Solutions Manual to Accompany

# Fundamentals of Microelectronics, 1st Edition

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2.1 (a)

$$k = 8.617 \times 10^{-5} \text{ eV/K}$$

$$n_i(T = 300 \text{ K}) = 1.66 \times 10^{15} (300 \text{ K})^{3/2} \exp\left[-\frac{0.66 \text{ eV}}{2\,(8.617 \times 10^{-5} \text{ eV/K})\,(300 \text{ K})}\right] \text{ cm}^{-3}$$

$$= \boxed{2.465 \times 10^{13} \text{ cm}^{-3}}$$

$$n_i(T = 600 \text{ K}) = 1.66 \times 10^{15} (600 \text{ K})^{3/2} \exp\left[-\frac{0.66 \text{ eV}}{2\,(8.617 \times 10^{-5} \text{ eV/K})\,(600 \text{ K})}\right] \text{ cm}^{-3}$$

$$= \boxed{4.124 \times 10^{16} \text{ cm}^{-3}}$$

Compared to the values obtained in Example 2.1, we can see that the intrinsic carrier concentration in Ge at $T = 300$ K is $\frac{2.465 \times 10^{13}}{1.08 \times 10^{10}} = 2282$ times higher than the intrinsic carrier concentration in Si at $T = 300$ K. Similarly, at $T = 600$ K, the intrinsic carrier concentration in Ge is $\frac{4.124 \times 10^{16}}{1.54 \times 10^{15}} = 26.8$ times higher than that in Si.

(b) Since phosphorus is a Group V element, it is a donor, meaning $N_D = 5 \times 10^{16} \text{ cm}^{-3}$. For an n-type material, we have:

$$n = N_D = \boxed{5 \times 10^{16} \text{ cm}^{-3}}$$

$$p(T = 300 \text{ K}) = \frac{[n_i(T = 300 \text{ K})]^2}{n} = \boxed{1.215 \times 10^{10} \text{ cm}^{-3}}$$

$$p(T = 600 \text{ K}) = \frac{[n_i(T = 600 \text{ K})]^2}{n} = \boxed{3.401 \times 10^{16} \text{ cm}^{-3}}$$

2. (a) Mobility of electrons in Si = 1350 $cm^2/V\text{-}s$
Mobility of holes in Si = 480 $cm^2/V\text{-}s$

$\Rightarrow$ velocity of electrons $= \mu_n E = \left(1350 \frac{cm^2}{V\text{-}s}\right)\left(0.1 \frac{V}{\mu m}\right)$

$= 1.35 \cdot 10^4 \; m/s$

velocity of holes $= \mu_p E = \left(480 \frac{cm^2}{V\text{-}s}\right)\left(0.1 \frac{V}{\mu m}\right)$

$= 4.8 \cdot 10^3 \; m/s$

(b) Given $E = 0.1 \; V/\mu m$ hole current negligible
$\mu_n = 1350 \; cm^2/V\text{-}s$ $\mu_p = 480 \; cm^2/V\text{-}s$

$$J_{tot} = 1 \; mA/\mu m^2 = q[\mu_n n E + \mu_p p E] \approx q \mu_n n E$$

$$\therefore n = \frac{J_{tot}}{q \mu_n E} = \frac{1 \; mA/\mu m^2}{(1.6 \cdot 10^{-19} C)(1350 \; cm^2/V\text{-}s)(0.1 \; V/\mu m)}$$

$$= 4.6 \cdot 10^{17} \; cm^{-3}$$

2.3 (a) Since the doping is uniform, we have no diffusion current. Thus, the total current is due only to the drift component.

$$\begin{aligned}
I_{tot} &= I_{drift} \\
&= q(n\mu_n + p\mu_p)AE \\
n &= 10^{17}\ \text{cm}^{-3} \\
p &= n_i^2/n = (1.08 \times 10^{10})^2/10^{17} = 1.17 \times 10^3\ \text{cm}^{-3} \\
\mu_n &= 1350\ \text{cm}^2/\text{V}\cdot\text{s} \\
\mu_p &= 480\ \text{cm}^2/\text{V}\cdot\text{s} \\
E = V/d &= \frac{1\ \text{V}}{0.1\ \mu\text{m}} \\
&= 10^5\ \text{V/cm} \\
A &= 0.05\ \mu\text{m} \times 0.05\ \mu\text{m} \\
&= 2.5 \times 10^{-11}\ \text{cm}^2
\end{aligned}$$

Since $n\mu_n \gg p\mu_p$, we can write

$$\begin{aligned}
I_{tot} &\approx qn\mu_n AE \\
&= \boxed{54.1\ \mu\text{A}}
\end{aligned}$$

(b) All of the parameters are the same except $n_i$, which means we must re-calculate $p$.

$$\begin{aligned}
n_i(T = 400\ \text{K}) &= 3.657 \times 10^{12}\ \text{cm}^{-3} \\
p = n_i^2/n &= 1.337 \times 10^8\ \text{cm}^{-3}
\end{aligned}$$

Since $n\mu_n \gg p\mu_p$ still holds (note that $n$ is 9 orders of magnitude larger than $p$), the hole concentration once again drops out of the equation and we have

$$\begin{aligned}
I_{tot} &\approx qn\mu_n AE \\
&= \boxed{54.1\ \mu\text{A}}
\end{aligned}$$

2.4 (a) From Problem 1, we can calculate $n_i$ for Ge.

$$\begin{aligned} n_i(T = 300\text{ K}) &= 2.465 \times 10^{13}\text{ cm}^{-3} \\ I_{tot} &= q(n\mu_n + p\mu_p)AE \\ n &= 10^{17}\text{ cm}^{-3} \\ p &= n_i^2/n = 6.076 \times 10^9\text{ cm}^{-3} \\ \mu_n &= 3900\text{ cm}^2/\text{V}\cdot\text{s} \\ \mu_p &= 1900\text{ cm}^2/\text{V}\cdot\text{s} \\ E &= V/d = \frac{1\text{ V}}{0.1\text{ μm}} \\ &= 10^5\text{ V/cm} \\ A &= 0.05\text{ μm} \times 0.05\text{ μm} \\ &= 2.5 \times 10^{-11}\text{ cm}^2 \end{aligned}$$

Since $n\mu_n \gg p\mu_p$, we can write

$$\begin{aligned} I_{tot} &\approx qn\mu_n AE \\ &= \boxed{156\text{ μA}} \end{aligned}$$

(b) All of the parameters are the same except $n_i$, which means we must re-calculate $p$.

$$\begin{aligned} n_i(T = 400\text{ K}) &= 9.230 \times 10^{14}\text{ cm}^{-3} \\ p &= n_i^2/n = 8.520 \times 10^{12}\text{ cm}^{-3} \end{aligned}$$

Since $n\mu_n \gg p\mu_p$ still holds (note that $n$ is 5 orders of magnitude larger than $p$), the hole concentration once again drops out of the equation and we have

$$\begin{aligned} I_{tot} &\approx qn\mu_n AE \\ &= \boxed{156\text{ μA}} \end{aligned}$$

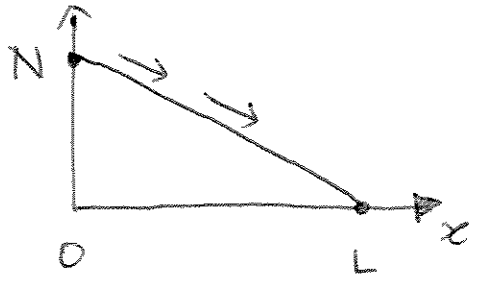
2.5 Since there's no electric field, the current is due entirely to diffusion. If we define the current as positive when flowing in the positive $x$ direction, we can write

$$
\begin{aligned}
I_{tot} &= I_{diff} = AJ_{diff} = Aq\left[D_n\frac{dn}{dx} - D_p\frac{dp}{dx}\right] \\
A &= 1\ \mu\text{m} \times 1\ \mu\text{m} = 10^{-8}\ \text{cm}^2 \\
D_n &= 34\ \text{cm}^2/\text{s} \\
D_p &= 12\ \text{cm}^2/\text{s} \\
\frac{dn}{dx} &= -\frac{5\times10^{16}\ \text{cm}^{-3}}{2\times10^{-4}\ \text{cm}} = -2.5\times10^{20}\ \text{cm}^{-4} \\
\frac{dp}{dx} &= \frac{2\times10^{16}\ \text{cm}^{-3}}{2\times10^{-4}\ \text{cm}} = 10^{20}\ \text{cm}^{-4} \\
I_{tot} &= \left(10^{-8}\ \text{cm}^2\right)\left(1.602\times10^{-19}\ \text{C}\right)\left[\left(34\ \text{cm}^2/\text{s}\right)\left(-2.5\times10^{20}\ \text{cm}^{-4}\right) - \left(12\ \text{cm}^2/\text{s}\right)\left(10^{20}\ \text{cm}^{-4}\right)\right] \\
&= \boxed{-15.54\ \mu\text{A}}
\end{aligned}
$$

6. Given Area = $a$

find total electrons stored.

$$n(x) = -\frac{N}{L}x + N$$




∴ total electrons stored

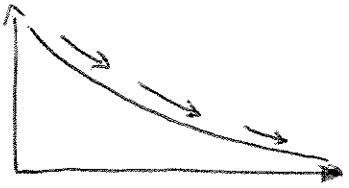
$$= \int a \cdot n(x)\, dx = \int_0^L a\left(-\frac{N}{L}x + N\right) dx$$

$$= aN\left(-\frac{x^2}{2L} + x\right)\Bigg|_0^L = \frac{aNL}{2}$$

7. Given Area = a

find total electrons stored.

$$n(x) = N \cdot \exp\left(\frac{-x}{L_d}\right)$$



∴ total electrons stored

$$= \int a\, n(x)\, dx = \int_0^\infty a \cdot N \cdot \exp\left(\frac{-x}{L_d}\right) dx$$

$$= aN\left(-L_d \cdot \exp\frac{-x}{L_d}\right)\Bigg|_0^\infty = aNL_d.$$

For the linear profile, the result depends on the length, L.

For the exponential profile, the result is constant (since $L_d$ is constant.)

2.8 Assume the diffusion lengths $L_n$ and $L_p$ are associated with the electrons and holes, respectively, in this material and that $L_n, L_p \ll 2$ μm. We can express the electron and hole concentrations as functions of $x$ as follows:

$$\begin{aligned}
n(x) &= Ne^{-x/L_n} \\
p(x) &= Pe^{(x-2)/L_p} \\
\#\text{ of electrons} &= \int_0^2 an(x)dx \\
&= \int_0^2 aNe^{-x/L_n}dx \\
&= -aNL_n\left(e^{-x/L_n}\right)\Big|_0^2 \\
&= -aNL_n\left(e^{-2/L_n}-1\right) \\
\#\text{ of holes} &= \int_0^2 ap(x)dx \\
&= \int_0^2 aPe^{(x-2)/L_p}dx \\
&= aPL_p\left(e^{(x-2)/L_p}\right)\Big|_0^2 \\
&= aPL_p\left(1-e^{-2/L_p}\right)
\end{aligned}$$

Due to our assumption that $L_n, L_p \ll 2$ μm, we can write

$$\begin{aligned}
e^{-2/L_n} &\approx 0 \\
e^{-2/L_p} &\approx 0 \\
\#\text{ of electrons} &\approx \boxed{aNL_n} \\
\#\text{ of holes} &\approx \boxed{aPL_p}
\end{aligned}$$

9. Drift is analogous to water flow in a river.

Water flows from top of mountain to bottom because of gravitational field; electron flows from one terminal to the other because of electric field.

| DRIFT | | WATER FLOW |
|---|---|---|
| electrons | ⟷ | water |
| electric field | ⟷ | gravitational field |
| drift/current | ⟷ | water flow |

2.10 (a)

$$n_n = N_D = \boxed{5 \times 10^{17}\ \text{cm}^{-3}}$$
$$p_n = n_i^2/n_n = \boxed{233\ \text{cm}^{-3}}$$
$$p_p = N_A = \boxed{4 \times 10^{16}\ \text{cm}^{-3}}$$
$$n_p = n_i^2/p_p = \boxed{2916\ \text{cm}^{-3}}$$

(b) We can express the formula for $V_0$ in its full form, showing its temperature dependence:

$$V_0(T) = \frac{kT}{q} \ln\left[\frac{N_A N_D}{(5.2 \times 10^{15})^2 T^3 e^{-E_g/kT}}\right]$$
$$V_0(T = 250\ \text{K}) = \boxed{906\ \text{mV}}$$
$$V_0(T = 300\ \text{K}) = \boxed{849\ \text{mV}}$$
$$V_0(T = 350\ \text{K}) = \boxed{789\ \text{mV}}$$

Looking at the expression for $V_0(T)$, we can expand it as follows:

$$V_0(T) = \frac{kT}{q}\left[\ln(N_A) + \ln(N_D) - 2\ln\left(5.2 \times 10^{15}\right) - 3\ln(T) + E_g/kT\right]$$

Let's take the derivative of this expression to get a better idea of how $V_0$ varies with temperature.

$$\frac{dV_0(T)}{dT} = \frac{k}{q}\left[\ln(N_A) + \ln(N_D) - 2\ln\left(5.2 \times 10^{15}\right) - 3\ln(T) - 3\right]$$

From this expression, we can see that if $\ln(N_A) + \ln(N_D) < 2\ln\left(5.2 \times 10^{15}\right) + 3\ln(T) + 3$, or equivalently, if $\ln(N_A N_D) < \ln\left[\left(5.2 \times 10^{15}\right)^2 T^3\right] - 3$, then $V_0$ will decrease with temperature, which we observe in this case. In order for this not to be true (i.e., in order for $V_0$ to increase with temperature), we must have either very high doping concentrations or very low temperatures.

2.11 Since the p-type side of the junction is undoped, its electron and hole concentrations are equal to the intrinsic carrier concentration.

$$\begin{aligned} n_n &= N_D = 3 \times 10^{16}\ \text{cm}^{-3} \\ p_p &= n_i = 1.08 \times 10^{10}\ \text{cm}^{-3} \\ V_0 &= V_T \ln\left(\frac{N_D n_i}{n_i^2}\right) \\ &= (26\ \text{mV}) \ln\left(\frac{N_D}{n_i}\right) \\ &= \boxed{386\ \text{mV}} \end{aligned}$$

2.12 (a)

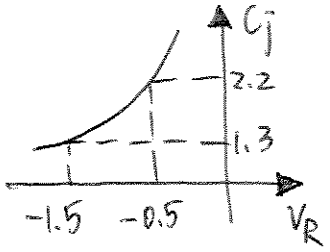
$$
\begin{aligned}
C_{j0} &= \sqrt{\frac{q\epsilon_{\text{Si}}}{2}\frac{N_A N_D}{N_A+N_D}\frac{1}{V_0}} \\
C_j &= \frac{C_{j0}}{\sqrt{1-V_R/V_0}} \\
N_A &= 2\times 10^{15}\ \text{cm}^{-3} \\
N_D &= 3\times 10^{16}\ \text{cm}^{-3} \\
V_R &= -1.6\ \text{V} \\
V_0 &= V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 701\ \text{mV} \\
C_{j0} &= 14.9\ \text{nF/cm}^2 \\
C_j &= 8.22\ \text{nF/cm}^2 \\
&= \boxed{0.082\ \text{fF/cm}^2}
\end{aligned}
$$

(b) Let's write an equation for $C_j'$ in terms of $C_j$ assuming that $C_j'$ has an acceptor doping of $N_A'$.

$$
\begin{aligned}
C_j' &= 2C_j \\
\sqrt{\frac{q\epsilon_{\text{Si}}}{2}\frac{N_A' N_D}{N_A'+N_D}\frac{1}{V_T\ln(N_A' N_D/n_i^2)-V_R}} &= 2C_j \\
\frac{q\epsilon_{\text{Si}}}{2}\frac{N_A' N_D}{N_A'+N_D}\frac{1}{V_T\ln(N_A' N_D/n_i^2)-V_R} &= 4C_j^2 \\
q\epsilon_{\text{Si}} N_A' N_D &= 8C_j^2(N_A'+N_D)(V_T\ln(N_A' N_D/n_i^2)-V_R) \\
N_A'\left[q\epsilon_{\text{Si}} N_D - 8C_j^2(V_T\ln(N_A' N_D/n_i^2)-V_R)\right] &= 8C_j^2 N_D(V_T\ln(N_A' N_D/n_i^2)-V_R) \\
N_A' &= \frac{8C_j^2 N_D(V_T\ln(N_A' N_D/n_i^2)-V_R)}{q\epsilon_{\text{Si}} N_D - 8C_j^2(V_T\ln(N_A' N_D/n_i^2)-V_R)}
\end{aligned}
$$

We can solve this by iteration (you could use a numerical solver if you have one available). Starting with an initial guess of $N_A' = 2\times 10^{15}\ \text{cm}^{-3}$, we plug this into the right hand side and solve to find a new value of $N_A' = 9.9976\times 10^{15}\ \text{cm}^{-3}$. Iterating twice more, the solution converges to $N_A' = 1.025\times 10^{16}\ \text{cm}^{-3}$. Thus, we must increase the $N_A$ by a factor of $N_A'/N_A = 5.125 \approx \boxed{5}$.

B.




$$\frac{C_{j0}}{\sqrt{1+\frac{0.5}{V_0}}} = 2.2 \quad \text{——①}$$

$$\frac{C_{j0}}{\sqrt{1+\frac{1.5}{V_0}}} = 1.3 \quad \text{——②}$$

① ÷ ②: $\quad \frac{1+1.5/V_0}{1+0.5/V_0} = \left(\frac{2.2}{1.3}\right)^2 \Rightarrow V_0 = 0.0365\ V$

Substitute $V_0$ into ①:

$$C_{j0} = 2.2\sqrt{1+\frac{0.5}{V_0}} \approx 8.43\ fF/\mu m^2$$

$$\Rightarrow \frac{N_A N_D}{N_A+N_D} = (C_{j0})^2 \cdot V_0 \cdot \frac{2}{\varepsilon_{si} q}$$

$$= \left(8.43\ \frac{fF}{\mu m^2}\right)^2 \times (0.0365V) \cdot \frac{2}{\varepsilon_{si} q} \approx 3.13 \cdot 10^{17}\ cm^{-3}$$

Fix a value for $N_A > \frac{N_A N_D}{N_A + N_D} \triangleq y$

$N_A = 2 \cdot 10^{18}\,cm^{-3} \Rightarrow N_D = \frac{y N_A}{N_A - y}$

$= \frac{(3.13 \cdot 10^{17}\,cm^{-3})(2 \cdot 10^{18}\,cm^{-3})}{(2 \cdot 10^{18} - 3.13 \cdot 10^{17})\,cm^{-3}}$

$\approx 3.71 \cdot 10^{17}\,cm^{-3}$

14 (a) In forward bias, $I_D = 1mA$, $V_D = 750mV$

$$\therefore I_S \approx I_D e^{-\frac{V_D}{V_T}} = (1mA)\exp[-750mV/26mV]$$

$$= 2.97 \cdot 10^{-16} A$$

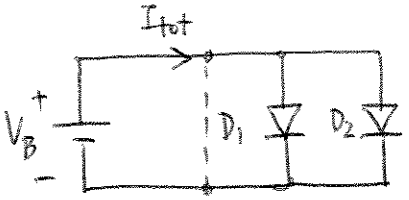
(b) Since $I_S \propto$ Area, doubling area implies doubling $I_S$. From (a),

$$I_D = 1\,mA = 2 \times I_S e^{V_D/V_T}$$

$$\therefore V_D = V_T \ln\left(\frac{I_D}{2I_S}\right) = (26mV)\ln\left(\frac{1mA}{2 \cdot 2.97 \times 10^{-16} A}\right)$$

$$= 0.732\,V$$

15 (a)




$$I_{tot} = I_{D_1} + I_{D_2} = I_{S_1}\left(e^{V_B/V_T} - 1\right) + I_{S_2}\left(e^{V_B/V_T} - 1\right)$$

$$= (I_{S_1} + I_{S_2})\left(e^{V_B/V_T} - 1\right)$$

Therefore, the parallel combination operates as an exponential device, with an equivalent saturation current of $I_{S_1} + I_{S_2}$.

(b) By KVL, $V_{D_1} = V_{D_2}$

$$\Rightarrow V_T \ln\left(\frac{I_{D_1}}{I_{S_1}}\right) = V_T \ln\left(\frac{I_{D_2}}{I_{S_2}}\right)$$

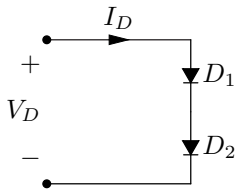
Also, $I_{tot} = I_{D_1} + I_{D_2} \quad \Rightarrow I_{D_2} = I_{tot} - I_{D_1}$

$$\therefore V_T \ln\left(\frac{I_{D_1}}{I_{S_1}}\right) = V_T \ln\left(\frac{I_{tot} - I_{D_1}}{I_{S_2}}\right)$$

$$\Rightarrow I_{D_1} = I_{tot}\left(\frac{I_{S_1}}{I_{S_1} + I_{S_2}}\right)$$

$$\Rightarrow I_{D_2} = I_{tot}\left(\frac{I_{S_2}}{I_{S_1} + I_{S_2}}\right)$$

2.16 (a) The following figure shows the series diodes.




Let $V_{D1}$ be the voltage drop across $D_1$ and $V_{D2}$ be the voltage drop across $D_2$. Let $I_{S1} = I_{S2} = I_S$, since the diodes are identical.

