement

- ✓ The spacing of transverse torsion reinforcement center-to-center is not to exceed the smaller of $p_h/8$ or 300 mm.
- ✓ The spacing of the stirrup is limited to ensure the development of the ultimate torsional strength of the beam and to control crack widths.

6.7.5.2 Longitudinal Reinforcement

- ✓ *ACI Code* requires that longitudinal torsion reinforcement be distributed around the perimeter of closed stirrups with a maximum spacing of 300 mm.
- ✓ One bar must be positioned in each corner of the stirrups to provide anchorage for the legs of the stirrups.
- ✓ The least longitudinal bar diameter to be used is the larger of 0.042 times the stirrup spacing, or 10 mm,

6.9 Summary of Design Procedure

1. Draw the shear force, bending moment and torque diagrams.
2. Select cross-sectional dimensions b and h based on factored **bending moment**, and determine the required area of reinforcement.
3. Check if torsion may be neglected. Torsion may be neglected if

$$T_u \leq \phi 0.083 \lambda \sqrt{f_c'} \frac{(A_{cp}^2)}{p_{cp}} \quad (6.17)$$

- If this is the case, proceed on with shear design, and choose flexural and shear reinforcement.
- If $T_u > \phi 0.083 \lambda \sqrt{f_c'} \frac{(A_{cp}^2)}{p_{cp}}$, torsion must be considered as shown in the following steps.
- The critical section for torsion is located at distance d from the face of the support if no torques are applied within this distance. If torques are applied within distance d from face of support, critical torsion is located at face of the support.

6.9 Summary of Design Procedure

4. **Check the adequacy of the size of the cross section** in terms of preventing brittle mode of failure

For a solid cross section to be adequate,

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A^2_{oh}}\right)^2} \leq \phi \left(\frac{V_c}{b_w d} + 0.66 \sqrt{f'_c} \right) \quad (6.18)$$

For a hollow cross section to be adequate,

$$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u p_h}{1.7 A^2_{oh}}\right) \leq \phi \left(\frac{V_c}{b_w d} + 0.66 \sqrt{f'_c} \right) \quad (6.19)$$

If *Eqn.* (6.18) or (6.19) is not satisfied, cross sectional dimensions need to be increased.

6.9 Summary of Design Procedure

5. **Determine the area of stirrups required for shear.** To facilitate the addition of stirrups for shear and torsion, the area of shear reinforcement is expressed in terms of A_v/S ,

$$\frac{A_v}{S} = \frac{V_s}{f_{yt} d} \quad (5.15)$$

If $V_s \leq 0.33 \sqrt{f'_c} b_w d$, maximum stirrup spacing S_{max} should not exceed the smaller of $d/2$ or 600 mm . If $0.66 \sqrt{f'_c} b_w d > V_s > 0.33 \sqrt{f'_c} b_w d$, maximum stirrup spacing is limited to the smaller of $d/4$ or 300 mm .

6. **Determine the required area of stirrups for torsion in terms of A_t/S .**

Since $T_u = \phi T_n$, Eqn. (6.26) when θ is taken as 45° can be written as

$$\frac{A_t}{S} = \frac{T_u}{2A_o \phi f_{yt}}$$

Besides, compute maximum stirrup spacing based on torsion. Maximum stirrup spacing is limited to the smaller of $p_h/8$ or 300 mm .

6.9 Summary of Design Procedure

7. **Determine combined area of stirrups** required for shear and torsion. A_t/S is expressed in terms of A_v/S .

