

Design of Reinforced Concrete Structures I

(ECIV 3316)

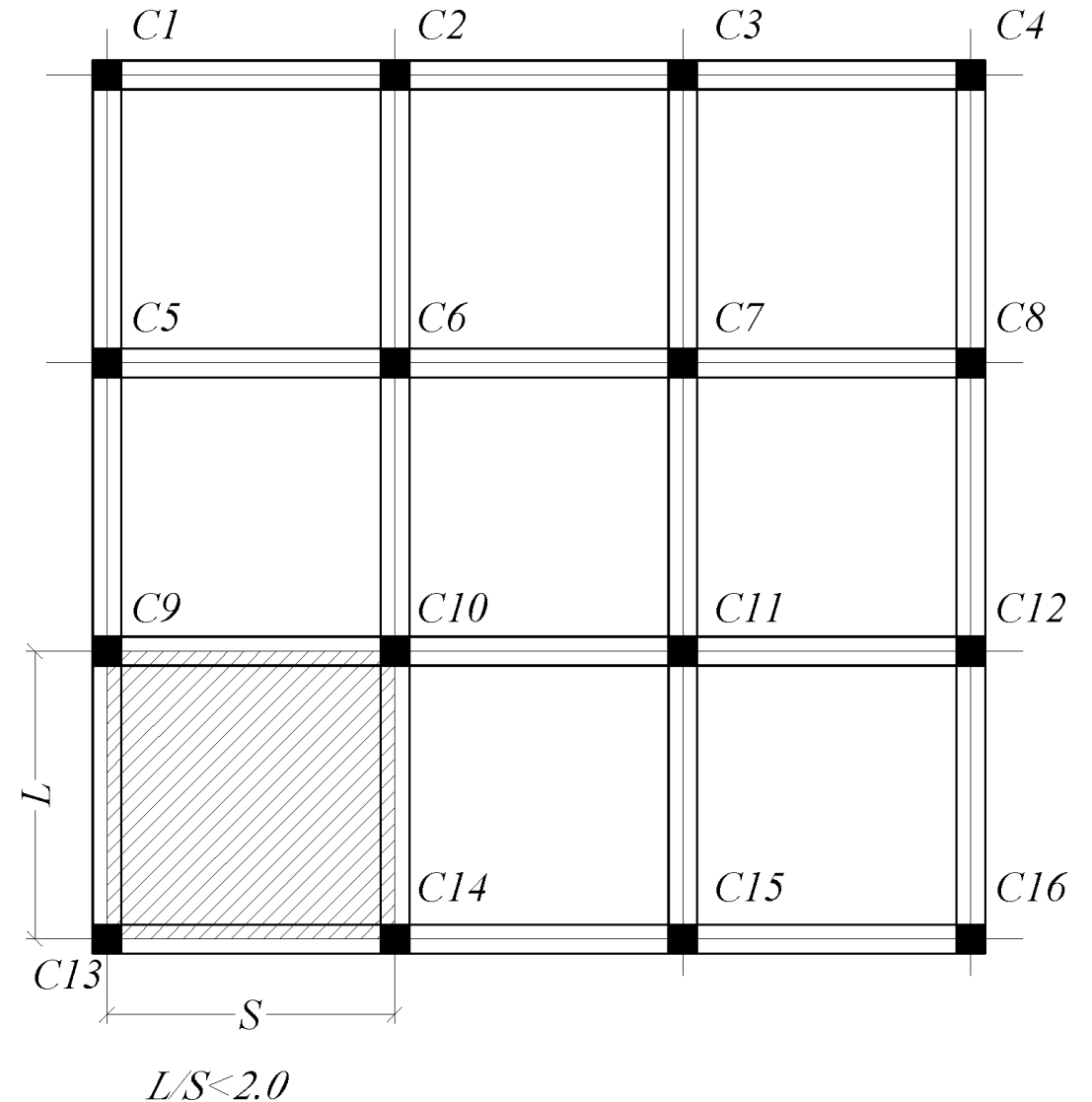
Chapter 10

Two Way Slabs

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10.1 Introduction

When the ratio (L/S) is less than 2.0, it is called two-way slab, shown in Figure 10.1. Bending will take place in the two directions in a dish-like form. Accordingly, main reinforcement is required in the two directions.



10.2 TYPES OF TWO-WAY SLABS

A. Flat Plates:

- ✓ Two-way slab system supported directly by columns, without beams, column capitals, or drop panels.
- ✓ This type of two-way slabs is suited for light loads associated with apartment construction and can be economically used for spans up to 6.0 m.
- ✓ Negative moment reinforcement is placed near the top of the plate close to the columns, while the positive moment reinforcement is placed near the bottom surface in the middle of the plate in both directions.
- ✓ Use of flat plates is limited by their punching shear capacities where large thickness is required in the case of heavy loads or large spans, which may make the system uneconomical.

10.2 TYPES OF TWO-WAY SLABS

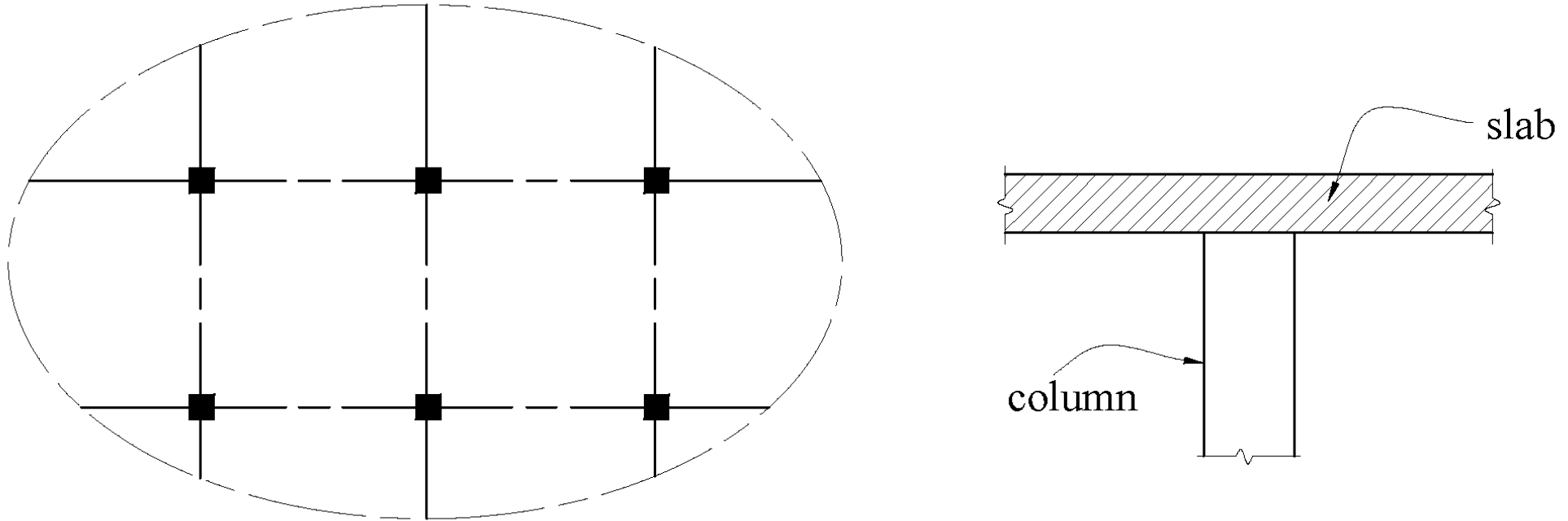


Figure 10.2: Flat plate

10.2 TYPES OF TWO-WAY SLABS

B. Flat Slabs:

- ✓ A two-way slab with column capitals or drop panels, or both, is shown in Figure 10.3.
- ✓ This system is appropriate for heavier loads encountered in office or industrial buildings.
- ✓ The system is used for spans up to 8.0 m, due to the increase in slab thickness around the columns.

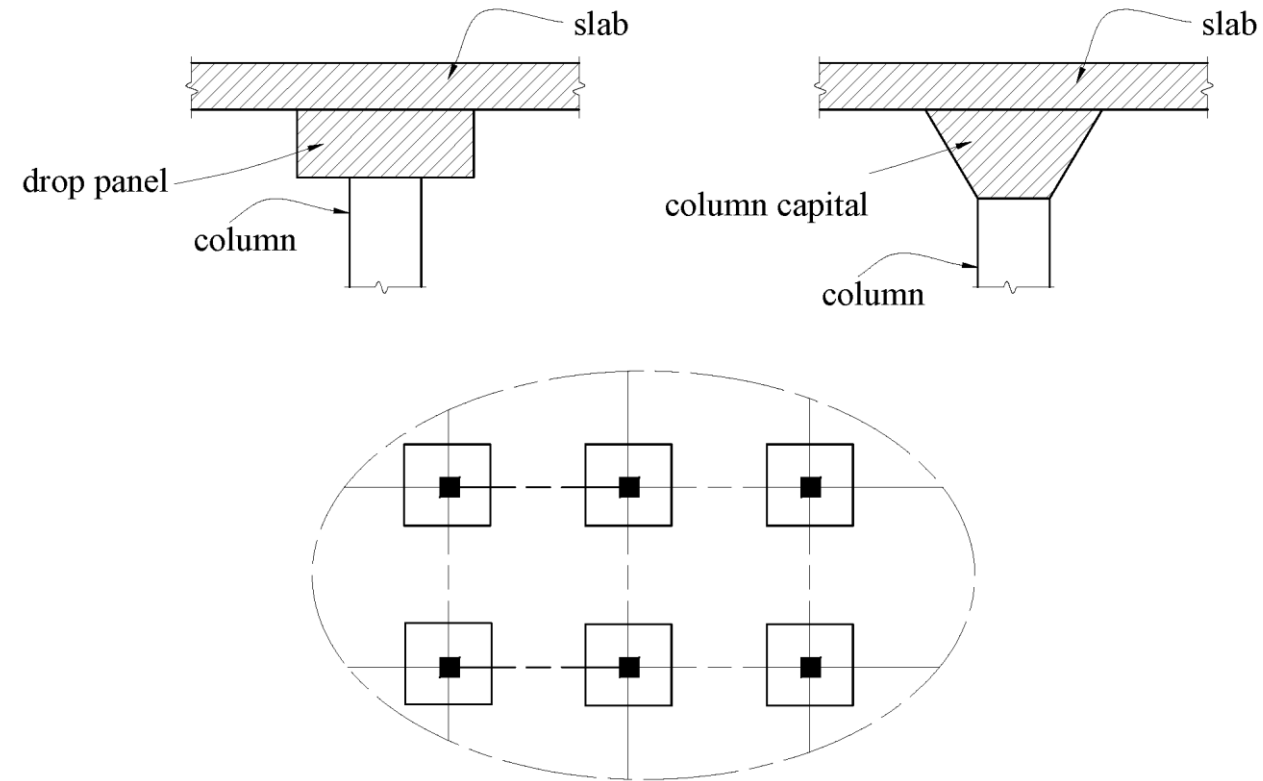


Figure 10.3: Flat Slab

10.2 TYPES OF TWO-WAY SLABS

C. Two-Way Edge-Supported Solid Slabs:

- ✓ In this system, shown in Figure 10.4, beams provide moment interaction with the columns especially when moment resisting frames are used to resist lateral loads.
- ✓ The system can be used economically for spans up to 7.0 meters.

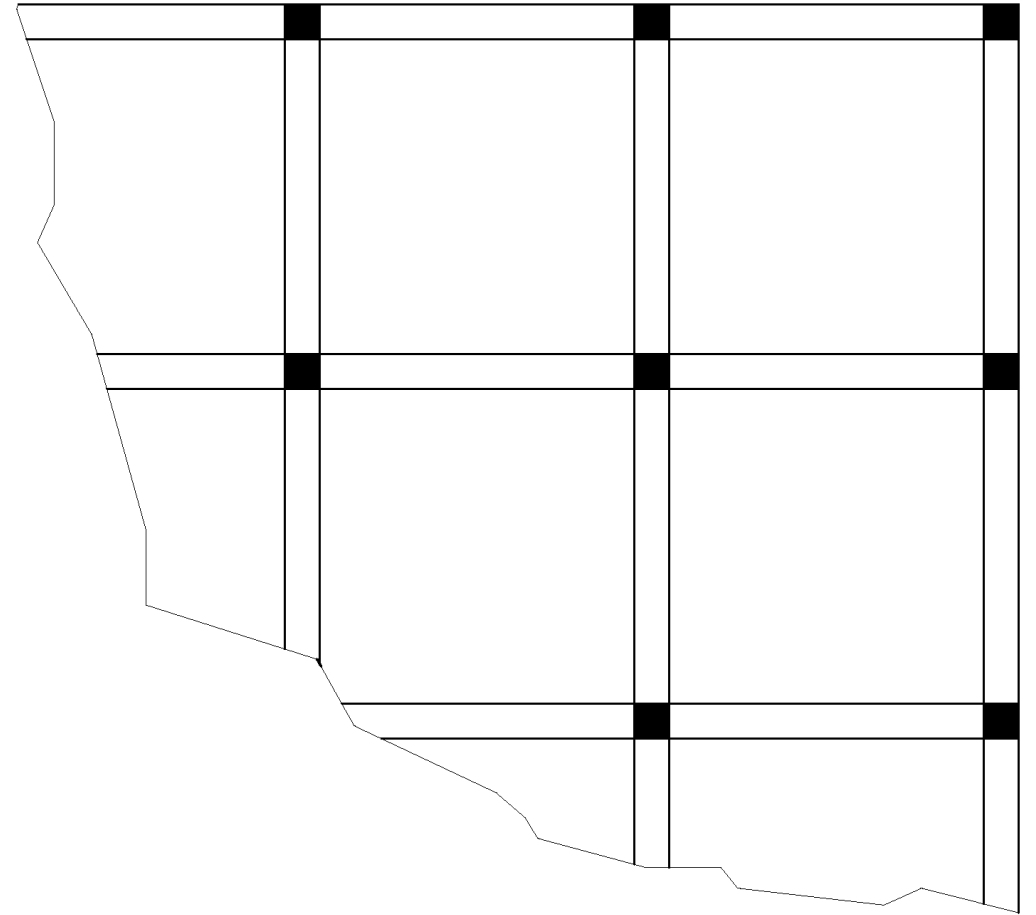


Figure 10.4: Two-way edge supported solid slab

10.2 TYPES OF TWO-WAY SLABS

D- Two-way Ribbed Slabs:

1. Waffle Slabs:

- ✓ The waffle slab is capable of providing the largest spans of the conventional concrete floor systems and can be economically used for spans up to 12.0 meters.
- ✓ This is attributed to the considerable reduction in dead load as compared to other floor systems. Waffle slab construction consists of orthogonal sets of ribs with solid parts at the columns, as shown in Figure 10.5. The ribs are formed with fiberglass or metal dome forms. The ribs are usually 0.60 to 0.90 meter on center. Shear is transferred to the columns by using beams or shear heads.

10.2 TYPES OF TWO-WAY SLABS

D- Two-way Ribbed Slabs:

1. Waffle Slabs:

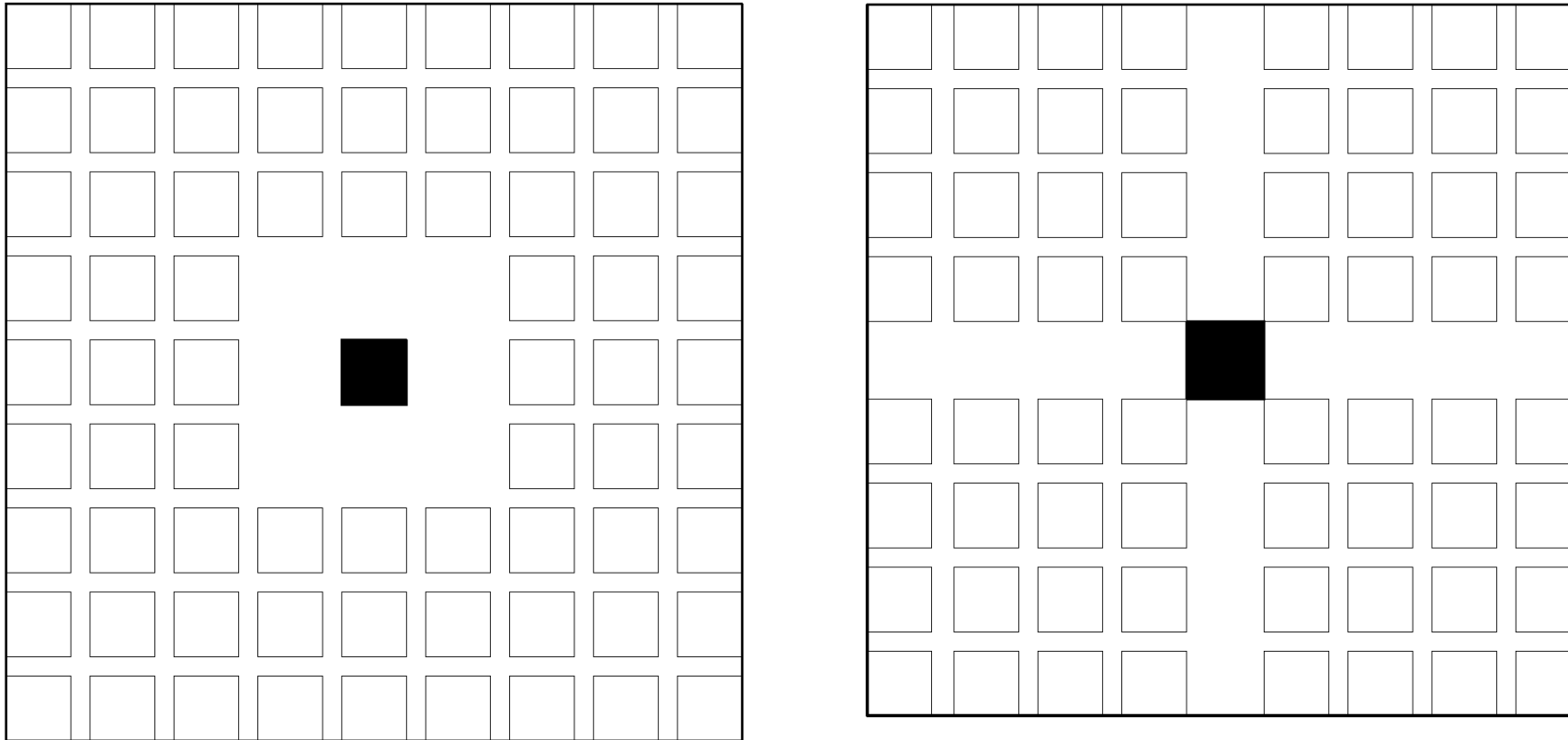


Figure 10.5: Waffle slabs

10.2 TYPES OF TWO-WAY SLABS

D- Two-way Ribbed Slabs:

2. Two-Way Edge-Supported Ribbed Slabs:

- ✓ This system, shown in Figure 10.6, can be economically used for spans up to 7.0 m.
- ✓ It is similar to the waffle slab, but the voids between ribs are filled with hollow blocks.
- ✓ Hidden or drop beams can be used with this system depending on their spans.

10.2 TYPES OF TWO-WAY SLABS

D- Two-way Ribbed Slabs:

2. Two-Way Edge-Supported Ribbed Slabs:

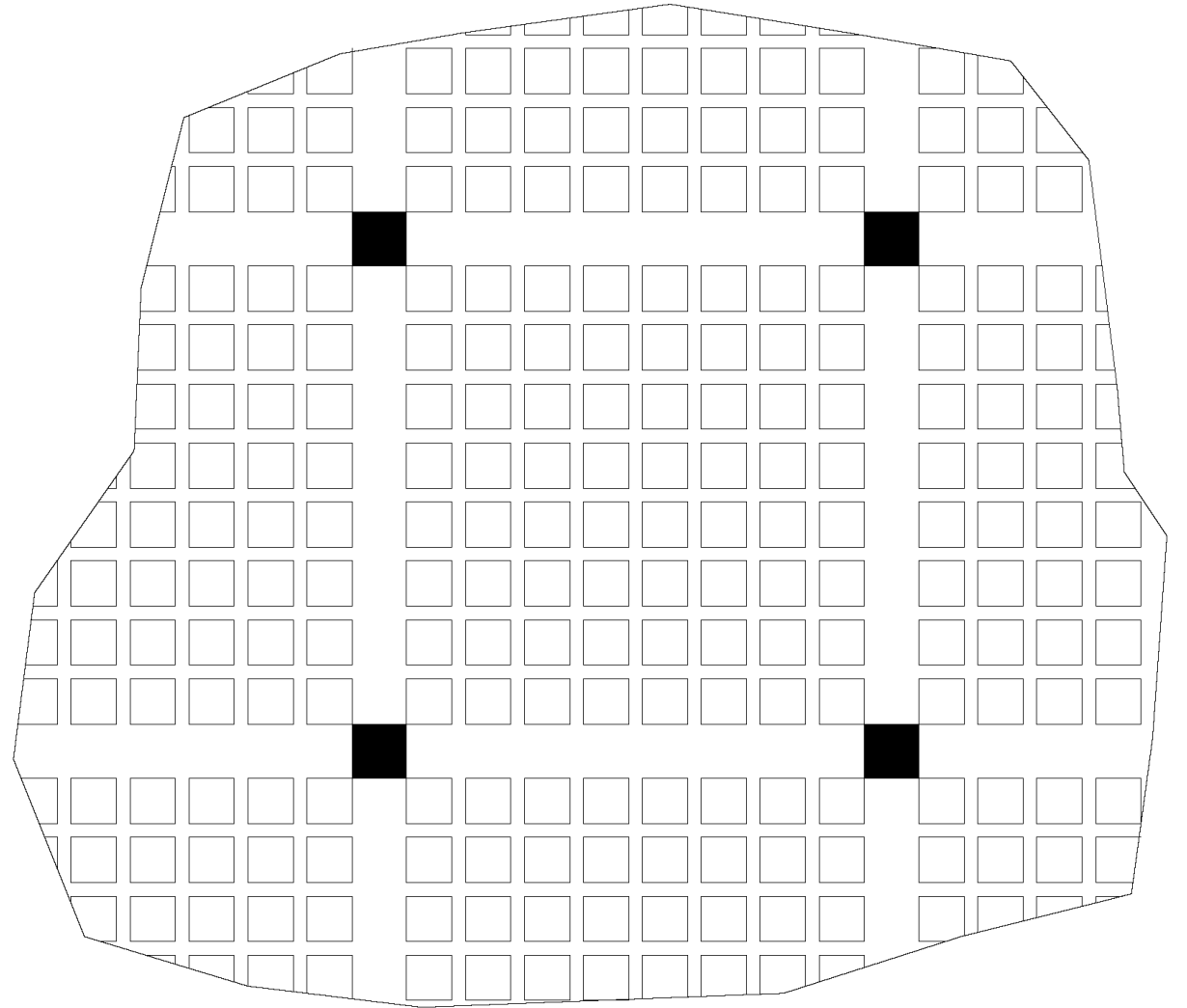


Figure 10.6: Two-way edge supported ribbed slabs

10.3 Design Methods

- ✓ **ACI 318-14 & earlier editions:** Provided two established methods for gravity-load analysis:
 - **Direct Design Method (DDM):** subject to specific limiting conditions.
 - **Equivalent Frame Method (EFM):** more general, but relatively complex.
- ✓ **ACI 318-19/22/25:** Provisions for DDM and EFM were removed from the Code as they represent only **two of several acceptable analysis methods**; both may still be used in accordance with **ACI 6.2.4.1**.
- ✓ **For Edge-Supported Slabs:** Simplified design methods can be used, based on Load distribution in the **short and long directions**, depending on:
 - (Long/Short span) **ratio**
 - **Edge support conditions**

10.4 SIMPLIFIED DESIGN METHODS

10.4.1 Grashoff-Rankine Method:

- ✓ Grashoff and Rankine formulated their method for simply-supported two-way slabs subject to uniformly distributed loads and when no provisions are made to prevent the corners of the slab from lifting, i.e., no provisions are made to resist torsion at the corners.
- ✓ The method is based on dividing the slab into intersecting parallel strips in the two directions. The load is distributed in the two directions with w_x in the short direction and w_y in the long direction, such that

$$w = w_x + w_y \quad (10.1)$$

Consider two strips of **unit width** in the directions of the shorter side of the panel, l_x and the longer side of the panel, l_y at the center of the slab.