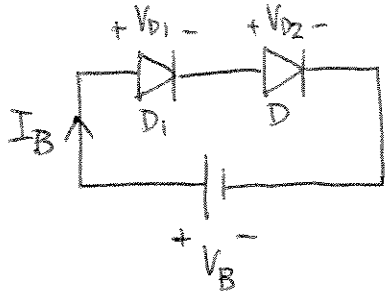
$$\begin{aligned} V_D &= V_{D1} + V_{D2} \\ &= V_T \ln\left(\frac{I_D}{I_S}\right) + V_T \ln\left(\frac{I_D}{I_S}\right) \\ &= 2V_T \ln\left(\frac{I_D}{I_S}\right) \\ I_D &= I_S e^{V_D/2V_T} \end{aligned}$$

Thus, the diodes in series act like a single device with an exponential characteristic described by $I_D = I_S e^{V_D/2V_T}$.

(b) Let $V_D$ be the amount of voltage required to get a current $I_D$ and $V_D'$ the amount of voltage required to get a current $10I_D$.

$$\begin{aligned} V_D &= 2V_T \ln\left(\frac{I_D}{I_S}\right) \\ V_D' &= 2V_T \ln\left(\frac{10I_D}{I_S}\right) \\ V_D' - V_D &= 2V_T \left[\ln\left(\frac{10I_D}{I_S}\right) - \ln\left(\frac{I_D}{I_S}\right)\right] \\ &= 2V_T \ln(10) \\ &= \boxed{120 \text{ mV}} \end{aligned}$$

17.




Find $I_B$, $V_{D_1}$, $V_{D_2}$ in terms of $V_B$, $I_{S_1}$, $I_{S_2}$

By KVL, $V_B = V_{D_1} + V_{D_2} = V_T \ln\left(\frac{I_B}{I_{S_1}}\right) + V_T \ln\left(\frac{I_B}{I_{S_2}}\right)$

$$\Rightarrow \quad V_B = V_T \ln\left(\frac{I_B^2}{I_{S_1} I_{S_2}}\right)$$

$$\therefore \quad I_B = \sqrt{I_{S_1} I_{S_2} \cdot \exp V_B/V_T} = \sqrt{I_{S_1} I_{S_2}} \cdot \exp\left(\frac{V_B}{2V_T}\right).$$

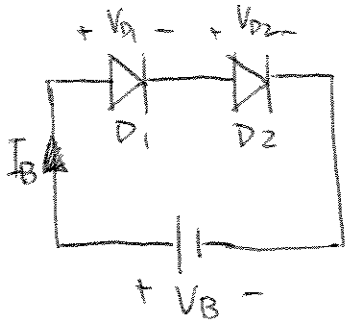
$$V_{D_1} = V_T \ln\left(\frac{I_B}{I_{S_1}}\right) = V_T \ln\left(\frac{\sqrt{I_{S_1} I_{S_2}}}{I_{S_1}} \cdot \exp\frac{V_B}{2V_T}\right)$$

$$= V_T \ln \sqrt{\frac{I_{S_2}}{I_{S_1}}} + \frac{V_B}{2}$$

$$V_{D_2} = V_T \ln\left(\frac{I_B}{I_{S_2}}\right) = V_T \ln\left(\sqrt{\frac{I_{S_1} I_{S_2}}{I_{S_2}}} \cdot \exp\frac{V_B}{2V_T}\right)$$

$$= V_T \ln \sqrt{\frac{I_{S_1}}{I_{S_2}}} + \frac{V_B}{2}$$

18.




$$V_B = V_T \ln \frac{I_B}{I_{S1}} + V_T \ln \frac{I_B}{I_{S2}} = V_T \ln\left(\frac{I_B^2}{I_{S1} I_{S2}}\right)$$

$$\Rightarrow I_B = \sqrt{I_{S1} I_{S2}} \cdot \exp \frac{V_B}{2V_T}$$

Increase $I_B$ by 10 times:

$$I_{B,new} = 10\, I_B$$

$$\Rightarrow V_{B,new} = V_T \ln\left(\frac{I_{B,new}^2}{I_{S1} I_{S2}}\right) = V_T \ln\left[\frac{(10 I_B)^2}{I_{S1} I_{S2}}\right]$$

$$= V_T \ln\left(\frac{I_B^2}{I_{S1} I_{S2}}\right) + V_T \ln 100$$

$$= V_B + V_T \ln 100 \approx V_B + 0.120\ V$$

$\therefore$ $V_B$ increases by 0.120 V.

2.19

$$\begin{aligned} V_X &= I_X R_1 + V_{D1} \\ &= I_X R_1 + V_T \ln\left(\frac{I_X}{I_S}\right) \\ I_X &= \frac{V_X}{R_1} - \frac{V_T}{R_1} \ln\left(\frac{I_X}{I_S}\right) \end{aligned}$$

For each value of $V_X$, we can solve this equation for $I_X$ by iteration. Doing so, we find

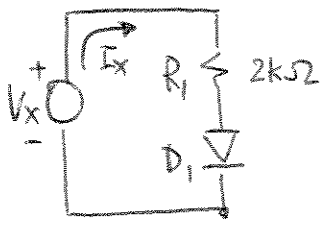
$$\begin{aligned} I_X(V_X = 0.5\text{ V}) &= 0.435\ \mu\text{A} \\ I_X(V_X = 0.8\text{ V}) &= 82.3\ \mu\text{A} \\ I_X(V_X = 1\text{ V}) &= 173\ \mu\text{A} \\ I_X(V_X = 1.2\text{ V}) &= 267\ \mu\text{A} \end{aligned}$$

Once we have $I_X$, we can compute $V_D$ via the equation $V_D = V_T \ln(I_X/I_S)$. Doing so, we find

$$\begin{aligned} V_D(V_X = 0.5\text{ V}) &= \boxed{499\text{ mV}} \\ V_D(V_X = 0.8\text{ V}) &= \boxed{635\text{ mV}} \\ V_D(V_X = 1\text{ V}) &= \boxed{655\text{ mV}} \\ V_D(V_X = 1.2\text{ V}) &= \boxed{666\text{ mV}} \end{aligned}$$

As expected, $V_D$ varies very little despite rather large changes in $I_D$ (in particular, as $I_D$ experiences an increase by a factor of over 3, $V_D$ changes by about 5 %). This is due to the exponential behavior of the diode. As a result, a diode can allow very large currents to flow once it turns on, up until it begins to overheat.

20.




Since $I_{S_1} \propto$ Area, $I_{D_1}$ becomes:

$$I_{D_1} = \frac{10 \times (2 \cdot 10^{-15} A)}{I_{S_1}'} \left(e^{\frac{V_{D1}}{V_T}} - 1\right)$$

**$V_x = 0.8$ V** Suppose $D_1$ is on. Assume $V_{D_1} = 0.7$ V

$V_{D_1} = 0.7$ V $\Rightarrow I_x = \frac{V_x - V_{D_1}}{R_1} = \frac{0.1\,V}{2\,k\Omega} = 0.05$ mA

$\Rightarrow V_{D_1} = V_T \ln(I_x / I_{S_1}') = (0.026\,V) \ln\left(\frac{0.05\,mA}{20 \cdot 10^{-15} A}\right)$

$= 0.563$ V

$V_{D_1} = 0.563$V $\Rightarrow I_x = \frac{(0.8 - 0.563)\,V}{2\,k\Omega} = 0.12$ mA

$\Rightarrow V_{D_1} = (0.026\,V) \ln\left(\frac{0.12\,mA}{20 \cdot 10^{-15} A}\right) \approx 0.585$ V

$V_{D_1} = 0.585$ V $\Rightarrow I_x = \frac{(0.8 - 0.585)V}{2\,k\Omega} = 0.11$ mA

$\Rightarrow V_{D_1} = (0.026\,V) \ln\left(\frac{0.11\,mA}{20 \cdot 10^{-15} A}\right) \approx 0.583$ V

$V_{D_1} = 0.583$ V $\Rightarrow I_x = \frac{(0.8 - 0.583)V}{2\,k\Omega} = 0.11$ mA

$\Rightarrow V_{D_1} = 0.583$ V

$\therefore$ $V_{D_1} \approx 0.583$ V

$I_x \approx 0.11$ mA.

$\boxed{V_X = 1.2\,V}$ Suppose $D_1$ is on. Use results from previous calculations as starting point.

$$V_{D_1} = 0.583\,V \Rightarrow I_X = \frac{(1.2 - 0.583)V}{2\ k\Omega} = 0.31\ mA$$

$$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{0.31mA}{20\times10^{-15}A}\right) \approx 0.610\,V$$

$$V_{D_1} = 0.610\,V \Rightarrow I_X = \frac{(1.2 - 0.610)V}{2\ k\Omega} = 0.30\,mA$$

$$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{0.30\,mA}{20\cdot10^{-15}A}\right) \approx 0.609\,V$$

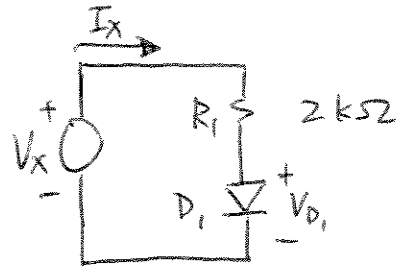
$$V_{D_1} = 0.609\,V \Rightarrow I_X = \frac{(1.2 - 0.609)V}{2\ k\Omega} = 0.30\,mA$$

$$\Rightarrow V_{D_1} = 0.609\,V$$

$\therefore V_{D_1} \approx 0.609\,V$
$I_X \approx 0.30\,mA.$

By increasing the cross-section area of $D_1$, intuitively this means $D_1$ can conduct same amount of current with less $V_{D_1}$. The results have shown that in this problem, $V_{D_1}$ is less and $I_X$ is more.

21.




Given: @ $V_X = 2V$, $V_{D_1} = 850mV$

$$\Rightarrow I_X = \frac{V_X - V_{D_1}}{R_1} = 0.58\ mA$$

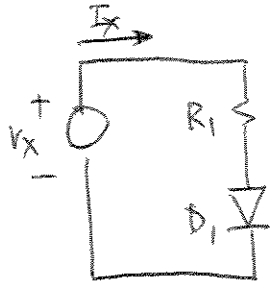
$$\therefore I_S = \frac{I_X}{(e^{V_{D_1}/V_T} - 1)} \approx I_X \exp[-V_{D_1}/V_T]$$

$$= (0.58\ mA) \exp[-0.85/0.026] \approx 3.64 \cdot 10^{-18} A$$

2.22

$$
\begin{aligned}
V_X/2 = I_X R_1 = V_{D1} &= V_T \ln(I_X/I_S) \\
I_X &= \frac{V_T}{R_1} \ln(I_X/I_S) \\
I_X &= 367\ \mu\text{A (using iteration)} \\
V_X &= 2 I_X R_1 \\
&= \boxed{1.47\ \text{V}}
\end{aligned}
$$

23.




Given $V_x = 1V \Rightarrow I_x = 0.2mA$

$V_x = 2V \Rightarrow I_x = 0.5mA$

Find $R_1$ and $I_s$.

By KVL, $V_{D_1} = V_x - I_x R_1 = V_T \ln\left(\frac{I_x}{I_s}\right)$

$\Rightarrow 1 - (0.2mA)R_1 = (0.026V)\ln\left(\frac{0.2mA}{I_s}\right)$ ——— ①

$2 - (0.5mA)R_1 = (0.026V)\ln\left(\frac{0.5mA}{I_s}\right)$ ——— ②

② − ① : $1 - (0.3mA)R_1 = (0.026V)\ln\left(\frac{0.5}{0.2}\right)$

$\Rightarrow R_1 = \frac{1 - (0.026)\ V}{0.3m\ A} = 3.25\ k\Omega$

Substitute $R_1$ into ①:

$$I_s = I_x \cdot \exp\left[-\frac{V_x - I_x R_1}{V_T}\right]$$

$$= (0.2mA)\exp\left[-\frac{1-(0.2m)(3.25k)}{0.026}\right] \approx 2.94 \cdot 10^{-10} A$$

$\therefore R_1 \approx 3.25\ k\Omega$

$I_s \approx 2.94 \cdot 10^{-10} A.$

24.




Given $I_S = 3 \cdot 10^{-16}\,A$, find $V_{D_1}$.

By KCL, $I_x = \frac{V_{D1}}{R_1} + I_{D_1} = \frac{V_T}{R_1} \ln\left(\frac{I_{D1}}{I_S}\right) + I_{D_1}$

Since $I_x$, $V_T$, $R_1$ and $I_S$ are known, this can be solved directly with special programs or graphing calculators. However, this can be also solved by iterations: Assume a $V_{D_1}$, calculate $I_{D_1}$, and re-iterate on $V_{D1}$.

Assume $V_{D_1} = 0.7\,V$ as starting point.

**$I_x = 1\,mA$**

$V_{D_1} = 0.7\,V \Rightarrow I_{D_1} = I_x - V_{D1}/R_1 = 1\,mA - \frac{0.7\,V}{1\,k\Omega} = 0.3\,mA$

$\Rightarrow V_{D_1} = V_T \ln\left(I_x/I_S\right)$

$= (0.026V) \ln\left(\frac{0.3\,mA}{3 \cdot 10^{-16}\,A}\right) \approx 0.718\,V$

$V_{D_1} = 0.718\,V \Rightarrow I_{D_1} = 1\,mA - \frac{0.718\,V}{1\,k\Omega} = 0.28\,mA$

$\Rightarrow V_{D_1} = (0.026\,V) \ln\left(\frac{0.28\,mA}{3 \cdot 10^{-16}\,A}\right) \approx 0.717\,V$

$V_{D_1} = 0.717\,V \Rightarrow I_{D_1} = 1mA - \frac{0.717V}{1k\Omega} = 0.28mA$

$\Rightarrow V_{D_1} = 0.717\,V$

$\therefore V_{D_1} \approx 0.717\,V.$

$\boxed{I_X = 2mA}$ Assume $V_{D_1} = 0.717\,V$ from previous result.

$V_{D_1} = 0.717\,V \Rightarrow I_{D_1} = 2mA - \frac{0.717V}{1k\Omega} = 1.28\,mA$

$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{1.28mA}{3\cdot10^{-16}A}\right) \approx 0.756V$

$V_{D_1} = 0.756\,V \Rightarrow I_{D_1} = 2mA - \frac{0.756V}{1k\Omega} = 1.24\,mA$

$\Rightarrow V_{D_1} = (0.026\,V)\ln\left(\frac{1.24mA}{3\cdot10^{-16}A}\right) \approx 0.755V$

$V_{D_1} = 0.755V \Rightarrow I_{D_1} = 2mA - \frac{0.755\,V}{1k\Omega} = 1.24\,mA$

$\Rightarrow V_{D_1} = 0.755V$

$\therefore V_{D_1} = 0.755\,V$

$\boxed{I_X = 4\,mA}$ Assume $V_{D_1} = 0.755\,V$ from previous result.

$$V_{D_1} = 0.755V \Rightarrow I_{D_1} = 4mA - \frac{0.755V}{1k\Omega} = 3.25\,mA$$

$$\Rightarrow V_{D_1} = (0.026)V \ln\left(\frac{3.25mA}{3\cdot10^{-16}A}\right) \approx 0.780\,V$$

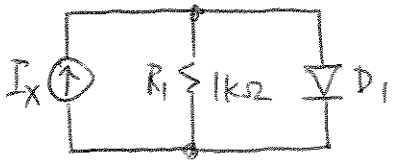
$$V_{D_1} = 0.780\,V \Rightarrow I_{D_1} = 4mA - \frac{0.780V}{1k\Omega} = 3.22\,mA$$

$$\Rightarrow V_{D_1} = (0.026\,V)\ln\left(\frac{3.22mA}{3\cdot10^{-16}A}\right) \approx 0.780V$$

$\therefore V_{D_1} \approx 0.780\,V.$

Note: As $I_X$ increases, $I_{D_1}$ increases, while $(V_{D_1}/R_1)$ stays relatively the same. Because of the exponential characteristic, the diode, once on, will absorb as much current as necessary to satisfy KCL.

25.




Given $I_{D_1} = 0.5\,mA$ when $I_X = 1.3\,mA$, find $I_S$.

This means $V_{D_1} = (I_X - I_{D_1}) R_1$

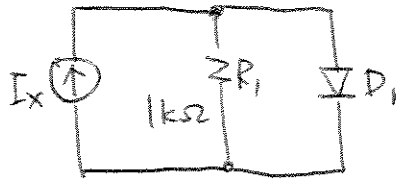
$= (0.8\,mA)\, 1k\Omega = 0.8\,V$

$$\Rightarrow I_S = I_{D_1} \cdot \exp\left[-V_{D_1}/V_T\right]$$

$$= (0.5\,mA)\exp\left[-0.8\,V / 0.026\,V\right]$$

$$\approx 2.17 \cdot 10^{-17}\,A$$

26




Given $I_{R_1} = I_x/2$

$I_S = 3 \cdot 10^{-16}$ A

find $I_x$.

$$V_{D_1} = \frac{I_x}{2} \cdot R_1 = V_T \ln\left(\frac{I_x/2}{I_S}\right)$$

This can be solved directly with special programs or graphing calculators. Alternatively, one can solve this iteratively by hand.

Assume $V_D = 0.8$ V.

$$V_D = 0.8\text{ V} \Rightarrow (I_x/2) = \frac{V_D}{R_1} = \frac{0.8\text{ V}}{1\text{ k}\Omega} = 0.8\text{ mA}$$

$$\Rightarrow V_D = V_T \ln\left(\frac{I_x/2}{I_S}\right) = (0.026\text{V}) \ln\left(\frac{0.8\text{ mA}}{3 \cdot 10^{-16}\text{A}}\right)$$

$$\approx 0.744\text{ V}$$

$$V_D = 0.744\text{ V} \Rightarrow I_x/2 = \frac{0.744\text{V}}{1\text{ k}\Omega} = 0.744\text{ mA}$$

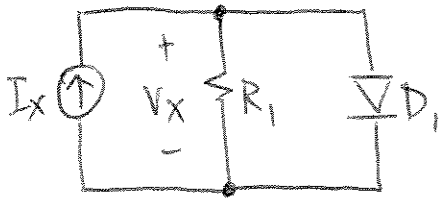
$$\Rightarrow V_D = (0.026\text{V}) \ln\left(\frac{0.744\text{ mA}}{3 \cdot 10^{-16}\text{A}}\right) \approx 0.742\text{ V}$$

$$V_D = 0.742V \quad \Rightarrow \quad I_X/2 = \frac{0.742\,V}{1\,k\Omega} = 0.742\ mA$$

$$\Rightarrow V_D = (0.026V)\ln\left(\frac{0.742mA}{3\cdot 10^{-16}A}\right) \approx 0.742V$$

$$\therefore I_X = 2(0.742\,mA) = 1.48\ mA$$

27.




Given $I_X = 1mA \rightarrow V_X = 1.2V$
$I_X = 2mA \rightarrow V_X = 1.8V$

find $R_1$ and $I_S$.

$$I_{D_1} = I_X - V_X/R_1 \quad \text{(KCL)}$$

By KVL, $$V_X = V_T \ln\left(\frac{I_{D_1}}{I_S}\right) = V_T \ln\left(\frac{I_X - V_X/R_1}{I_S}\right)$$

$$\Rightarrow (1.2\,V) = (0.026\,V) \ln\left[\frac{(1mA) - (1.2V)/R_1}{I_S}\right] \quad \text{——— ①}$$

$$(1.8\,V) = (0.026\,V) \ln\left[\frac{(2mA) - (1.8V)/R_1}{I_S}\right] \quad \text{——— ②}$$

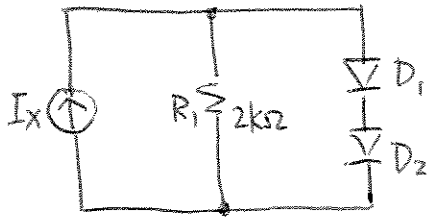
② − ①: $$0.6V = (0.026V) \ln\left(\frac{2mA - 1.8V/R_1}{1mA - 1.2V/R_1}\right)$$

$$\Rightarrow R_1 = \frac{1.2 \cdot \exp\left[0.6/0.026\right] - 1.8}{1mA \cdot \exp\left[0.6/0.026\right] - 2mA} \approx 1.2\,k\Omega$$

$$I_S = I_D \exp\left[-V_X/V_T\right] = \left(2mA - \frac{1.8V}{1.2k\Omega}\right) \exp\left[\frac{-1.8V}{0.026V}\right]$$

$$\approx 4.29 \cdot 10^{-34}\,A.$$

28.




Given $D_1 = D_2$ with $I_S = 5 \cdot 10^{-16} A$

Find $V_{R_1}$ for $I_X = 2mA$.

Current through the diodes $= I_D$
$= I_X - \frac{V_{R_1}}{R_1}$ where $V_{R_1}$ = voltage across $R_1$

$$\Rightarrow V_{R_1} = 2 \cdot V_T \ln\left(\frac{I_D}{I_S}\right) = 2\left[V_T \ln\left(\frac{I_X}{I_S} - \frac{V_{R_1}}{I_S R_1}\right)\right]$$

This can be solved directly with special programs or graphing calculators or by hand iteratively.