For two-legged closed stirrups,

$$\left(\frac{A_{v+t}}{S}\right)_T = \left(\frac{A_v}{S}\right) + \left(\frac{2A_t}{S}\right)$$

where $\left(\frac{A_{v+t}}{S}\right)_T$ should be equal or larger than $\frac{0.062\sqrt{f_c'} b_w S}{f_{yt}} \geq \frac{0.35b_w S}{f_{yt}}$

8. **Select stirrup size**, and compute stirrup spacing based on the amount determined in step 7. Maximum stirrup spacing must not exceed the smaller of the two values evaluated in **steps 5 and 6**

6.9 Summary of Design Procedure

9. Calculate the longitudinal reinforcement required for torsion.

$$A_l = \left(\frac{A_t}{S} \right) p_h \left(\frac{f_{yt}}{f_y} \right) \cot^2 \theta \quad (6.34)$$

If $f_{yt} = f_y$, and θ is taken as 45° , $A_l = \left(\frac{A_t}{S} \right) p_h$

The longitudinal reinforcement A_l is not to be less than the lesser value evaluated from *Eqn.* (5.35) and (3.36), or

$$A_{l,min} = \frac{0.42 \sqrt{f'_c} A_{cp}}{f_y} - \left(\frac{A_t}{S} \right) p_h \frac{f_{yt}}{f_y} \quad (6.35)$$

and

$$A_{l,min} = \frac{0.42 \sqrt{f'_c} A_{cp}}{f_y} - \left(\frac{0.175 b_w}{f_{yt}} \right) p_h \frac{f_{yt}}{f_y} \quad (6.36)$$

6.9 Summary of Design Procedure

10. Size combined longitudinal reinforcement.

- $d_b \geq 0.042$ stirrup spacing or 10 mm, whichever is larger.
- Flexural reinforcement determined in step 2 is to be added to the longitudinal reinforcement required for torsion at the flexural steel level
- If the vertical distance, center-to-center, between the top and bottom reinforcement exceeds 300 *mm*, middle layers are added until the spacing is satisfied.

11. Torsion reinforcement must be extended at least $(b_t + d)$ beyond the point where no torsional reinforcement is required.

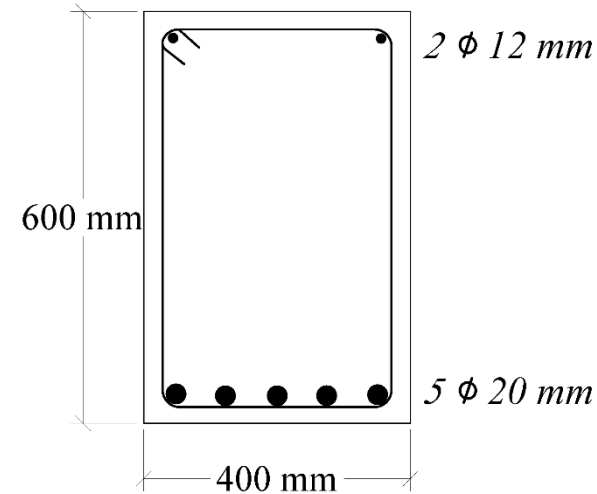
12. Draw neat sketches showing reinforcement details.

Example (6.1):

For the cross section shown in Figure 6.13, design the torsional reinforcement, if required, given that:

$$T_u = 40 \text{ kN.m and } V_u = 160 \text{ kN}$$

$$f'_c = 25 \text{ MPa and } f_y = 420 \text{ MPa}$$



Solution:

1. Draw the shear force, bending moment, and torque diagrams.

The section is already designed for bending moment. The magnitudes of shear force and torsion are given.

2. Select cross sectional dimensions and determine area of flexural reinforcement.

Cross sectional dimensions in addition to flexural reinforcement are given.

Example (6.1):

3. Check if torsion may be neglected.

Torsion may be neglected if T_u is smaller than

$$T_u \leq \phi \cdot 0.083 \cdot \lambda \cdot \sqrt{f_c'} \cdot \frac{(A_{cp}^2)}{p_{cp}}$$

$$A_{cp} = 600 \times 400 = 240 \times 10^3 \text{ mm}^2$$

$$p_{cp} = 2(400 + 600) = 2000 \text{ mm}$$

$$\frac{0.75 \times 0.083 \times \sqrt{25} (240 \times 10^3)^2}{(2000) \times 10^6} = 9.0 \text{ kN.m} < T_u = 40 \text{ kN.m}$$

Thus, torsion reinforcement must be considered.