10.4 SIMPLIFIED DESIGN MET

Mid-point deflection of the short direction,

$$\Delta_x = \frac{5w_x l_x^4}{384 EI} \quad (10.2)$$

Mid-point deflection of the long direction,

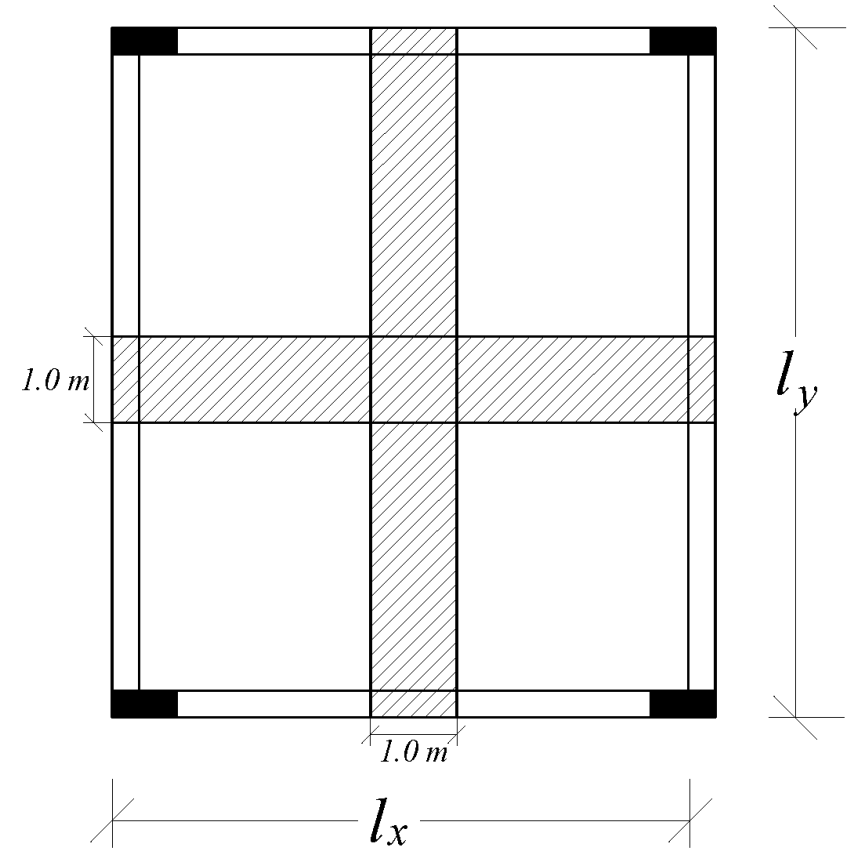
$$\Delta_y = \frac{5w_y l_y^4}{384 EI} \quad (10.3)$$

Equating *Eqns.* (10.1) and (10.3) gives

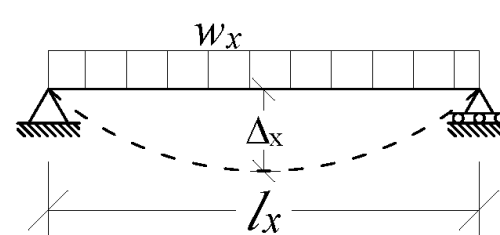
$$\frac{5w_x l_x^4}{384 EI} = \frac{5w_y l_y^4}{384 EI}$$

or

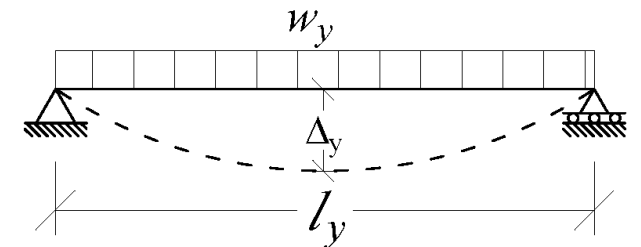
$$\frac{w_x}{w_y} = \frac{l_y^4}{l_x^4} \quad (10.4)$$



(a)



(b)



(c)

10.4 SIMPLIFIED DESIGN METHODS

Eqn. (10.4) may be written as

$$\frac{w_x}{w_x + w_y} = \frac{l_y^4}{l_x^4 + l_y^4} \quad (10.5)$$

Letting $r = \frac{l_y}{l_x}$, and using *Eqn. (10.1)*, *Eqn. (10.5)* is written as

$$\begin{aligned} \frac{w_x}{w} &= \frac{r^4}{r^4 + 1}, \text{ or} \\ w_x &= w \left(\frac{r^4}{r^4 + 1} \right) \end{aligned} \quad (10.6)$$

But $w_y = w - w_x$, or

$$w_y = w \left(\frac{1}{r^4 + 1} \right) \quad (10.7)$$

10.4 SIMPLIFIED DESIGN METHODS

The maximum bending moment per unit width in the short direction is

$$M_x = w_x \left(\frac{l_x^2}{8} \right) = \alpha_x w l_x^2 \quad (10.8)$$

The maximum bending moment per unit width in the long direction is

$$M_y = w_y \left(\frac{l_y^2}{8} \right) = \alpha_y w l_x^2 \quad (10.9)$$

where

$$\alpha_x = \frac{1}{8} \left(\frac{r^4}{r^4 + 1} \right)$$
$$\alpha_y = \frac{1}{8} \left(\frac{r^2}{r^4 + 1} \right)$$

10.4 SIMPLIFIED DESIGN METHODS

The maximum shearing force per unit width in the short direction is given as

$$V_x = w_x \left(\frac{l_x}{2} \right) = \frac{w l_x}{2} \left(\frac{r^4}{r^4 + 1} \right) \quad (10.10)$$

The maximum shearing force per unit width in the long direction is

$$V_y = w_y \left(\frac{l_y}{2} \right) = \frac{w l_y}{2} \left(\frac{1}{r^4 + 1} \right) \quad (10.11)$$

10.4 SIMPLIFIED DESIGN METHODS

10.4.2 Marcus Method:

For the slab with corners held down, Marcus has modified the Grashoff-Rankine method coefficients by multiplying them with correction factors.

The modification factors for simply supported slabs,

$$C_x = C_y = 1 - \frac{5}{6} \left(\frac{r^2}{1 + r^4} \right) \quad (10.12)$$

Thus, the modified values of moments taking into account torsion effects are

$$M'_x = C_x M_x \text{ and } M'_y = C_y M_y.$$

The Grashoff-Rankine moments are conservative compared with Marcus method since that the load on the slab is carried not only by bending but also by twisting

10.4 SIMPLIFIED DESIGN METHODS

10.4.3 Simplified Coefficient Method by Egyptian code

- ✓ Grashoff-Rankine and Marcus methods have basically dealt with analysis of simply supported slab panels.
- ✓ The Egyptian Code has extended these methods for analysis of continuous slabs where individual panels have different supporting conditions.
- ✓ To account for the various supporting conditions, the rectangularity ratio $r = L/S$ is modified by multiplying the nominator and the dominator of this expression by the ratio of distance between inflection points to the length between supports for longer and shorter spans, respectively, or

$$r' = \frac{L}{S} \frac{m_l}{m_s} = r \times \frac{m_1}{m_2}$$

- ✓ For simply supported spans, $m = 1.0$, for spans continuous from one side, $m = 0.87$, and for spans continuous from both sides, $m = 0.76$.
- ✓ In table 10.1 $\alpha = 0.5r - 0.15$ $\beta = 0.35/r^2$

10.4 SIMPLIFIED DESIGN

$$w_x = \alpha \times w$$
$$w_y = \beta \times w$$

TABLE 10.1
Coefficients for two-way solid slabs with beams ($L \leq 5.0 \text{ kN/m}^2$)

r'	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
α	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
β	0.35	0.29	0.25	0.21	0.18	0.16	0.14	0.12	0.11	0.09	0.08

TABLE 10.2
Coefficients for two-way ribbed slabs with beams ($L \leq 5.0 \text{ kN/m}^2$)

r'	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
a	0.396	0.473	0.543	0.606	0.660	0.706	0.746	0.778	0.806	0.830	0.849
β	0.396	0.333	0.262	0.212	0.172	0.140	0.113	0.093	0.077	0.063	0.053

TABLE 10.3
Coefficients for two-way ribbed slabs with beams ($L > 5.0 \text{ kN/m}^2$)

r'	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
α	0.500	0.595	0.672	0.742	0.797	0.834	0.867	0.893	0.914	0.928	0.941
β	0.500	0.405	0.328	0.258	0.203	0.165	0.133	0.107	0.086	0.072	0.059

10.4 SIMPLIFIED DESIGN METHODS

10.4.4 Pieper-Marten Coefficients (Eurocode 2 Annex I)

- ✓ The Pieper-Martens method is an elastic-plastic analysis approach for two-way reinforced concrete slabs, formally adopted by Eurocode 2 (EN1992-1-1, Annex I).
- ✓ This method provides simplified moment coefficients to determine bending moments in rectangular slabs under uniform loads, based on the assumption of equal or proportionally distributed reinforcement in both directions.
- ✓ It is applicable to simply supported slabs (where corners are prevented from lifting), and to continuous/fixed-edge slabs (with full or partial restraint).
- ✓ While the method was originally developed for isotropic conditions, Eurocode 2 adapts it for practical use with typical orthotropic reinforcement by providing separate coefficients for the two principal directions.

10.4 SIMPLIFIED DESIGN METHODS

10.4.4 Pieper-Marten Coefficients (Eurocode 2 Annex I)

a) Simply -supported Slabs:

When simply-supported slabs do not have adequate provision to resist torsion at the corners, and to prevent the corners from lifting, such as slabs not cast monolithically with the supporting beams or slabs supported by concrete block walls, the maximum moments per unit width are given by the following equations:

$$m_{sx} = \alpha_{sx} w l_x^2 \quad (10.13)$$

$$m_{sy} = \alpha_{sy} w l_x^2 \quad (10.14)$$

where

$$\alpha_{sx} = \frac{(l_y/l_x)^4}{8(1+(l_y/l_x)^4)}$$

$$\alpha_{sy} = \frac{(l_y/l_x)^2}{8(1+(l_y/l_x)^4)}$$

10.4 SIMPLIFIED DESIGN METHODS

10.4.4 Pieper-Marten Coefficients (Eurocode 2 Annex I)

a) Simply -supported Slabs:

The values of α_{sx} and α_{sy} which are the same as those given by Grashoff-Rankine coefficients, are given in Table 10.3 for l_y/l_x ranging from 1.0 to 2.0

TABLE 10.4

Moment coefficients for two-way slabs, simply supported on four sides

l_y/l_x	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
α_{sx}	0.062	0.074	0.084	0.093	0.099	0.104	0.113	0.118
β_{sy}	0.062	0.061	0.059	0.055	0.051	0.046	0.037	0.029

10.4 SIMPLIFIED DESIGN METHODS

10.4.4 Pieper-Marten Coefficients (Eurocode 2 Annex I)

a) Simply -supported Slabs:

For shear Force (The same as those given by Grashoff-Rankine)

The maximum shearing force per unit width in the short direction is given as

$$V_{sx} = \alpha_{vx} \left(\frac{wl_x}{2} \right)$$

The maximum shearing force per unit width in the long direction is

$$V_{sy} = \alpha_{vy} \left(\frac{wl_y}{2} \right)$$

Where

$$\alpha_{vx} = \frac{r^4}{1 + r^4} \quad \& \quad \alpha_{vy} = \frac{1}{1 + r^4}$$

10.4 SIMPLIFIED DESIGN METHODS

10.4.4 Pieper-Marten Coefficients (Eurocode 2 Annex I)

b) Restrained Slabs:

- ✓ For restrained slabs where the corners are prevented from lifting, and provision for torsion is made, such as slabs cast monolithically with their supporting beams,
- ✓ The maximum design moments per unit width are given by the following equation:

$$m_{sx} = \beta_{sx} w_u L_x^2 \text{ and } m_{sy} = \beta_{sy} w_u L_x^2 \quad (10.15)$$

Bending moment coefficients β_{sx} and β_{sy} are given in **Table 10.5** w_u is the uniformly distributed factored load on the slab, and L_x is the short span.

- ✓ For calculating the shear forces in restrained slabs. The shear force is calculated as

$$V_{sx} = \beta_{vx} w_u L_x \text{ and } V_{sy} = \beta_{vy} w_u L_x \quad (10.16)$$

where β_{vx} and β_{vy} are the shear force coefficients as given in the **Table 10.6**

10.4 SIMPLIFIED DESIGN METHODS

- Case 1: Interior panel
- Case 2: One short edge discontinuous
- Case 3: One long edge discontinuous
- Case 4: Two adjacent edges discontinuous
- Case 5: Two short edges discontinuous
- Case 6: Two long edges discontinuous
- Case 7: Two short edges and one long edge discontinuous
- Case 8: Two long edges and one short edge discontinuous
- Case 9: All edges simply supported

$$m_{sx} = \beta_{sx} w_u L_x^2$$
$$m_{sy} = \beta_{sy} w_u L_x^2$$

TABLE 10.5
Moment coefficients for two-way slabs

Case	Location	Short span L_x coefficients: $\beta_{sx} \times 10^3$								Long span L_y $\beta_{sy} \times 10^3$
		Side ratio L_y/L_x								For all ratios
		1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
1	Edge	−31	−37	−42	−46	−50	−53	−59	−63	−32
	Mid-span	24	28	32	35	37	40	44	48	24
2	Edge	−39	−44	−48	−52	−55	−58	−63	−67	−37
	Mid-span	29	33	36	39	41	43	47	50	28
3	Edge	−39	−49	−56	−62	−68	−73	−82	−89	−37
	Mid-span	30	36	42	47	51	55	62	67	28
4	Edge	−47	−56	−63	−69	−74	−78	−87	−93	−45
	Mid-span	36	42	47	51	55	59	65	70	34
5	Edge	−46	−50	−54	−57	−60	−62	−67	−70	−
	Mid-span	34	38	40	43	45	47	50	53	34
6	Edge	−	−	−	−	−	−	−	−	−45
	Mid-span	34	46	56	65	72	78	91	100	34
7	Edge	−57	−65	−71	−76	−81	−84	−92	−98	−
	Mid-span	43	48	53	57	60	63	69	74	44
8	Edge	−	−	−	−	−	−	−	−	−58
	Mid-span	42	54	63	71	78	84	96	105	44
9	Mid-span	55	65	74	81	87	92	103	111	56

10.4 SIMPLIFIED DESIGN METHODS

- Case 1: Interior panel
- Case 2: One short edge discontinuous
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- Case 5: Two short edges discontinuous
- Case 4: Two adjacent edges discontinuous
- Case 6: Two long edges discontinuous
- Case 7: Two short edges and one long edge discontinuous
- Case 8: Two long edges and one short edge discontinuous
- Case 9: All edges simply supported

$$V_{sx} = \beta_{vx} w_u L_x$$
$$V_{sy} = \beta_{vy} w_u L_x$$

TABLE 10.6
Shear force coefficients for two-way slabs

		Short span L_x coefficients: $\beta_{vx} \times 10^2$								Long span: $\beta_{vy} \times 10^2$
		Side ratio L_y/L_x								
Case	Type of edge	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	For all ratios
1	Continuous	33	36	39	41	43	45	48	50	33
2	Continuous	36	39	42	44	45	47	50	52	36
	Discontinuous	—	—	—	—	—	—	—	—	24
3	Continuous	36	40	44	47	49	51	55	59	36
	Discontinuous	24	27	29	31	32	34	36	38	—
4	Continuous	40	44	47	50	52	54	57	60	40
	Discontinuous	26	29	31	33	34	35	38	40	26
5	Continuous	40	43	45	47	48	49	52	54	—
	Discontinuous	—	—	—	—	—	—	—	—	26
6	Continuous	—	—	—	—	—	—	—	—	40
	Discontinuous	26	30	33	36	38	40	44	47	—
7	Continuous	45	48	51	53	55	57	60	63	—
	Discontinuous	30	32	34	35	36	37	39	41	29
8	Continuous	—	—	—	—	—	—	—	—	45
	Discontinuous	29	33	36	38	40	42	45	48	30
9	Discontinuous	33	36	39	41	43	45	48	50	33

10.4 SIMPLIFIED DESIGN METH

Case 1: Interior panel

Case 2: One short edge discontinuous

Case 3: One long edge discontinuous

Case 4: Two adjacent edges discontinuous

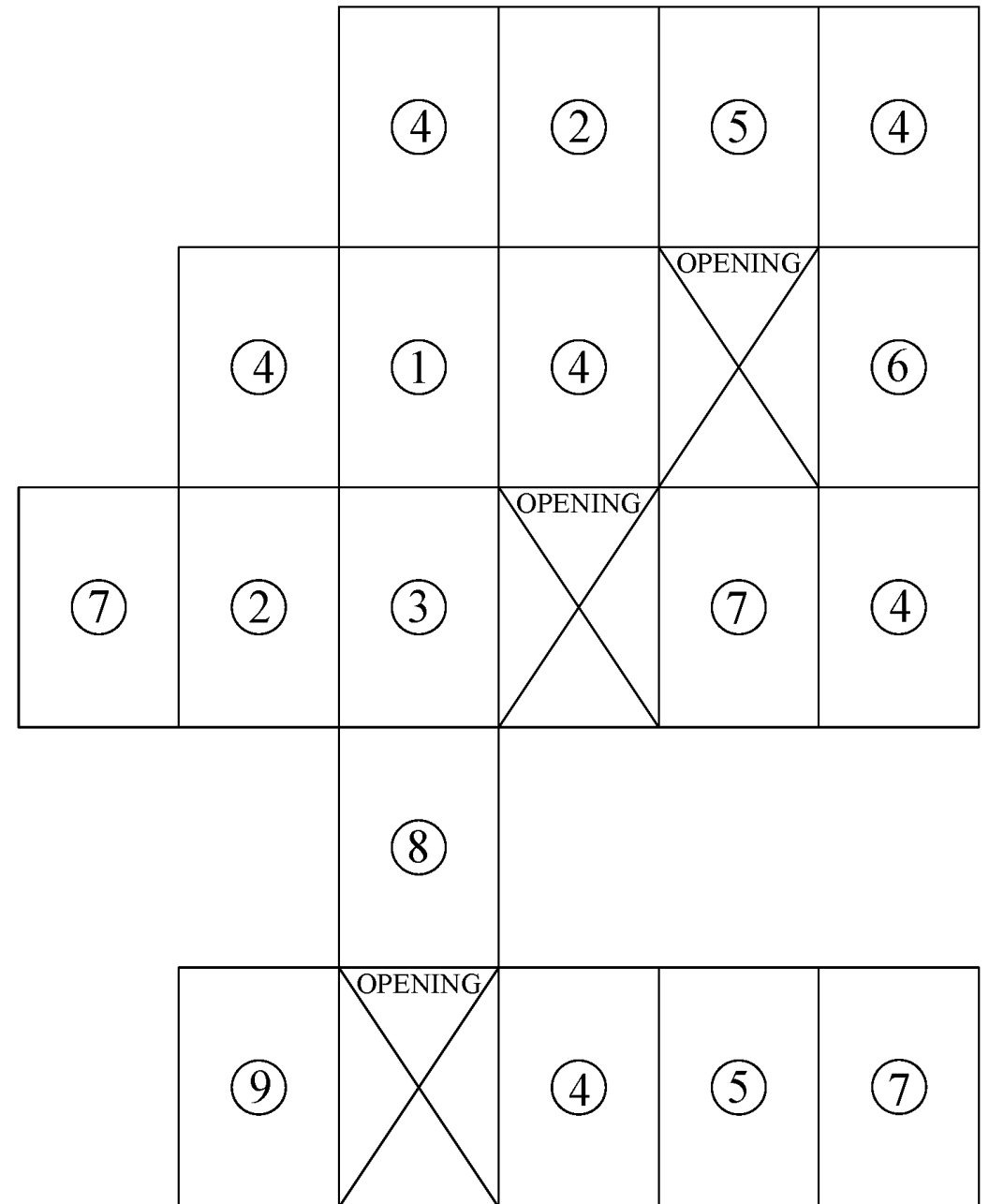
Case 5: Two short edges discontinuous

Case 6: Two long edges discontinuous

Case 7: Two short edges and one long edge discontinuous

Case 8: Two long edges and one short edge discontinuous

Case 9: All edges simply supported



10.4 SIMPLIFIED DESIGN METHODS

The following rules apply to Table 10.5 and Table 10.6:

- ✓ In each direction, slabs are divided into a middle strip and two edge strips. The middle strip occupies three-quarters of the panel width, and each edge strip occupies one-eighth.
- ✓ Maximum design moments apply only to the middle strips; no redistribution is permitted.
- ✓ Reinforcement in an edge strip, parallel to the edge, shall not exceed the maximum area of tension reinforcement.

c) Analysis of Ribbed Slabs

Two-way ribbed slabs may be designed as two-way solid slabs or as flat slabs, whichever is the more appropriate.

10.4 SIMPLIFIED DESIGN METHODS

10.4.5 Design of Beams in Two-Way Slabs by Approximate Methods

- ✓ The load supported by each beam is considered to be the entire load on tributary areas bounded by 45-degree lines drawn from the corners of the panel, shown in Figure 10.9(a) and (b). Triangular or trapezoidal distributions are possible.
- ✓ If available analysis software cannot cope with such load distributions, as shown in Figure 10.9(c), it is possible to replace these load distributions by equivalent uniform loads that give same maximum shear and moment as the original loads do.
- ✓ For **triangular load distribution**, the equivalent shear force coefficient C_s is equal to **0.50**, and the equivalent bending moment coefficient C_b is equal to **2/3**.

10.4 SIMPLIFIED DESIGN METHODS

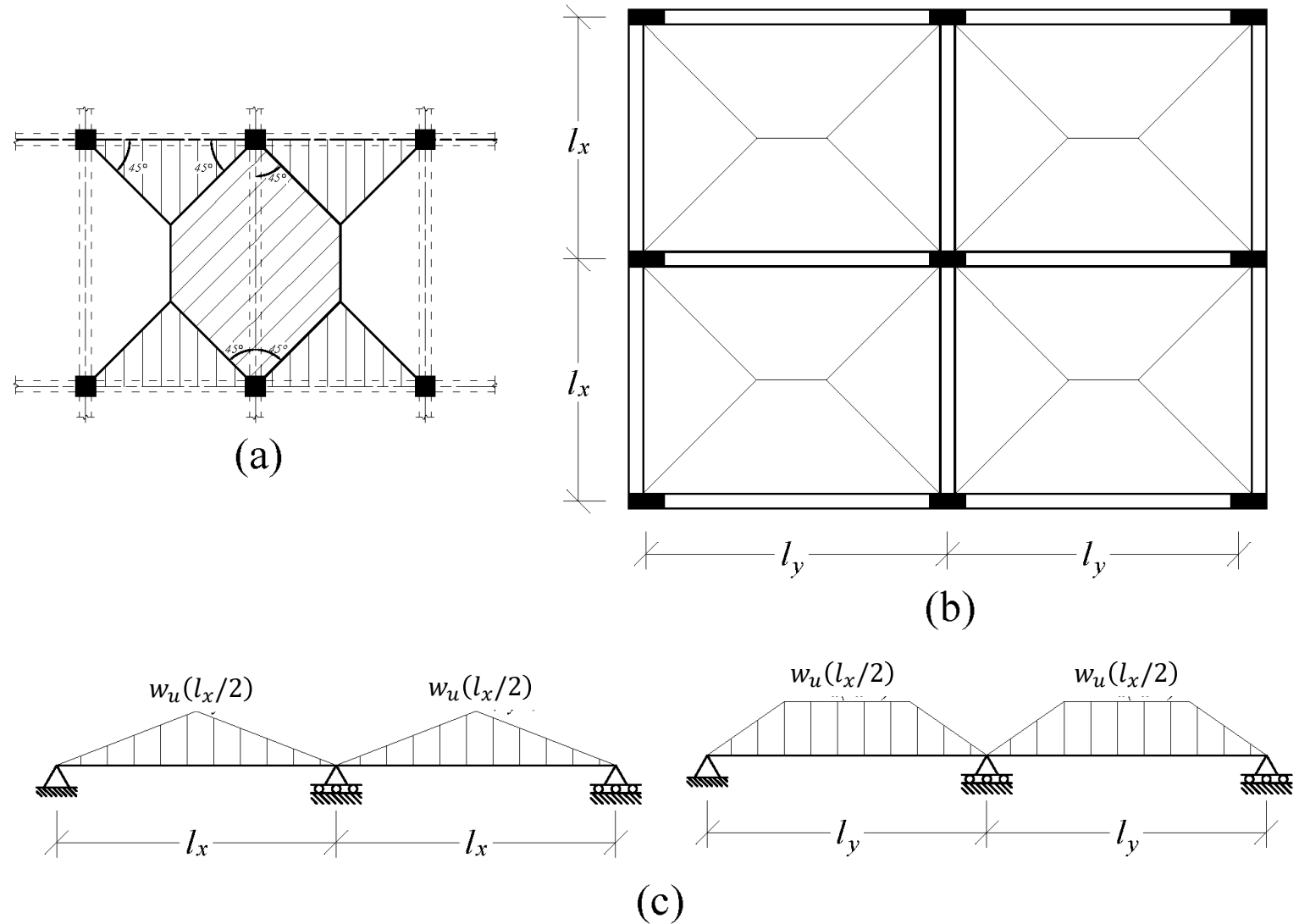


Figure 10.9: Loads supported by beams in two-way slabs
(a) Tributary areas for slab loads on beams; (b) tributary areas for loads on continuous beams; (c) loads on beams

10.4 SIMPLIFIED DESIGN METHODS

- ✓ For trapezoidal load distribution, C_s and C_b are given in Table 10.7, where,
$$C_s = 1 - \frac{1}{2r}, \text{ and } C_b = 1 - \frac{1}{3r^2}.$$
- ✓ The total load on a given beam consists of its own weight, weight of partitions, if any, and the loads resulting from the slab, as explained earlier.

TABLE 10.7

Shear and moment equivalent load coefficients for trapezoidal load distribution

r	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
C_s	0.500	0.545	0.583	0.615	0.643	0.667	0.688	0.706	0.722	0.737	0.750
C_b	0.667	0.725	0.769	0.803	0.830	0.852	0.870	0.885	0.897	0.908	0.917

10.4 SIMPLIFIED DESIGN METHODS

Eurocode 2 provide almost the same principle in distributing the slab load to the supporting beams.

- ✓ They suggest distributing slab loads to beams based on tributary areas and load dispersion at 45° from the corners as shown in Figure 10.9.
- ✓ It is possible to replace these load distributions by equivalent uniform distributed loads (UDL) as follows.

For triangular loads $w_{eq} = \frac{w_{slab}L_x}{3}$

For trapezoidal loads $w_{eq} = \frac{w_{slab}L_x}{2} \left(1 - \frac{1}{2\beta^2}\right)$, where $\beta = \frac{L_y}{L_x}$

- ✓ **An adjustment** for beam edge continuity shall be applied. If the beam **is continuous**, increase the w_{eq} by 10% to attract more loads and **decrease it by 10%** if the beam is simply supported.