Assume a $V_{R_1}$, calculate $I_D$, and re-iterate on new $V_{R_1} = (2 \times V_{D_1})$. From experience, most diodes conduct at $V_D \approx 0.7\,V$. Assume $V_{R_1} = 1.4\,V$.

$$V_{R_1} = 1.4\,V \Rightarrow I_D = I_X - \frac{V_{R_1}}{R_1} = 2mA - \frac{1.4V}{2k\Omega} = 1.3mA$$

$$\Rightarrow V_{R_1} = 2 V_T \ln\left(\frac{I_D}{I_S}\right)$$

$$= 2(0.026V) \ln\left(\frac{1.3mA}{5 \cdot 10^{-16} A}\right) \approx 1.49\,V$$

$V_{R_1} = 1.49\,V \Rightarrow I_D = 2mA - \frac{1.49}{2k\Omega} = 1.26\,mA$

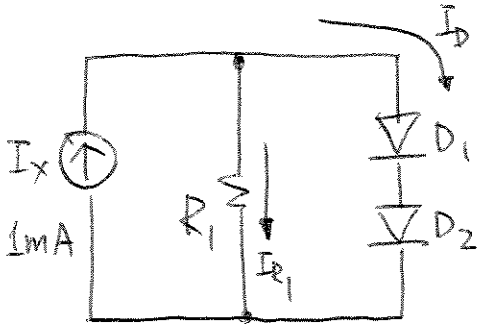
$\Rightarrow V_{R_1} = 2(0.026V)\ln\left(\frac{1.26\,mA}{5\cdot10^{-16}A}\right) \approx 1.48\,V$

$V_{R_1} = 1.48\,V \Rightarrow I_D = 2mA - \frac{1.48\,V}{2k\Omega} = 1.26\,mA$

$\Rightarrow V_{R_1} = 1.48\,V$

$\therefore$ voltage across $R_1 = 1.48\,V$

29.




Given $I_{R_1} = 0.5\,mA$,
$I_S = 5 \cdot 10^{-16}\,A$ for each diode.

Find $R_1$.

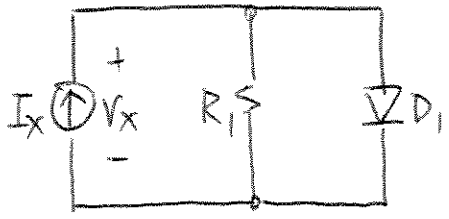
By KCL, $I_D = I_x - I_{R_1} = 0.5\,mA$

$$\Rightarrow V_{D_1} = V_{D_2} = V_T \ln\left(\frac{I_D}{I_S}\right) = 0.026 \ln\left(\frac{0.5mA}{5 \cdot 10^{-16}A}\right)$$

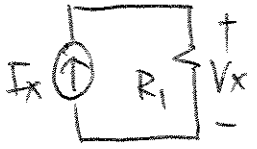
$$\approx 0.718\ V$$

$$\therefore R_1 = \frac{V_{R_1}}{I_{R_1}} = \frac{2V_{D_1}}{I_{R_1}} = \frac{2(0.718V)}{0.5mA} = 2.87\ k\Omega$$

30.

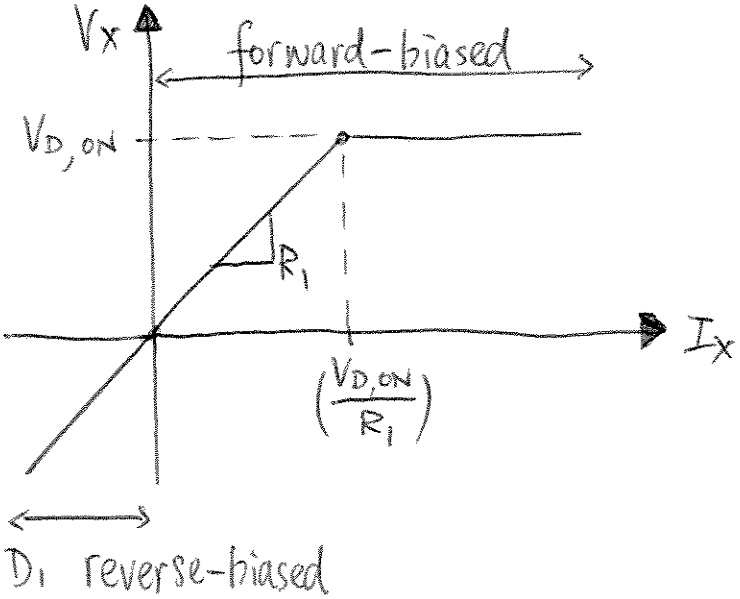



(a) Constant-voltage model:
Consider, first, the extreme cases: when $D_1$ is off, we have the following:

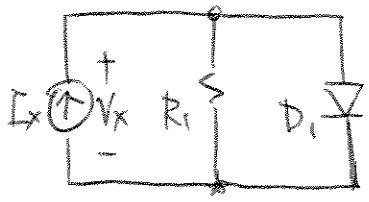



This implies $V_x$ is linearly proportional to $I_x$

When $D_1$ is on, $V_x$ is <u>fixed</u> (by KVL) by $D_1$ ($= V_{D,ON}$). This implies that any additional current from $I_x$ cannot flow through $R_1$, which means $D_1$ will absorb all the currents to satisfy KVL.



(b) exponential model:



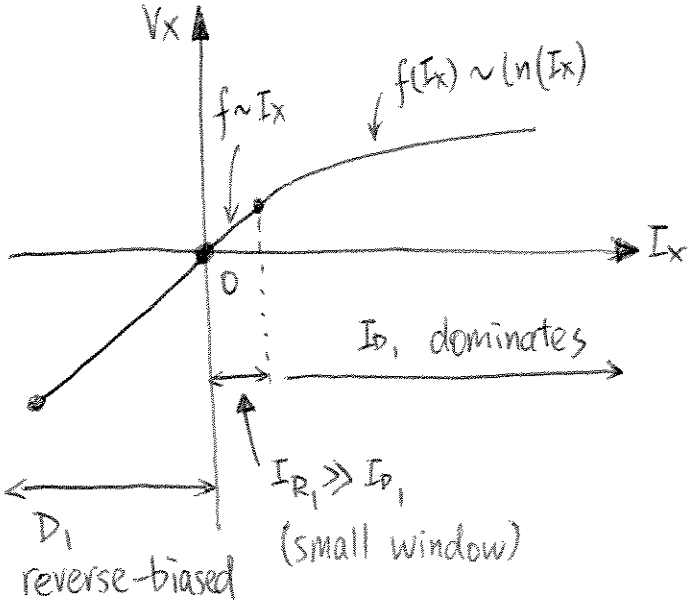

Assume $I_S$ negligible.

When $D_1$ is off, most of $I_X$ flows through $R_1$. When $D_1$ is on, $V_{D_1}$ $(= V_X)$ follows this relationship:

$$V_{D_1} = V_X = V_T \ln\left(\frac{I_{D_1}}{I_S}\right) = V_T \ln\left(\frac{I_X}{I_S} - \frac{V_X}{I_X R_1}\right)$$

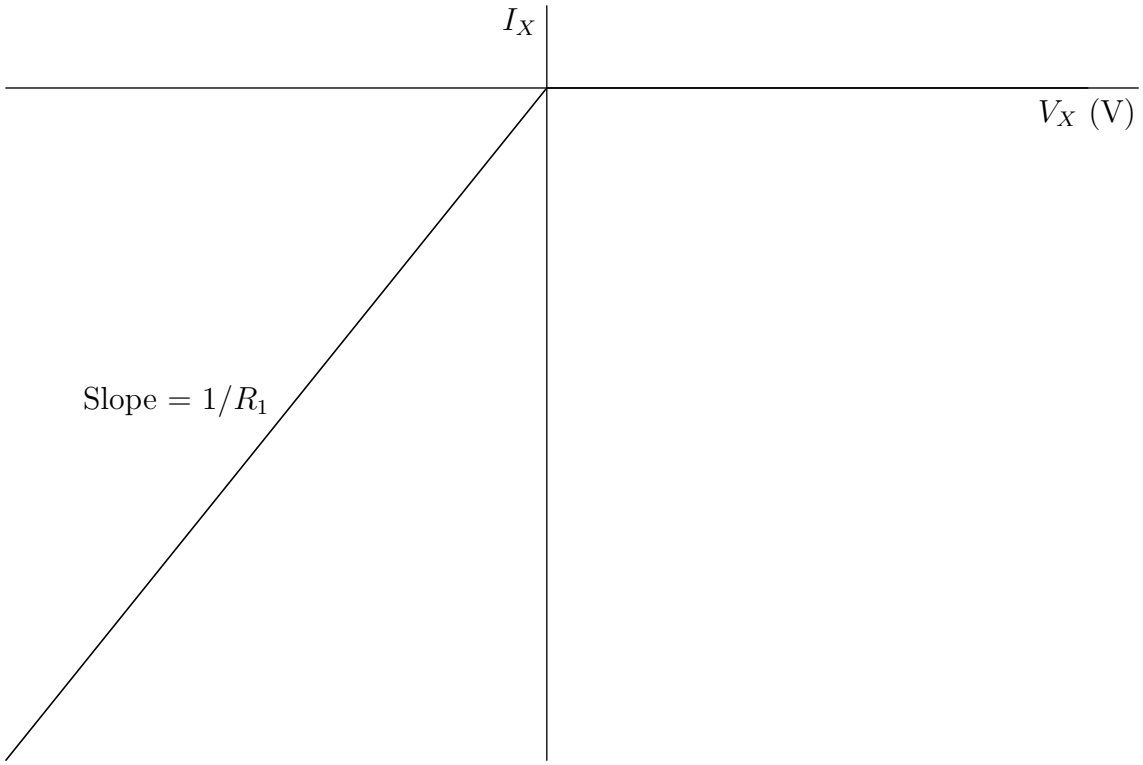
$$\Rightarrow I_X = I_S \exp(V_X/V_T) + V_X/R_1$$
$$\approx I_S \exp(V_X/V_T) \quad \text{when } D_1 \text{ is forward-biased } (V_X > V_T)$$

i.e. $V_X \approx V_T \ln(I_X/I_S)$



3.1 (a)

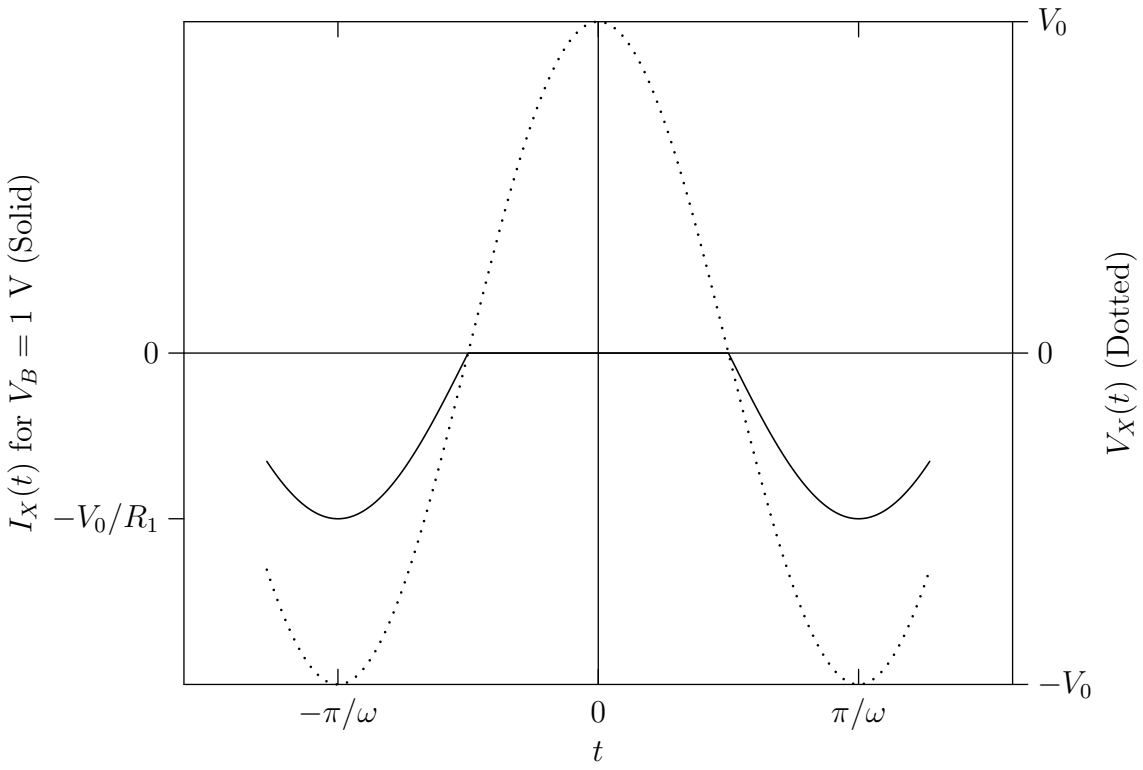
$$I_X = \begin{cases} \frac{V_X}{R_1} & V_X < 0 \\ 0 & V_X > 0 \end{cases}$$



3.2

$$I_X = \begin{cases} \frac{V_X}{R_1} & V_X < 0 \\ 0 & V_X > 0 \end{cases}$$

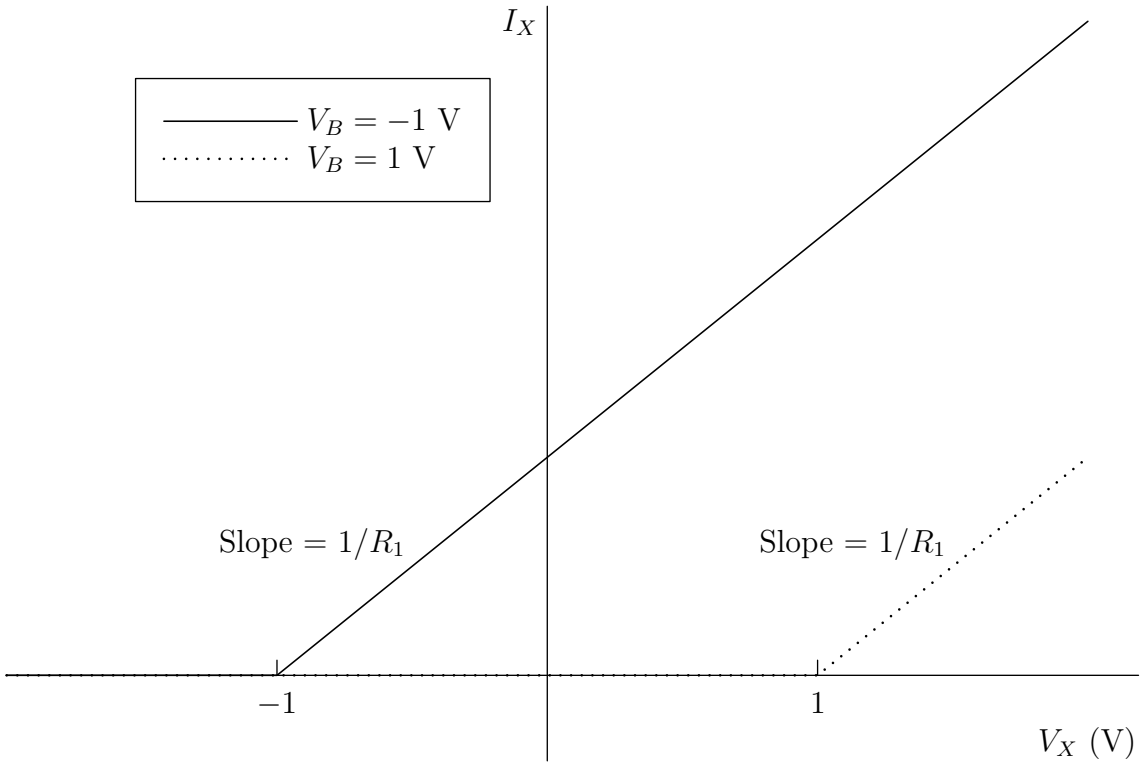
Plotting $I_X(t)$, we have



3.3

$$I_X = \begin{cases} 0 & V_X < V_B \\ \frac{V_X - V_B}{R_1} & V_X > V_B \end{cases}$$

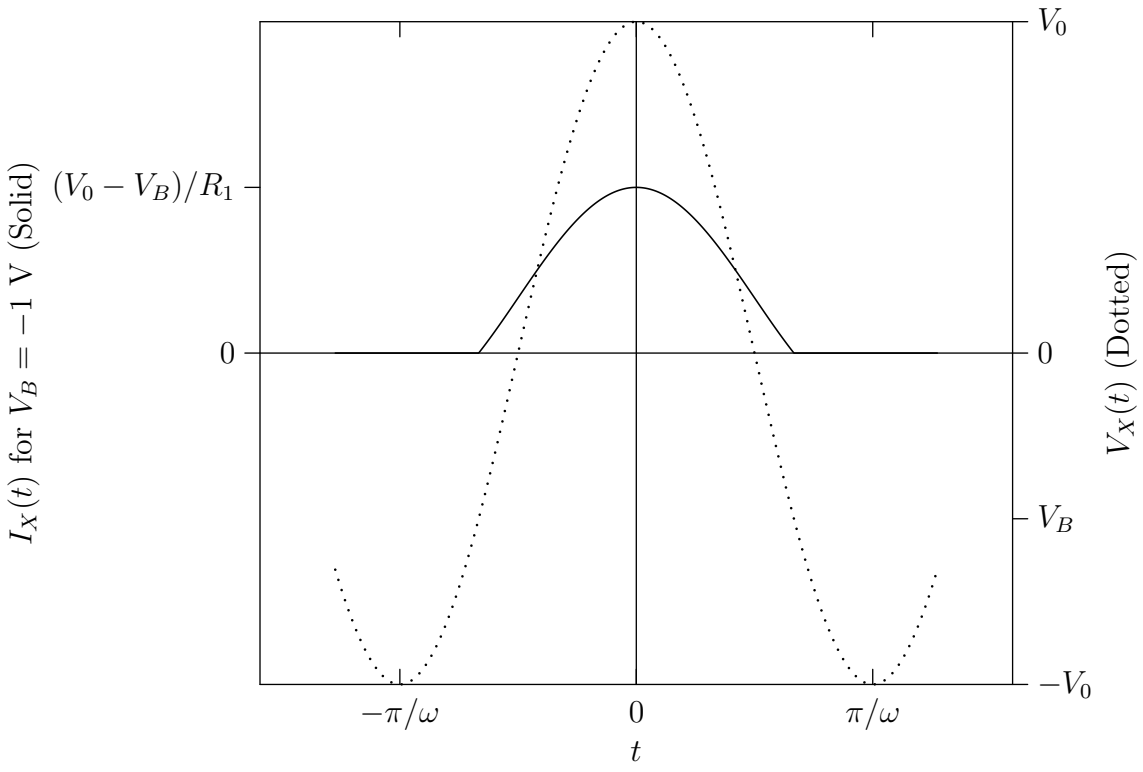
Plotting $I_X$ vs. $V_X$ for $V_B = -1$ V and $V_B = 1$ V, we get:



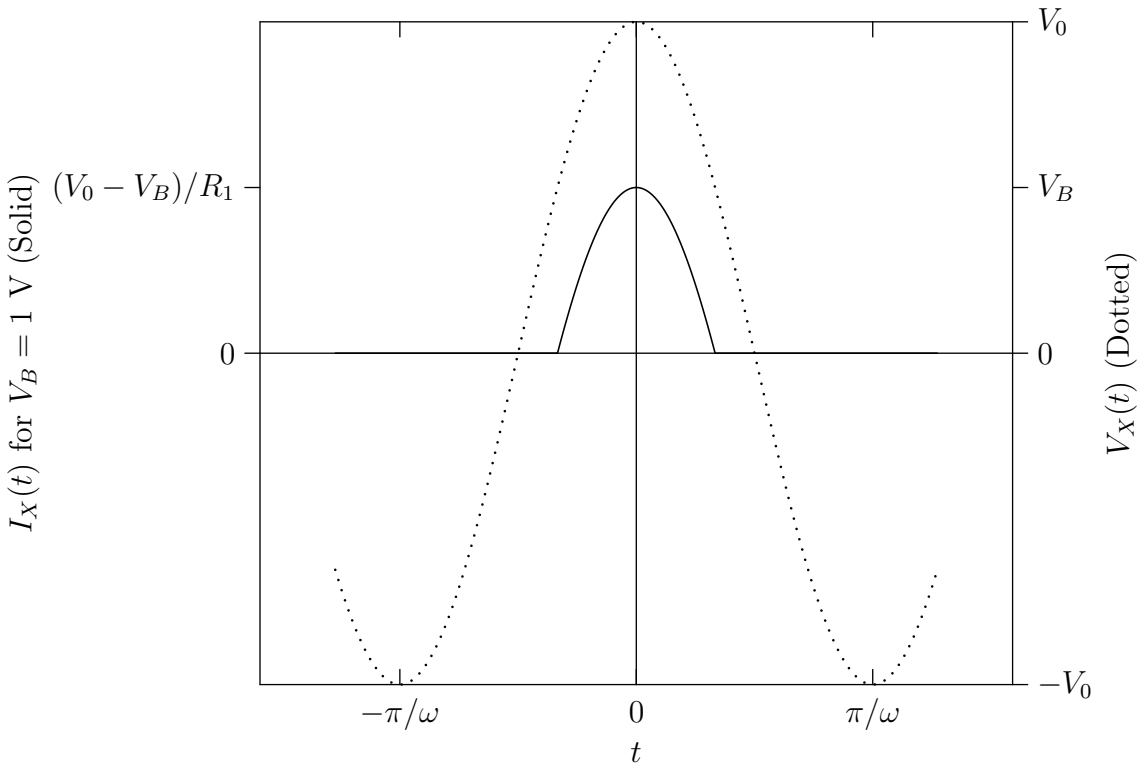
3.4

$$I_X = \begin{cases} 0 & V_X < V_B \\ \frac{V_X - V_B}{R_1} & V_X > V_B \end{cases}$$