4. Check for probability of brittle failure of concrete.

The dimensions of the cross section are adequate to prevent a brittle mode of failure if the following equation is satisfied

Example (6.1):

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} \leq \phi \left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right)$$

Assuming that $\phi 8 \text{ mm}$ stirrups are used,

$$d = 600 - 40 - 8 - 10.0 = 542 \text{ mm}$$

$$x_o = 400 - 2 \times 40 - 8.0 = 312 \text{ mm}$$

$$y_o = 600 - 2 \times 40 - 8.0 = 512 \text{ mm}$$

$$p_h = 2 (312 + 512) = 1648 \text{ mm}$$

$$A_{oh} = (312 \times 512) = 159744 \text{ mm}^2$$

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} = \sqrt{\left(\frac{160 \times 10^3}{400 (542)}\right)^2 + \left(\frac{40 \times 10^6 \times 1648}{1.70 (159744)^2}\right)^2} = 1.69 \text{ MPa}$$

$$\phi \left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right) = \left(0.17\sqrt{f_c'} + 0.66\sqrt{f_c'}\right) = 0.83\phi\sqrt{f_c'} = 0.83\phi\sqrt{25} = 3.1 \text{ MPa}$$

i.e. cross sectional dimensions are adequate for resisting brittle failure.

Example (6.1):

5. Determine the required area of stirrup reinforcement for shear.

$$V_c = 0.17 \sqrt{f'_c} b_w d$$

$$V_s = \frac{V_u}{\phi} - V_c$$

$$V_s = \frac{16.0}{0.75} - \frac{0.17 \sqrt{25} (400) (542.0)}{1000} = 29kN$$

$$\frac{A_v}{S} = \frac{V_s}{f_{yt} d} = \frac{29 (1000)}{420 (542)} = 0.127 \text{ mm}^2/\text{mm}$$

Since $V_s \leq 0.66 \sqrt{f'_c} b_w d$, maximum stirrup spacing S_{max} should not exceed the smaller of $d/2 = 542.0/2 = 271.0 \text{ mm}$ or 600 mm , thus S_{max} is not to exceed 270 mm .

Example (6.1):

6. Determine the required area of stirrup reinforcement for torsion.

The torsional stirrup reinforcement per unit length A_t/S , is given by

$$\frac{A_t}{S} = \frac{T_u}{2\phi f_{yt} A_o \cot^2(45^\circ)} = \frac{40 \times 10^6}{2 (0.75) (420) (0.85 \times 159744)} = 0.47 \text{ mm}^2/\text{mm}$$

S_{max} is the smaller of $p_h/8$ or 300 mm , where $p_h/8 = 1648.0/8 = 206.0 \text{ mm}$, thus S_{max} is not to exceed 200 mm .

7. Determine the combined area of stirrup reinforcement for shear and torsion.

$$\left(\frac{A_{v+t}}{S} \right)_{total} = 0.139 + 2 (0.47) = 1.079 \text{ mm}^2/\text{mm}$$

$$\frac{0.35 b_w}{f_{yt}} = \frac{0.35 (400)}{420} = 0.33 \text{ mm}^2/\text{mm} < 1.079 \text{ cm}^2 \text{ O.K}$$

Example (6.1):

8. Select stirrup size and spacing.

Try $\phi 8 \text{ mm}$ closed stirrups

$$\frac{2 (50.0)}{S} = 1.079 \text{ and } S = 92.6 \text{ mm.}$$

Try $\phi 10 \text{ mm}$ closed stirrups

$$\frac{2 (78.5)}{S} = 1.079 \text{ and } S = 145.5 \text{ mm.}$$

Therefore, maximum stirrup spacing based on shear and torsion combined is 200 mm , which is larger than 140 mm . Use two-legged, $\phi 10 \text{ mm}$ closed stirrups spaced at 140 mm .