10.4 SIMPLIFIED DESIGN METHODS

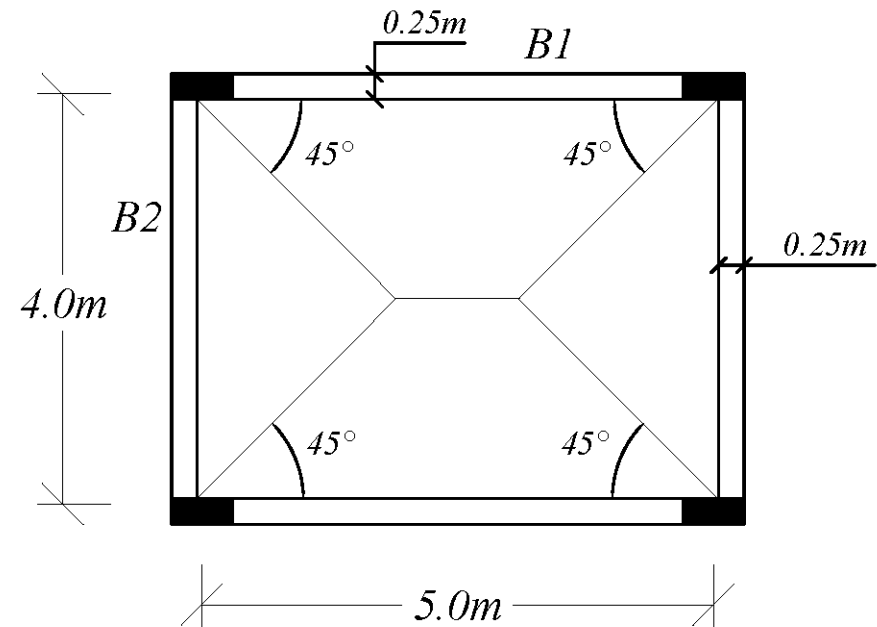
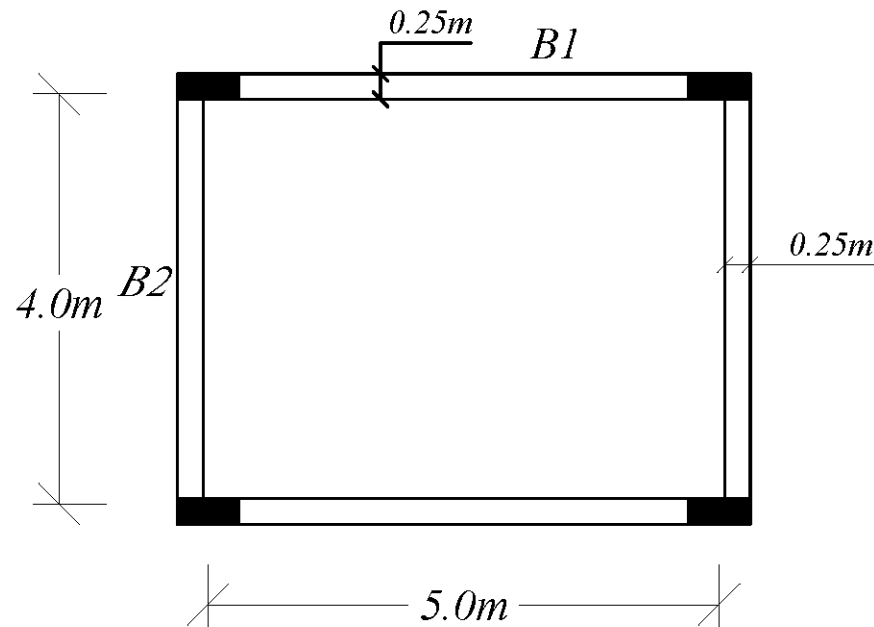
Summary of Design Procedure for Two-way Slabs using the Simplified Methods

1. Slab thickness is evaluated based on deflection control requirement.
2. The total factored load supported by the slab is calculated.
3. The loads distributed in the short and long directions of each panel are determined as discussed in earlier sections.
4. Representative $1m$ wide strips in the short and long directions are selected.
5. The shearing force and bending moment diagrams for each of the strips are drawn.
6. Strength of each of the strips in shear is checked.
7. Design the reinforcement at critical negative and positive bending moment sections for each of the strips.
8. Prepare design drawings showing the dimensions and the provided reinforcement.

Example 10.1

Using Eurocode 2 coefficients, design the simply supported solid slab shown in Figure 10.10. Assume that the slab thickness is 130 mm and the beam webs are 250 mm wide. Weight of flooring materials is 2 kN/m^2 , and the live load is 5 kN/m^2 . Use $f'_c = 25 \text{ MPa}$ and $f_y = 420 \text{ MPa}$. Consider the following cases:

- (a) No adequate provision to resist torsion at the corners, and to prevent the corners from lifting.
- (b) Corners are prevented from lifting and adequate provision is made for torsion.



Example 10.1

Solution:

Part (a): No adequate provision to resist torsion at the corners

1. Determine the total factored load per unit area of the slab:

Since $r = 5/4 = 1.25 < 2.0$, the slab is classified as being two way

Own weight of slab $= 0.13 \times 25.0 = 3.25 \text{ kN/m}^2$

$$w_u = 1.20(3.25 + 2.0) + 1.60(5.0) = \mathbf{14.3 \text{ kN/m}^2}$$

2 Evaluate load distribution in both directions: $l_y/l_x = 1.25$

Using Eurocode 2 (Pieper-Marten) Coefficients,

For Bending Moment $\alpha_{sx} = \frac{(l_y/l_x)^4}{8(1+(l_y/l_x)^4)} = 0.0885$ and $\alpha_{sy} = \frac{(l_y/l_x)^2}{8(1+(l_y/l_x)^4)} = 0.057$

For shear Force $\alpha_{vx} = \frac{r^4}{1+r^4} = \frac{(1.25)^4}{(1.25)^4+1} = 0.709$, $\alpha_{vy} = \frac{1}{1+r^4} = \frac{1}{(1.25)^4+1} = 0.291$

Example 10.1

3 Select representative 1m wide design strips to span in the short and long directions:

Two strips are used, one in the short direction and the other in the long direction.

4. Evaluate shear and bending moment at critical section locations:

$$\text{Maximum shear force in the short direction } V_{sx} = 14.3 \times 0.709 \times \frac{4.0}{2} = 20.3 \text{ kN}$$

$$\text{Maximum shear force in the long direction } V_{sy} = 14.3 \times 0.291 \times \frac{5.0}{2} = 10.4 \text{ kN}$$

$$\text{Maximum moment in the short direction } m_{sx} = 0.0885(14.3) (4)^2 = 20.2 \text{ kN.m.}$$

$$\text{Maximum moment in the long direction } m_{sy} = 0.057(14.3) (4)^2 = 13.0 \text{ kN.m}$$

5. Check slab thickness for beam shear:

a. Short Direction:

Average effective depth $d = 130 - 20 - 12 = 98 \text{ mm}$, assuming $\phi 12 \text{ mm}$ bars

$$\phi V_c = 0.75(0.17)\sqrt{25}(1000)(98.0)/1000 = 62.5 \text{ kN}$$

i.e. , **slab thickness is adequate** in terms of resisting beam shear in the shorter direction.

Example 10.1

b. Long direction:

$$V_{u,max} < \phi V_c$$

i.e. , **slab thickness is adequate** in terms of resisting beam shear in the longer direction.

6. Design the reinforcement:

a. Short direction:

$$\rho = \frac{0.85(25)}{420} \left[1 - \sqrt{1 - \frac{2.353 \times 10^6 (20.2)}{(0.9)1000 (98)^2 (25)}} \right] = 0.0060$$

Area of flexural reinforcement $A_s = 0.006 (1000) (98) = 588 \text{ mm}^2/\text{m}$

Use $\phi 12 \text{ mm}$ @ 175 mm at the bottom side of the slab.

For main reinforcement, spacing between bars is not to exceed the smaller of

$$S_b \leq \begin{cases} 3t_{slab} = 3 \times 130 = 390\text{mm} \\ 450\text{mm} \end{cases} \quad \text{which is already satisfied}$$

Example 10.1

b. Long direction:

$$\rho = \frac{0.85(25)}{420} \left[1 - \sqrt{1 - \frac{2.353 \times 10^6 (13.0)}{(0.9)1000 (98)^2 (25)}} \right] = 0.0037$$

Area of flexural reinforcement $A_s = 0.0037 (1000) (98) = 363 \text{ mm}^2/\text{m}$

Use $\phi 12 \text{ mm @ } 300 \text{ mm}$ at the bottom side of the slab.

Example 10.1

Part (b): Corners are prevented from lifting and adequate provision is made for torsion.

2 Evaluate load distribution in both directions:

Rectangularity ratio $l_y/l_x = 1.25$ and for four edges continuous case

Using Eurocode Coefficients Table 10.5 and 10.6

For Bending Moment $\beta_{sx} = 0.077$ and $\beta_{sy} = 0.056$

For shear Force $\beta_{vx} = 0.40$ and $\beta_{vy} = 0.33$

4- Select representative 1-*m* wide design strips in the short and long directions:

Two strips are used, one in the short direction and the other in the long direction.

a) Evaluate shear and bending moment at critical section locations:

Maximum shear force in the short direction $V_{sx} = \beta_{vx} w l_x = 0.40(14.3)(4) = 22.9 \text{ kN}$

Maximum shear force in the long direction $V_{sy} = \beta_{vy} w l_x = 0.33(14.3)(4) = 18.9 \text{ kN}$

Example 10.1

Maximum moment in the short direction $m_{sx} = 0.077(14.3) (4)^2 = 17.6 \text{ kN.m}$

Maximum moment in the long direction $m_{sy} = 0.056(14.3) (4)^2 = 12.8 \text{ kN.m}$

5) Check slab thickness for beam shear:

a. Short Direction:

Average effective depth $d = 130 - 20 - 12 = 98 \text{ mm}$, assuming $\phi 12 \text{ mm}$ bars in both directions.

$$\phi V_c = 0.75(0.17)\sqrt{25}(1000)(98.0)/1000 = 62.5 \text{ kN}$$

i.e. , slab thickness is adequate in terms of resisting beam shear in the shorter direction.

b. Long direction:

$$V_{u,max} < \phi V_c$$

i.e. , slab thickness is adequate in terms of resisting beam shear in the longer direction.

Example 10.1

6) Design the reinforcement:

a. Short direction:

$$\rho = \frac{0.85(25)}{420} \left[1 - \sqrt{1 - \frac{2.353 \times 10^6 (17.6)}{(0.9)(1000) (98)^2 (25)}} \right] = 0.0052$$

Area of flexural reinforcement $A_s = 0.0052 (1000) (98) = 510 \text{ mm}^2/\text{m}$

Use $\phi 12 \text{ mm @ } 200 \text{ mm}$ at the bottom side of the slab.

For main reinforcement, spacing between bars is not to exceed the smaller of $3(130) = 390 \text{ mm}$ and 450 mm , which is already satisfied.

a. Long direction:

$$\rho = \frac{0.85(25)}{420} \left[1 - \sqrt{1 - \frac{2.353 \times 10^6 (12.8)}{(0.9)(1000) (98)^2 (25)}} \right] = 0.0037$$

Area of flexural reinforcement $A_s = 0.0037 (1000) (980) = 3630 \text{ mm}^2/\text{m}$

Use $\phi 12 \text{ mm @ } 300 \text{ mm}$ at the bottom side of the slab.

Example 10.1

Design of beam B1:

$$h_{min} = l/16 = 5000/16 = 312.5 \text{ mm}$$

Try 250 mm × 500 mm cross section.

Factored own weight of beam =

$$1.2 (0.25) (0.5) (25) = 3.75 \text{ kN/m}$$

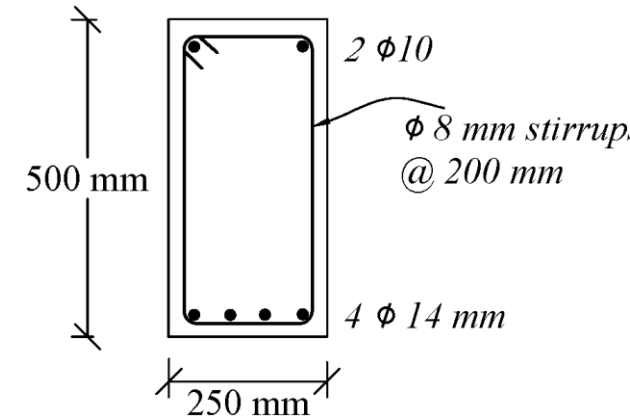
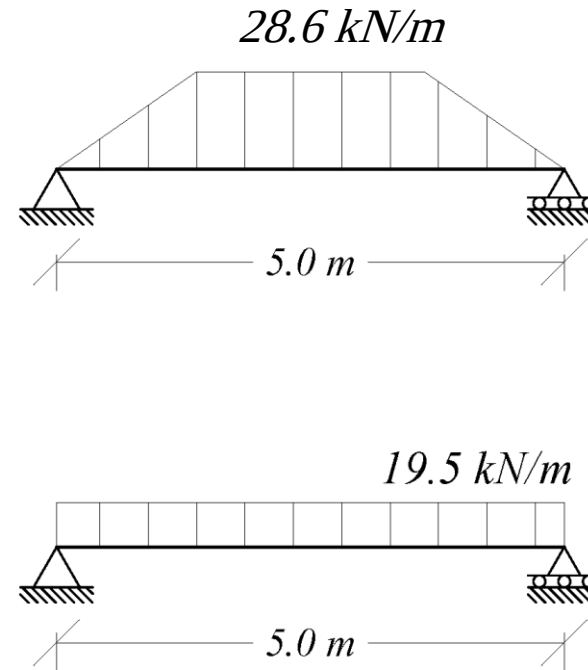
Maximum factored moment due to own weight of beam =

$$3.75 (5)^2/8 = 11.72 \text{ kN.m}$$

Maximum factored shear due to own weight of beam = $3.75 (5)/2 = 9.37 \text{ kN}$

Equivalent distributed load = $w_{eq} = \frac{w_{slab} L_x}{2} \left(1 - \frac{1}{2\beta^2}\right)$, where $\beta = \frac{L_y}{L_x}$

$$w_{eq} = \frac{14.3 \times 4}{2} \left(1 - \frac{1}{2(1.25)^2}\right) = 19.5 \text{ kN/m}$$



Example 10.1

$$\text{Maximum factored shear} = 19.5 \times (5)/2 = 48.75 \text{ kN}$$

$$\text{Maximum factored moment} = 19.5 (5)^2/8 = 60.9 \text{ kN.m}$$

$$\text{Total factored shear} = 9.37 + 48.75 = 58.12 \text{ kN}$$

$$\text{Total factored moment} = 11.72 + 60.9 = 72.62 \text{ kN.m}$$

Assuming that $\phi 8 \text{ mm}$ stirrups and $\phi 12 \text{ mm}$ reinforcing bars are used,

$$d = 500 - 40.0 - 8.0 - 6.0 = 446 \text{ mm}$$

$$\rho = \frac{0.85(25)}{420} \left[1 - \sqrt{1 - \frac{2.353 \times 10^6 (72.62)}{(0.9)(250) (446)^2 (25)}} \right] = 0.00402 > \rho_{min}$$

$$A_s = 0.00402 (250) (446) = 448 \text{ mm}^2, \text{ use } 4 \phi 14 \text{ mm bars.}$$

Check section for shear:

$$\phi V_c = 0.75(0.17)\sqrt{25}(250)(446)/1000 = 71 \text{ kN}$$

Use $\phi 8 \text{ mm}$ @ 200 mm as minimum shear reinforcement.

Example 10.1

Design of beam B2:

$$h_{min} = l/16 = 400/16 = 25.0 \text{ cm}$$

Try 250 mm × 500 mm cross section.

Factored own weight of beam =

$$1.2 (0.25) (0.5) (25) = 3.75 \text{ kN/m}$$

Maximum factored moment due to own weight of beam =

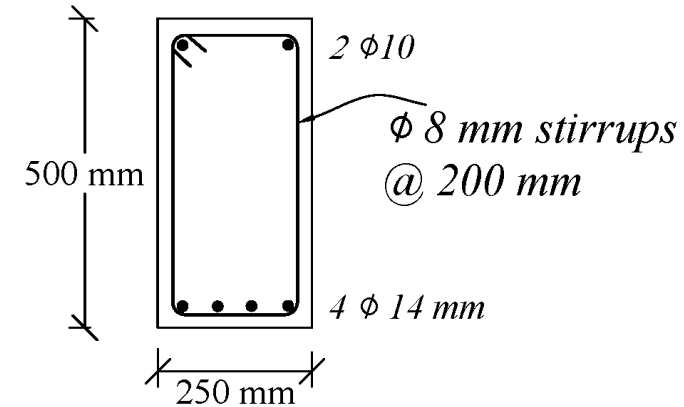
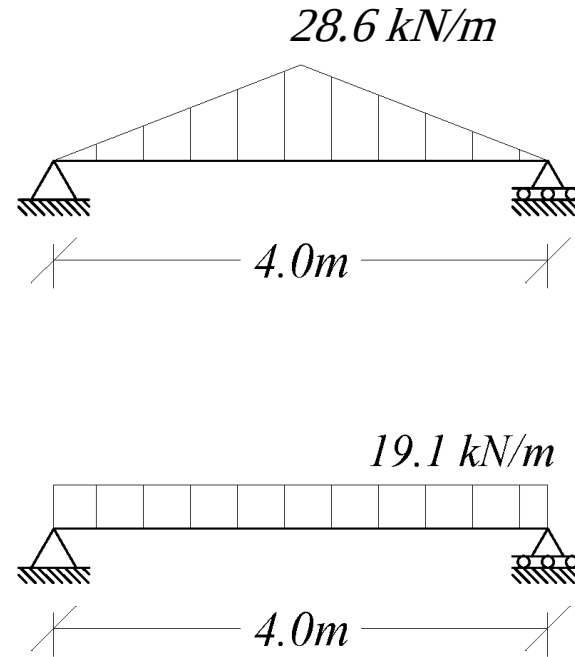
$$3.75 (4)^2/8 = 7.5 \text{ kN.m}$$

$$\text{Maximum factored shear due to own weight of beam} = 3.75 \frac{4}{2} = 7.5 \text{ kN}$$

$$w_{eq}' = \frac{w_u L_x}{3} = \frac{14.3 \times 4}{3} = 19.1 \text{ kN/m}$$

$$\text{Maximum factored shear} = 19.1 (4)/2 = 38.2 \text{ kN}$$

$$\text{Maximum factored moment} = 19.1 (4)^2/8 = 38.2 \text{ kN.m}$$



Example 10.1

Total factored shear = $7.5 + 38.2 = 45.7 \text{ kN}$

Total factored moment = $7.5 + 38.2 = 45.7 \text{ kN.m}$

Assuming that $\phi 8 \text{ mm}$ stirrups and $\phi 12 \text{ mm}$ reinforcing bars are used,

$$d = 500 - 40 - 8.0 - 6.0 = 446 \text{ mm}$$

$$\rho = \frac{0.85(25)}{420} \left[1 - \sqrt{1 - \frac{2.353 \times 10^6 (45.7)}{(0.9)(250) (446)^2 (25)}} \right] = 0.00250 < \rho_{\min}$$

$$A_s = 0.0033 (250) (446) = 368 \text{ mm}^2, \text{ use } 4 \phi 12 \text{ mm bars.}$$

Check section for shear:

$$\phi V_c = 0.75(0.17)\sqrt{25}(250)(446)/1000 = 71.0 \text{ kN}$$

Use $\phi 8 \text{ mm @ } 200 \text{ mm}$ as minimum shear reinforcement.

Example 10.1

7. Prepare design drawings showing the dimensions and the provided reinforcement

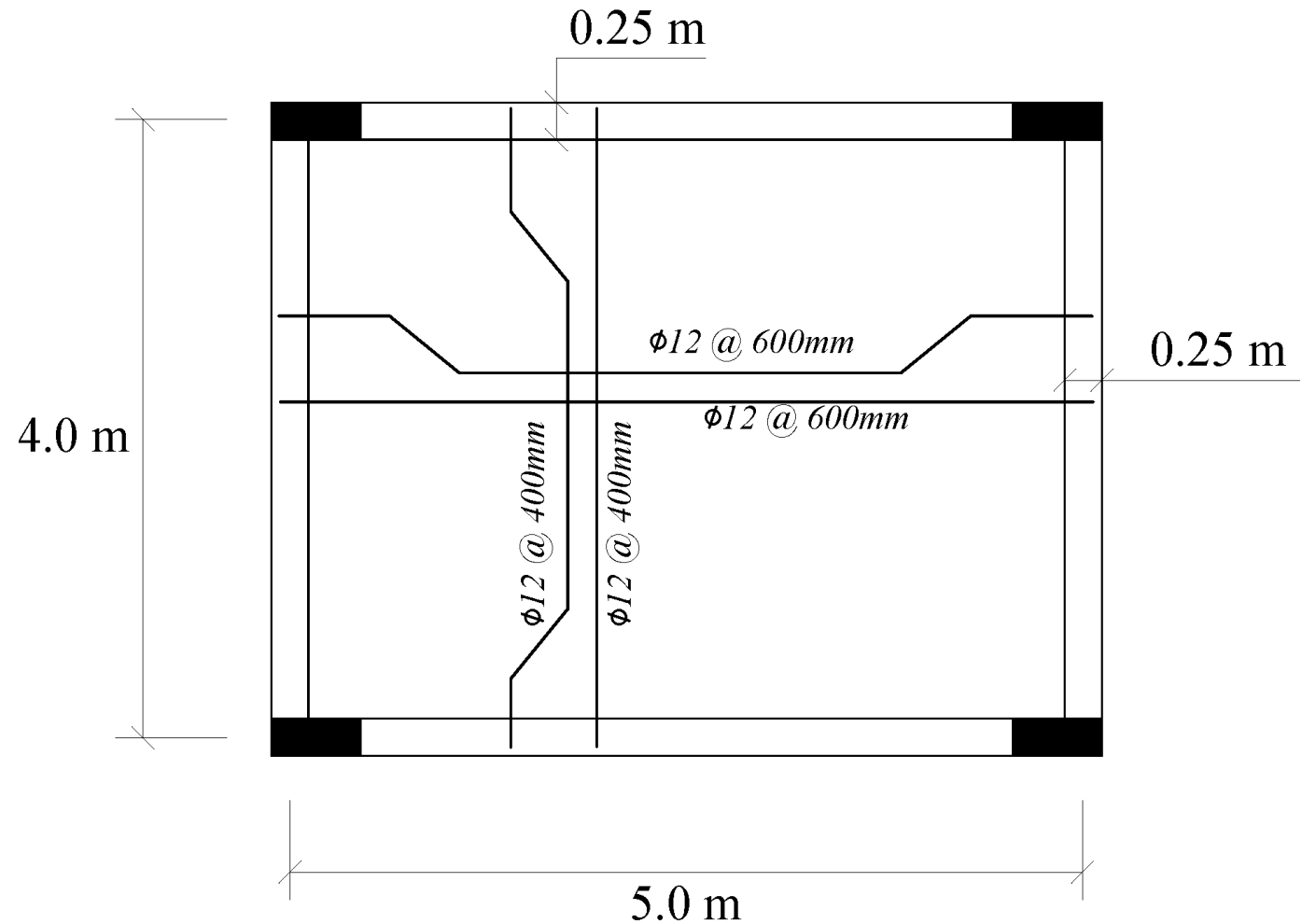


Figure 10.13: slab reinforcement details

Example 10.2

Design the two-way solid slab shown in Figure 10.14, using the simplified coefficient method by Egyptian code.

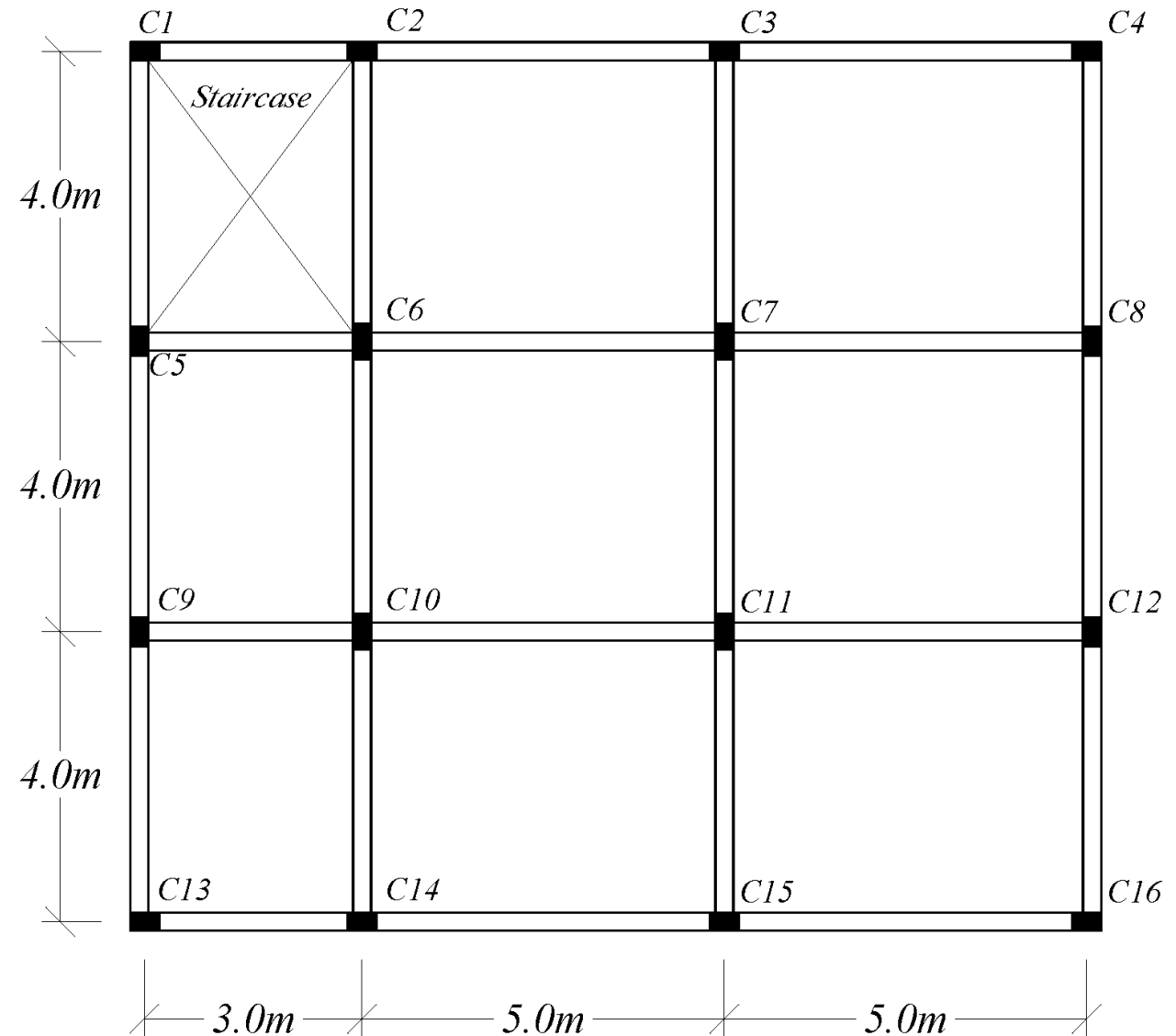
Assume the slab thickness is 140 mm.

The covering materials weigh 2.58 kN/m^2 , and the live load is 3.0 kN/m^2 .

Also, calculate the factored load acting on beam C3-C7-C11-C15 due to slab loads.