Let's assume $V_0 > 1$ V. Plotting $I_X(t)$ for $V_B = -1$ V, we get



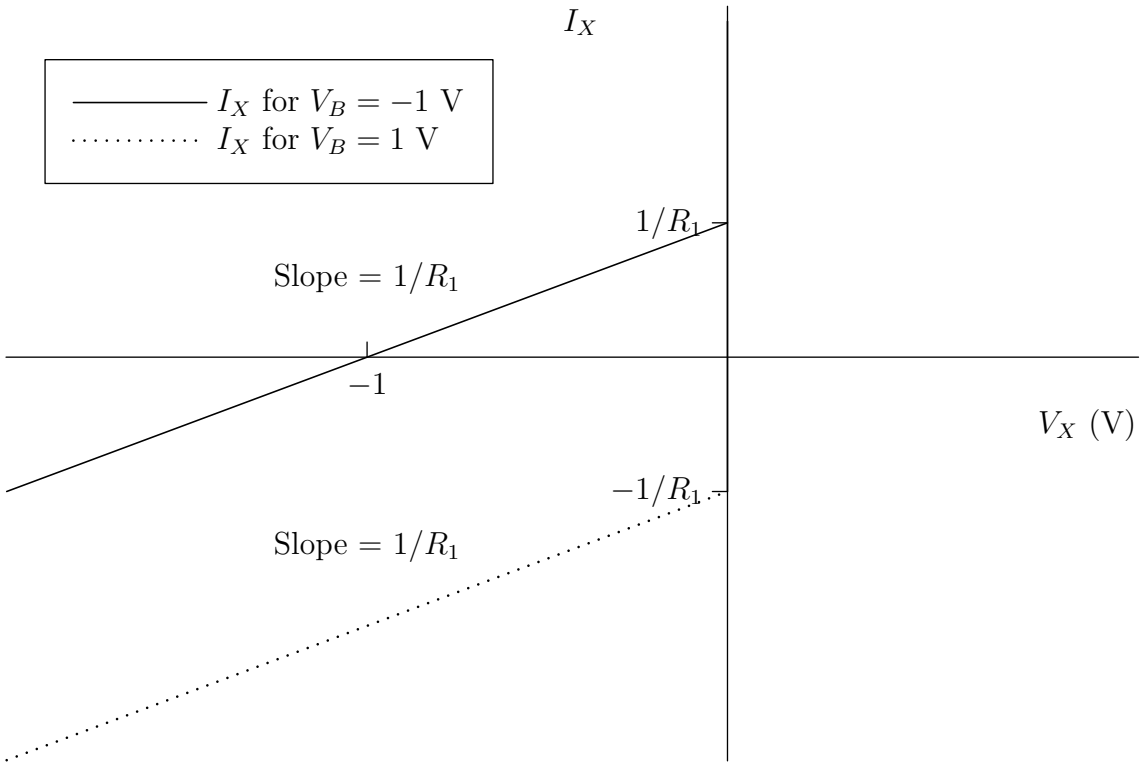

Plotting $I_X(t)$ for $V_B = 1$ V, we get


$V_0$
$(V_0 - V_B)/R_1$
$V_B$
0
0
$-V_0$
$I_X(t)$ for $V_B = 1$ V (Solid)
$V_X(t)$ (Dotted)
$-\pi/\omega$
0
$\pi/\omega$
$t$

3.5

$$I_X = \begin{cases} \frac{V_X - V_B}{R_1} & V_X < 0 \\ \infty & V_X > 0 \end{cases}$$

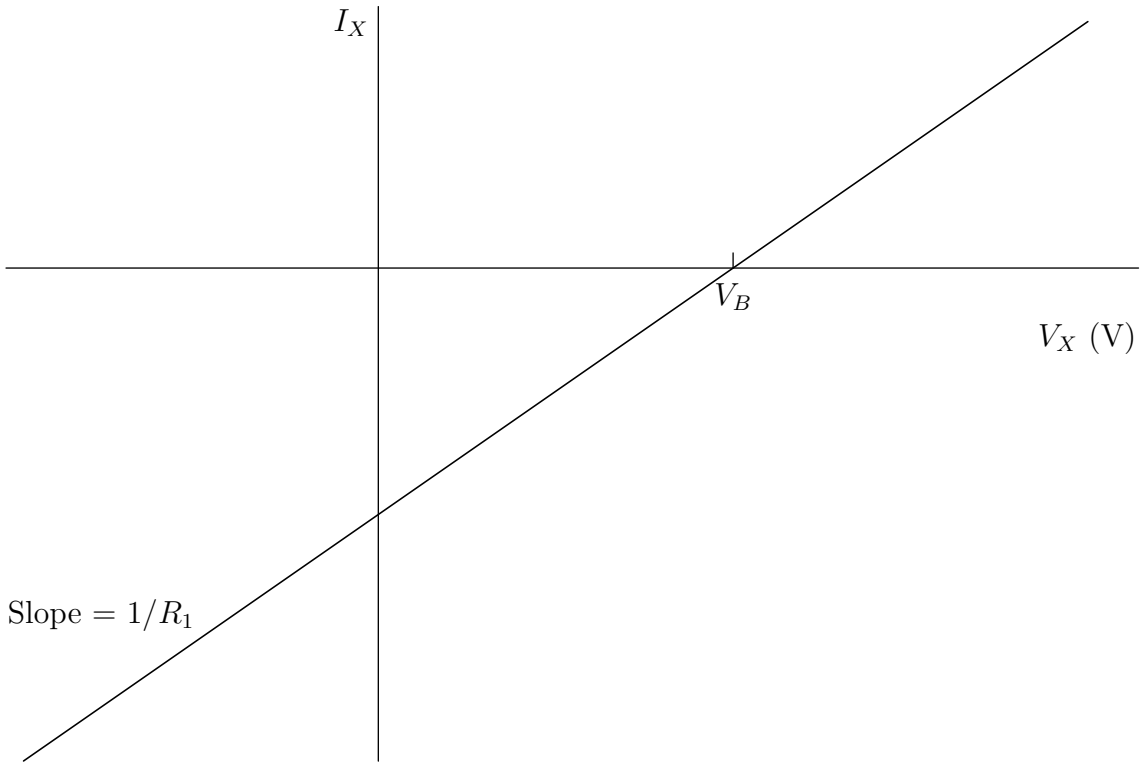
Plotting $I_X$ vs. $V_X$ for $V_B = -1$ V and $V_B = 1$ V, we get:



3.6 First, note that $I_{D1} = 0$ always, since $D_1$ is reverse biased by $V_B$ (due to the assumption that $V_B > 0$). We can write $I_X$ as

$$I_X = (V_X - V_B)/R_1$$

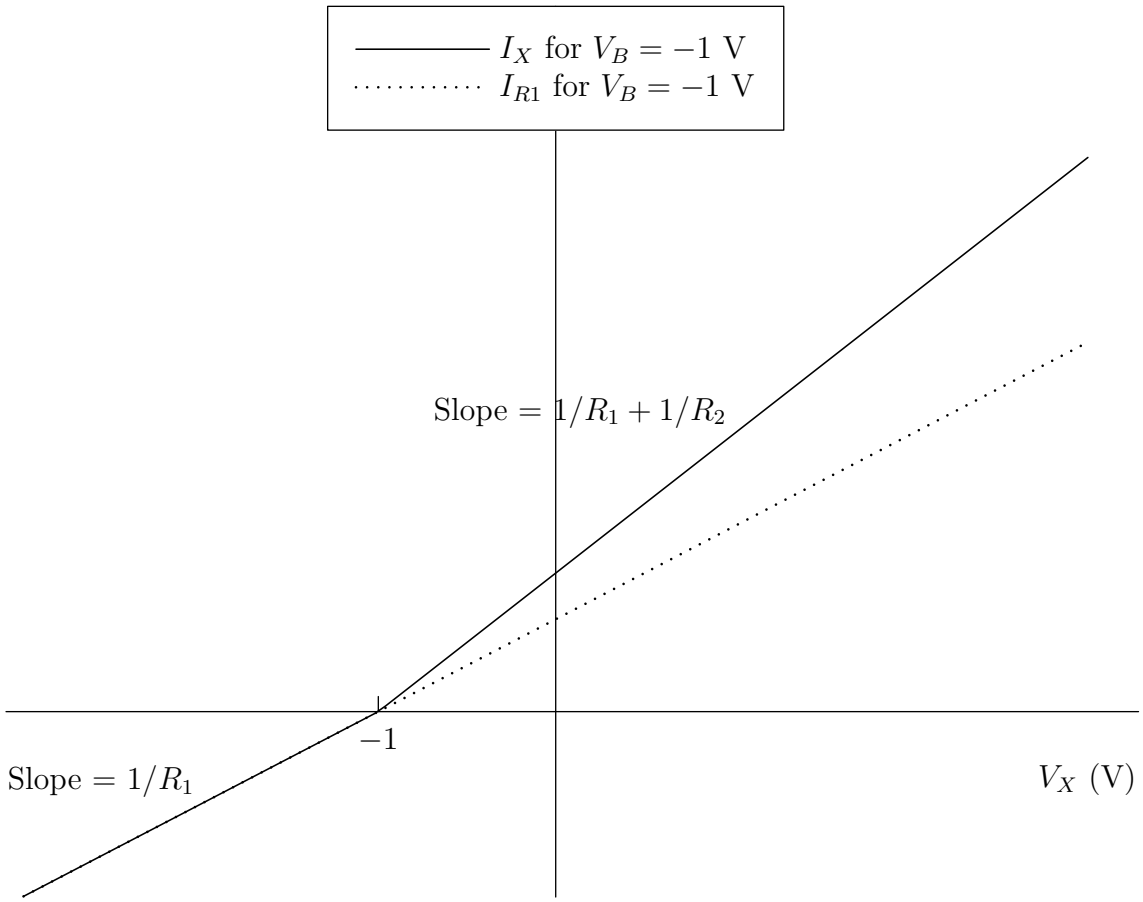
Plotting this, we get:



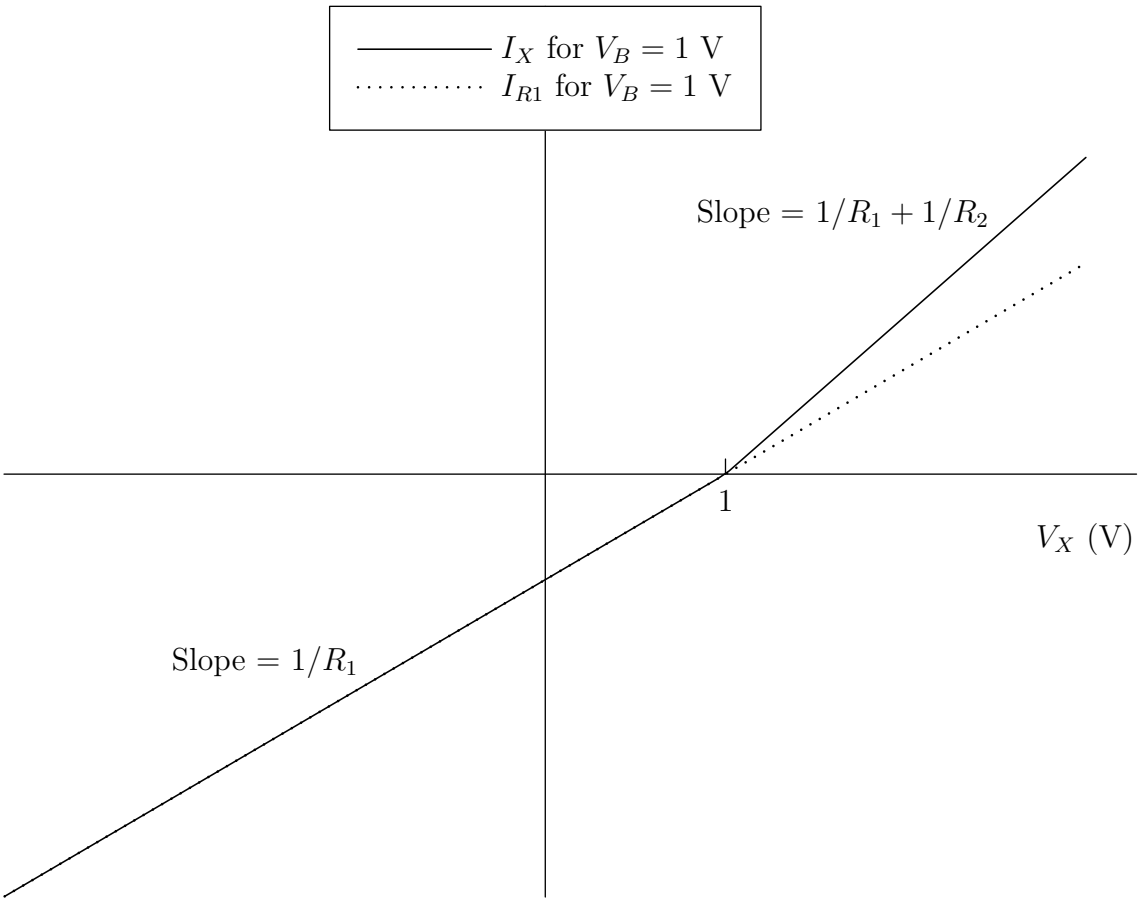
3.7

$$I_X = \begin{cases} \frac{V_X - V_B}{R_1} & V_X < V_B \\ \frac{V_X - V_B}{R_1 \| R_2} & V_X > V_B \end{cases}$$
$$I_{R1} = \frac{V_X - V_B}{R_1}$$

Plotting $I_X$ and $I_{R1}$ for $V_B = -1$ V, we get:




Plotting $I_X$ and $I_{R1}$ for $V_B = 1$ V, we get:

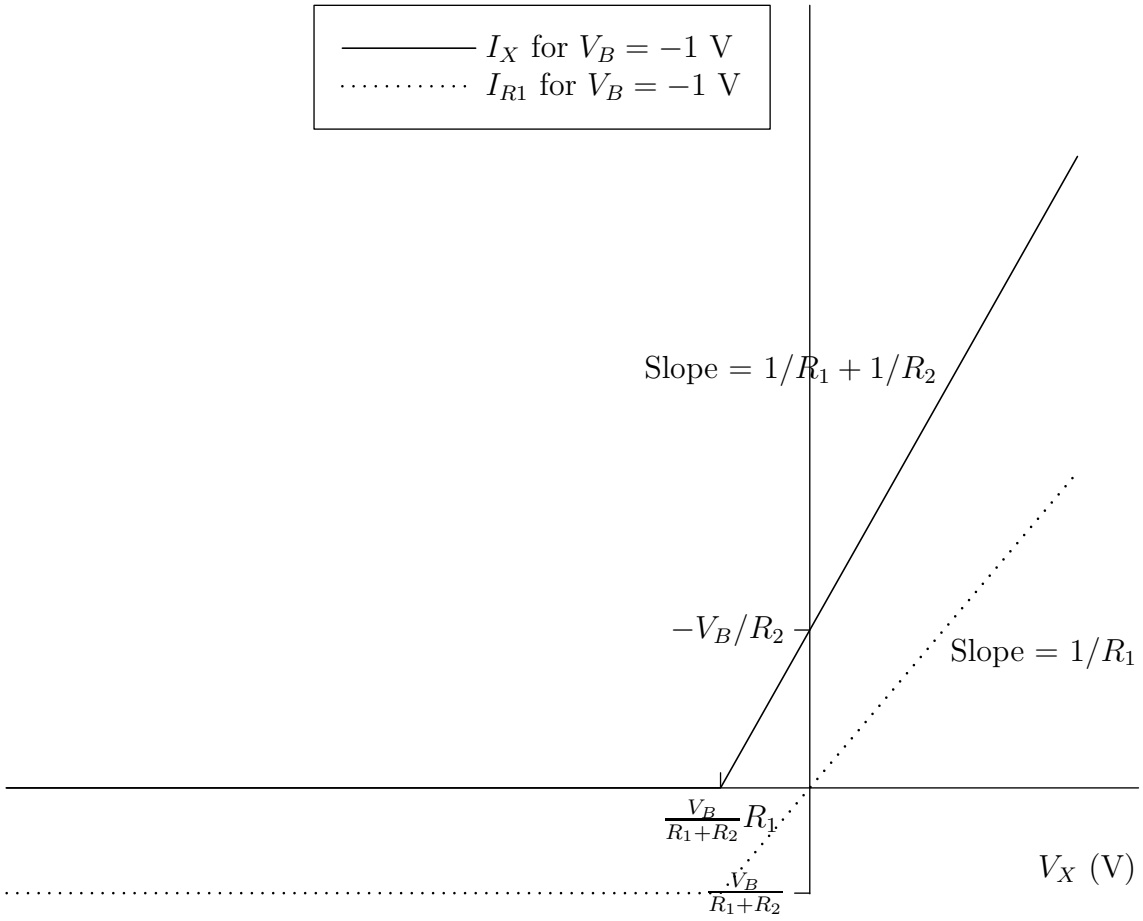

$I_X$ for $V_B = 1$ V
$I_{R1}$ for $V_B = 1$ V
Slope $= 1/R_1 + 1/R_2$
1
$V_X$ (V)
Slope $= 1/R_1$

3.8

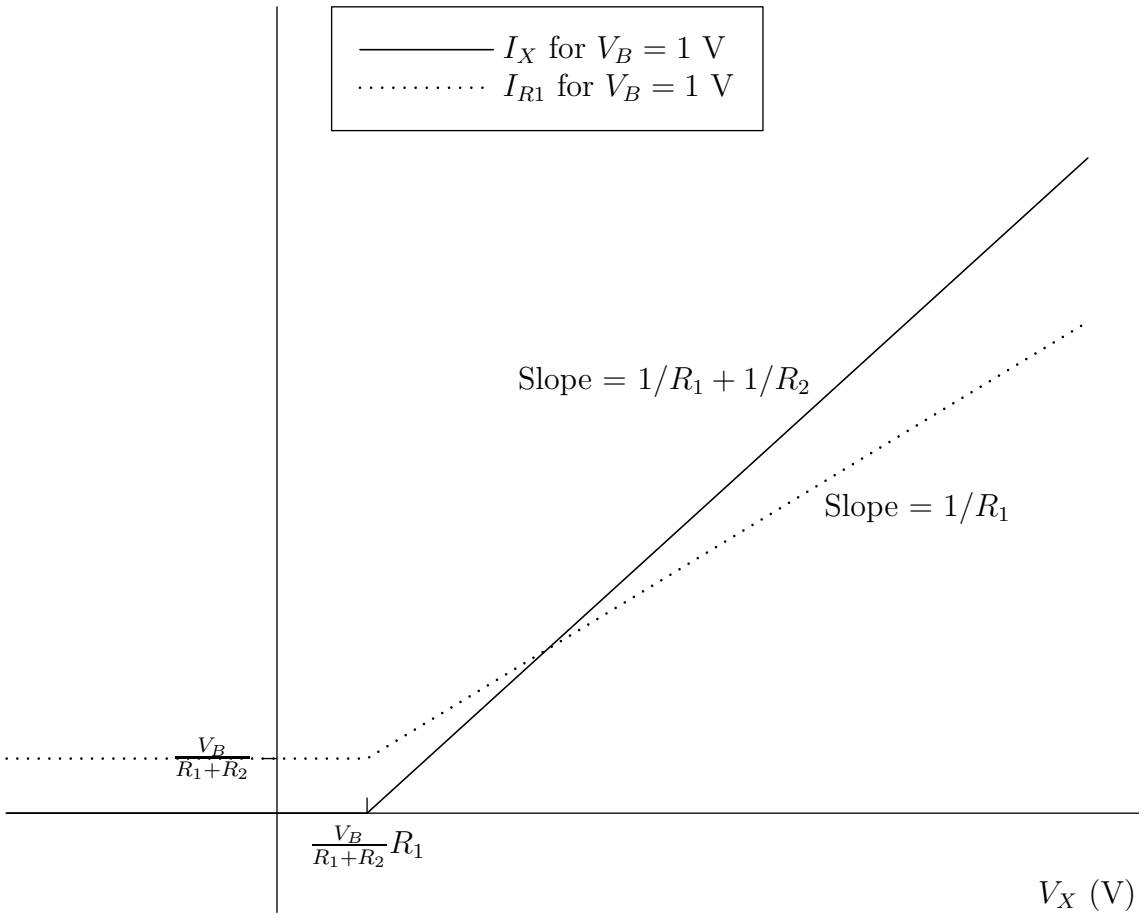
$$I_X = \begin{cases} 0 & V_X < \frac{V_B}{R_1+R_2}R_1 \\ \frac{V_X}{R_1} + \frac{V_X - V_B}{R_2} & V_X > \frac{V_B}{R_1+R_2}R_1 \end{cases}$$

$$I_{R1} = \begin{cases} \frac{V_B}{R_1+R_2} & V_X < \frac{V_B}{R_1+R_2}R_1 \\ \frac{V_X}{R_1} & V_X > \frac{V_B}{R_1+R_2}R_1 \end{cases}$$