9. Calculate the longitudinal reinforcement required for torsion.

Since $f_{yt} = f_y$ and for $\theta = 45^\circ$,

$$A_l = \left(\frac{A_t}{S} \right) p_h = (0.47) (1648.0) = 775 \text{ mm}^2$$

Example (6.1):

The longitudinal reinforcement is not to be less than the lesser value obtained from the following equations

$$A_{l,min} = \frac{0.42\sqrt{f'_c}A_{cp}}{f_y} - \left(\frac{A_t}{S}\right) p_h \frac{f_{yt}}{f_y} = \frac{0.42\sqrt{25}(400)(600)}{420} - 775 = 425 \text{ mm}^2$$

and

$$A_{l,min} = \frac{0.42\sqrt{f'_c}A_{cp}}{f_y} - \left(\frac{0.175 b_w}{f_{yt}}\right) p_h \frac{f_{yt}}{f_y} = \frac{0.42\sqrt{25}(400)(600)}{420} - \frac{(0.175)(400)(1648)}{420} = 925.3 \text{ mm}^2$$

10. Size combined longitudinal reinforcement.

Distance between the centerlines of tension and compression reinforcement exceeds 300 *mm*, so a middle layer of longitudinal reinforcement is needed.

$$\frac{A_l}{3} = \frac{775}{3} = 258 \text{ mm}^2$$

Minimum bar diameter is $0.042S = 0.042(140) = 5.88 \text{ mm}$ or 10 *mm*, whichever is larger, and 10 *mm* is the least diameter to be used.

Example (6.1):

Total area of top reinforcement = $226 + 258 = 484 \text{ mm}^2$, use 5 $\phi 12 \text{ mm}$.

Area of bottom reinforcement = $258 + 1571 = 1829 \text{ mm}^2$, use 5 $\phi 22 \text{ mm}$.

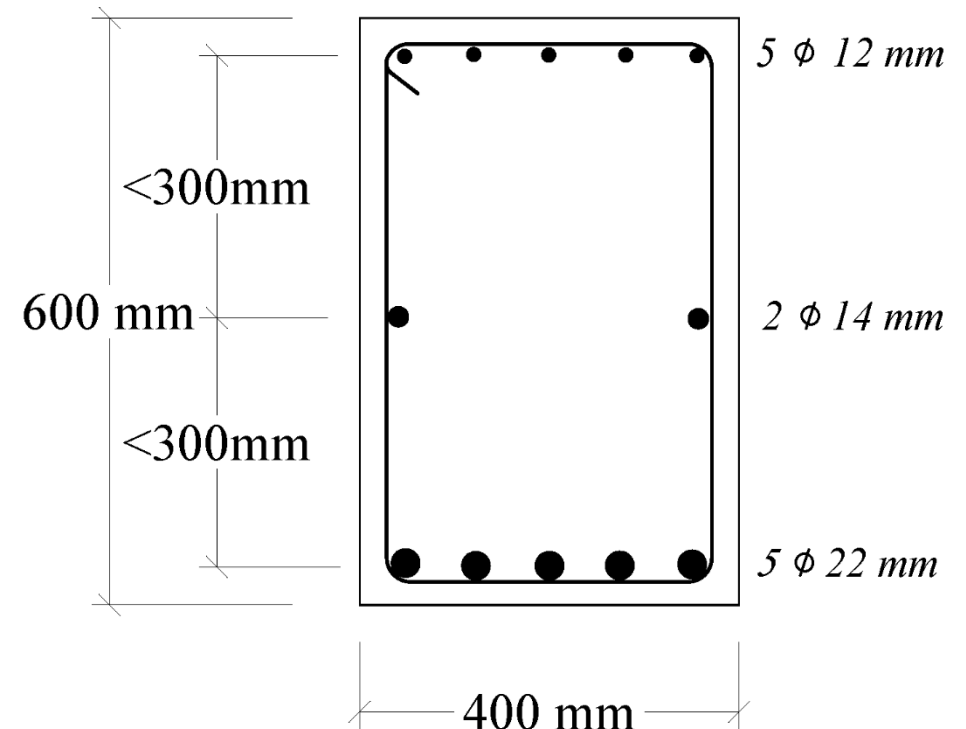
Area of middle reinforcement = 258 mm^2 , use 2 $\phi 14 \text{ mm}$.