Use $f'_c = 25 \text{ MPa}$ and $f_y = 420 \text{ MPa}$.

All beams are 250 mm wide.



Example 10.2

Solution:

1. Determine the total factored load per unit area of the slab:

$$w_u = 1.2 [0.14 (25) + 2.58] + 1.6 (3.0) = 12.1 \text{ kN/m}^2$$

2. Evaluate load distribution in both directions:

Load distributions, are determined based on Egyptian code load coefficient given in Table 10.1

$$\text{For panels } S1, S2 \text{ and } S8: r' = \frac{5.0 (0.87)}{4.0 (0.87)} = 1.25, \alpha = 0.475 \text{ and } \beta = 0.230.$$

$$\text{For panels } S3 \text{ and } S6: r' = \frac{4.0 (0.87)}{3.0 (0.87)} = 1.33, \alpha = 0.517 \text{ and } \beta = 0.190.$$

$$\text{For panel } S4: r' = \frac{5.0 (0.76)}{4.0 (0.76)} = 1.25, \alpha = 0.475 \text{ and } \beta = 0.230.$$

$$\text{For panel } S5: r' = \frac{5.0 (0.87)}{4.0 (0.76)} = 1.43, \alpha = 0.565 \text{ and } \beta = 0.174.$$

$$\text{For panel } S7: r' = \frac{5.0 (0.76)}{4.0 (0.87)} = 1.092, \alpha = 0.396 \text{ and } \beta = 0.295.$$

Example 10.2

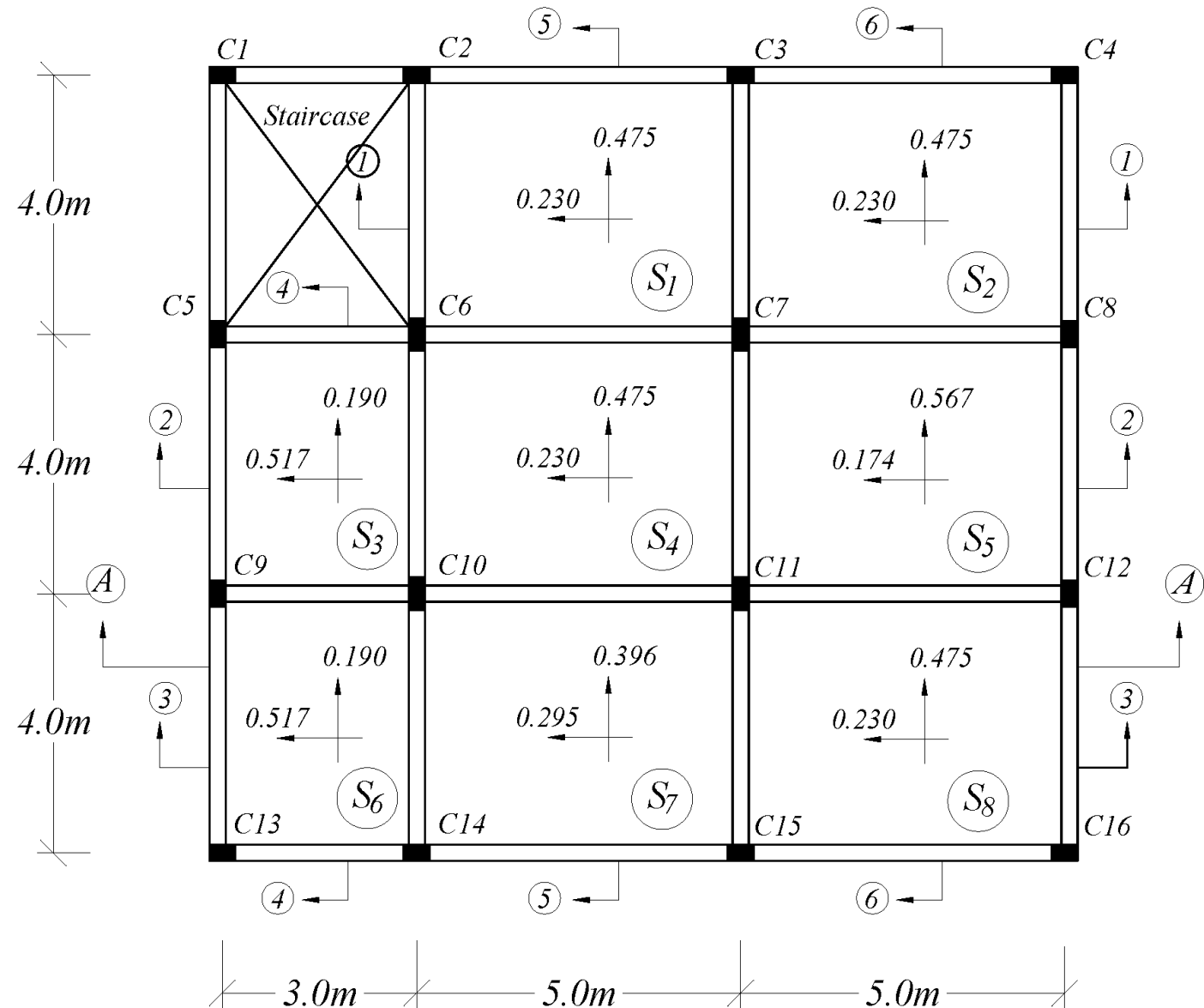
1. Select representative 1-*m* wide design strips to span in the short and long directions:

Six design strips are used, three in the short direction and three in the long direction.

$$\begin{aligned}d_{avg} &= 140 - 20 - 12 \\ &= 108 \text{ mm, assuming } \phi 12 \text{ mm} \\ &\text{reinforcing bars.}\end{aligned}$$

1. Check slab thickness for shear:

$$\begin{aligned}\phi V_c &= (0.75)(0.17)\sqrt{25} (1000) (108) \\ &/1000 = 68.9 \text{ kN O.K}\end{aligned}$$



Example 10.2

5. Design the reinforcement:

Strip 1-1:

For $M_u = -8.7 \text{ kN.m}$, as shown in

$$\rho = \frac{0.85(25)}{420} \left[1 - \sqrt{1 - \frac{2.353 \times 10^6 (8.7)}{(0.9)(1000)(108)^2 (25)}} \right]$$
$$= 0.002$$

$$A_s = 0.002 (1000) (108) = 216 \text{ mm}^2/\text{m}$$

$$A_{s,min} = 0.0018(1000)(140) = 252 \text{ mm}^2/\text{m}$$

use $\phi 10 \text{ mm @ } 300 \text{ mm}$.

For $M_u = 4.9 \text{ kN.m}$

$$A_{s,min} = 0.0018(1000)(140) = 252 \text{ mm}^2/\text{m},$$

use $\phi 10 \text{ mm @ } 300 \text{ mm}$.

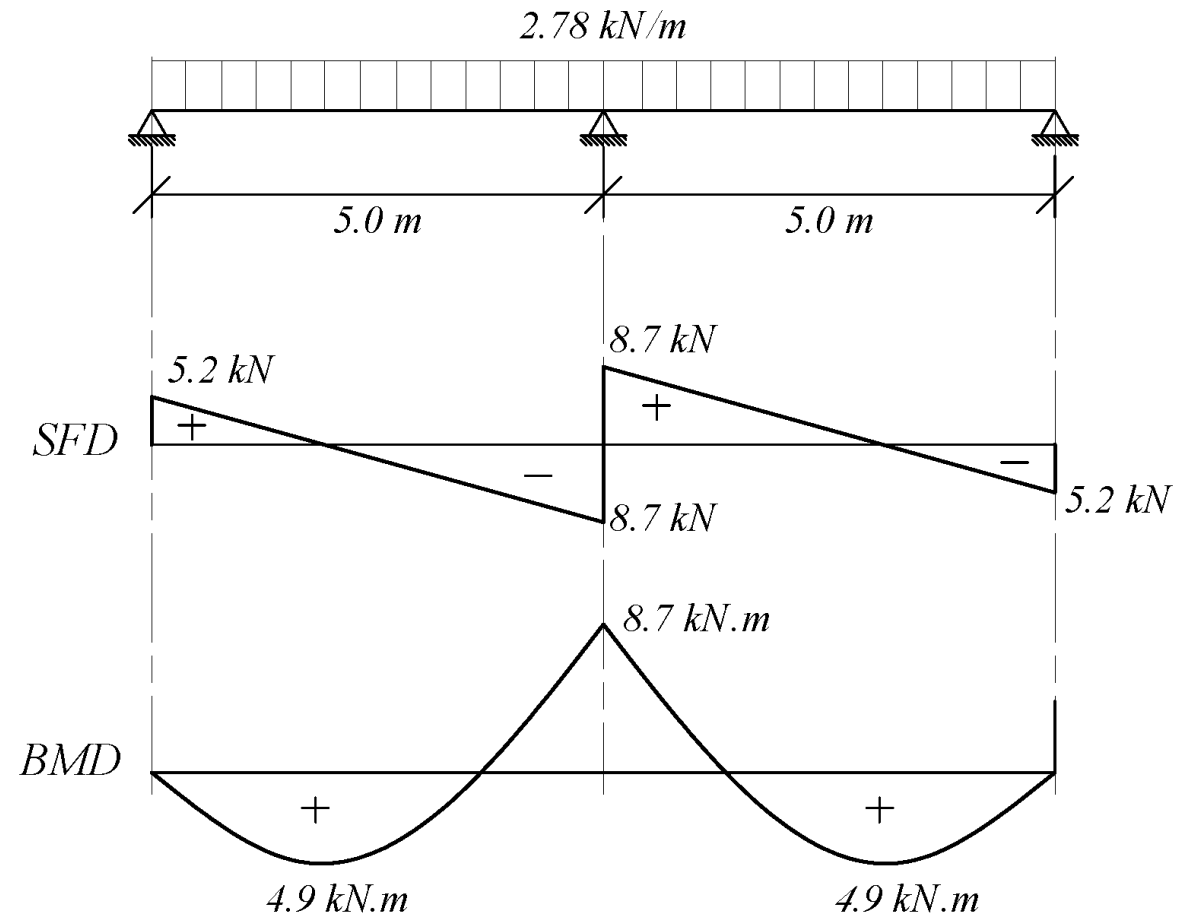


Figure 10.16: S.F.D and B.M.D for strip 1-1

Example 10.2

Strip 2-2:

For $M_u = -6.2 \text{ kN.m}$,

$$A_{s,min} = 0.0018(1000)(140) \\ = 252 \text{ mm}^2/\text{m},$$

use $\phi 10 \text{ mm @ } 300 \text{ mm}$.

For $M_u = 4.3 \text{ kN.m}$

$$A_{s,min} = 0.0018(1000)(140) \\ = 252 \text{ cm}^2/\text{m},$$

use $\phi 10 \text{ mm @ } 300 \text{ mm}$.

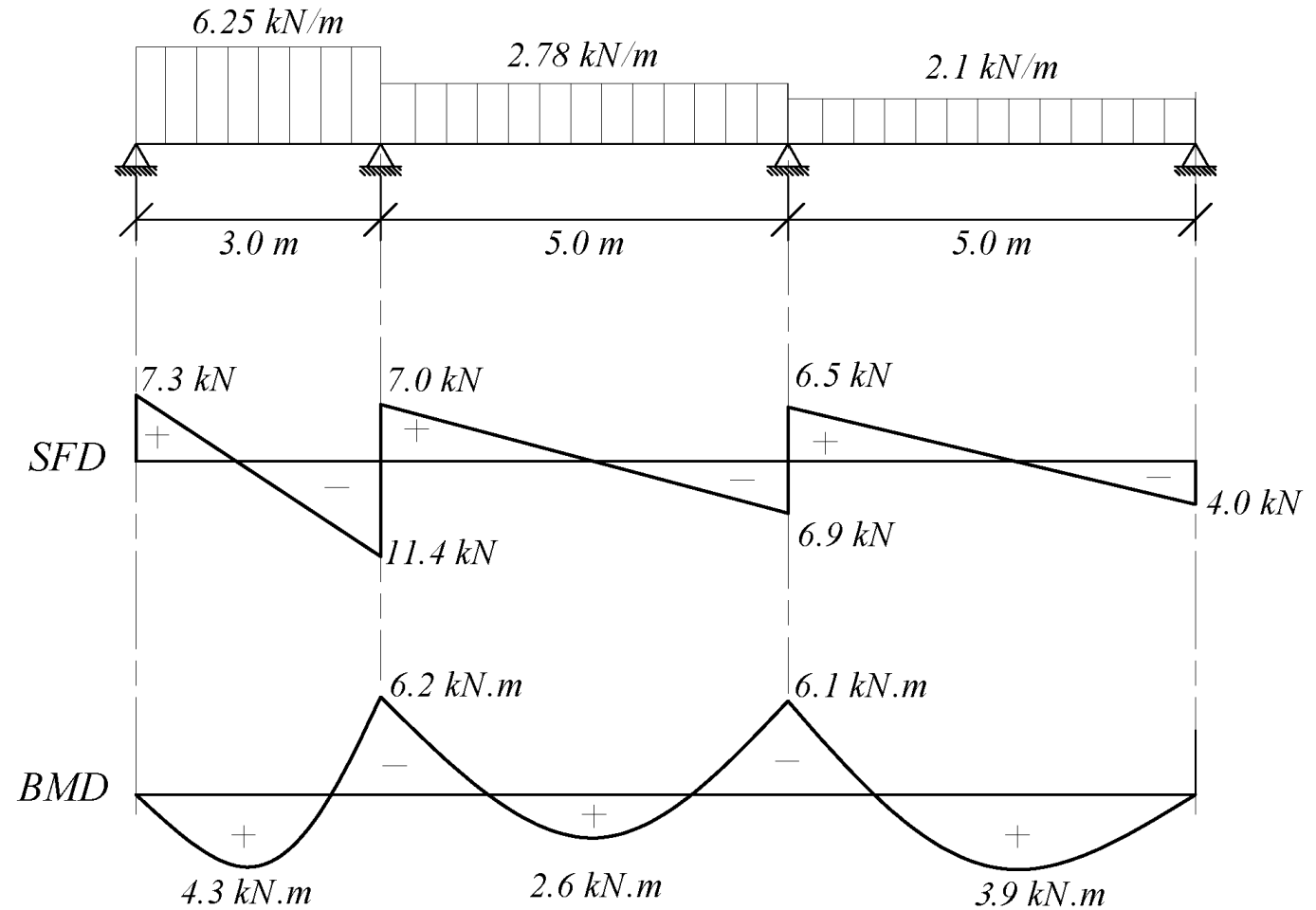


Figure 10.17: S.F.D and B.M.D for strip 2-2

Example 10.2

Strip 2-2:

For $M_u = -6.2 \text{ kN.m}$,

$$A_{s,min} = 0.0018(1000)(140) \\ = 252 \text{ mm}^2/\text{m},$$

use $\phi 10 \text{ mm @ } 300 \text{ mm}$.

For $M_u = 4.3 \text{ kN.m}$

$$A_{s,min} = 0.0018(1000)(140) \\ = 252 \text{ cm}^2/\text{m},$$

use $\phi 10 \text{ mm @ } 300 \text{ mm}$.

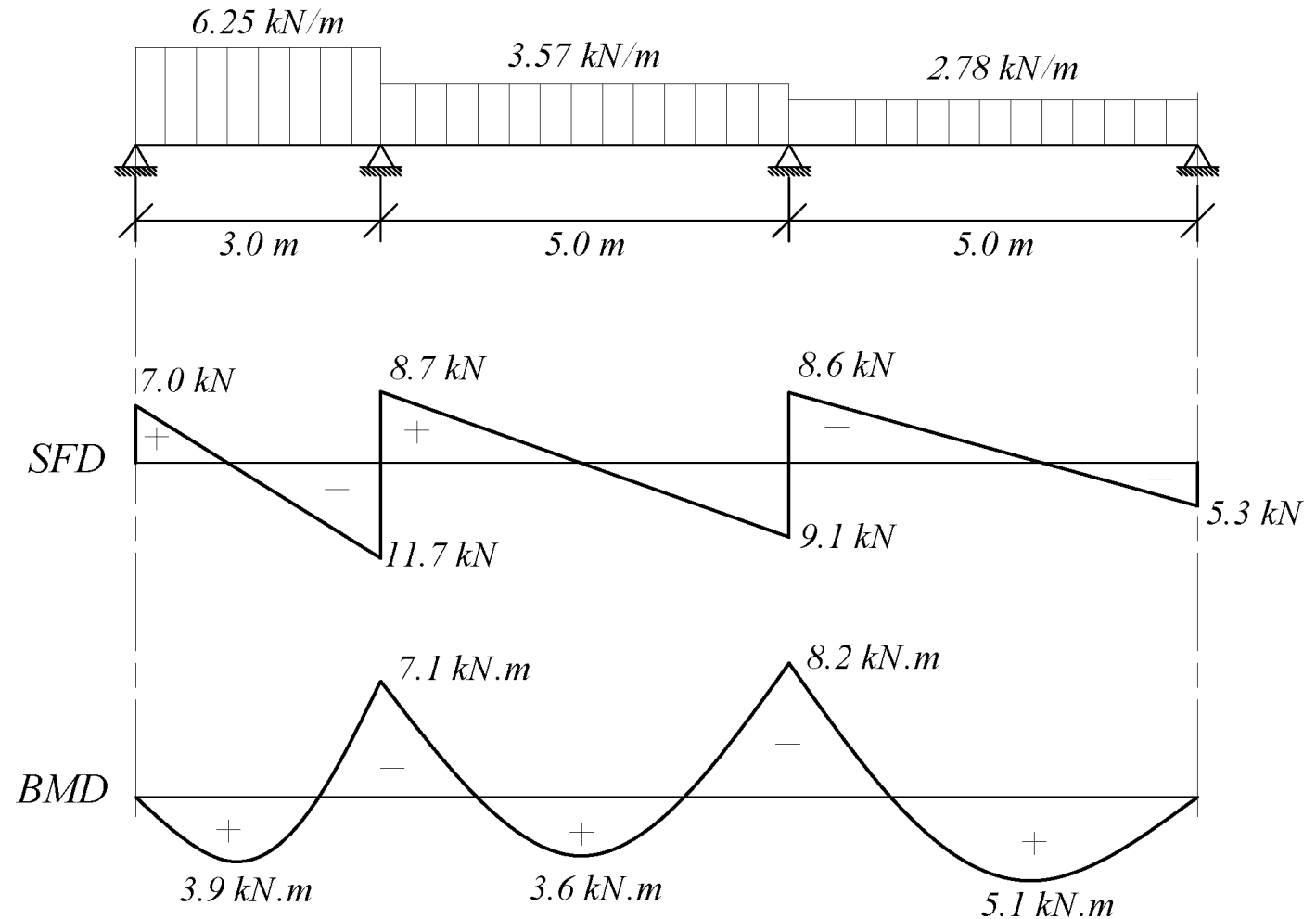


Figure 10.17: S.F.D and B.M.D for strip 3-3

Example 10.2

Strip 3-3:

For $M_u = -8.2 \text{ kN.m}$,

$$\rho = \frac{0.85(25)}{420} \left[1 - \sqrt{1 - \frac{2.353 \times 10^6 (8.2)}{(0.9)(1000)(108)^2 (25)}} \right]$$
$$= 0.00189$$

$A_{s,min} = 0.0018(1000)(140) = 252 \text{ mm}^2$
/m, use $\phi 10 \text{ mm @ } 300 \text{ mm}$.

For $M_u = 5.1 \text{ kN.m}$

$A_{s,min} = 0.0018(1000)(1400) = 252 \text{ mm}^2$
/m, use $\phi 10 \text{ mm @ } 300 \text{ mm}$.

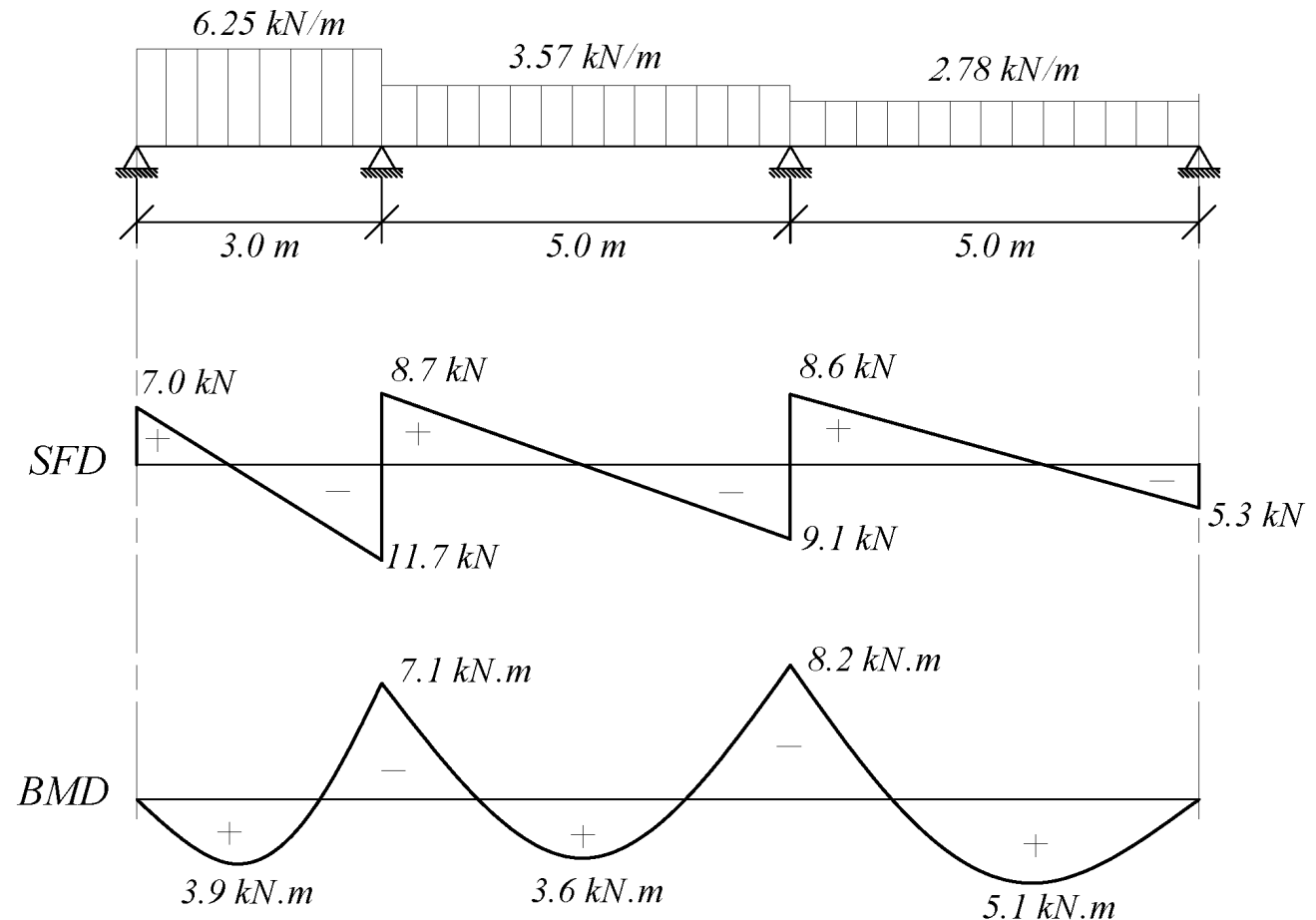


Figure 10.18: S.F.D and B.M.D for strip 3-3

Example 10.2

Strip 4-4:

For $M_u = -4.6 \text{ kN.m}$,

$A_{s,min} = 0.0018(1000)(140) = 252 \text{ mm}^2$
/m, use ϕ 10 mm @ 300 mm.

For $M_u = 2.6 \text{ kN.m}$

$A_{s,min} = 0.0018(100)(140) = 252 \text{ mm}^2$ /m
use ϕ 10 mm @ 300 mm.

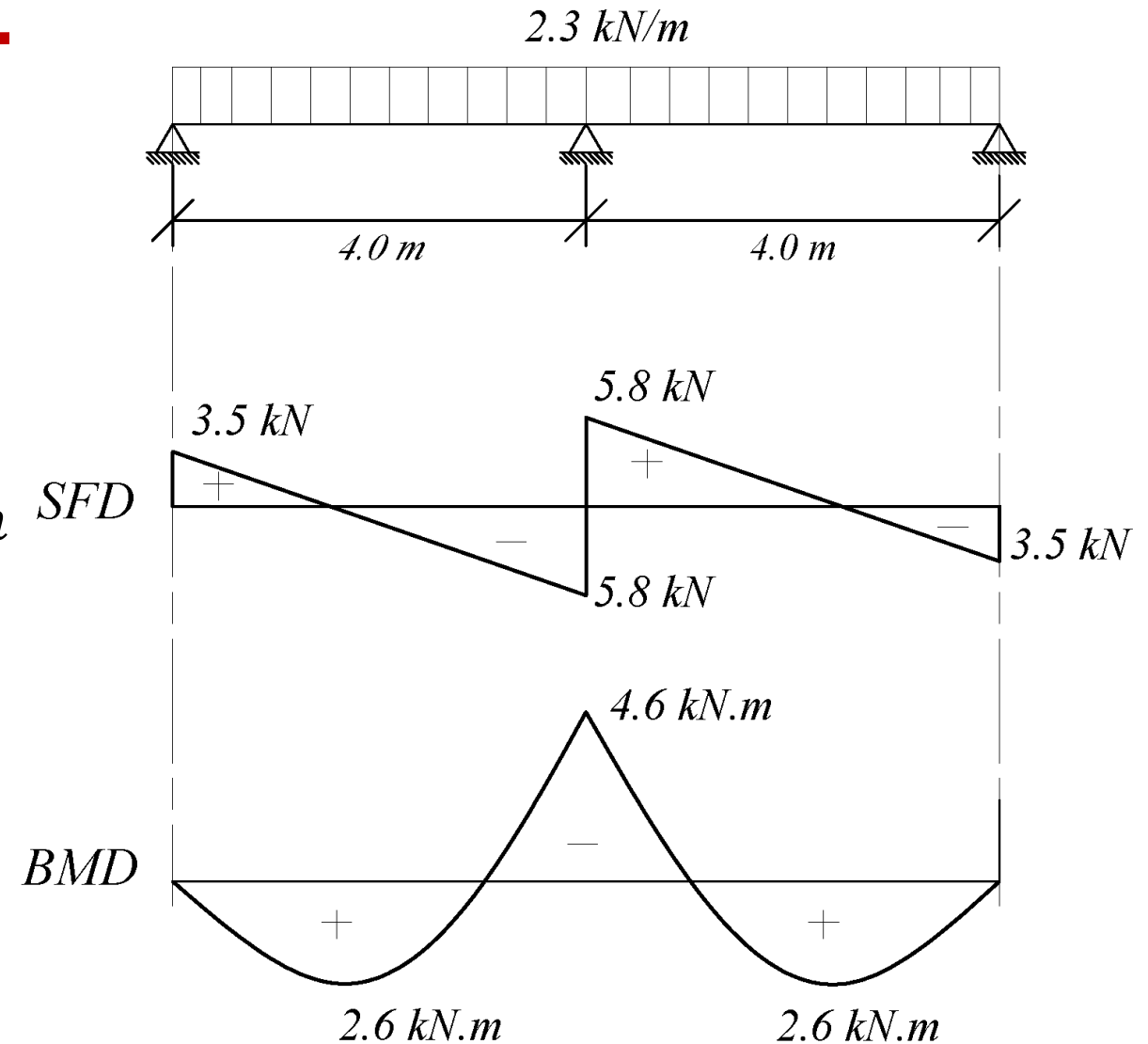


Figure 10.19: S.F.D and B.M.D for strip 4-4

Example 10.2

Strip 5-5:

For $M_u = -9.5 \text{ kN.m}$,

ρ

$$= \frac{0.85(25)}{420} \left[1 - \sqrt{1 - \frac{2.353 \times 10^6(9.5)}{(0.9)(1000)(108)^2(25)}} \right]$$

$$= 0.0022$$

$$A_s = 0.0022 (1000) (108) = 237 \text{ mm}^2/\text{m}$$

$$A_{s,min} = 0.0018(1000)(140) = 252 \text{ mm}^2/\text{m},$$

use ϕ 10 mm @ 300 mm.