Plotting $I_X$ and $I_{R1}$ for $V_B = -1$ V, we get:

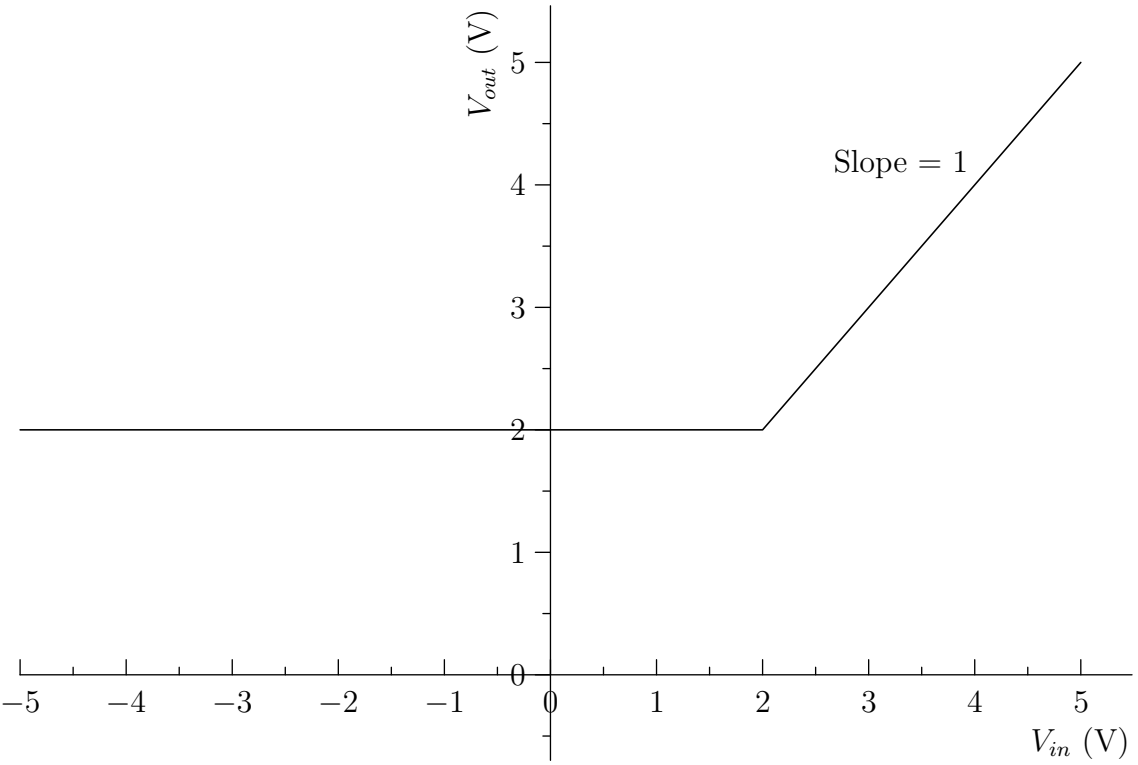



Plotting $I_X$ and $I_{R1}$ for $V_B = 1$ V, we get:


$I_X$ for $V_B = 1$ V
$I_{R1}$ for $V_B = 1$ V
Slope $= 1/R_1 + 1/R_2$
Slope $= 1/R_1$
$\frac{V_B}{R_1+R_2}$
$\frac{V_B}{R_1+R_2}R_1$
$V_X$ (V)

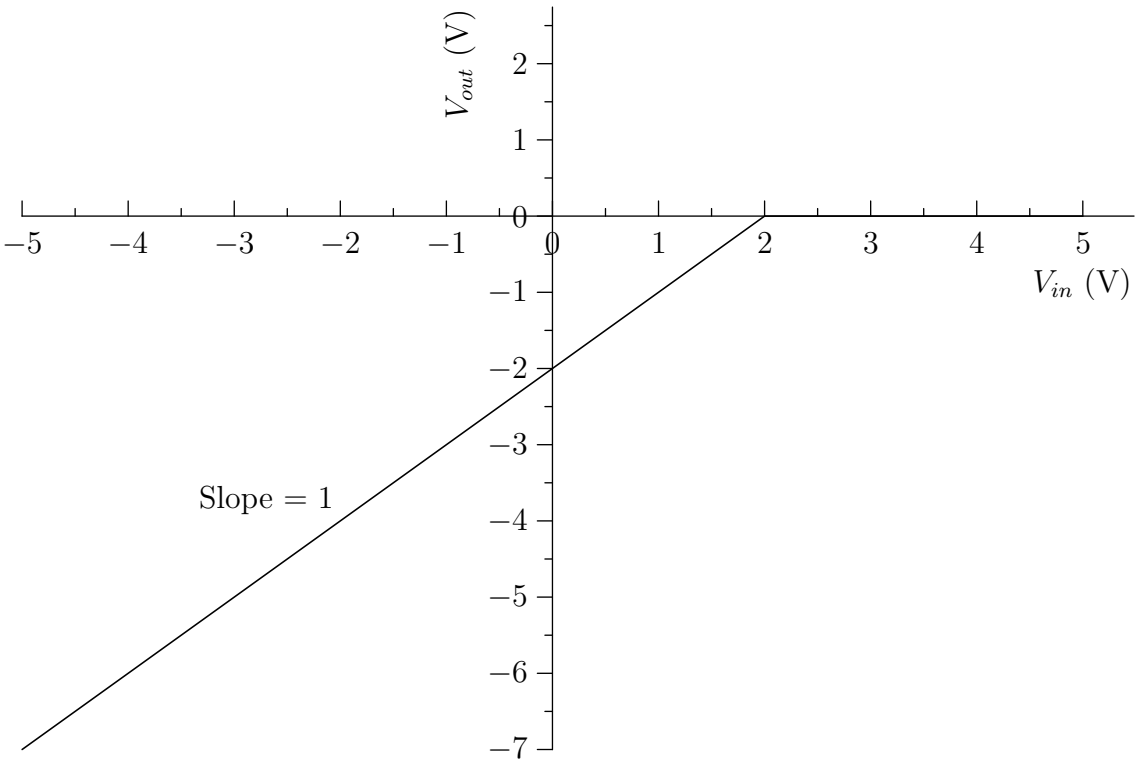
3.9 (a)

$$V_{out} = \begin{cases} V_B & V_{in} < V_B \\ V_{in} & V_{in} > V_B \end{cases}$$



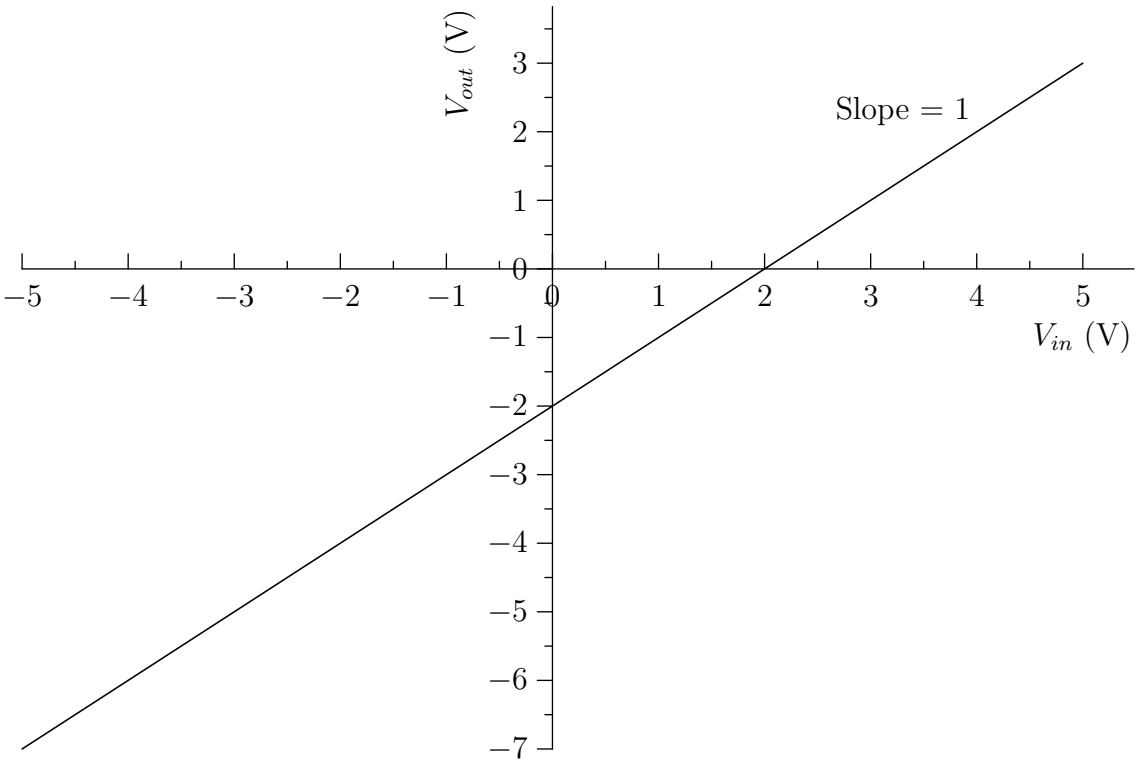

(b)

$$V_{out} = \begin{cases} V_{in} - V_B & V_{in} < V_B \\ 0 & V_{in} > V_B \end{cases}$$



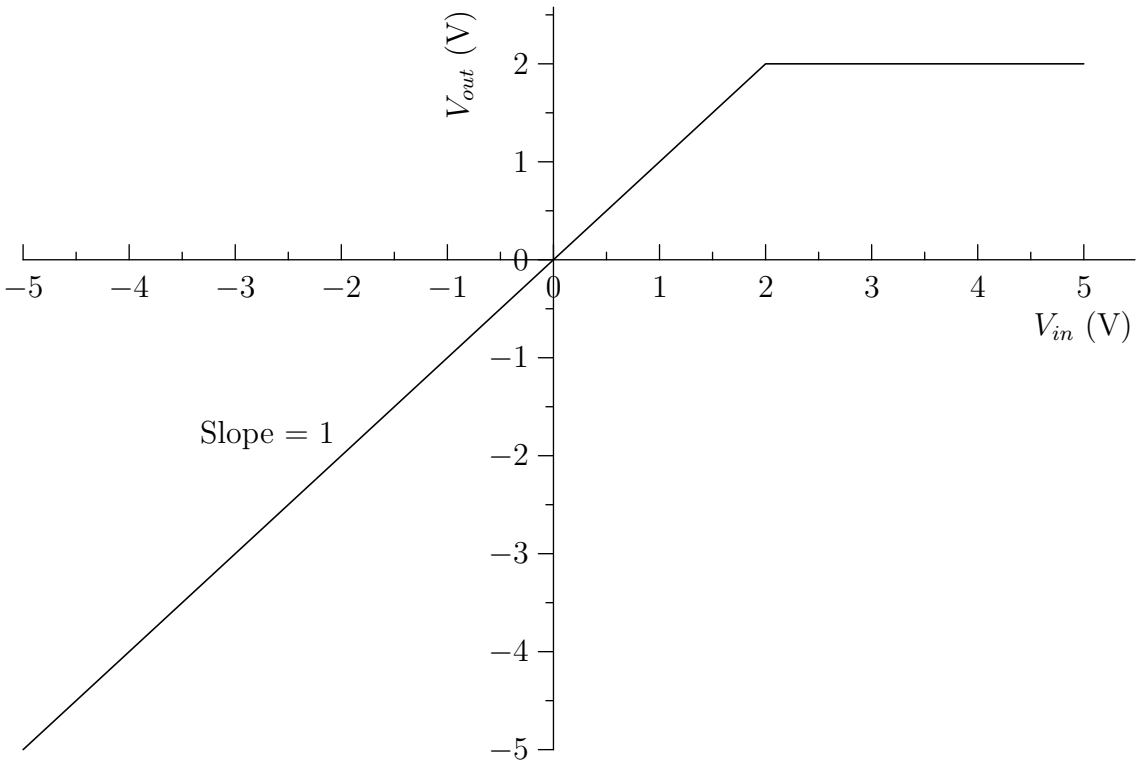

(c)

$$V_{out} = V_{in} - V_B$$



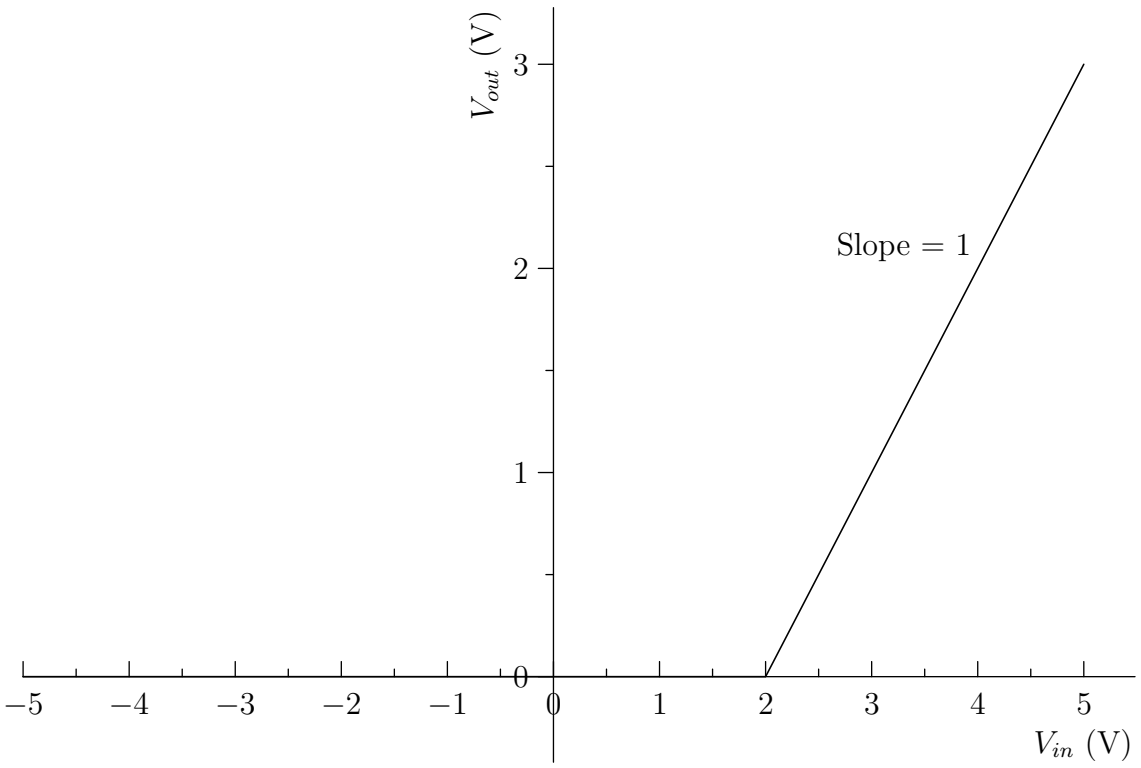
(d)

$$V_{out} = \begin{cases} V_{in} & V_{in} < V_B \\ V_B & V_{in} > V_B \end{cases}$$

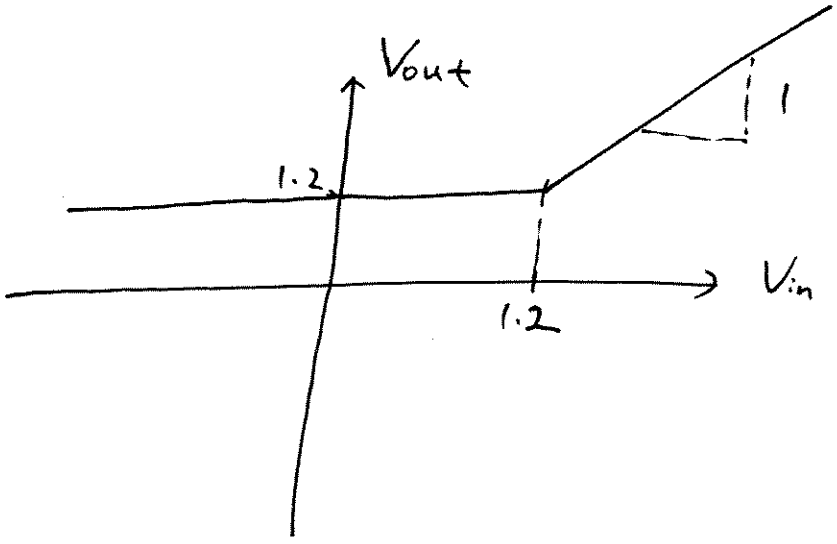



(e)

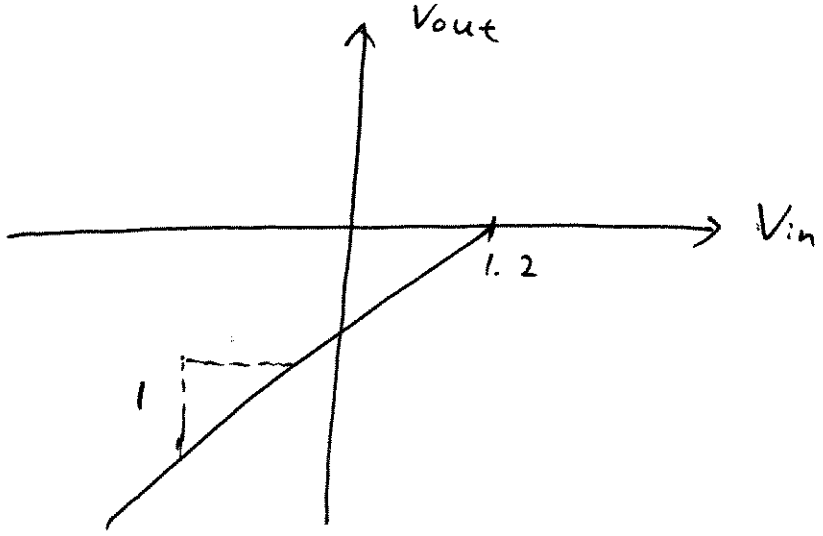
$$V_{out} = \begin{cases} 0 & V_{in} < V_B \\ V_{in} - V_B & V_{in} > V_B \end{cases}$$


$V_{out}$ (V)
3
2
1
0
Slope = 1
−5
−4
−3
−2
−1
0
1
2
3
4
5
$V_{in}$ (V)

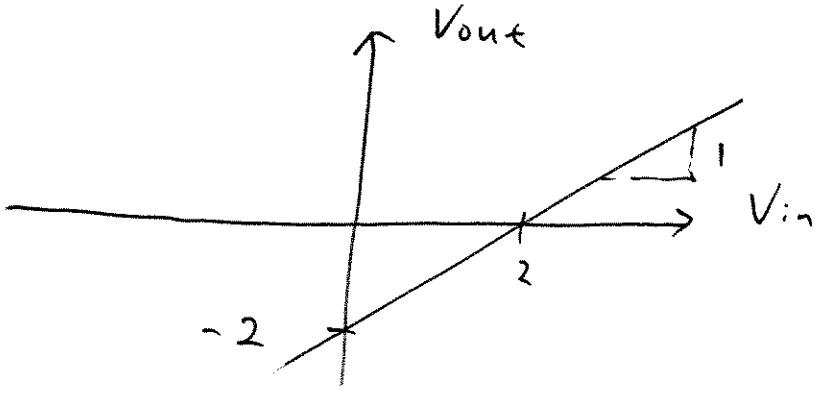
(10) a)



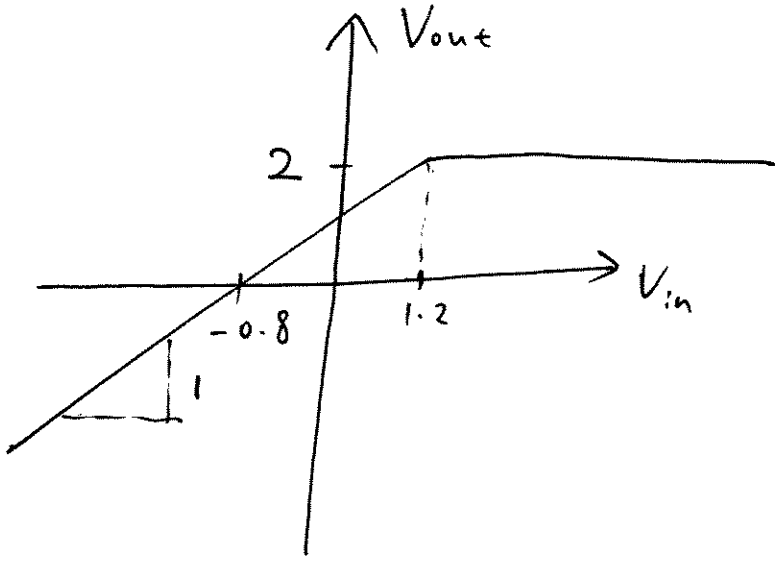

b)