11. Develop torsional reinforcement.

Torsion reinforcement must be extended at least $(b_t + d) = (400 + 542) \cong 950 \text{ mm}$ beyond the section where no torsion reinforcement is required.

12. Draw neat sketches showing reinforcement details.

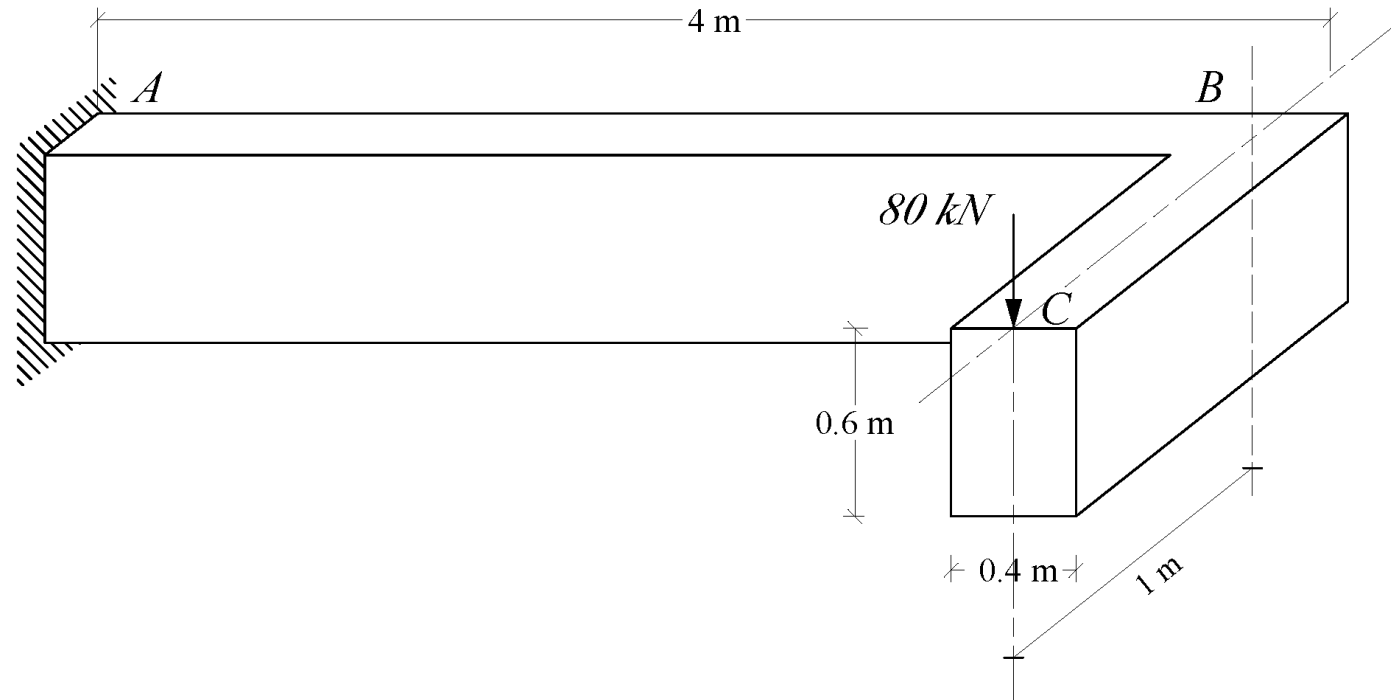
Figure 6.14 shows cross sectional dimensions and reinforcement details.