For $M_u = 7.3 \text{ kN.m}$

$$A_{s,min} = 0.0018(1000)(140) = 252 \text{ mm}^2/\text{m},$$

use ϕ 10 mm @ 300 mm.

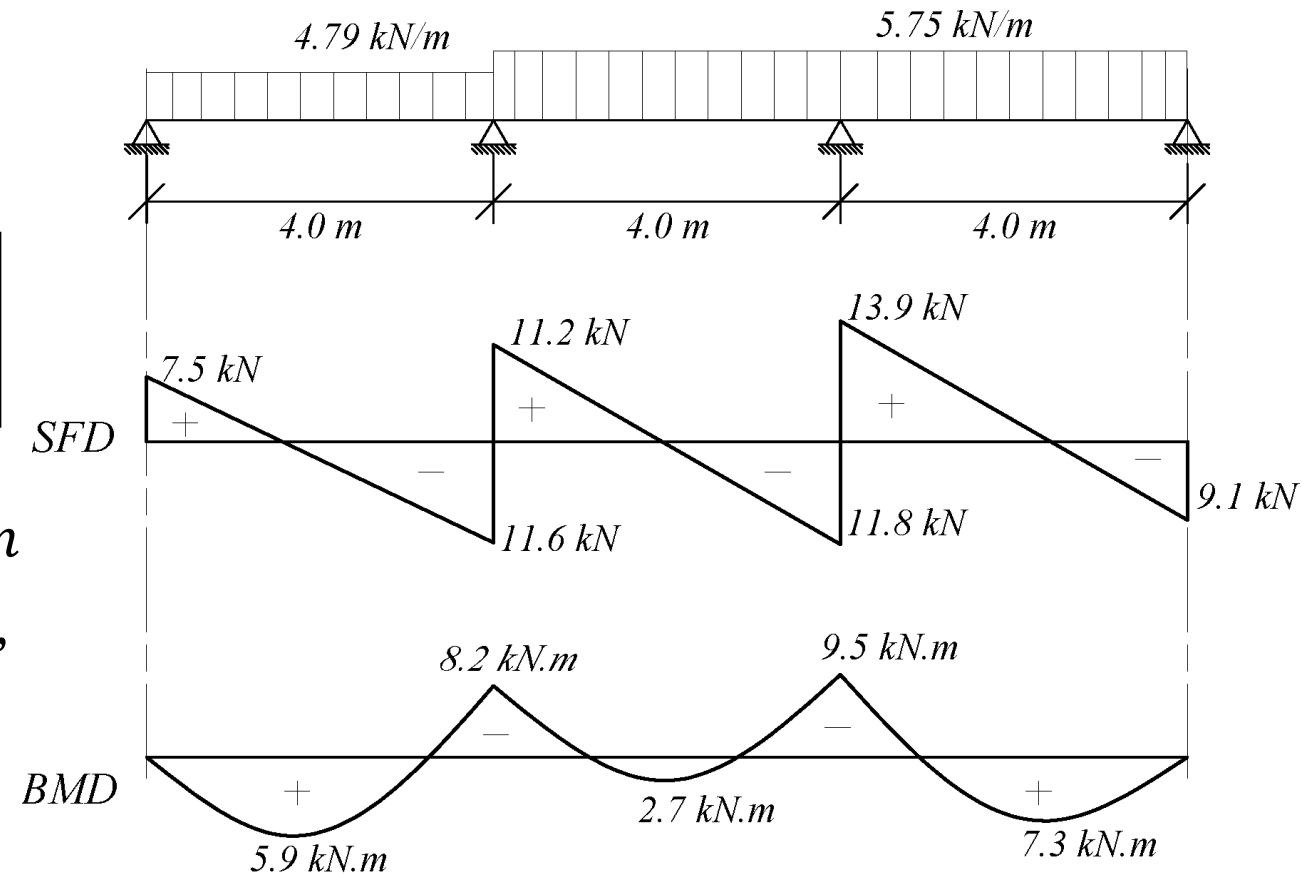


Figure 10.20: S.F.D and B.M.D for strip 5-5

Example 10.2

Strip 5-5:

For $M_u = -10.1 \text{ kN.m}$,

$$\rho = \frac{0.85(25)}{420} \left[1 - \sqrt{1 - \frac{2.353 \times 10^6(10.1)}{(0.9)(1000)(108)^2(25)}} \right]$$

$$= 0.00234$$

$$A_s = 0.00234 (1000) (108) = 253 \text{ mm}^2/\text{m}$$

$$A_{s,min} = 0.0018(1000)(140) = 252 \text{ mm}^2/\text{m}, \text{ use } \phi 10 \text{ mm @ } 300 \text{ mm}.$$

For $M_u = 7.0 \text{ kN.m}$

$$A_{s,min} = 0.0018(1000)(140) = 252 \text{ mm}^2/\text{m}, \text{ use } \phi 10 \text{ mm @ } 300 \text{ mm}.$$

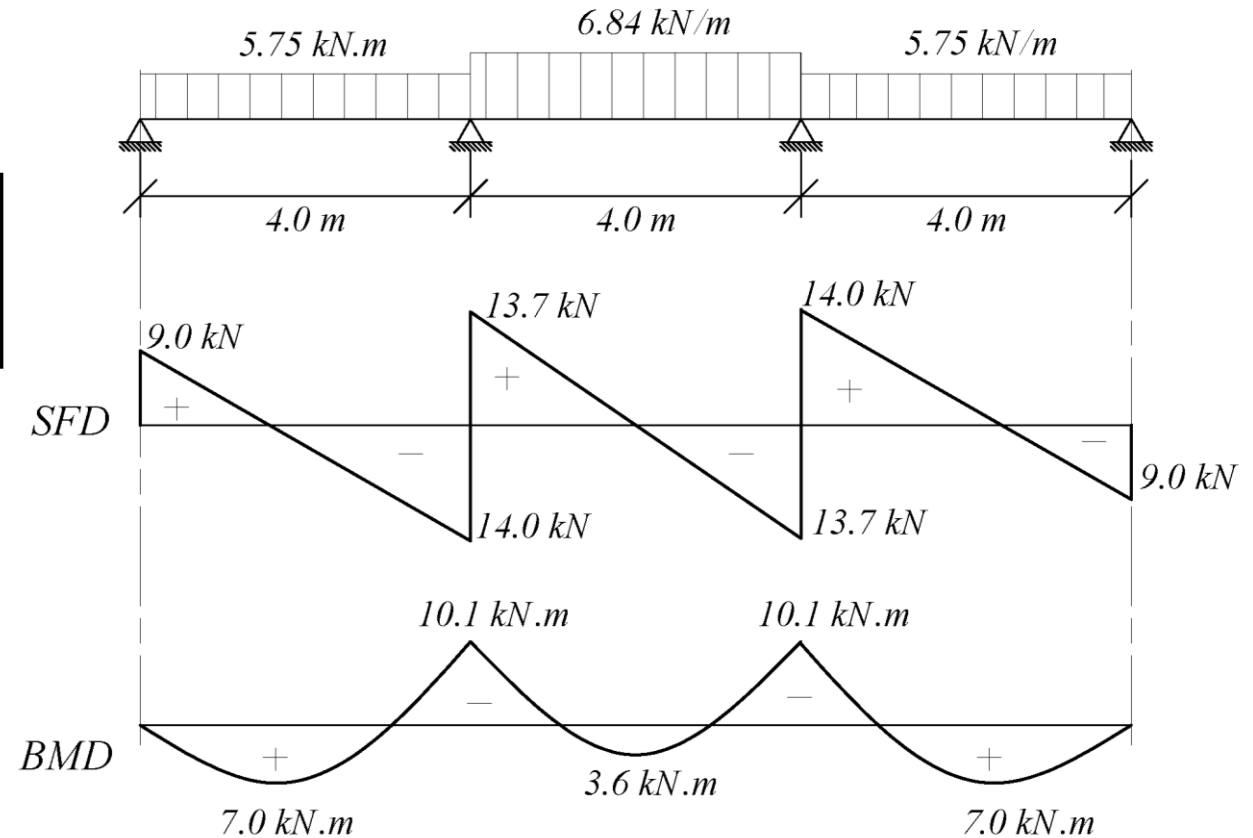


Figure 10.21: S.F.D and B.M.D for strip 6-6

Example 10.2

6. Prepare design drawings showing the dimensions and the provided reinforcement:

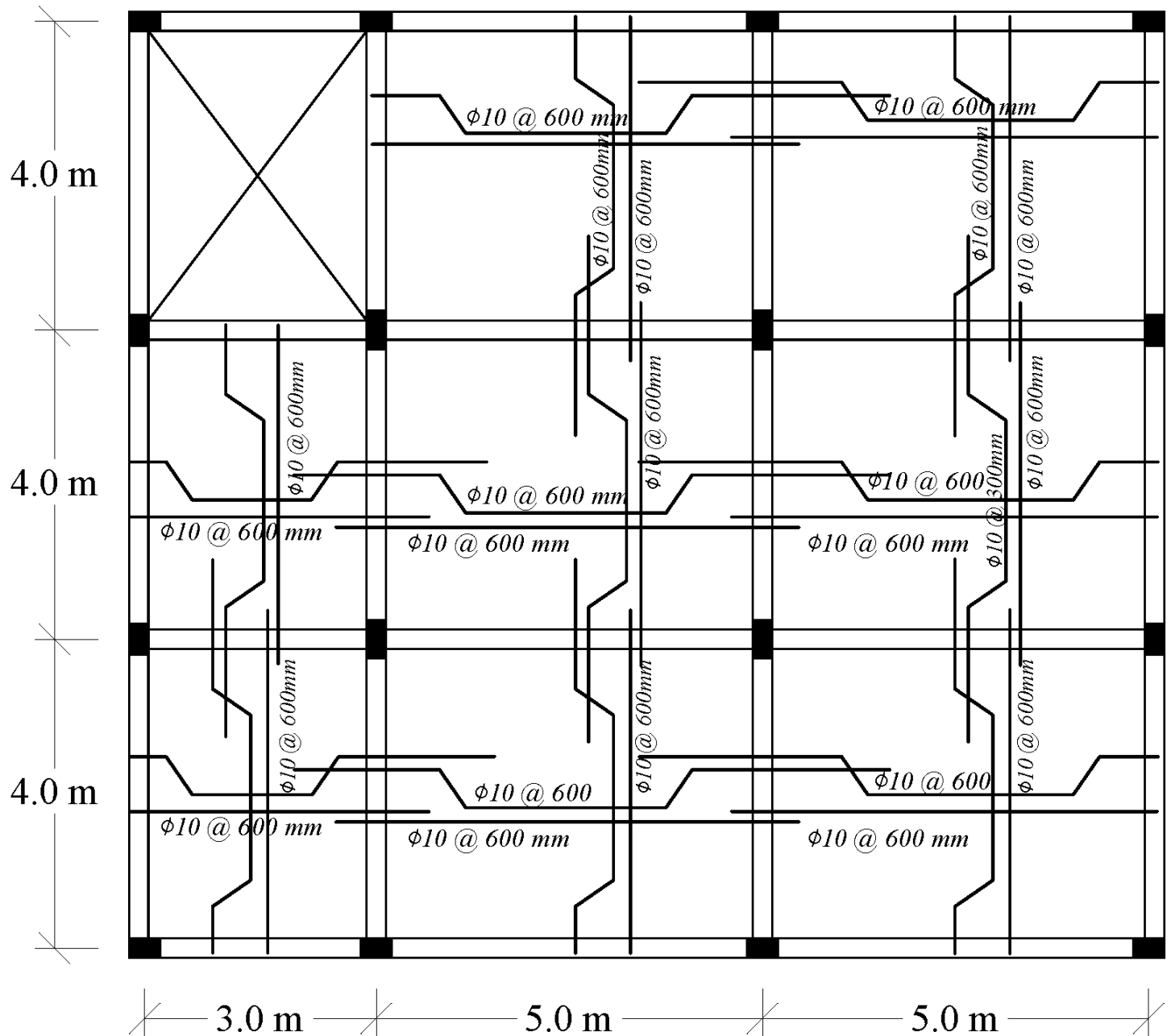


Figure 10.22: Reinforcement layout

Example 10.2

Load on beam *C3-C7-C11-C15*:

$w_u = 12.1 (4/2) (2) = 48.4 \text{ kN/m}$, as shown in Figure 10.23

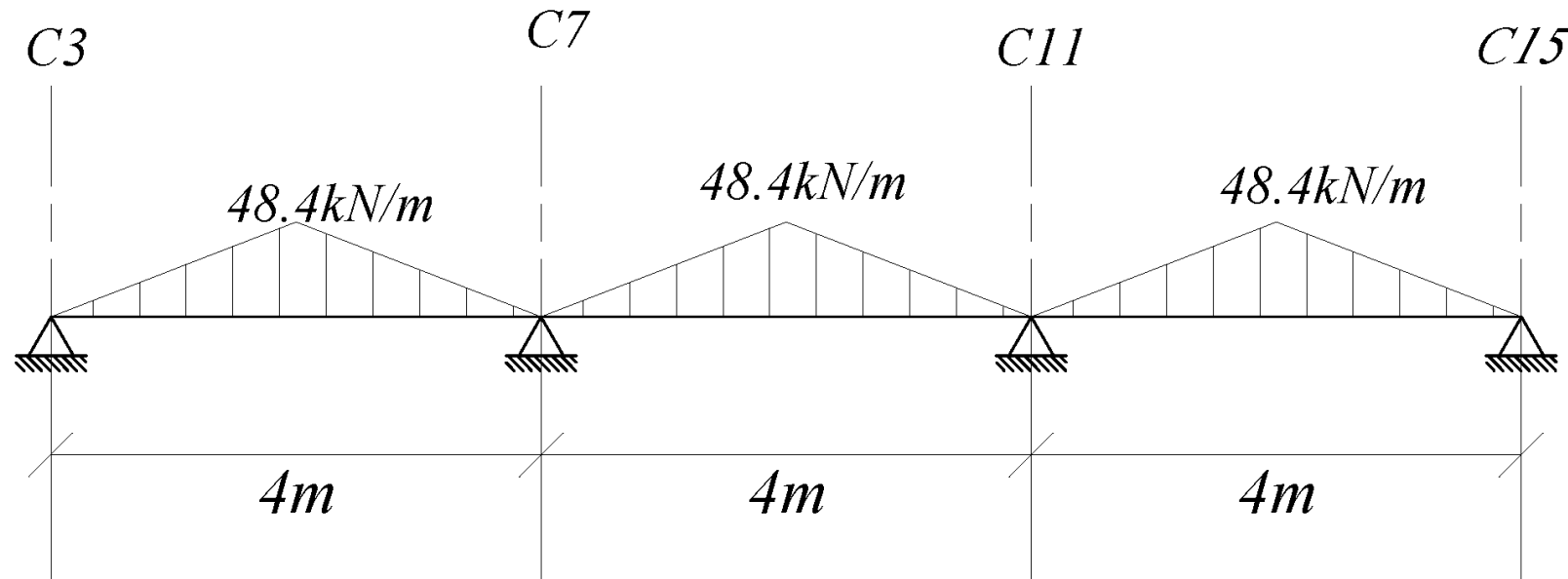


Figure 10.23: Tributary loads on beam C3-C7-C11-C13

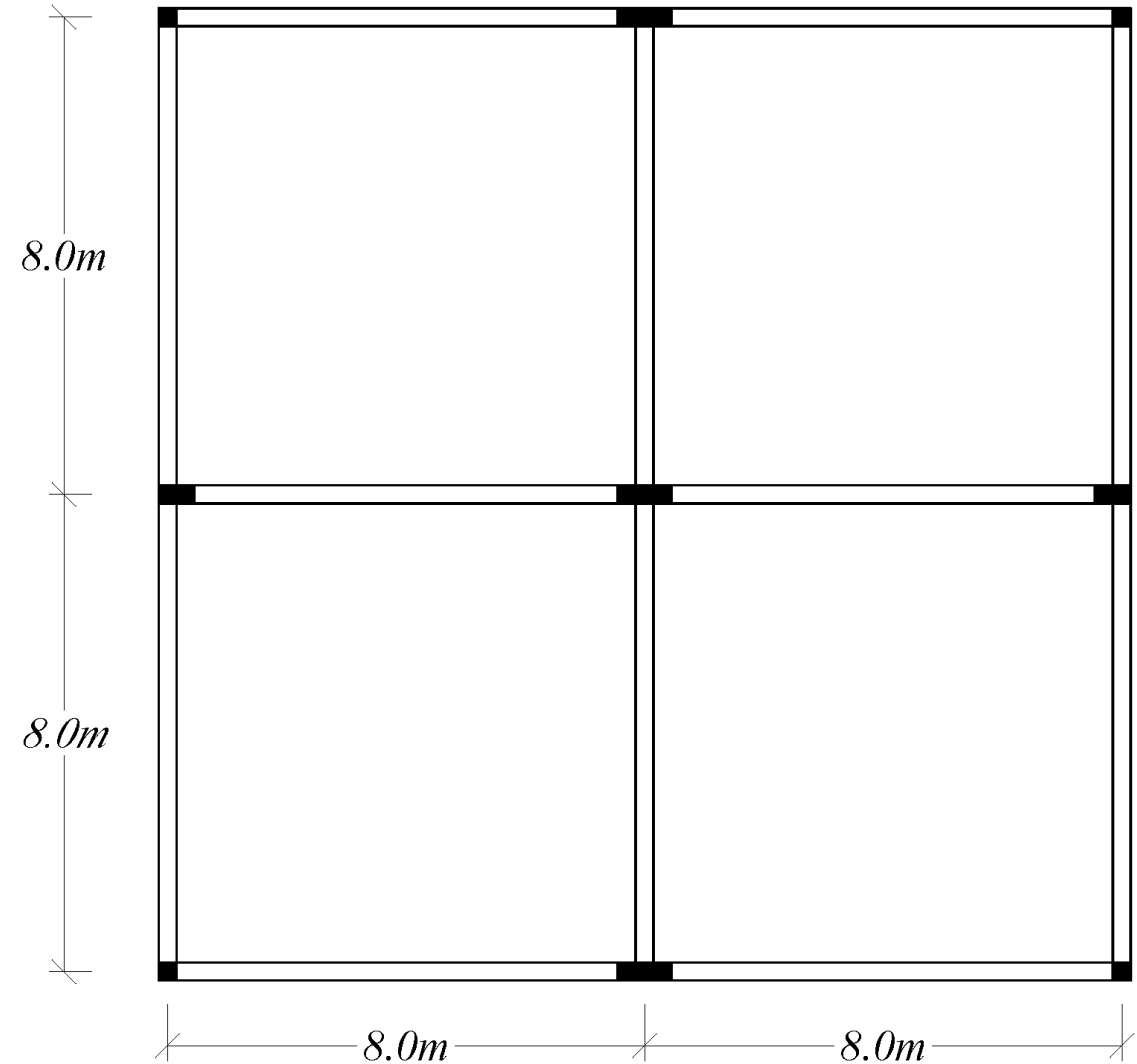
Example 10.3

Design the two-way ribbed slab shown in Figure 10.24. Assume the slab thickness is 230mm. The covering materials weigh 1.50 kN/m^2 , equivalent partition load is equal to 0.73 kN/m^2 , concrete hollow blocks are $400 \times 250 \times 170 \text{ mm}$ in dimension, each 0.17 kN in weight and the live load is 4 kN/m^2 .

Use $f'_c = 25 \text{ MPa}$ and $f_y = 420 \text{ MPa}$.

All beams are 300 mm wide.

Note: Live-load patterning need not be considered; assume that the specified live load is uniformly applied to all spans.



Example 10.3

Solution:

1. Evaluate overall slab thickness and key ribbed slab components:

Use an overall slab thickness of 230 *mm*.

Topping slab thickness = $230 - 170 = 60$ *mm*

Let width of web be equal to 120 *mm*

Area of shrinkage reinforcement A_s
 $= 0.0018(1000) (60) = 108$ mm^2/m

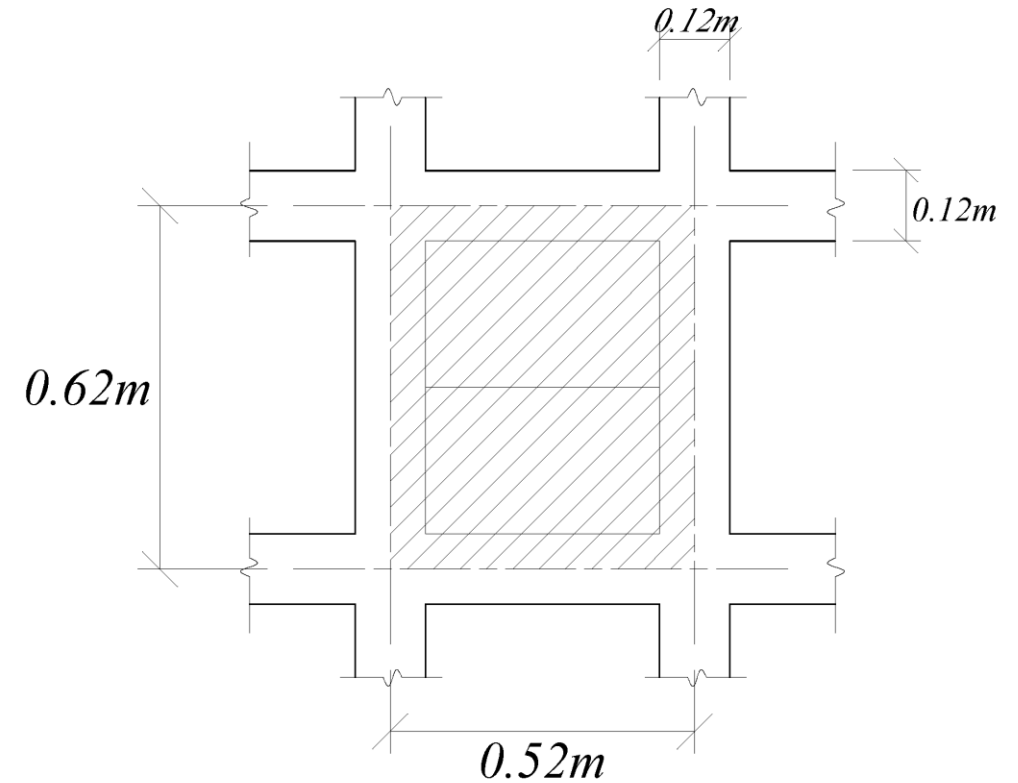
Use 5 ϕ 6 *mm/m* in both directions.