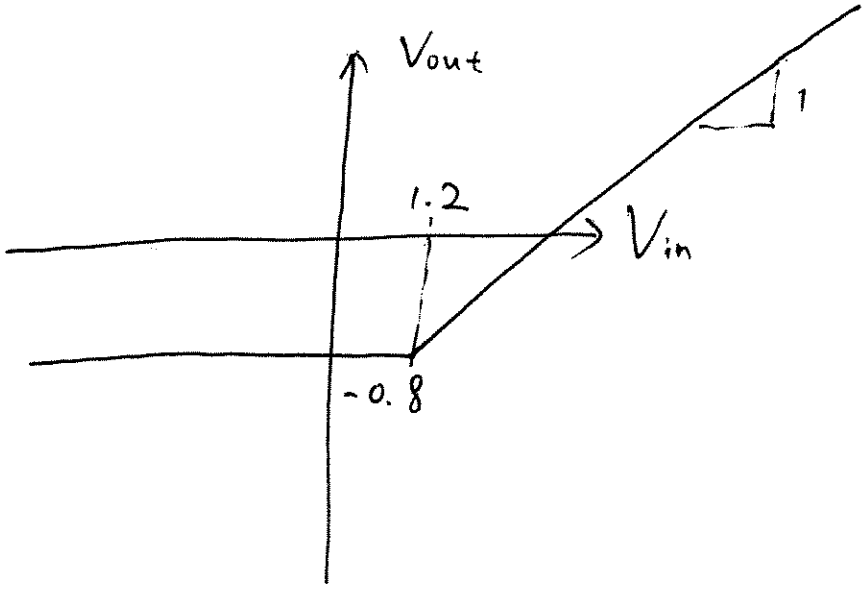

c)



d)



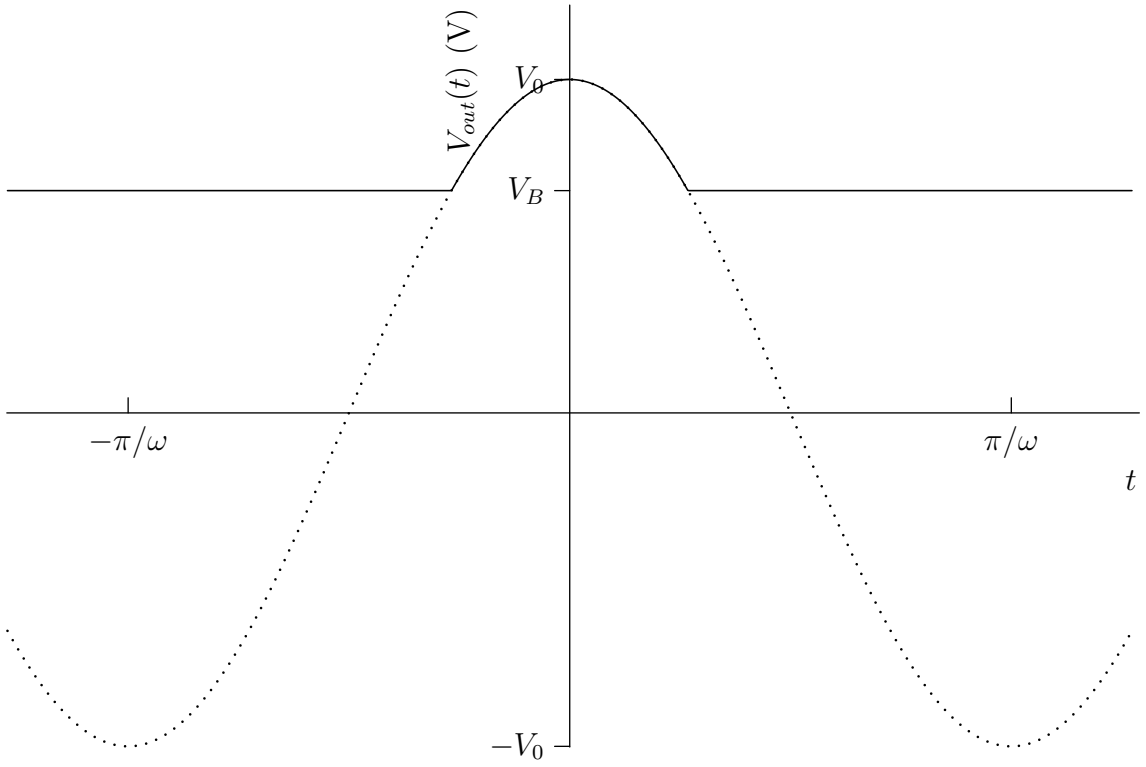

e)



3.11 For each part, the dotted line indicates $V_{in}(t)$, while the solid line indicates $V_{out}(t)$. Assume $V_0 > V_B$.

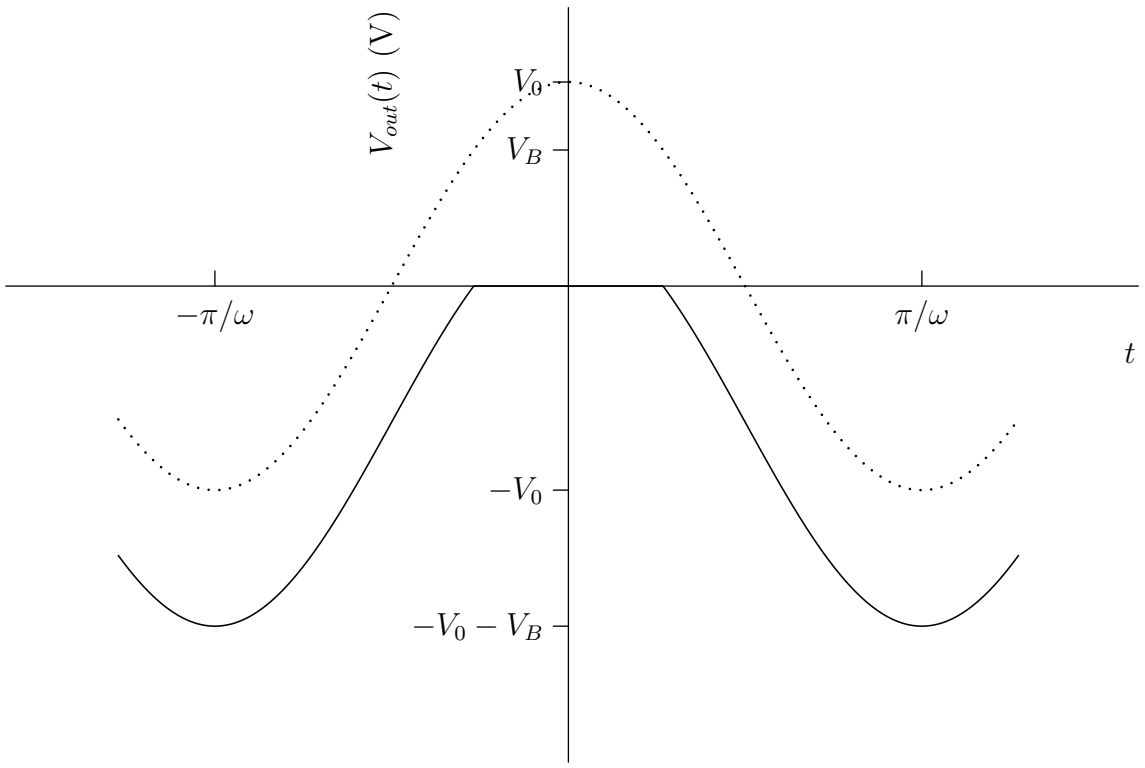
(a)

$$V_{out} = \begin{cases} V_B & V_{in} < V_B \\ V_{in} & V_{in} > V_B \end{cases}$$



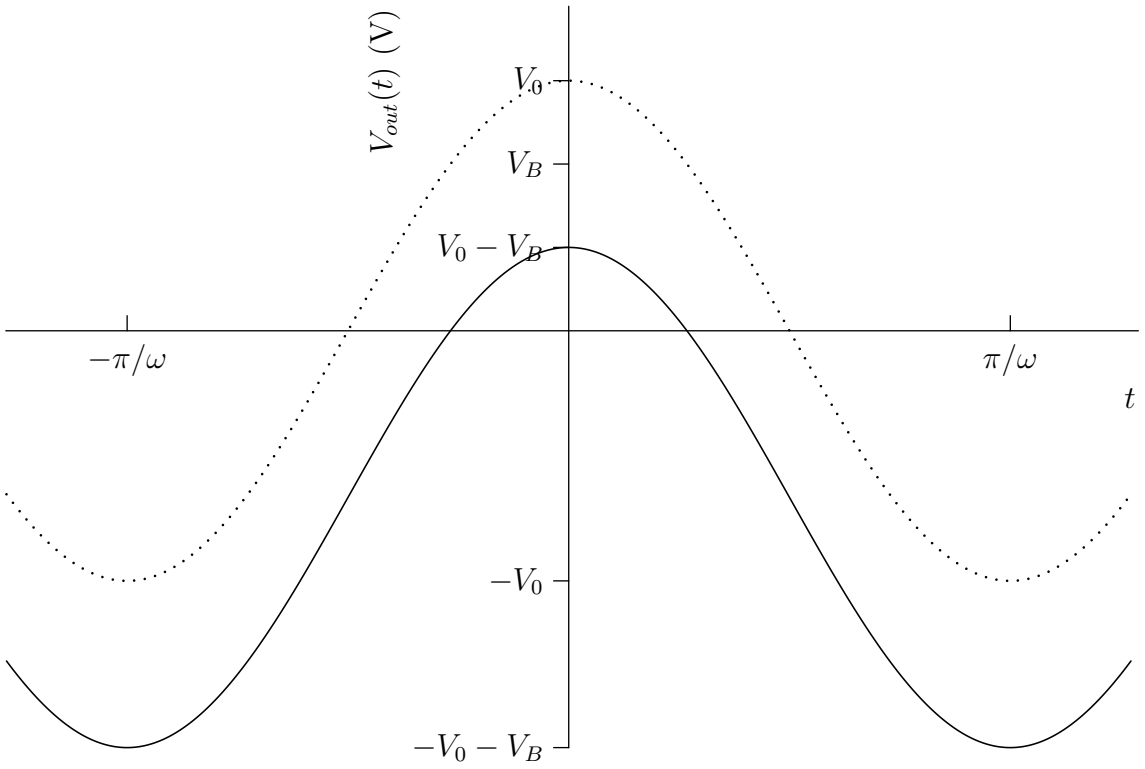

(b)

$$V_{out} = \begin{cases} V_{in} - V_B & V_{in} < V_B \\ 0 & V_{in} > V_B \end{cases}$$


$V_{out}(t)$ (V)
$V_0$
$V_B$
$-\pi/\omega$
$\pi/\omega$
$t$
$-V_0$
$-V_0 - V_B$

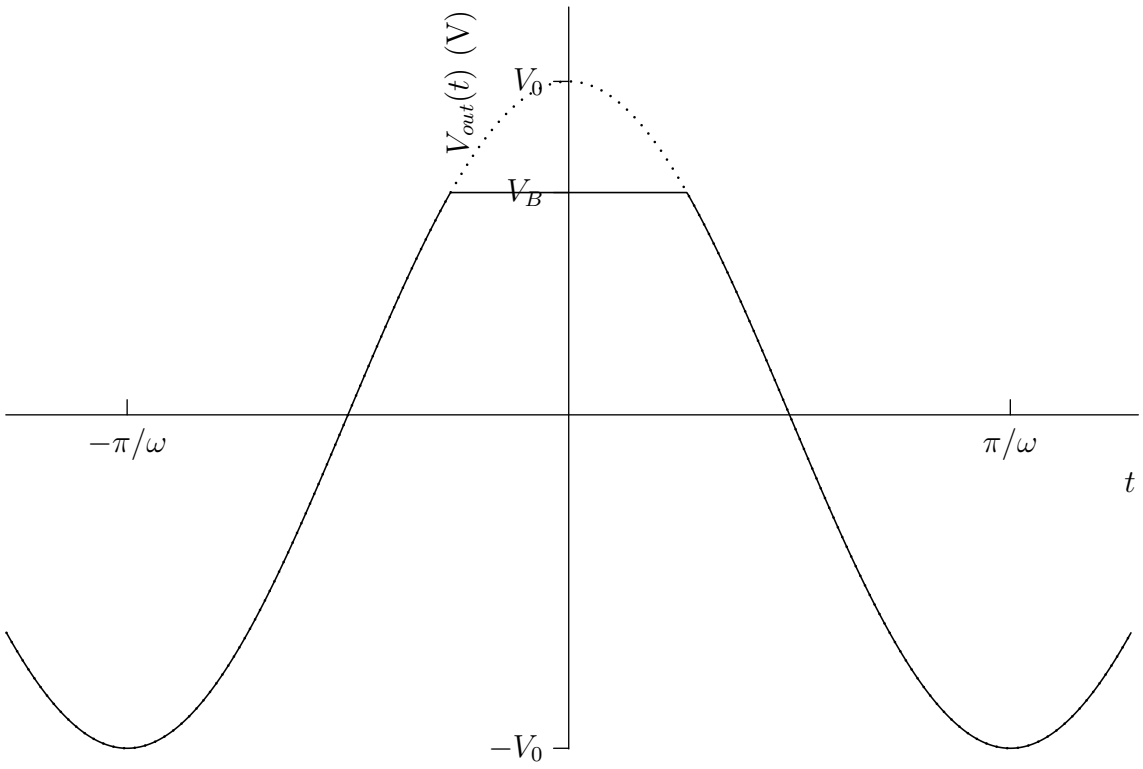
(c)

$$V_{out} = V_{in} - V_B$$


$V_{out}(t)$ (V)
$V_0$
$V_B$
$V_0 - V_B$
$-\pi/\omega$
$\pi/\omega$
$t$
$-V_0$
$-V_0 - V_B$

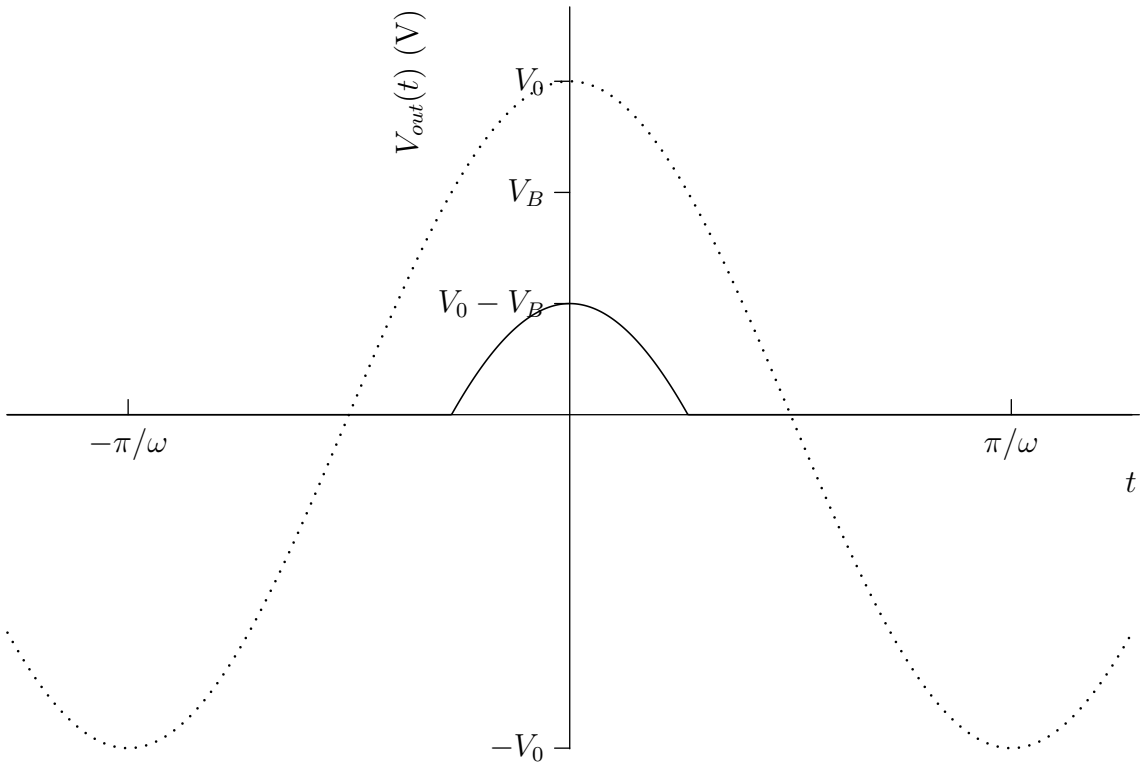
(d)

$$V_{out} = \begin{cases} V_{in} & V_{in} < V_B \\ V_B & V_{in} > V_B \end{cases}$$




(e)

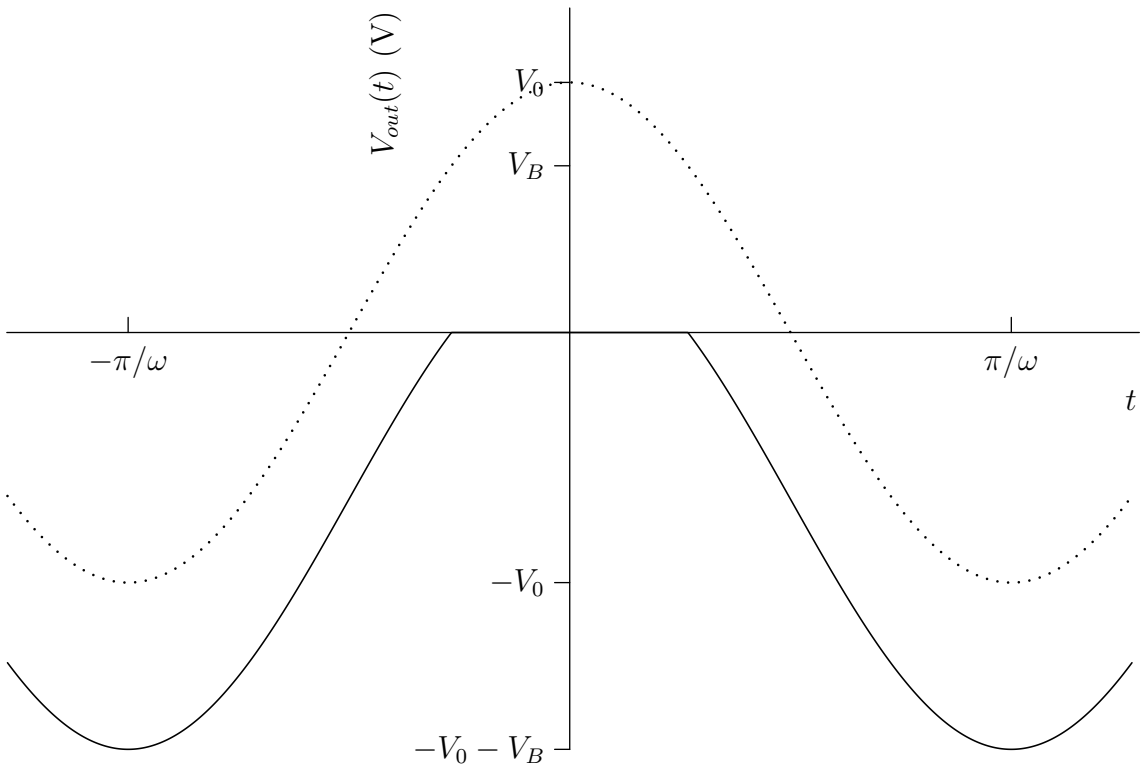
$$V_{out} = \begin{cases} 0 & V_{in} < V_B \\ V_{in} - V_B & V_{in} > V_B \end{cases}$$


$V_{out}(t)$ (V)
$V_0$
$V_B$
$V_0 - V_B$
$-\pi/\omega$
$\pi/\omega$
$t$
$-V_0$

3.12 For each part, the dotted line indicates $V_{in}(t)$, while the solid line indicates $V_{out}(t)$. Assume $V_0 > V_B$.