Example (6.2):

The cantilever beam shown in Figure 6.15 supports a factored concentrated load of 80 kN applied at point C. Neglecting own weights of members AB and BC, design member AB for moment, shear, and torsion.

$$f'_c = 30 \text{ MPa} \text{ and } f_y = 420 \text{ MPa}$$

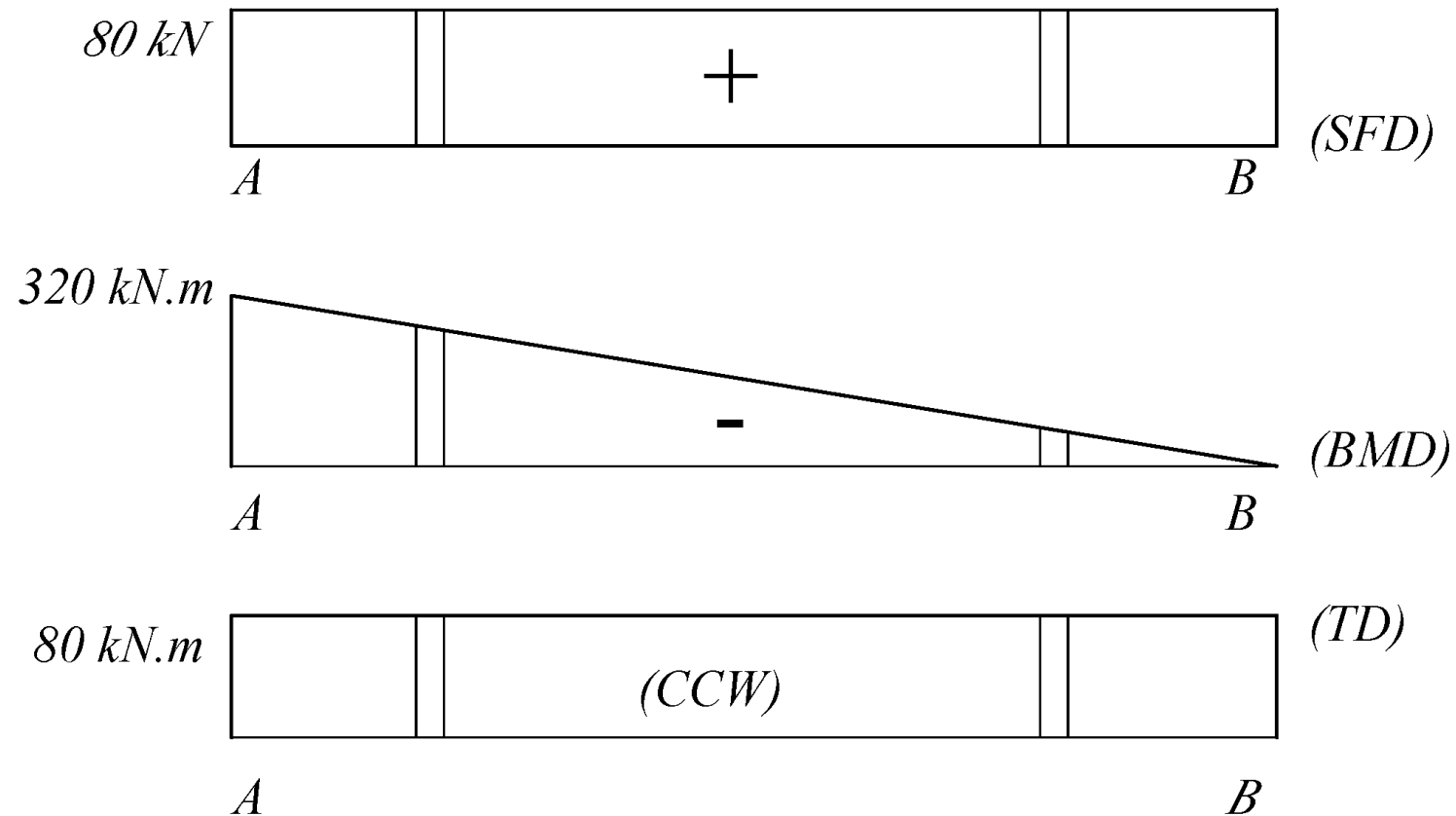


Example (6.2):