2. Determine the total factored load on the slab:

Load per rib, shown in Figure 10.25

Total volume (hatched) = $0.52 \times 0.62 \times 0.23 = 0.074152$ m^3

Volume of hollow blocks = $0.4 \times 0.17 \times 0.5 = 0.034$ m^3



Example 10.3

$$\text{Net concrete volume} = 0.074152 - 0.034 = 0.040152 \text{ m}^3$$

$$\text{Weight of concrete /m}^2 = \frac{(0.040152)(25)}{(0.52)(0.62)} = 3.11 \text{ kN/m}^2$$

$$\text{Weight of hollow blocks /m}^2 = \frac{2(0.17)}{0.52 \times 0.62} = 1.05 \text{ kN/m}^2$$

$$w_u = 1.20(3.11 + 1.05 + 0.73 + 1.5) + 1.60(4.0) = 14.07 \text{ kN/m}^2$$

1. Determine load distributions in the two principal directions:

Load distribution according to simplified coefficients method:

$$r = (8 \times 0.87) / (8 \times 0.87) = 1.0$$

$$\alpha = \beta = 0.396$$

$$w_1 = w_2 = 0.396 (14.07) = 5.57 \text{ kN/m}^2$$

$$\text{Load per rib in the first direction: } w_u/\text{rib} = 5.57 (62/100) = 3.45 \text{ kN/m}$$

$$\text{Load per rib in the second direction: } w_u/\text{rib} = 5.57 (52/100) = 2.9 \text{ kN/m}$$

Example 10.3

4- Select representative design strips in the two principal directions:

5- Draw shear force and bending moment diagrams:

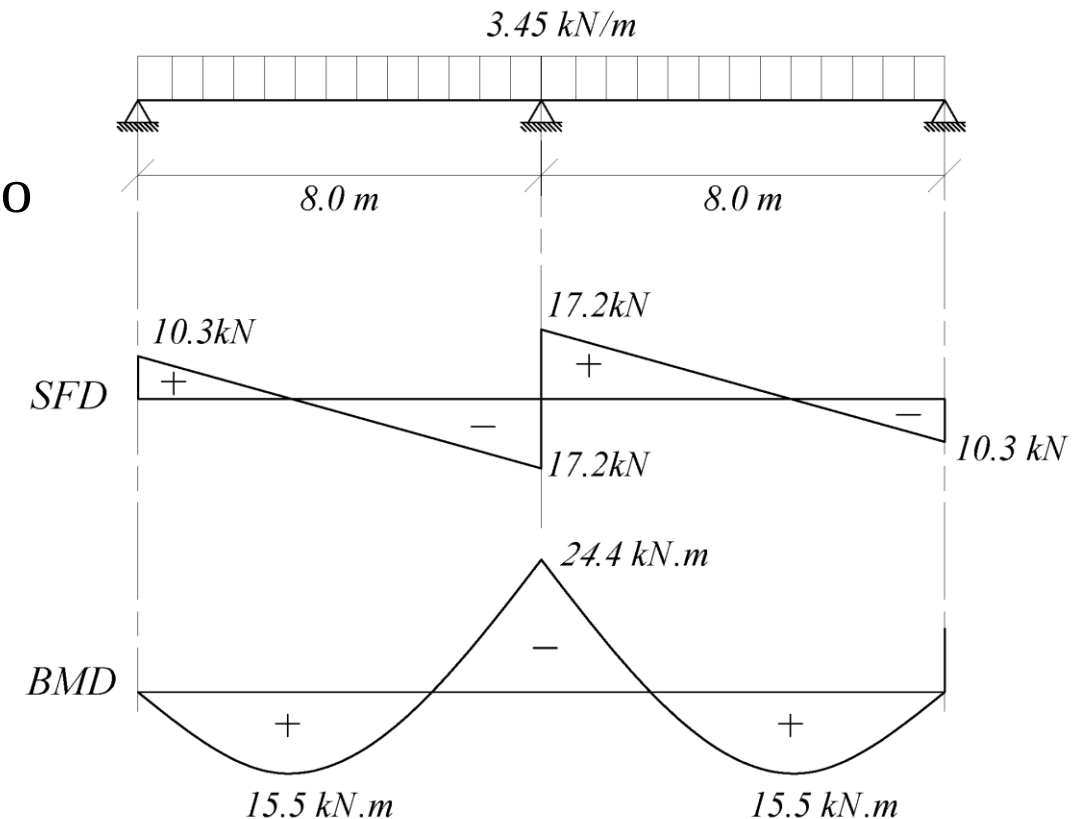
Using structural analysis software, one gets the shown SFD and BMD for a rib in **Horizontal direction**

6. Check web width for beam shear:

For reinforcement in the first direction (the closest to the outside surface of concrete):

Effective depth $d = 230 - 20 - 6.0 - 8 = 196 \text{ mm}$,
assuming $\phi 16 \text{ mm}$ reinforcing bars and $\phi 6 \text{ mm}$ stirrups.

$$\begin{aligned} 1.1\phi V_c &= (1.1)(0.17) \phi \lambda \sqrt{f'_c} b_w d \\ &= (1.1)(0.17)(0.75)(1.0)\sqrt{30}(120)(196)/1000 \\ &= 18.1 \text{ kN} > 17.2 \text{ kN} \quad \text{OK} \end{aligned}$$



Example 10.3

7- Design rib reinforcement:

Horizontal direction

Positive moment reinforcement:

Assuming that $a < 60 \text{ mm}$, as shown in Figure 10.27. The compressive force in concrete can be calculated as follows:

$$C = 0.85 (f'_c) (b_e) (a)$$

$$C = 0.85 (30) (620) (a) / 1000 = 15.81 a \text{ kN}$$

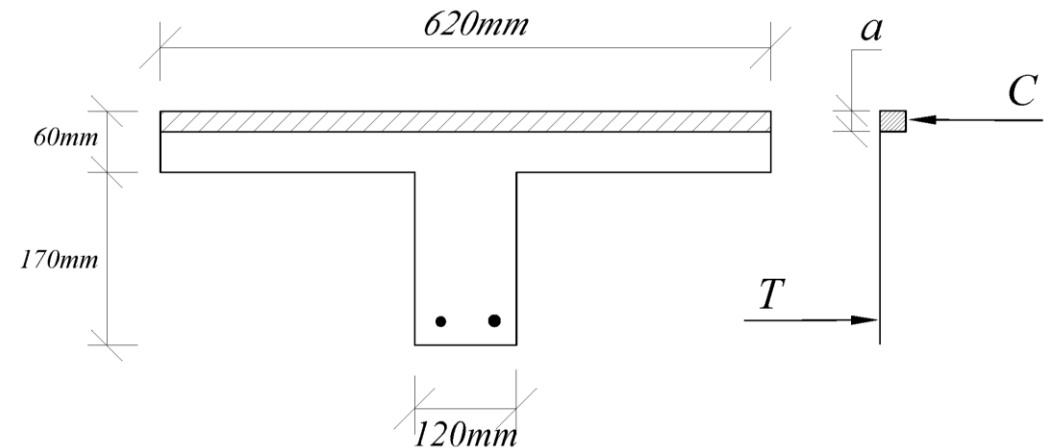
$$M_u = \phi M_n = \phi (C) (d - a/2)$$

$$15.5 \times 1000 = 0.9 (15.81 a) (196 - a/2)$$

$$a^2 - 392 a + 544.66 = 0$$

Solving this equation gives $a = 5.64 \text{ mm}$, which means that the assumption made before is valid, and the other root is too large to be considered.

From equilibrium, $C = T$



Example 10.3

Area of flexural reinforcement

$$A_s = \frac{15.81 (5.64) (1000)}{420} = 212.31 \text{ mm}^2$$

$$A_{s,min} = 0.0033 \times 120 \times 196 = 77.6 \text{ mm}^2 < A_s$$

Use 2 ϕ 12 mm reinforcing bars in each rib.

Negative moment reinforcement:

Since the factored negative moment creates compression in the web, the section at corresponding location is to be designed as a rectangular section.

For $M_u = -24.4 \text{ kN.m}$

$$\rho = \frac{0.85(30)}{420} \left[1 - \sqrt{1 - \frac{2.353 \times 10^6 (27.6)}{(0.9)(120)(196)^2 (30)}} \right] = 0.0187$$

$$A_s = 0.0187 (120) (196) = 439.8 \text{ mm}^2, \text{ use 1 } \phi 16\text{mm and 1 } \phi 18\text{mm per rib.}$$

Example 10.3

8- Design rib **Vertical direction**

Using structural analysis software, one gets the shown SFD and BMD

Positive moment reinforcement:

$$C = 0.85 (f'_c) (b_e) (a)$$

$$C = 0.85 (30) (520) (a) / 1000 = 13.26 a \text{ kN}$$

$$M_u = \phi M_n = \phi (C) (d - a/2)$$

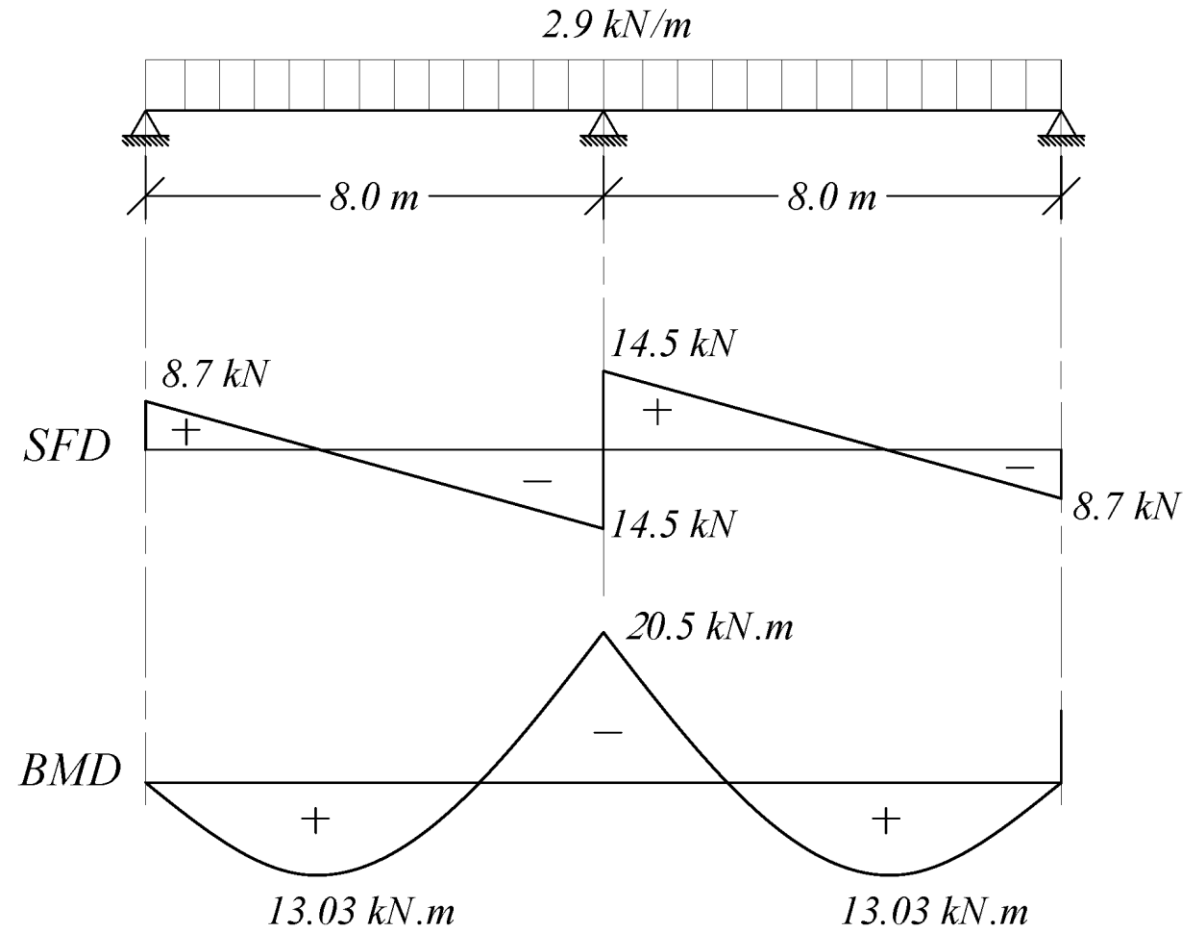
$$13.03 \times 1000 = 0.9 (13.26 a) (196 - a/2)$$

$$a^2 - 392 a + 2183.68 = 0$$

Solving this equation gives

$$a = 5.65 \text{ mm} < 60 \text{ mm. OK}$$

From equilibrium, $C = T$



Example 10.3

Area of flexural reinforcement

$$A_s = \frac{13.26 (5.65) (1000)}{420} = 178.4 \text{ mm}^2$$

$$A_{s,min} = 0.0033 \times 120 \times 196 = 77.6 \text{ mm}^2 < A_s$$

Use 2 ϕ 12 mm reinforcing bars in each rib.

Negative moment reinforcement:

Since the factored negative moment creates compression in the web, the section at corresponding location is to be designed as a rectangular section.

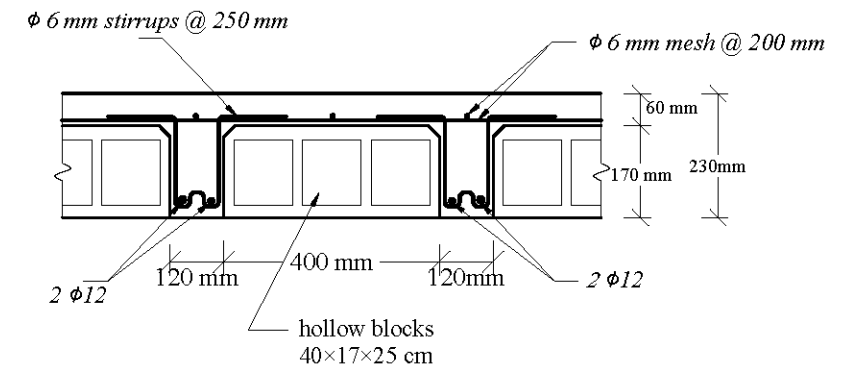
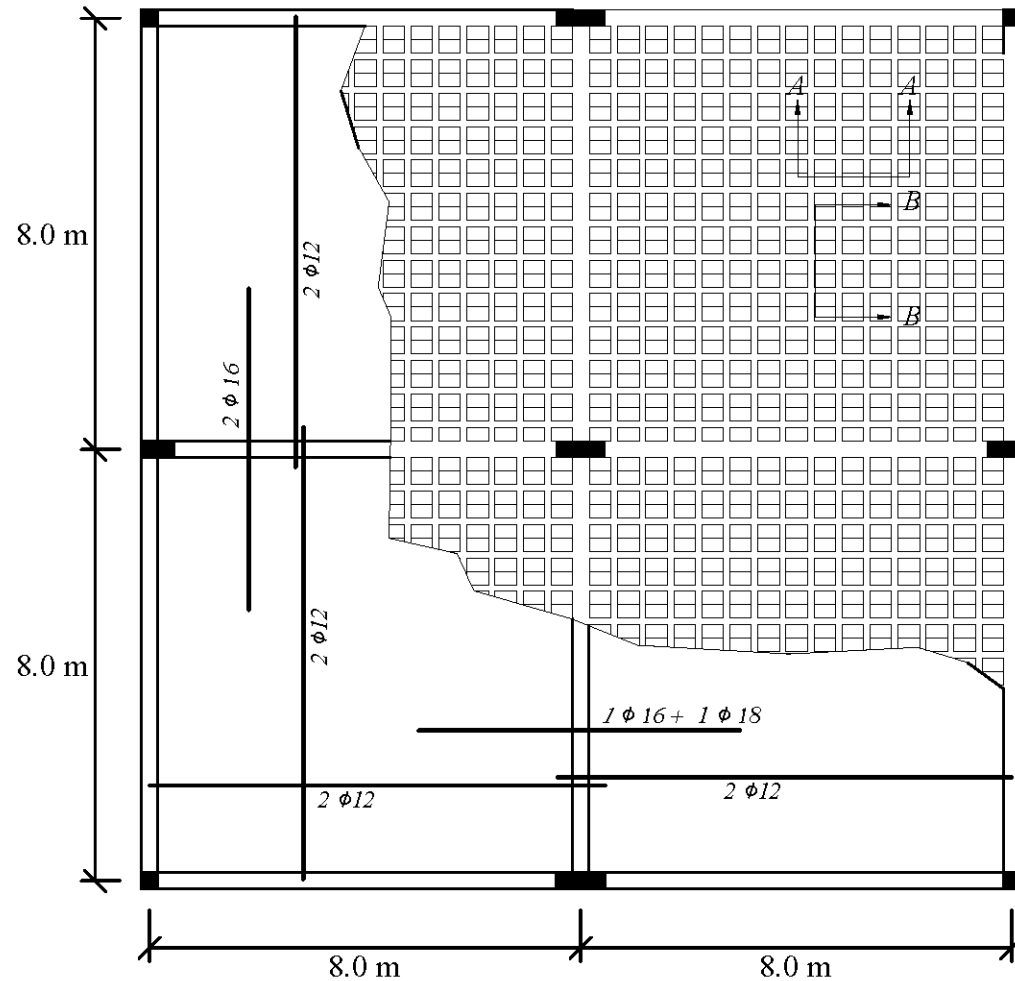
For $M_u = -20.5 \text{ kN.m}$

$$\rho = \frac{0.85(30)}{420} \left[1 - \sqrt{1 - \frac{2.353 \times 10^6 (20.5)}{(0.9)(120)(196)^2 (30)}} \right] = 0.0136$$

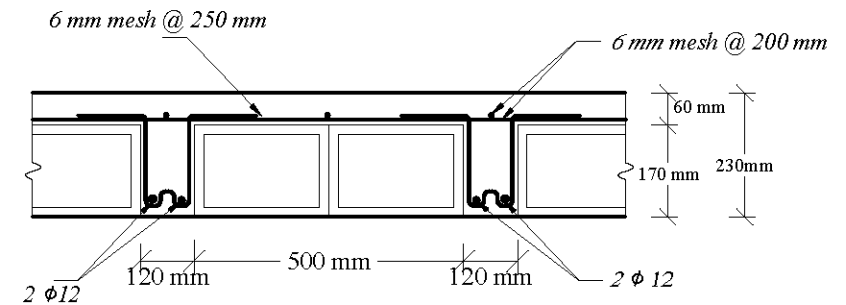
$$A_s = 0.0136 (120) (196) = 316.6 \text{ mm}^2, \text{ use } 2 \phi 16 \text{ mm per rib.}$$

Example 10.3

8- Prepare design drawings:



Section A-A



Section B-B

10.3.2 Minimum Slab Thickness of Two-way Edge-supported Slabs

As specified by *ACI Code 8.3*, for slabs with beams spanning between the supports on all sides, the minimum thickness shall be as follows:

For α_{f_m} equal to or less than 0.20, minimum slab thickness is determined from provisions on slabs without interior beams spanning between the supports, shown in Table 10.8, but not less than:

- Slabs without drop panels 125 mm
- Slabs with drop panels 100 mm

TABLE 10.8
Minimum thickness of slabs without interior beams

f_y MPa	Without drop panels			With drop panels		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams		Without edge beams	With edge beams	
280	$l_n/33$	$l_n/36$	$l_n/36$	$l_n/36$	$l_n/40$	$l_n/40$
420	$l_n/30$	$l_n/33$	$l_n/33$	$l_n/33$	$l_n/36$	$l_n/36$
520	$l_n/27$	$l_n/30$	$l_n/30$	$l_n/30$	$l_n/33$	$l_n/33$

10.3.2 Minimum Slab Thickness of Two-way Edge-supported Slabs

l_n is the length of clear span in the long direction measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.

α_f = ratio of the flexural stiffness of the beam section to the flexural stiffness of a width of slab bounded laterally by centerlines of adjacent panels: $\alpha_f = \frac{E_{cb}I_b}{E_{cs}I_s}$

- a) For α_{f_m} greater than 0.20 but not greater than 2.0, the thickness shall not be less than

$$h = \frac{l_n (0.80 + f_y/1400)}{36 + 5\beta (\alpha_{f_m} - 0.20)} \geq 125 \text{ mm} \quad (10.17)$$

- b) For α_{f_m} greater than 2.0, the thickness shall not be less than

$$h = \frac{l_n (0.80 + f_y/1400)}{36 + 9\beta} \geq 90 \text{ mm} \quad (10.18)$$

- c) At discontinuous edges, an edge beam shall be provided with a stiffness ratio α_f not less than 0.80 or the minimum thickness required by *Eqn. (10.17)* or *Eqn. (10.18)* shall be increased by at least 10 % in the panel with a discontinuous edge.

10.3.2 Minimum Slab Thickness of Two-way Edge-supported Slabs

ACI Code 318-89 (Revised 92) included three different equations for determination of minimum slab thickness. Two of those equations will be given here for their simplicity, as they require no prior evaluation of α . The smallest thickness h_{min} is given as

$$h_{min} = \frac{l_n (0.80 + f_y/1400)}{36 + 5\beta (1 + \beta_s)} \quad (10.19)$$

The largest thickness h_{max} is given as

$$h_{max} = \frac{l_n (0.80 + f_y/1400)}{36} \quad (10.20)$$

where $\beta = \frac{L_n (Long\ side)}{L_n (Short\ side)}$

β_s = ratio of length of the continuous edges to the total perimeter of the panel.

Example (10.4):

Determine the minimum and maximum slab thickness according to ACI 318-89 for the slabs in Examples 10.1 through Example 10.3.

a) Slab in Example 10.1

$$l_n = 5.0 - 0.25 = 4.75 \text{ m}$$

Since $r = 5/4 = 1.25 < 2.0$, the slab is classified as being two way.

The smallest slab thickness is given by

$$h_{min} = \frac{l_n (0.80 + f_y/140)}{36 + 5\beta (1 + \beta_s)} = \frac{4750 (0.80 + 420/140)}{36 + 5(4.75/3.75)(1 + 0)} = 123.4 \text{ mm}$$

The largest slab thickness is given by

$$h_{max} = \frac{l_n (0.80 + f_y/1400)}{36} = \frac{4750 (0.80 + 420/1400)}{36} = 145.1 \text{ mm}$$

Slab thickness will be taken as 130.0 mm.

Example (10.4):

b) Slab in Example 10.2

The smallest slab thickness is given by

$$\beta = (5.0 - 0.25)/(4 - 0.25) = 1.27$$

$$h_{min} = \frac{l_n (0.80 + f_y/1400)}{36 + 5\beta (1 + \beta_s)} = \frac{4750 (0.80 + 420/1400)}{36 + 5(1.27)(1 + 0)} = 123.4 \text{ mm}$$

The largest slab thickness is given by

$$h_{max} = \frac{l_n (0.80 + f_y/1400)}{36} = \frac{4750(0.80 + 420/1400)}{36} = 145.1 \text{ mm}$$

Use a slab thickness of 140.0 mm.

Example (10.4):

b) Slab in Example 10.3

The smallest slab thickness is given by

$$h_{min} = \frac{l_n (0.80 + f_y/1400)}{36 + 5\beta (1 + \beta_s)} = \frac{7700 (0.80 + 420/14000)}{36 + 5(1.0) (1 + 0.5)} = 194.7 \text{ mm}$$

The largest slab thickness is given by

$$h_{max} = \frac{l_n (0.80 + f_y/1400)}{36} = \frac{7700 (0.80 + 420/1400)}{36} = 235.3 \text{ mm}$$

Use an overall slab thickness of 230.0 *mm*.

10.3.3 Direct Design Method DDM (Based on ACI 318-14)

The direct design method (DDM) consists of certain steps for distributing moments to slab and beam sections to satisfy safety requirements and most serviceability requirements simultaneously. Three basic steps are involved:

- a) Determination of the total factored static moment;
- b) Distribution of the total factored static moment to negative and positive sections;
- c) Distribution of negative and positive factored moments to the column and middle strips and to the beams if any.

10.3.3 Direct Design Method DDM (Based on ACI 318-14)

10.3.3.1 Limitations

The slab system, to be designed using the direct design method, should conform to the following limitations as given by the ACI code:

1. There shall be three continuous spans in each direction.
2. Slab panels shall be rectangular, with the **ratio of longer to shorter panel dimensions** measured center-to-center of supports, **not to exceed 2.0**.
3. Successive **span lengths** measured center-to-center of supports, in each direction shall **not differ by more than one-third** of the longer span.
4. **Column offset shall not exceed 10 % of the span** in the direction of offset from either axis between centerlines of successive columns.
5. **All loads shall be due to gravity only and uniformly** distributed over an entire panel.
6. Unfactored live load shall not exceed 2 times the unfactored dead load.

10.3.3 Direct Design Method DDM (Based on ACI 318-14)

10.3.3.1 Limitations _ Continue

7. For a panel with beams between supports on all sides, the relative stiffness of beams in two perpendicular directions $\frac{\alpha_{f_1} l_2^2}{\alpha_{f_2} l_1^2}$ is not to be less than 0.20 and not greater than 5.0.

where

$$\alpha_{f_1} = \frac{\alpha'_1 + \alpha'_3}{2} \text{ in direction of } l_1$$

$$\alpha_{f_2} = \frac{\alpha'_2 + \alpha'_4}{2} \text{ in direction of } l_2$$

as shown in Figure 10.29

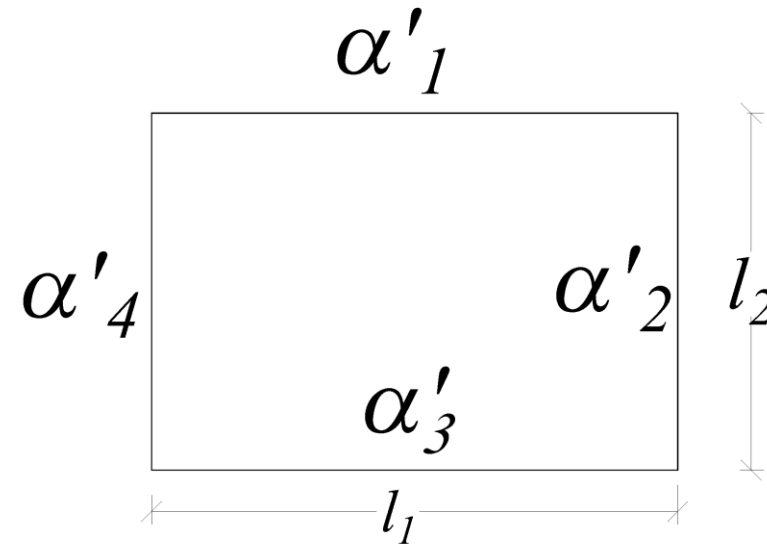


Figure 10.29: Panel bounded by four beams

10.3.3 Direct Design Method DDM (Based on ACI 318-14)

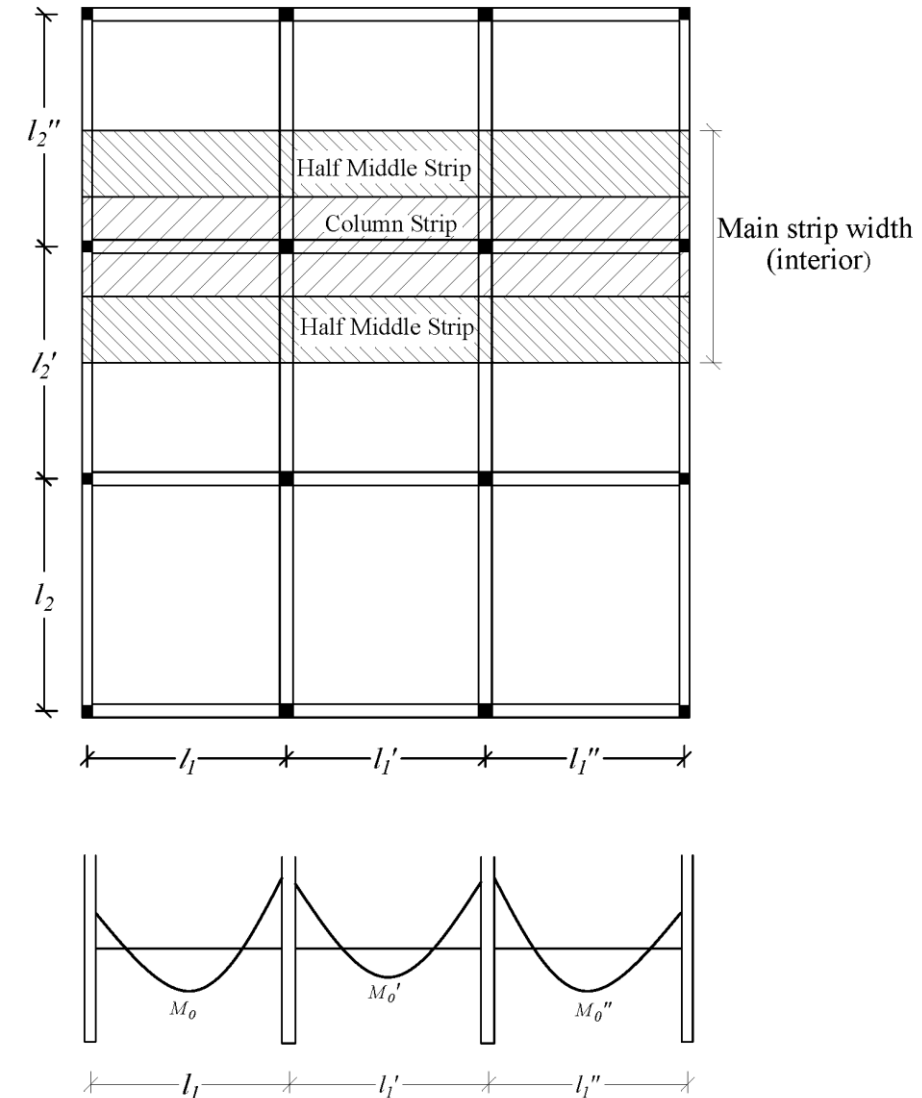
10.3.3.1 Design Procedure

1 Determination of the total factored static moment:

- ✓ **Total factored** static moment, M_o for a span is determined for a strip bounded laterally by the panel centerline on each side of the centerline of supports.
- ✓ **The absolute sum of positive and average negative** factored moments in each direction is not to be less than

$$M_o = q_u l_2 l_n^2 / 8$$

- ✓ Where l_n is **clear span** in the direction moments are being considered, measured face-to-face of supports
- ✓ l_2 taken as the average of adjacent transverse spans.