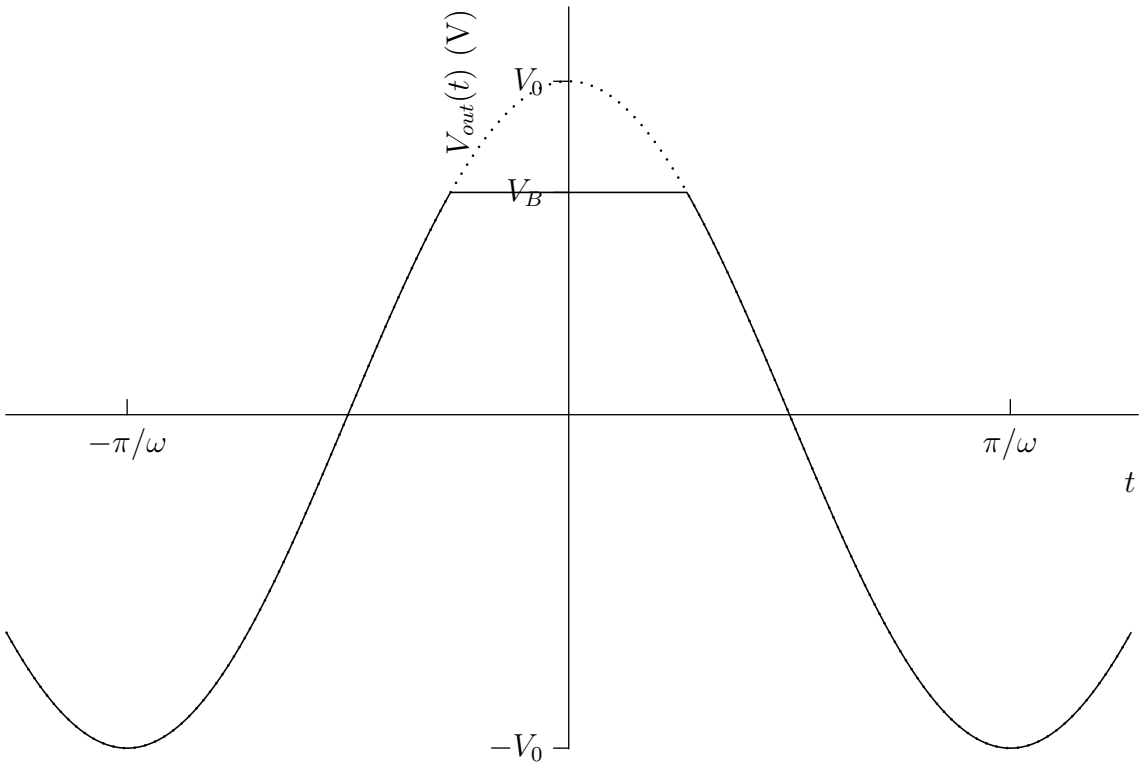
(a)

$$V_{out} = \begin{cases} V_{in} - V_B & V_{in} < V_B \\ 0 & V_{in} > V_B \end{cases}$$



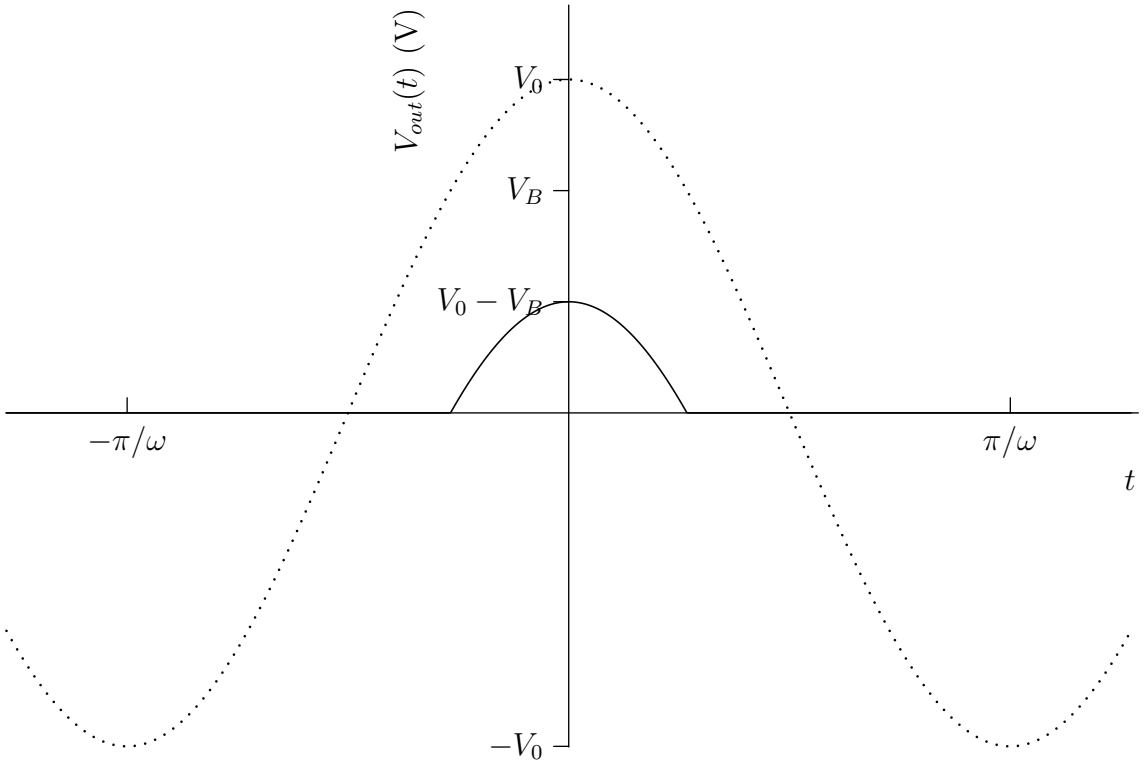

(b)

$$V_{out} = \begin{cases} V_{in} & V_{in} < V_B \\ V_B & V_{in} > V_B \end{cases}$$



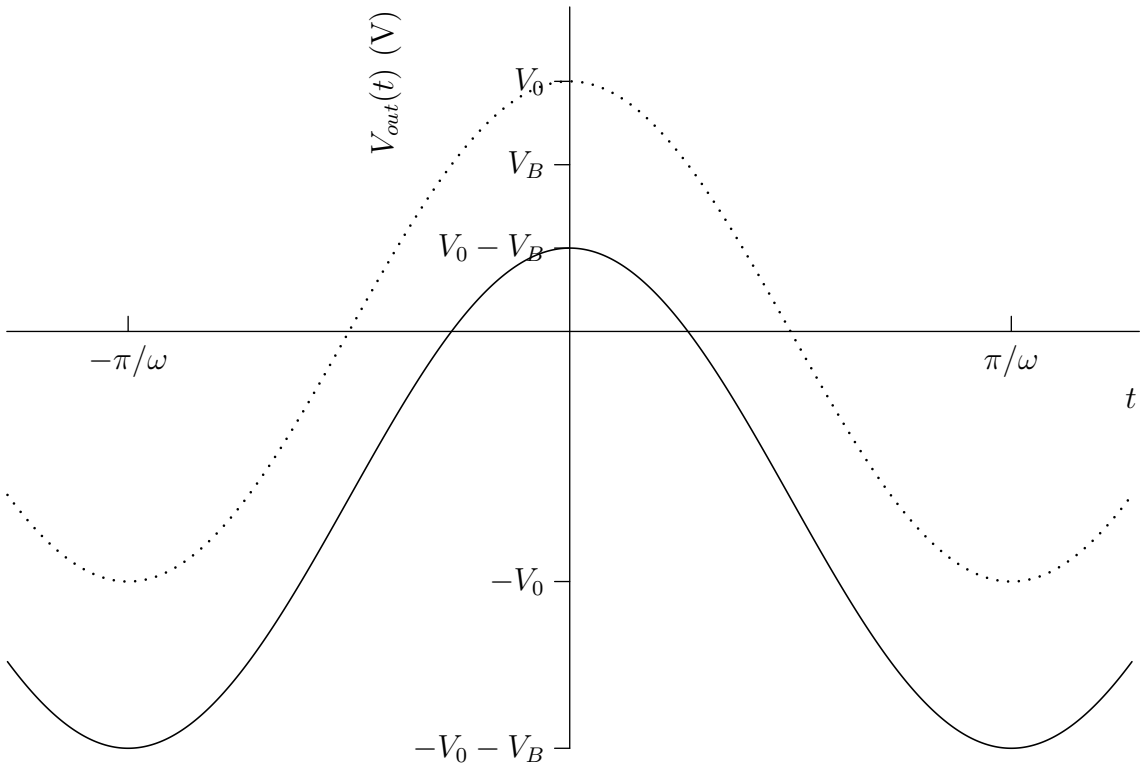

(c)

$$V_{out} = \begin{cases} 0 & V_{in} < V_B \\ V_{in} - V_B & V_{in} > V_B \end{cases}$$



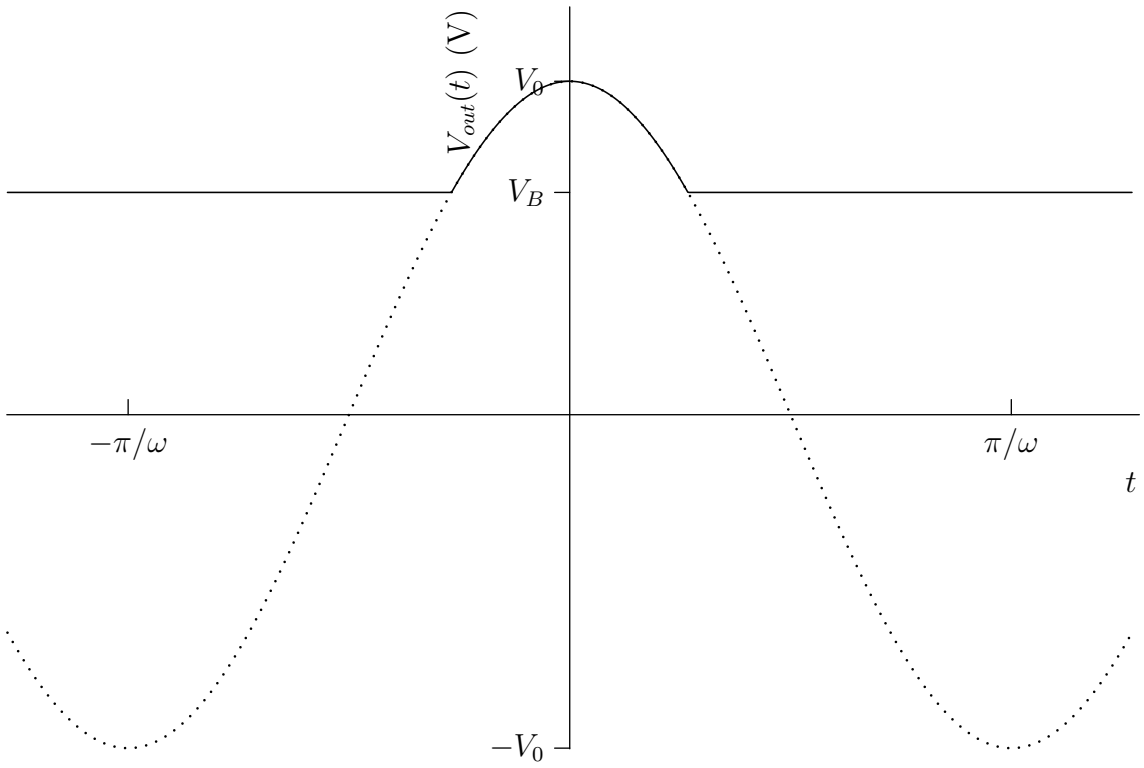
(d)

$$V_{out} = V_{in} - V_B$$

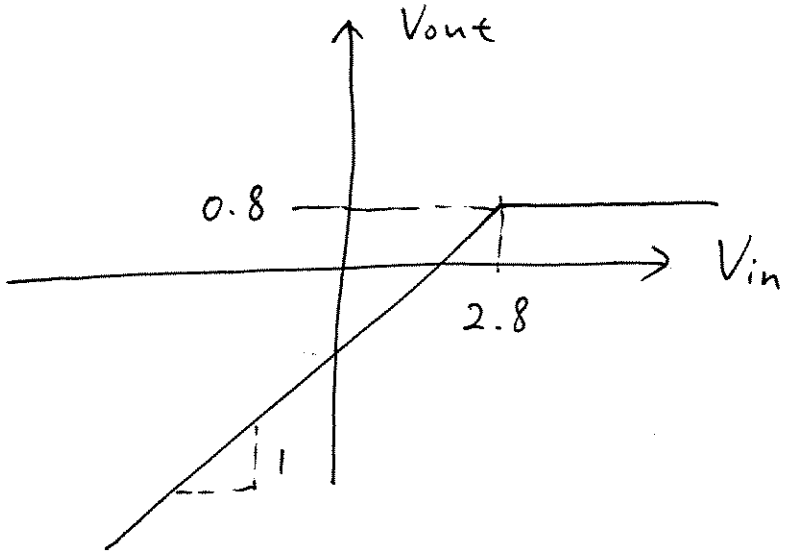



(e)

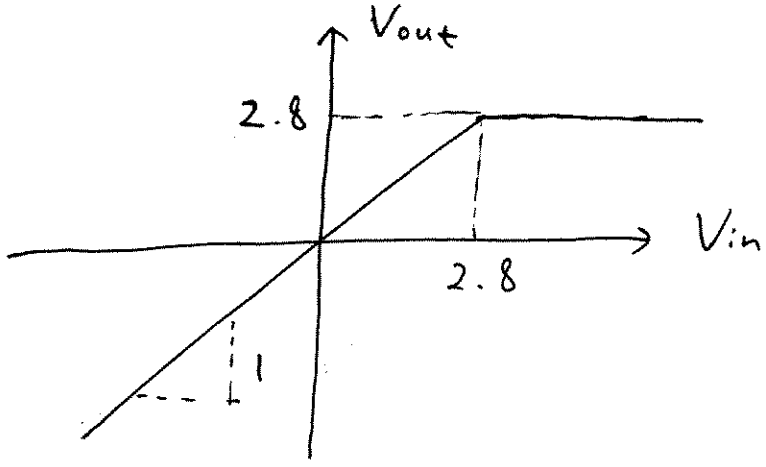
$$V_{out} = \begin{cases} V_B & V_{in} < V_B \\ V_{in} & V_{in} > V_B \end{cases}$$


$V_{out}(t)$ (V)
$V_0$
$V_B$
$-V_0$
$-\pi/\omega$
$\pi/\omega$
$t$

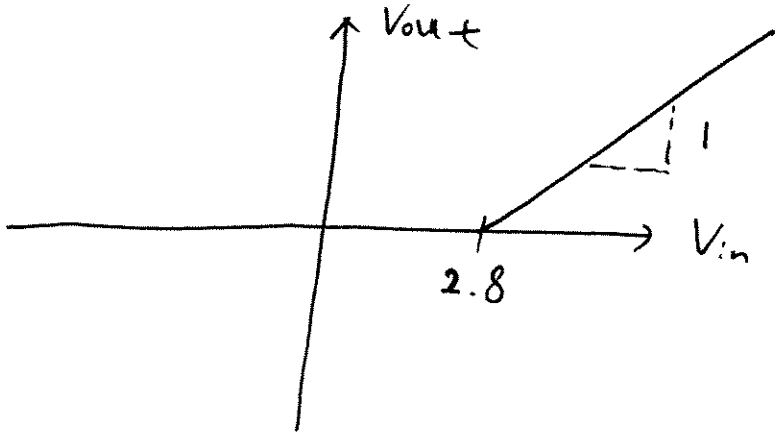
(13) a)



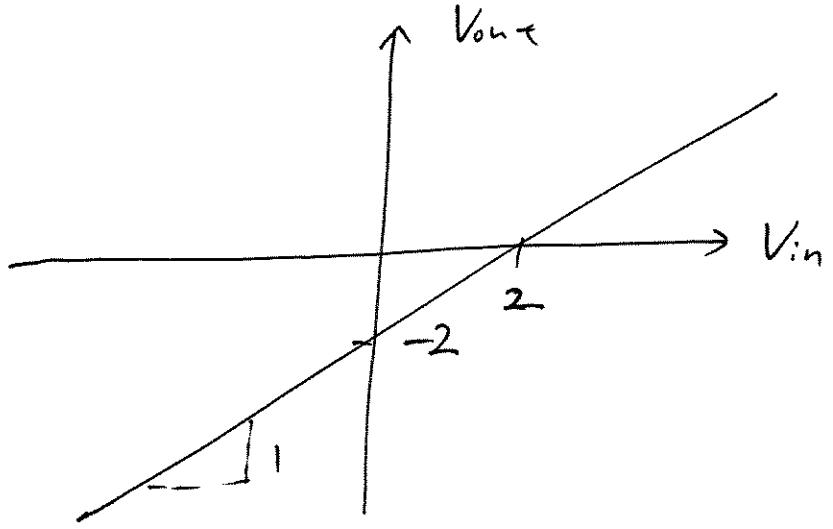

b)



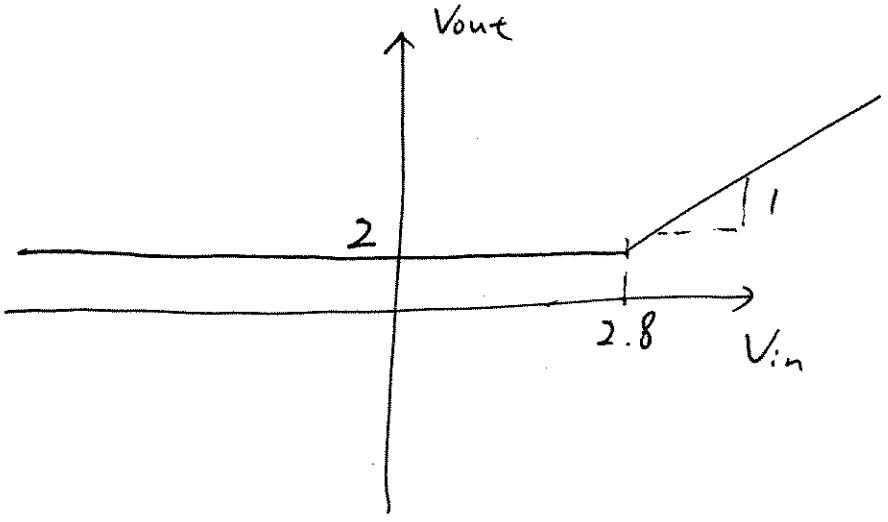

c)



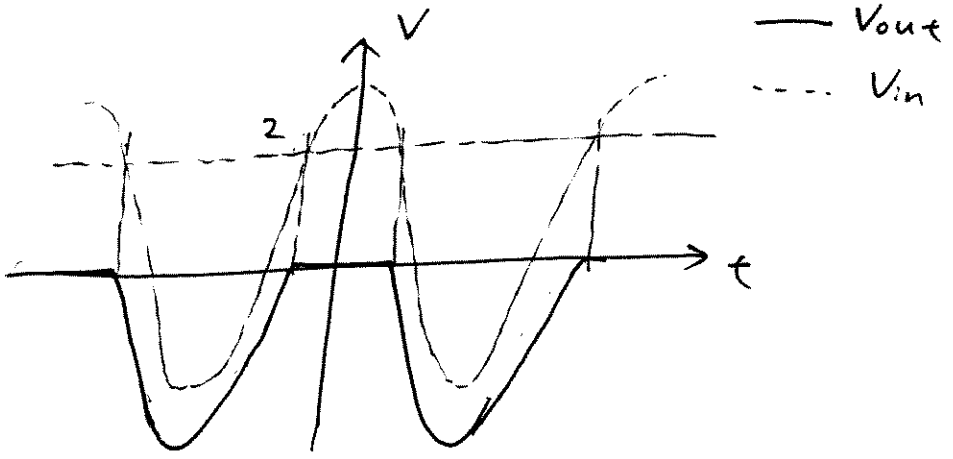
d)



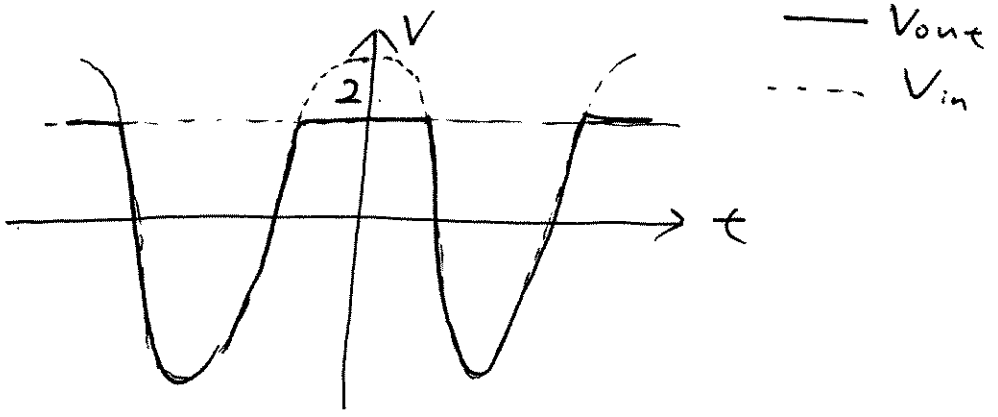

e)



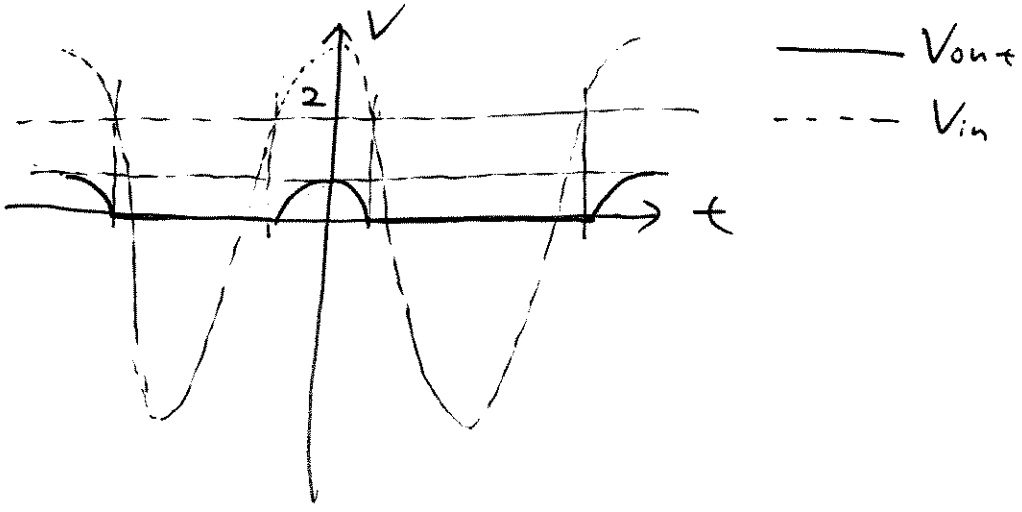
(14) a)