Solution:

- 1- Draw the shear force, bending moment, and torque diagrams.

$$M_{u,max} = 320 \text{ kN.m}, V_{u,max} = 80 \text{ kN}, \text{ and } T_{u,max} = 80 \text{ kN.m}$$



Example (6.2):

1. Select cross sectional dimensions and determine area of flexural reinforcement.

Minimum depth of member AB based on deflection requirement is equal to

$l/8 = 4000/8 = 500 \text{ mm}$. Use a $400 \times 600 \text{ mm}$ cross section.

$d = 600 - 40 - 8.0 - 10.0 = 542.0 \text{ mm}$, assuming $\phi 20 \text{ mm}$ reinforcing bars, and $\phi 8 \text{ mm}$ stirrups.

The reinforcement ratio at section of maximum factored moment is given by

$$\rho = \frac{0.85 (30)}{420} \left[1 - \sqrt{1 - \frac{2.353 (10)^6 (320)}{0.9 (400)(542)^2 (30)}} \right] = 0.00769$$

$$A_s = 0.00769 (400) (542.0) = 1666.7 \text{ mm}^2$$

Example (6.2):

2. Check if torsion may be neglected.

Torsion may be neglected if T_u is smaller than

$$T_u \leq \phi 0.083 \lambda \sqrt{f'_c} \frac{(A_{cp}^2)}{p_{cp}} = \frac{0.75 \times 0.083 \times \sqrt{30} (240 \times 10^3)^2}{(2000) \times 10^6} = 9.8 < T_u$$

Therefore, **torsion must be considered.**

3. Check for probability of brittle failure of concrete.

The dimensions of the cross section are adequate to insure a ductile mode of failure if this condition is satisfied

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} \leq \phi \left(\frac{V_c}{b_w d} + 0.66 \sqrt{f'_c}\right)$$

$$x_o = 400 - 2 \times 40 - 8.0 = 312 \text{ mm}$$

$$y_o = 600 - 2 \times 40 - 8.0 = 512 \text{ mm}$$

$$p_h = 2 (312 + 512) = 16480 \text{ mm}$$

$$A_{oh} = (312 \times 512) = 159744 \text{ mm}^2$$

Example (6.2):

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.70 A_{oh}^2}\right)^2} = \sqrt{\left(\frac{80 \times 10^3}{400 (542.0)}\right)^2 + \left(\frac{80 \times 10^6 (1648.0)}{1.70 (159744)^2}\right)^2}$$
$$= \sqrt{0.136 + 9.2363} = \sqrt{9.372} = 3.06 \text{ MPa}$$

$$\phi \left(\frac{V_c}{b_w d} + 0.66 \sqrt{f'_c} \right) = \phi \left(0.17 \sqrt{f'_c} + 0.66 \sqrt{f'_c} \right) = 0.83 \phi \sqrt{f'_c} = 3.4 \text{ MPa}$$

i.e. cross sectional dimensions **are adequate** in terms of ensuring ductile failure.