10.3.3 Direct Design Method DDM (Based on ACI 318-14)

2 Distribution of the total factored static moment to negative and positive moments:

For an interior span, total factored static moment is distributed in such a way that the negative factored moment at face of support is taken as $0.65M_o$, and positive factored moment at midspan is taken as $0.35M_o$.

In end spans, total factored static moment M_o is distributed as shown in Table 10.9 and Figure 10.31.

	(1)	(2)	(3)		(4)
			Slabs without beams between interior supports		
	Exterior edge unrestrained	Beams between all supports	Without an edge beam	With an edge beam	Exterior edge fully restrained
Interior negative factored moment	0.75	0.70	0.70	0.70	0.65
Positive factored moment	0.63	0.57	0.52	0.50	0.35
Exterior negative factored moment	0.00	0.16	0.26	0.30	0.65

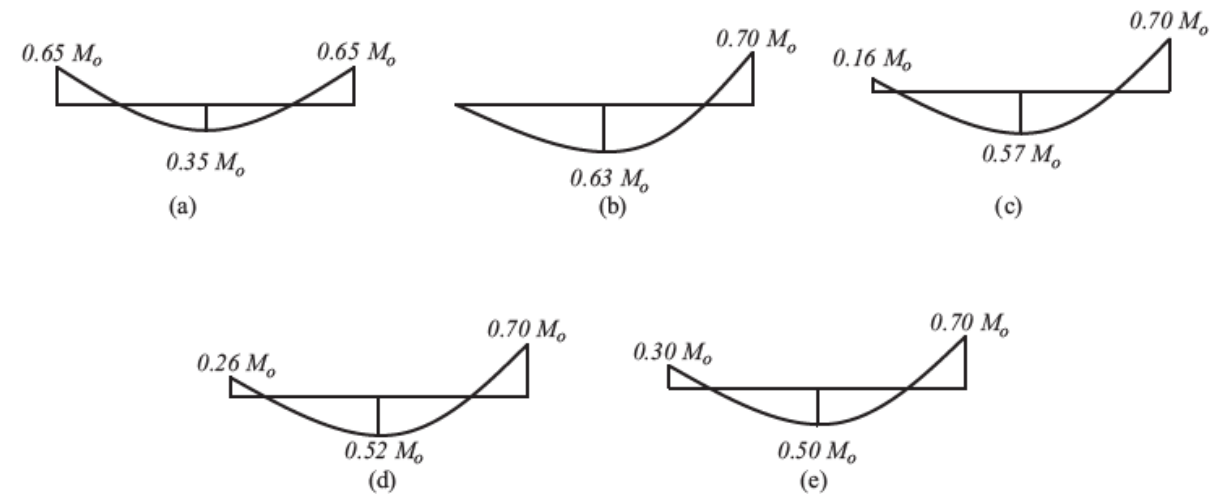


Figure 10.31: Distribution of factored moment M_o .

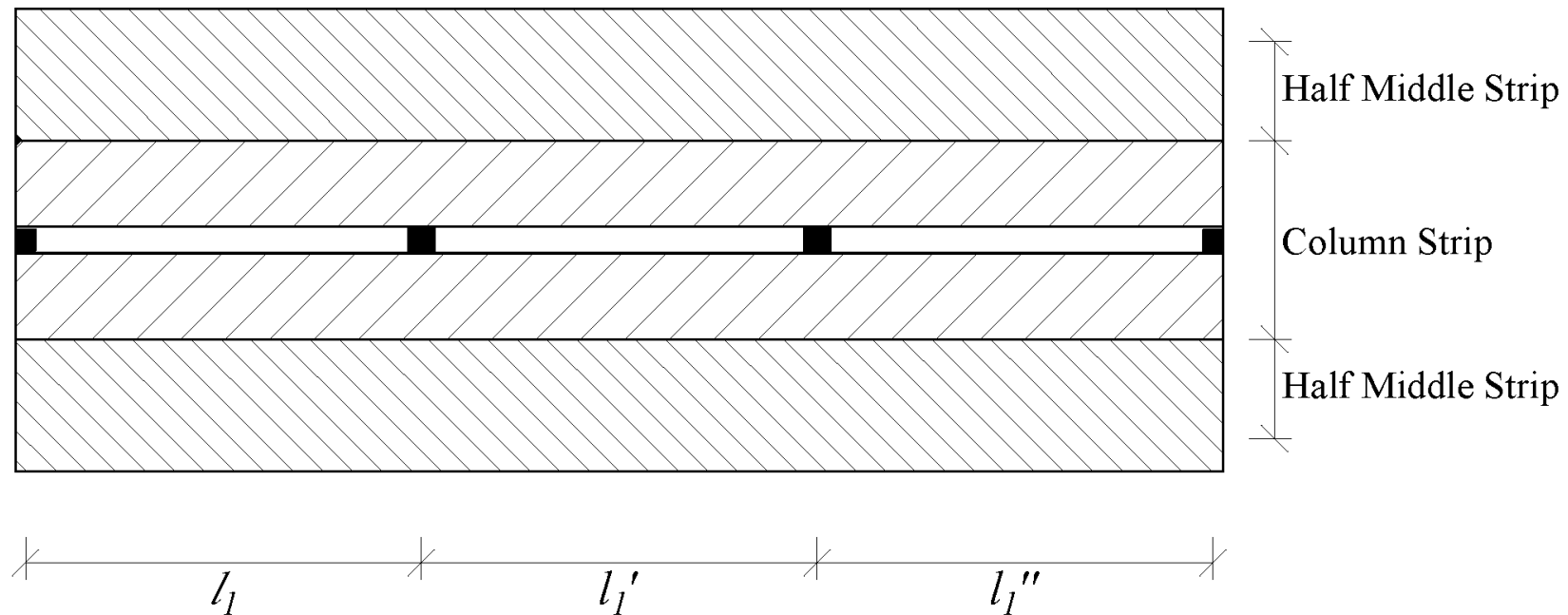
(a) Exterior edge fully restrained; (b) exterior edge unrestrained; (c) beams between all supports; (d) No beams between interior supports and without edge beam; (e) No beams between interior supports and with edge beam

10.3.3 Direct Design Method DDM (Based on ACI 318-14)

3. Distribution of the positive and negative factored moments to the column and middle strips:

Column strip: A column strip is a design strip with a width on each side of a column centerline equal to $0.25 l_1$ or $0.25 l_2$, whichever is less. Column strip includes beams, if any.

Middle strip: Middle strip is a design strip bounded by two column strips.



10.3.3 Direct Design Method DDM (Based on ACI 318-14)

a. Factored moments in column strips:

Column strip moments, as percentages of total factored positive and negative moments, are given in Table 10.10.

β_t is defined as ratio of torsional stiffness of edge beam section to flexural stiffness of a width of slab equal to span length of beam, center-to-center of supports, $\beta_t = E_{cb} C / (2E_{cs}l_s)$

where C is cross-sectional constant to define torsional properties given by

$$C = \sum \left(1 - 0.63 \frac{x}{y} \right) \frac{x^3 y}{3}$$

where x is the shorter overall dimension of rectangular part of cross section and y is the longer overall dimension of rectangular part of cross section.

TABLE 10.10
Column strip factored moments (percentage)

		l_2/l_1	0.50	1.0	2.0
Exterior negative factored moment	$\alpha_{fl_2}/l_1 = 0$	$\beta_t = 0$	100	100	100
		$\beta_t \geq 2.5$	75	75	75
	$\alpha_{fl_2}/l_1 \geq 1$	$\beta_t = 0$	100	100	100
		$\beta_t \geq 2.5$	90	75	45
Positive factored moment	$\alpha_{fl_2}/l_1 = 0$		60	60	60
	$\alpha_{fl_2}/l_1 \geq 1$		90	75	45
Interior negative factored moment	$\alpha_{fl_2}/l_1 = 0$		75	75	75
	$\alpha_{fl_2}/l_1 \geq 1$		90	75	45

10.3.3 Direct Design Method DDM (Based on ACI 318-14)

b. Factored moments in beams caused by slab loads:

- ✓ Beams are to be designed to resist 85 % of column strip moments if $\alpha_{f_1} l_2 / l_1 \geq 1$.
- ✓ For values of $\alpha_{f_1} l_2 / l_1$ between 1.0 and zero, proportion of column strip moments resisted by beams is obtained by linear interpolation between 85 and zero.
- ✓ In addition to these moments, beams are to be proportioned to resist moments caused by factored loads applied directly on the beams, including the weight of the web above and below the slab.

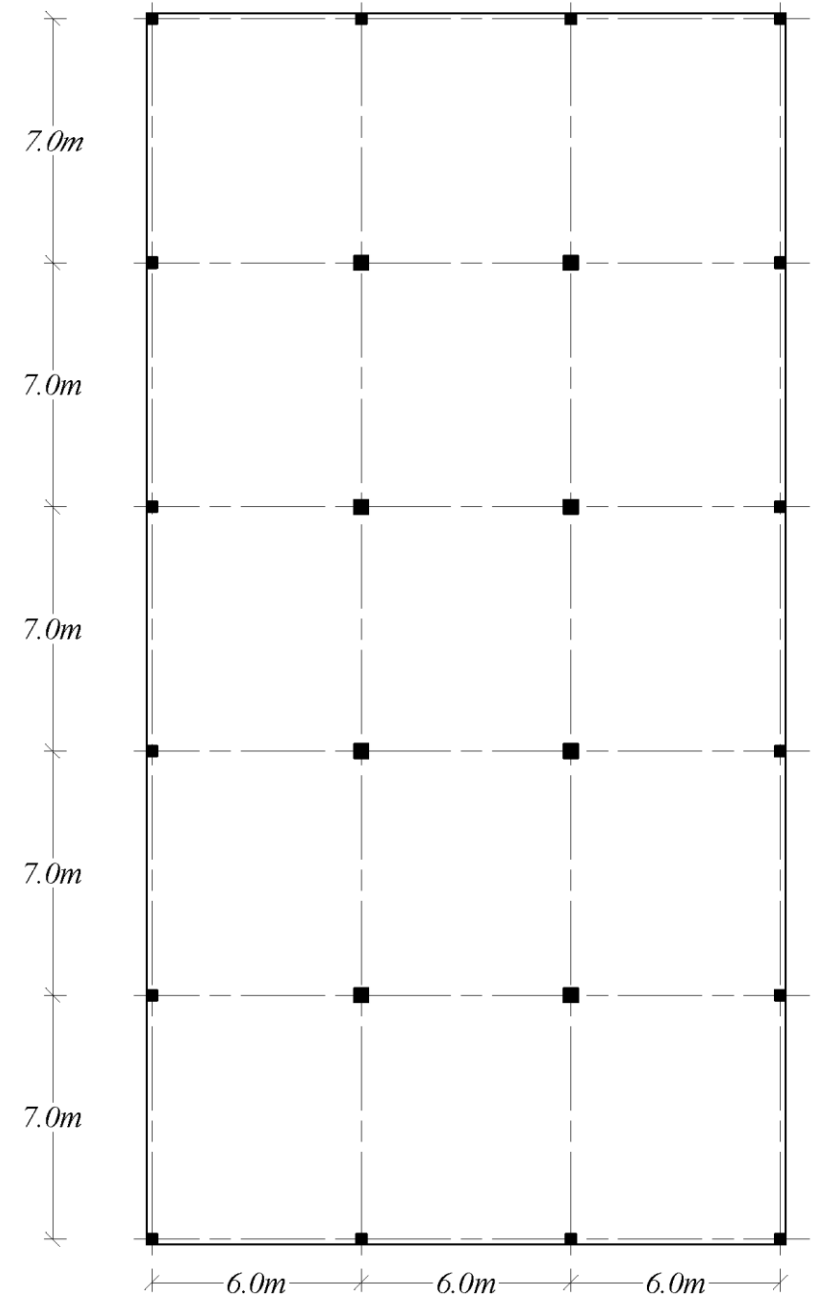
c. Factored shears in beams caused by slab loads:

- ✓ Beams with $\alpha_{f_1} l_2 / l_1 \geq 1$ are to be designed for shear caused by factored loads on tributary areas which are bounded by 45 degree lines drawn from the corners of the panels and the centerlines of the adjacent panels parallel to the long sides.
- ✓ For values of $\alpha_{f_1} l_2 / l_1$ less than 1.0, linear interpolation assuming beams carry no load at $\alpha_{f_1} = 0$ is permitted to be used.
- ✓ In addition to these shears, beams are to be proportioned to resist shears caused by factored loads applied directly on these beams.

Example (10.5):

Design the two-way flat plate with no edge (spandrel) beams, shown in Figure 10.34, given the following: interior columns are $400\text{ mm} \times 400\text{ mm}$, exterior columns are $300\text{ mm} \times 300\text{ mm}$, covering materials weigh 2.29 kN/m^2 , equivalent partition load 0.50 kN/m^2 and the live load is 3 kN/m^2 .

Use $f'_c = 28\text{ MPa}$ and $f_y = 420\text{ MPa}$.



Example (10.5):

Solution:

1. Evaluate slab thickness:

$$l_{n1} = 7000 - 200 - 150 = 6650 \text{ mm}$$

$$l_{n2} = 7000 - 200 - 200 = 6600 \text{ mm}$$

Using Table 10.8 For flat plates with no edge beams, minimum slab thickness

$$h_{min} = 6650/30 = 221.6 \text{ mm, taken as } 220 \text{ mm.}$$

2. Check limitations for slab analysis by the direct design method:

The first six conditions are satisfied, while the seventh condition does not apply due to the nonexistence of beams.

3. Calculate the factored load on the slab:

$$q_u = 1.2 [0.22 (25) + 2.29 + 0.5] + 1.6 (3.0) = 14.75 \text{ kN/m}^2$$

Example (10.5):

3. Calculate the factored static moment:

Strip in the shorter direction:

Width of intermediate strip = 7000 mm and width of column strip is the smaller of $(l_1/2)$ and $(l_2/2)$, taken as $(6000/2) = 3000$ mm,

Total factored static moment:

Clear span for exterior panels =

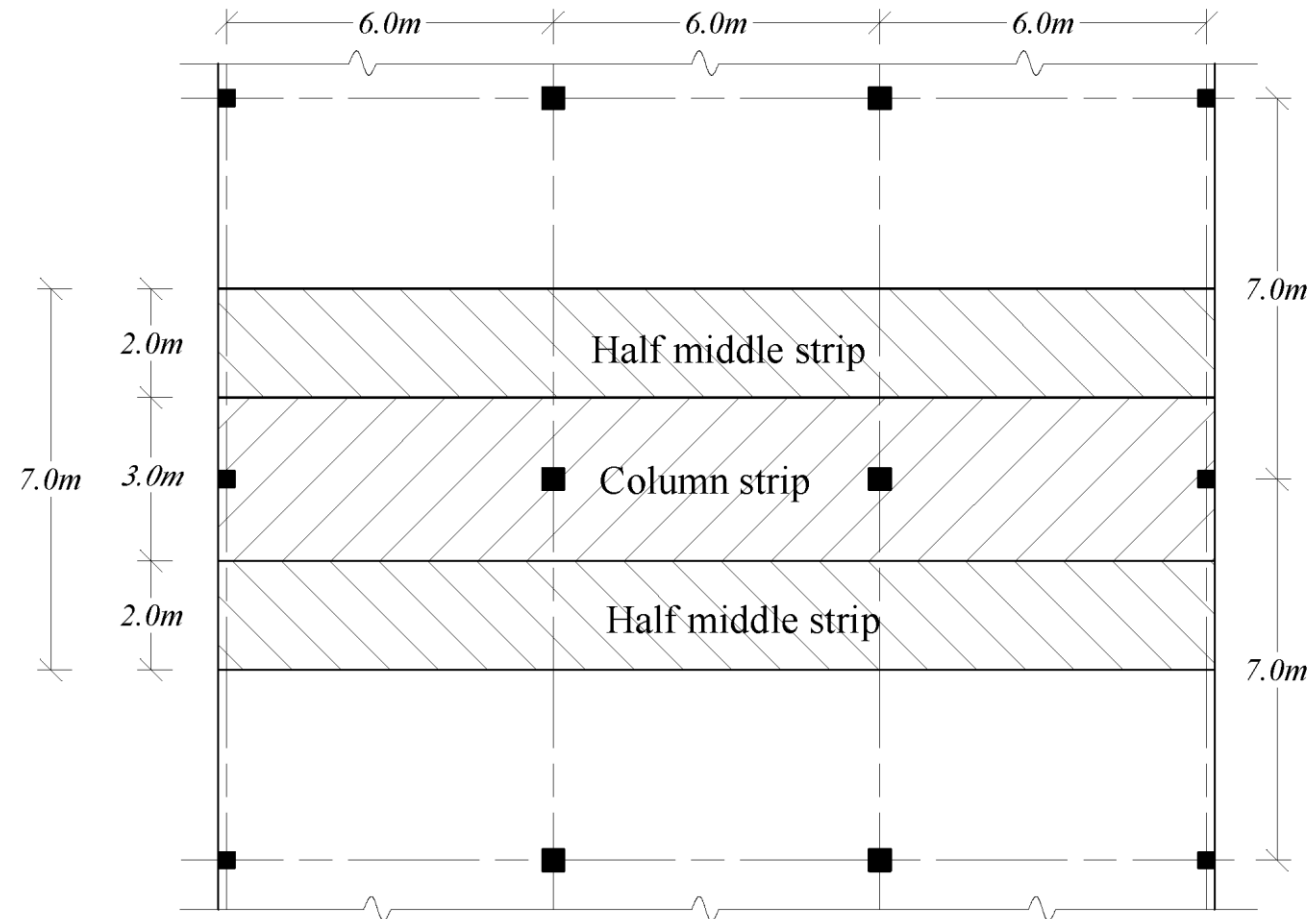
$$6000 - 150 - 200 = 5650 \text{ mm}$$

Clear span for interior panels =

$$6000 - 200 - 200 = 5600 \text{ mm}$$

The larger of the two values will be used in moment calculations.

$$M_o = 14.75(7)(5.65)^2/8 = 412.0 \text{ kN.m}$$



Example (10.5):

4. Distribute the total factored static moment into positive and negative moments:

Slab moment (kN.m)	End spans			Interior spans	
	Exterior negative	Positive	Interior negative	Positive	Negative
Total moments (M_T)	$0.26 = 107.1$	$0.52M_o = 214.2$	$0.70M_o = 288.4$	$0.35M_o = 144.2$	$0.65M_o = 267.8$

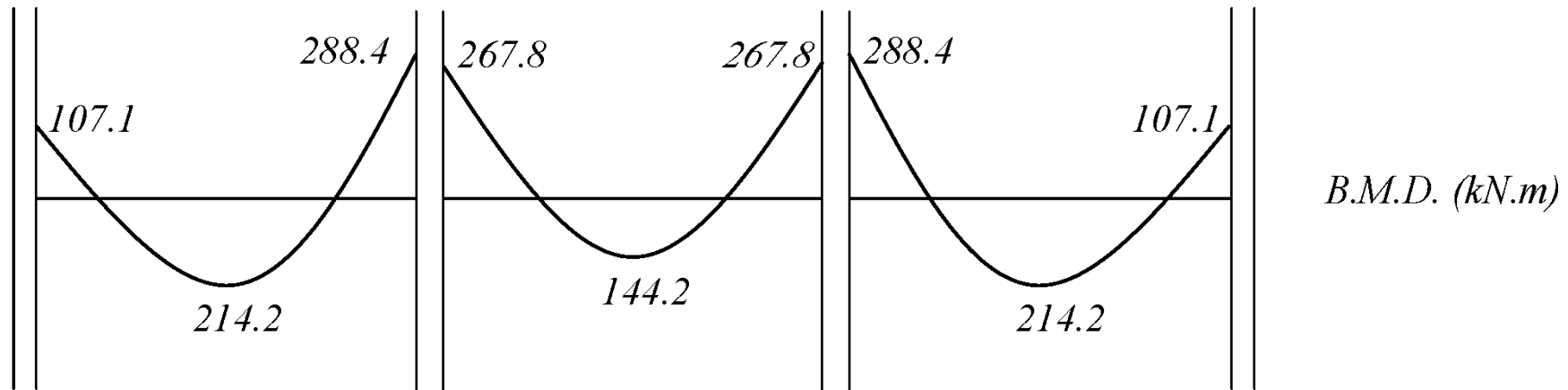


Figure 10.36a: Strips bending moment diagrams for column strip and middle strip

Example (10.5):

5. Distribute the positive and negative moments to the column and middle strips:

Slab moment (kN.m)	End spans			Interior spans	
	Exterior negative	Positive	Interior negative	Positive	Negative
Total moments (M_T)	$0.26 = 107.1$	$0.52M_o = 214.2$	$0.70M_o = 288.4$	$0.35M_o = 144.2$	$0.65M_o = 267.8$
Column strip moments	$1.0M_T=107.1$	$0.60M_T= 128.5$	$0.75M_T= 216.3$	$0.60M_T= 86.5$	$0.75M_T= 200.8$
Middle strip moments	0.00	85.7	72.1	57.7	67.0

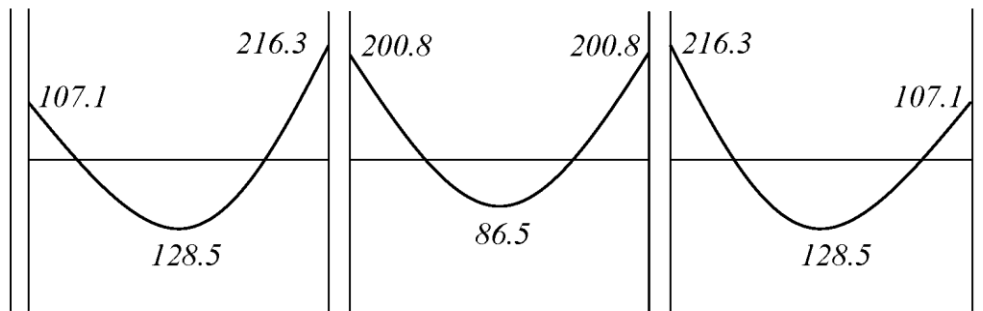


Figure 10.36 (b) Bending moment diagram for column strip

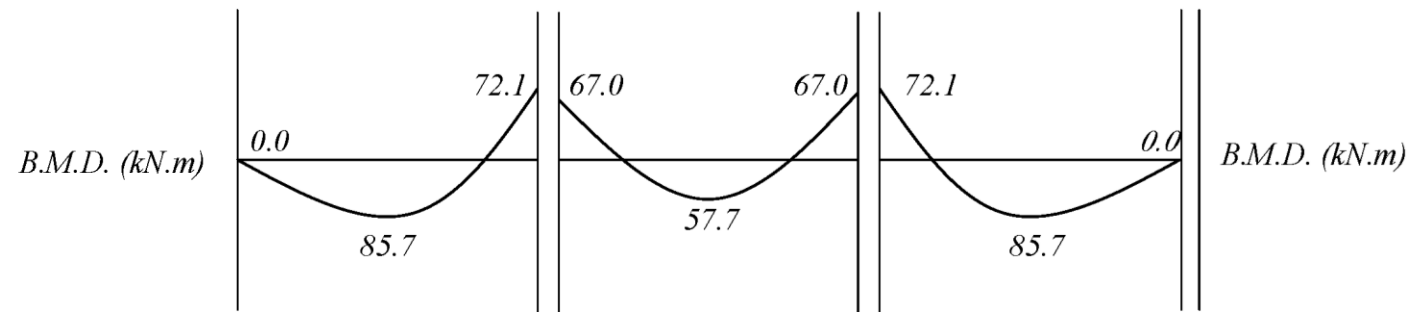


Figure 10.36 (c) bending moment diagram for middle strip

Example (10.5):

7. Design the reinforcement:

a. Column strip reinforcement:

Design sections at maximum positive and negative moments as rectangular sections, as shown in Table 10.12, where, $d = 180 \text{ mm}$, and $b = 3000 \text{ mm}$,

M, kN.m	ρ	A_s	Reinforcement	
			Straight	Bent-up
107.1	0.0030	16.20	6 $\phi 12 \text{ mm}$	9 $\phi 12 \text{ mm}$
128.5	0.0036	19.44	9 $\phi 12 \text{ mm}$	9 $\phi 12 \text{ mm}$
216.3	0.0062	33.48	15 $\phi 12 \text{ mm}$	9 $\phi 12 + 6 \phi 12$

Example (10.5):

7. Design the reinforcement:

b. Middle strip reinforcement:

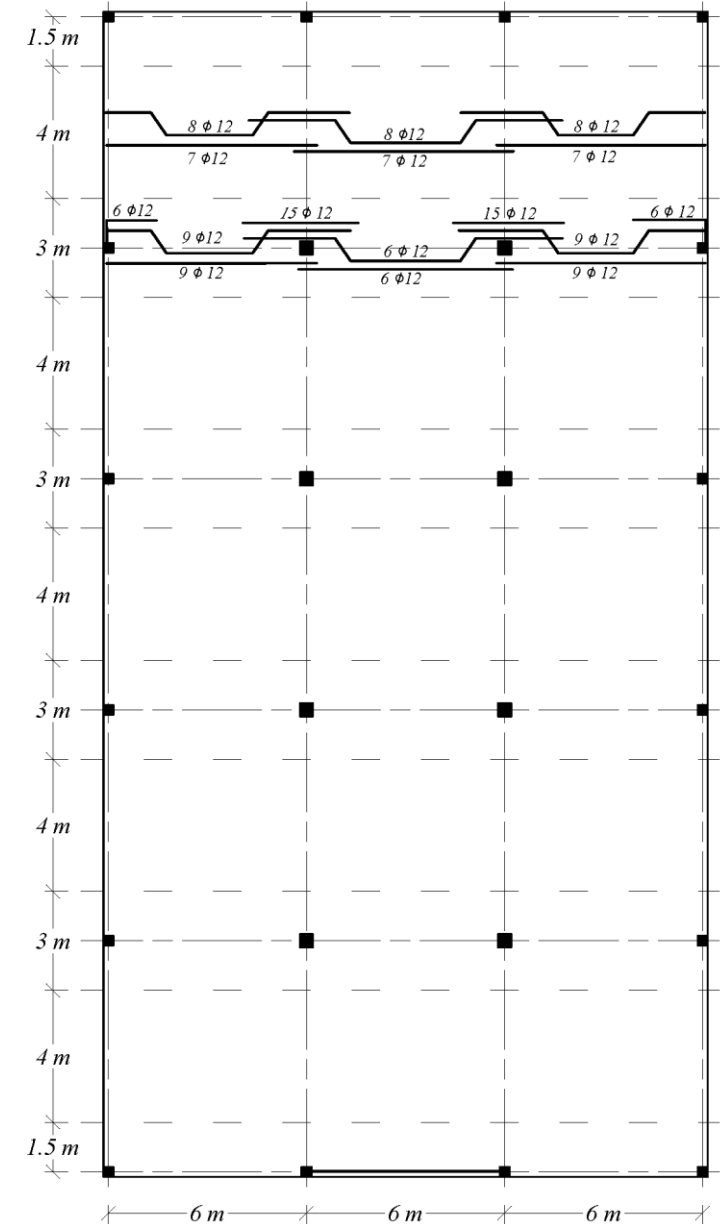
Design sections at maximum positive and negative moments as rectangular sections, as shown in Table 10.13 where, $d = 180 \text{ mm}$, and $b = 4000 \text{ mm}$,

M, kN.m	ρ	A_s	Reinforcement	
			Straight	Bent-up
85.7	0.0018	15.84	7 ϕ 12 mm	8 ϕ 12 mm
72.1	0.0018	15.84	---	8 ϕ 12 + 8 ϕ 12
57.7	0.0018	15.84	7 ϕ 12 mm	8 ϕ 12 mm

Example (10.5):

8- Prepare design drawings:

Figure 10.37 shows reinforcement detail.



Example (10.5):

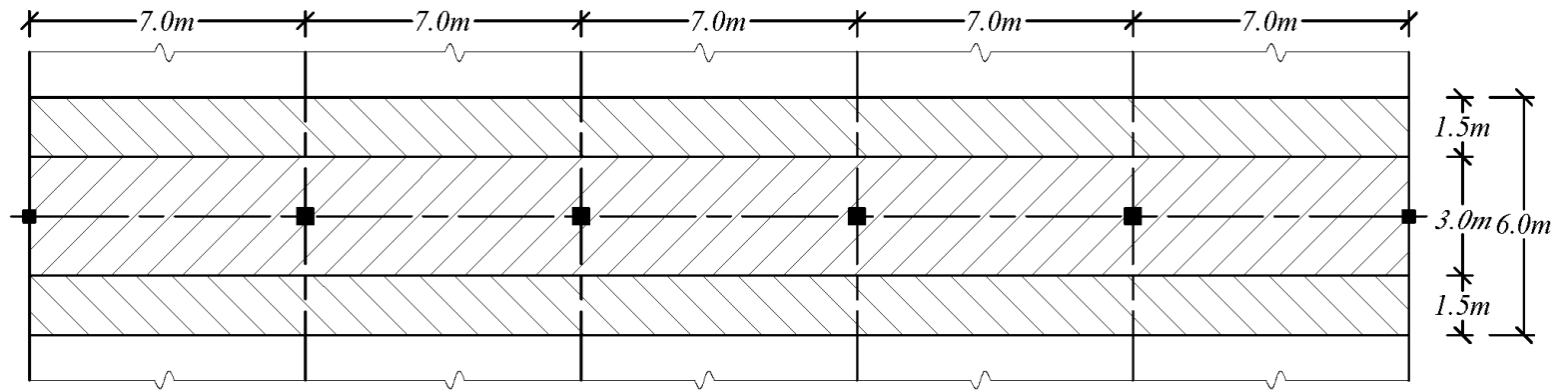
Continue from step 4 for the strip in the longer direction

4. Calculate the factored static moment:

Strip in the longer direction:

Width of intermediate strip =
 6000 mm

and width of column strip is
the smaller of $(l_1/2)$ and
 $(l_2/2)$, taken as $(600/2) = 3000 \text{ mm}$.



Total factored static moment:

Clear span for exterior panels = $7000 - 150 - 200 = 6650 \text{ mm}$

Clear span for interior panels = $7000 - 200 - 200 = 6600 \text{ mm}$

The larger of the two values will be used in moment calculations.

$$M_o = 14.75 (6) (6.65)^2 / 8 = 489.2 \text{ kN.m}$$

Example (10.5):

- 4. Distribute the total factored static moment into positive and negative moments:
- 5. Distribute the positive and negative moments to the column and middle strips:

TABLE 10.14

Negative and positive factored moments

Slab moment (kN·m)	End spans			Interior spans	
	Exterior negative	Positive	Interior negative	Positive	Negative
Total moments (M_T)	$0.26M_s = 127.2$	$0.52M_s = 254.4$	$0.7M_s = 342.4$	$0.35M_s = 171.2$	$0.65M_s = 318.0$
Column strip moments	$1.0M_T = 127.2$	$0.6M_T = 152.6$	$0.75M_T = 256.8$	$0.6M_T = 102.7$	$0.75M_T = 238.5$
Middle strip moments	0.00	101.8	85.6	68.5	79.5

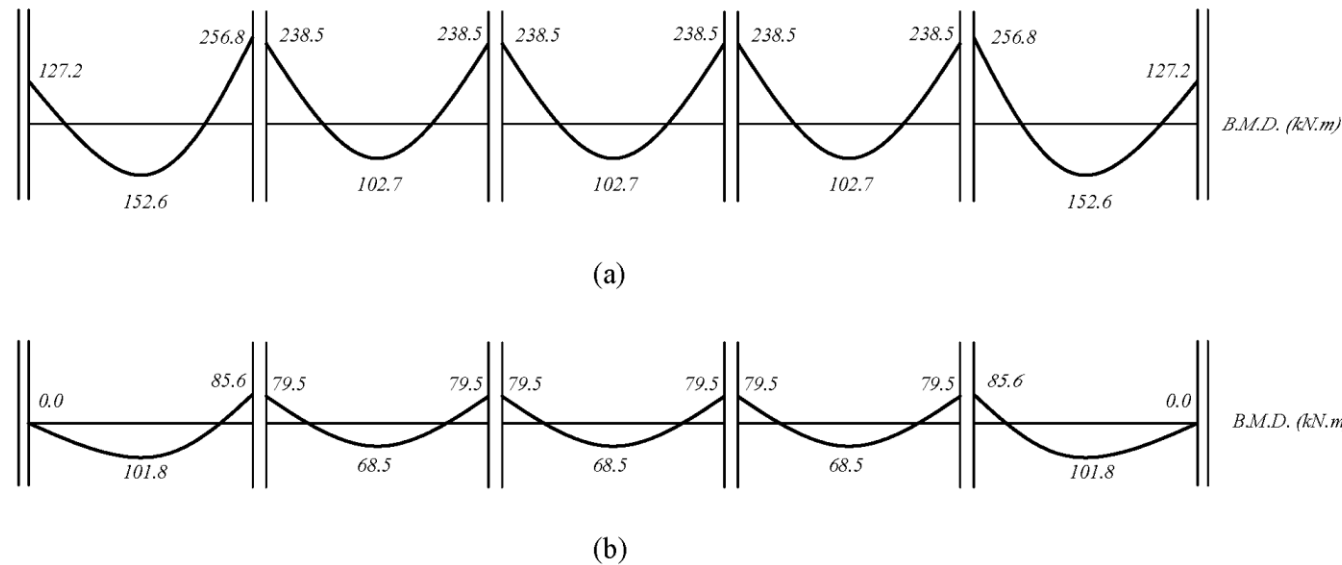


Figure 10.39: Strips bending moment diagrams (a) Bending moment diagram for column strip; (b) bending moment diagram for middle strip

Example (10.5):

7- Design the reinforcement:

Column strip reinforcement:

Design sections at maximum positive and negative moments as rectangular sections, are shown in Table 10.15, where, $d = 180 \text{ mm}$, and $b = 3000 \text{ mm}$.

Half middle strip reinforcement:

Design sections at maximum positive and negative moments as rectangular sections are shown in Table 10.16, where: $d = 180 \text{ mm}$, and $b = 3000 \text{ mm}$

TABLE 10.15

Column strip reinforcement

M, kN·m	ρ	A_s	Reinforcement	
			Straight	Bent-up
127.2	0.00357	19.28	8 ϕ 12 mm	10 ϕ 12 mm
152.6	0.00431	23.27	11 ϕ 12 mm	10 ϕ 12 mm
256.8	0.00747	40.34	19 ϕ 12 mm	7 ϕ 12 + 10 ϕ 12
238.5	0.00690	37.26	19 ϕ 12 mm	7 ϕ 12 + 7 ϕ 12
102.7	0.00286	15.46	7 ϕ 12 mm	7 ϕ 12 mm

TABLE 10.16

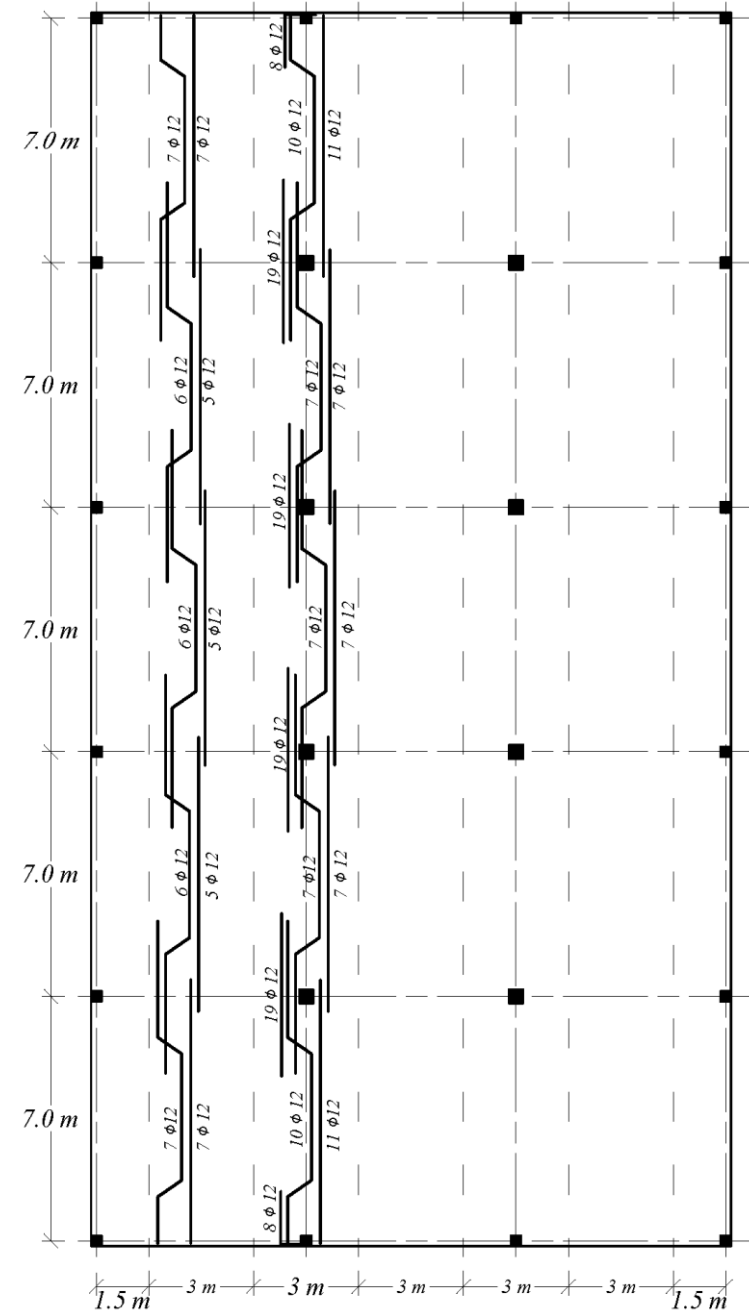
Middle strip reinforcement

M, kN·m	ρ	A_s	Reinforcement	
			Straight	Bent-up
101.8	0.0028	15.12	7 ϕ 12 mm	7 ϕ 12 mm
85.6	0.0024	12.96	-----	6 ϕ 12 + 7 ϕ 12
79.5	0.0022	11.88	-----	6 ϕ 12 + 6 ϕ 12
68.5	0.0019	11.88	5 ϕ 12 mm	6 ϕ 12 mm

Example (10.5):

8- Prepare design drawings:

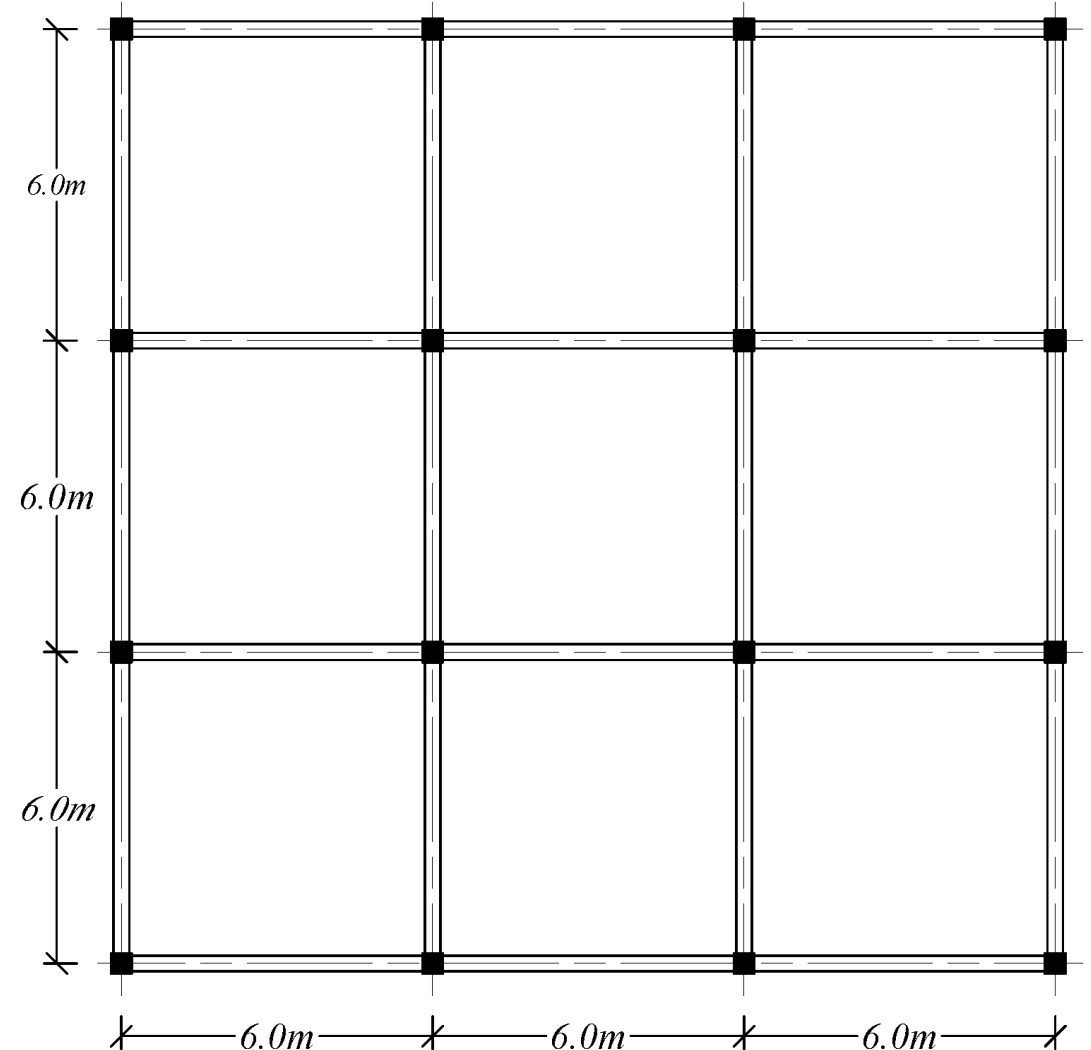
Figure 10.39 shows reinforcement detail.



Example (10.6):

For the two-way solid slab with beams on all column lines, shown in Figure 10.41, evaluate the moments acting on any of the internal beams, using the direct design method. All columns are $300\text{ mm} \times 300\text{ mm}$ in cross section, all beams are $300\text{ mm} \times 600\text{ mm}$ in cross section, slab thickness is equal to 140 mm , covering materials weigh 1.83 kN/m^2 and the live load is 4 kN/m^2 .

Use $f'_c = 28\text{ MPa}$ and $f_y = 420\text{ MPa}$



Example (10.6):

Solution:

Evaluate the constants α and β_t :

For calculation of α and β_t of T-section from (Chapter 6)

$$b_e = b_w + 2(h - t) \leq b_w + 8t$$

$$b_e = 300 + 2(600 - 140) = \mathbf{1220mm} \leq 1440mm$$

Beam sectional properties are shown in Figure 10.42.

a. Internal beams:

Figure 10.43, relative stiffness of beam is calculated as follows:

$$y_t = 395.9 \text{ mm}, \quad I_b = 9581.98 \times 10^6 \text{ mm}^4$$

$$\alpha = \frac{I_b}{I_s} = \frac{9581.98 \times 10^6}{6000(140)^3/12} = 6.984$$

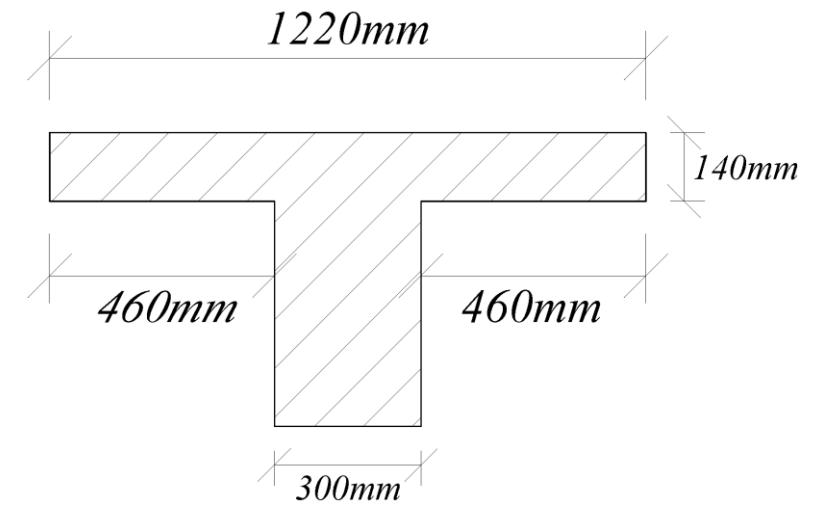


Figure 10.42: Beam sectional properties for relative stiffness calculations

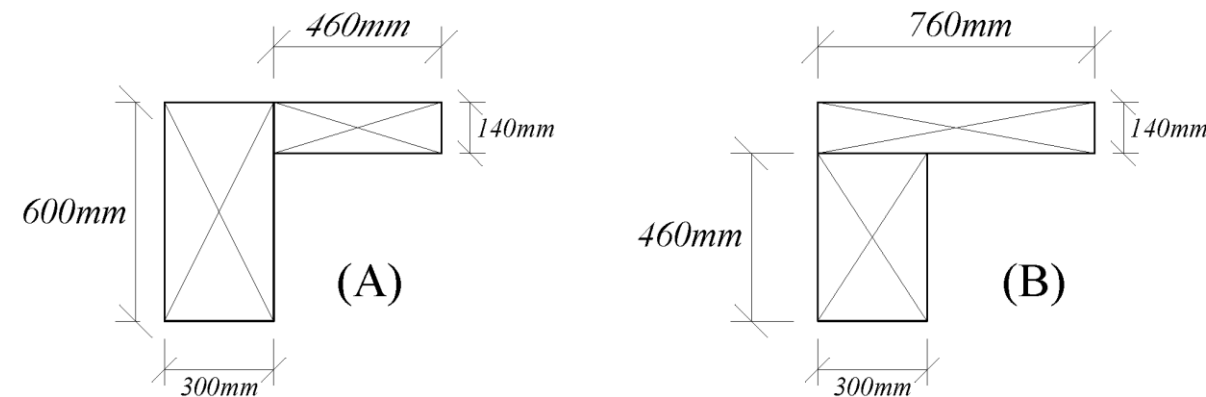


Figure 10.43: Beam sectional properties for relative torsional stiffness calculations

Example (10.6):

Calculation of torsional constant β_t

$$C = \sum \left(1 - 0.63 \frac{x}{y} \right) \left(\frac{x^3 y}{3} \right)$$

$$C_A = (1 - 0.63 (140/760)) (140)^3 (760/3) + (1 - 0.63 (300/460)) (300)^3 (460/3) \\ = 3053.47 \times 10^6 \text{ mm}^4$$

$$C_B = (1 - 0.63 (30/60)) (30)^3 (60/3) + (1 - 0.63 (14/46)) (14)^3 (46/3) \\ = 4039.07 \times 10^6 \text{ mm}^4$$

$$C_{max} = C_B = 4039.07 \times 10^6 \text{ mm}^4$$

$$\beta_t = \frac{E_{cb} C}{2E_{cs} I_s} = \frac{4039.07 \times 10^6}{2 (6000) (140)^3 / 12} = 1.472$$

$$q_u = 1.2(0.14 (25) + 1.83) + 1.6(4.0) \\ = 12.8 \text{ kN/m}^2$$

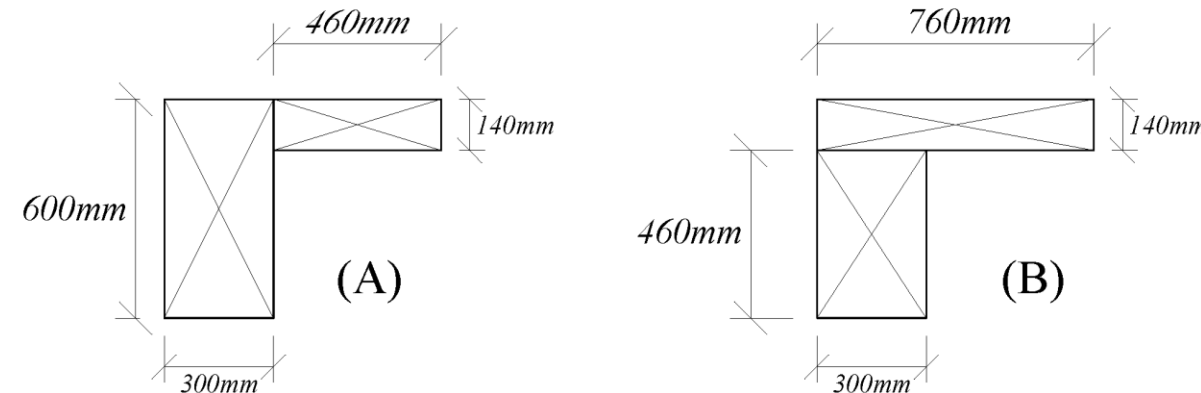


Figure 10.43: Beam sectional properties for relative torsional stiffness calculations

Example (10.6):

2. Determine the total factored static moment (internal strip):

Total factored static moment:

$$M_o = q_u l_2 l_{1n}^2 / 8 = 12.8 (6.0) (5.7)^2 / 8 = 311.9 \text{ kN.m}$$

Negative and positive factored moments:

$$\alpha_{f_1} l_2 / l_1 = 6.984 > 1.0 \text{ and from Table 10.10}$$

For $\beta_t = 0.0$, exterior negative moment is 100 % of total strip moment

For $\beta_t \geq 2.5$, exterior negative moment is 75 % of total strip moment

By linear interpolation, for $\beta_t = 1.472$,exterior negative moment is **85.28 %** of total strip moment.

Example (10.6):

3- Evaluate the moments in one of the internal beams:

The results of moments in one of the internal Beams are shown in Table 10.17:

Bending moment (kN.m)	End spans			Interior spans	
	Exterior negative	Positive	Interior negative	Positive	Negative
Total moments (M_T)	$0.16M_o =$ 49.90	$0.57M_o =$ 177.78	$0.70M_o =$ 218.33	$0.35M_o =$ 109.16	$0.65M_o =$ 202.73
Column strip mom. (M_C)	$0.85M_T =$ 42.41	$0.75M_T =$ 133.33	$0.75M_T =$ 163.75	$0.75M_T =$ 81.87	$0.75M_T =$ 152.05
Beam moments	$0.85M_C =$ 36.0	$0.85M_C =$ 113.3	$0.85M_C =$ 139.2	$0.85M_C =$ 69.6	$0.85M_C =$ 129.2