4. Determine the required area of stirrup reinforcement for shear.

$$V_c = 0.17 \sqrt{f'_c} b_w d$$

$$V_s = \frac{V_u}{\phi} - V_c$$

$$V_s = \frac{80}{0.75} - \frac{0.17 \sqrt{30} (400) (542.0)}{1000} = 106.67 - 201.87 = -95.2 \text{ kN}$$

This means that shear reinforcement is not required, and $\frac{A_v}{s} = 0.0$

Example (6.2):

5. Determine the required area of stirrup reinforcement for torsion.

The torsional stirrup reinforcement per unit length $\frac{A_t}{s}$, is given as

$$\frac{A_t}{s} = \frac{T_u}{2\phi f_{yt} A_o \cot(45^\circ)} = \frac{80.0 \times 10^6}{2 (0.75) (420) (0.85 \times 159744)} = 0.935 \text{ mm}^2/\text{mm}$$

S_{max} is the smaller of $p_h/8$ or 300 mm, where $p_h/8 = 1648.0/8 = 206.0$ mm, thus S_{max} is not to exceed 200 mm.

6. Determine the combined area of stirrup reinforcement for shear and torsion.

$$\left(\frac{A_{v+t}}{s} \right)_{total} = 0.0 + 2 (0.935) = 1.87 \text{ mm}^2/\text{mm}$$

$$\frac{0.35 b_w}{f_{yt}} = \frac{0.35 (400)}{420} = 0.33 \text{ mm}^2/\text{mm} < 1.87 \text{ mm}^2 \quad \text{O.K}$$

Example (6.2):

7. Select stirrup size and spacing.

Try $\phi 10 \text{ mm}$ closed stirrups

$\frac{2 (7.85)}{S} = 1.87$ and $S = 84.0 \text{ mm}$. This spacing is rather small,

so try $\phi 12 \text{ mm}$ closed stirrups, $\frac{2 (11.3)}{S} = 1.87$ and $S = 120.9 \text{ mm}$.

Use $\phi 12 \text{ mm}$ closed stirrups spaced at 120 mm .

8. Calculate the longitudinal reinforcement required for torsion.

Since $f_{yt} = f_y$ and for $\theta = 45^\circ$,

$$A_l = \left(\frac{A_t}{S} \right) p_h = (0.935) (1648.0) = 1541 \text{ mm}^2$$

The longitudinal reinforcement is not to be less than the lesser value obtained from the following equations

Example (6.2):

$$A_{l,min} = \frac{0.42\sqrt{f'_c}A_{cp}}{f_y} - \left(\frac{A_t}{S}\right) p_h \frac{f_{yt}}{f_y} = \frac{0.42\sqrt{30}(400)(600)}{420} - 1541 = 1314.5 - 1541 \\ = -226.5 \text{ mm}^2 \text{ (not applicable)}$$

$$A_{l,min} = \frac{0.42\sqrt{f'_c}A_{cp}}{f_y} - \left(\frac{0.175 b_w}{f_{yt}}\right) p_h \frac{f_{yt}}{f_y} = \frac{0.42\sqrt{30}(400)(600)}{420} - \frac{(0.175)(400)(1648)}{420} \\ = 1039.9 \text{ mm}^2$$

9. Size combined longitudinal reinforcement.

Distance between centerlines of tension and compression reinforcement exceeds 300 *mm*, so a middle layer of longitudinal reinforcement is needed.

$$\frac{A_l}{3} = \frac{1541}{3} = 514 \text{ mm}^2$$

Minimum bar diameter is $0.042S = 0.042(120) = 5.04 \text{ mm}$ or 10 *mm*, whichever is larger, and 10 *mm* is the least diameter to be used.

Example (6.2):

Total area of top reinforcement = $1667 + 514 = 2181 \text{ mm}^2$ use 5 $\phi 25 \text{ mm}$.

Area of bottom reinforcement = 514 mm^2 , use 4 $\phi 14 \text{ mm}$.

Area of middle reinforcement = 514 mm^2 , use 4 $\phi 14 \text{ mm}$.

11- Develop torsional reinforcement.

Torsional reinforcement is needed for the entire length of part AB, since the torque is constant in magnitude.

12- Draw neat sketches showing reinforcement details.

Figure 6.17 shows cross sectional dimensions and designed reinforcement.