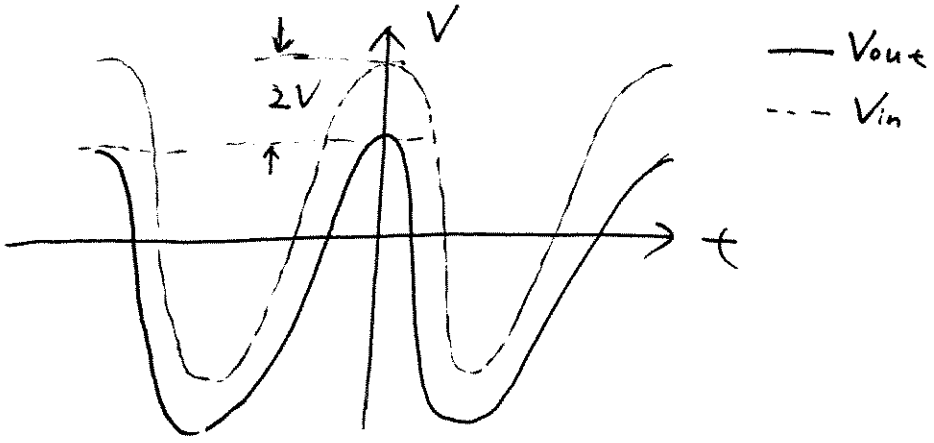

b)



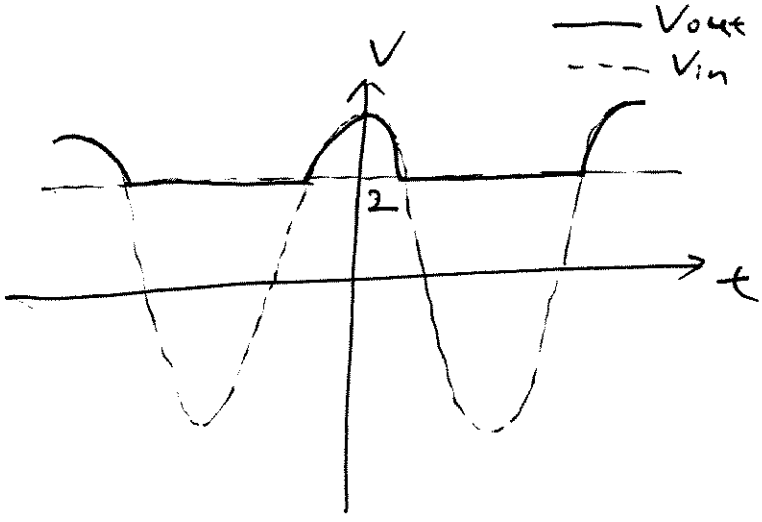

c)



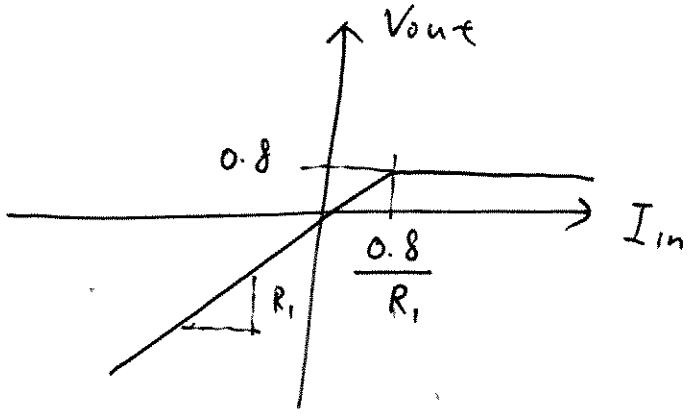
d)



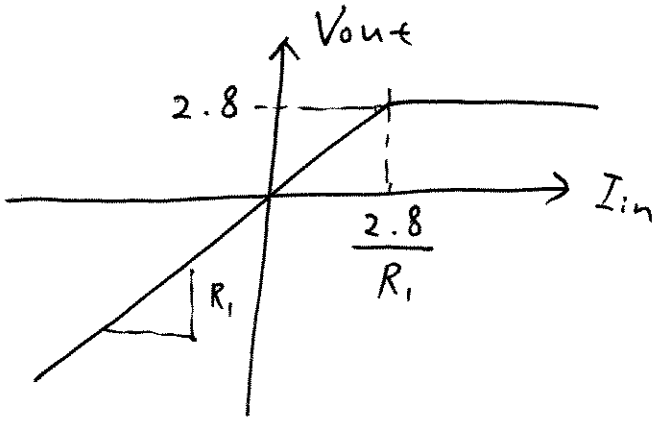

e)



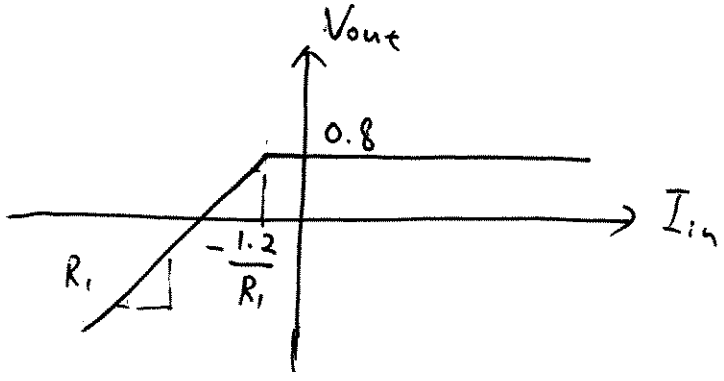
(15) a)


Vout
0.8
$I_{in}$
$\frac{0.8}{R_1}$
$R_1$

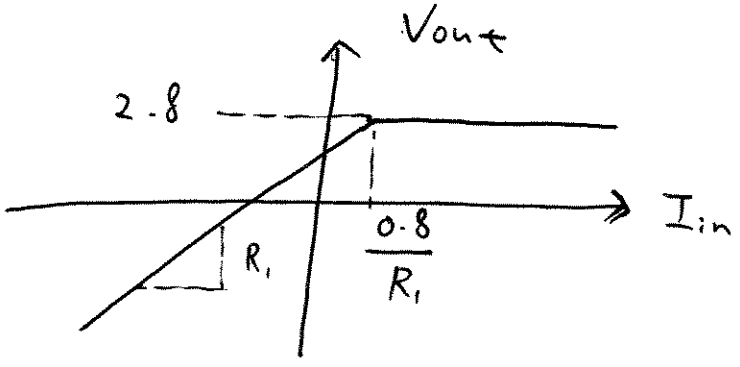
b)


Vout
2.8
$I_{in}$
$\frac{2.8}{R_1}$
$R_1$

c)

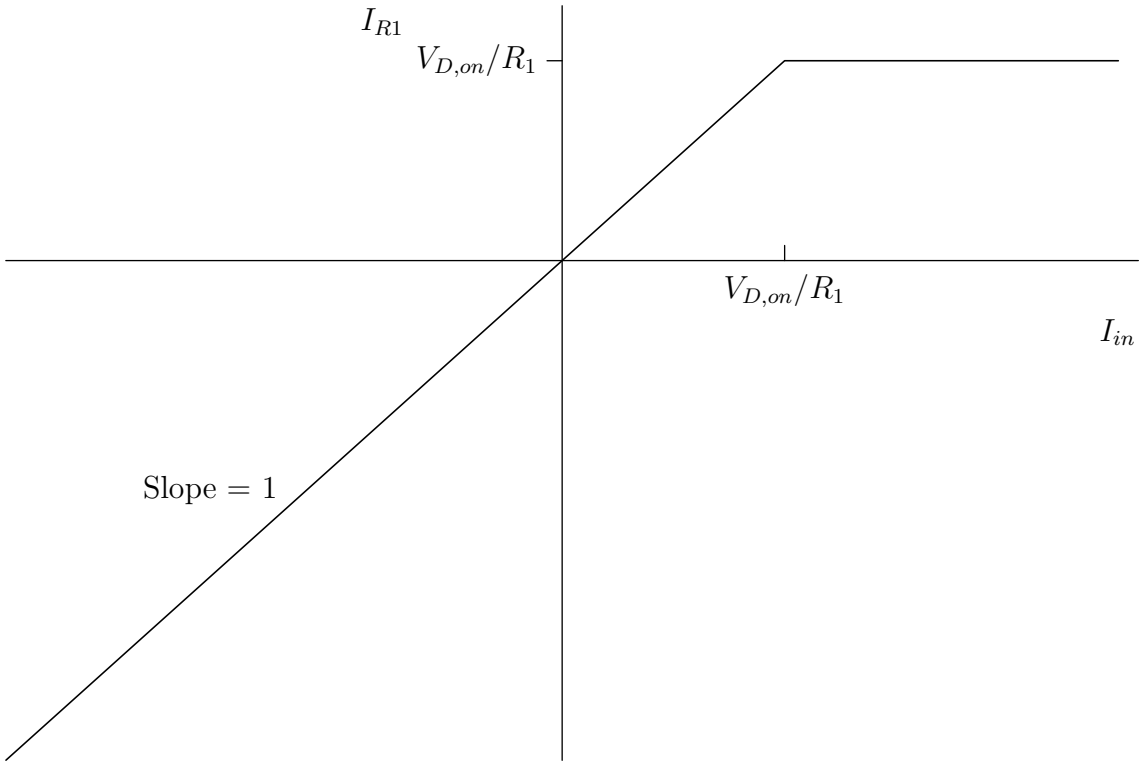

Vout
0.8
$I_{in}$
$-\frac{1.2}{R_1}$
$R_1$

d)


Vout
2.8
$I_{in}$
$\frac{0.8}{R_1}$
$R_1$

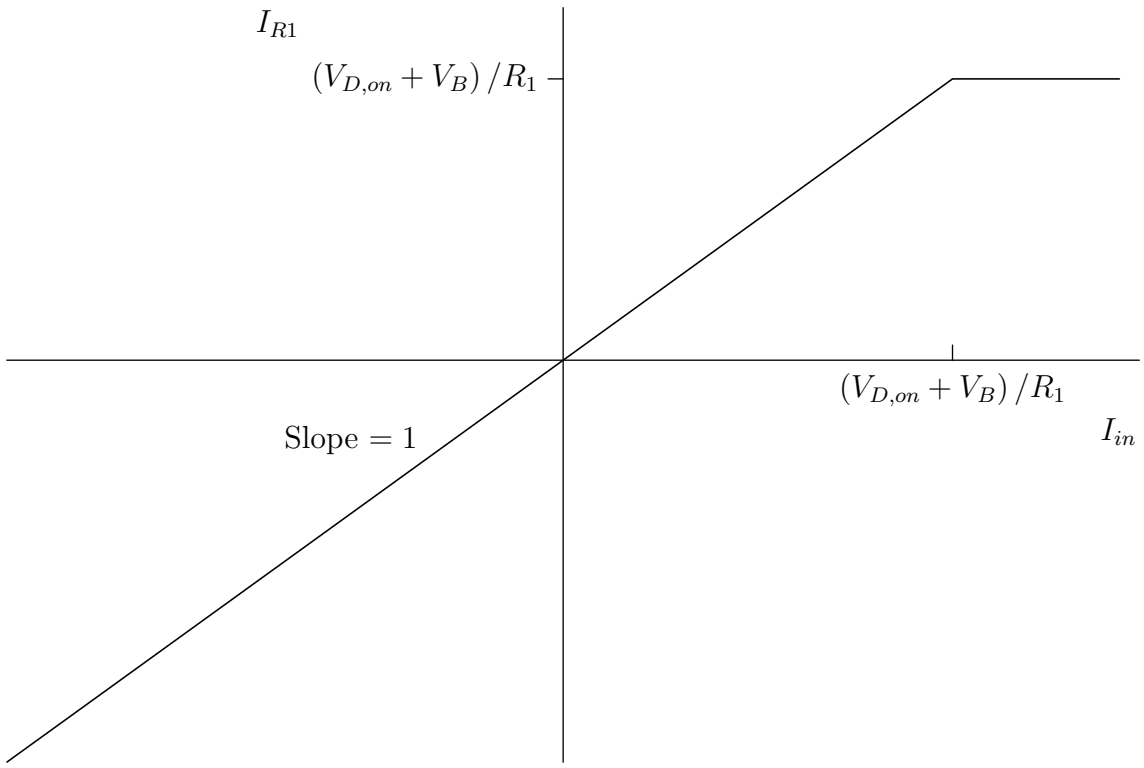
3.16 (a)

$$I_{R1} = \begin{cases} I_{in} & I_{in} < \frac{V_{D,on}}{R_1} \\ \frac{V_{D,on}}{R_1} & I_{in} > \frac{V_{D,on}}{R_1} \end{cases}$$



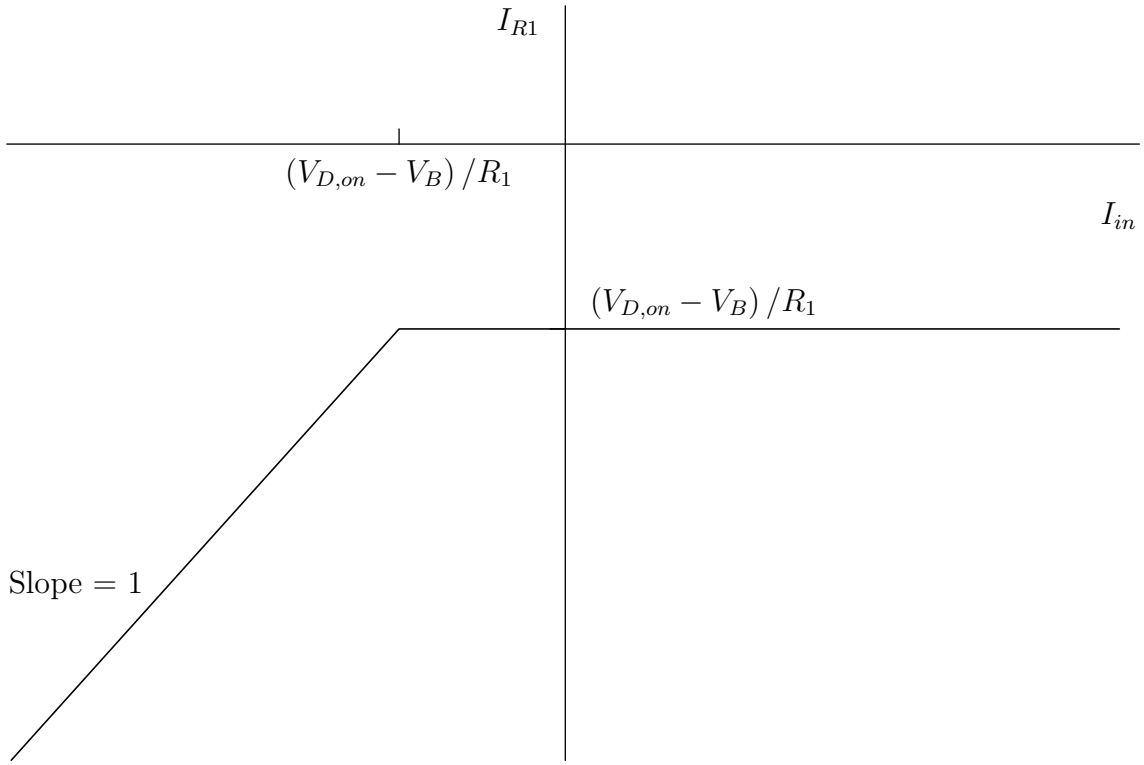

(b)

$$I_{R1} = \begin{cases} I_{in} & I_{in} < \frac{V_{D,on}+V_B}{R_1} \\ \frac{V_{D,on}+V_B}{R_1} & I_{in} > \frac{V_{D,on}+V_B}{R_1} \end{cases}$$



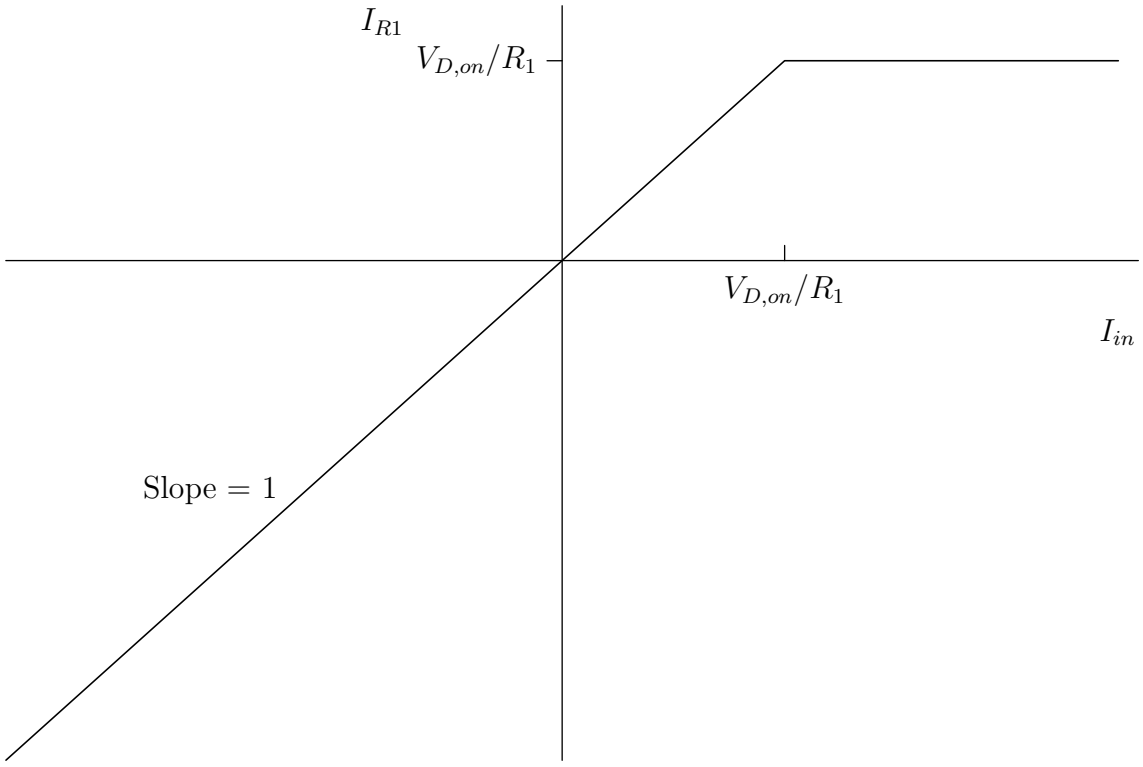

(c)

$$I_{R1} = \begin{cases} I_{in} & I_{in} < \frac{V_{D,on} - V_B}{R_1} \\ \frac{V_{D,on} - V_B}{R_1} & I_{in} > \frac{V_{D,on} - V_B}{R_1} \end{cases}$$



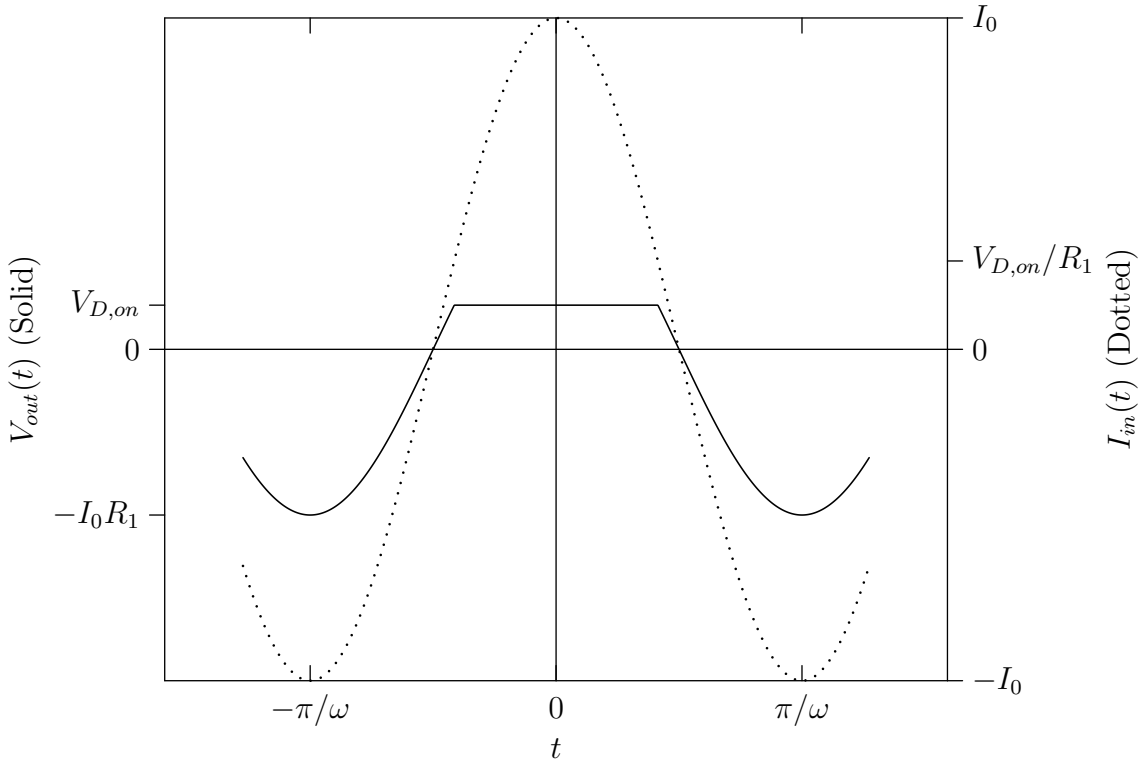
(d)

$$I_{R1} = \begin{cases} I_{in} & I_{in} < \frac{V_{D,on}}{R_1} \\ \frac{V_{D,on}}{R_1} & I_{in} > \frac{V_{D,on}}{R_1} \end{cases}$$



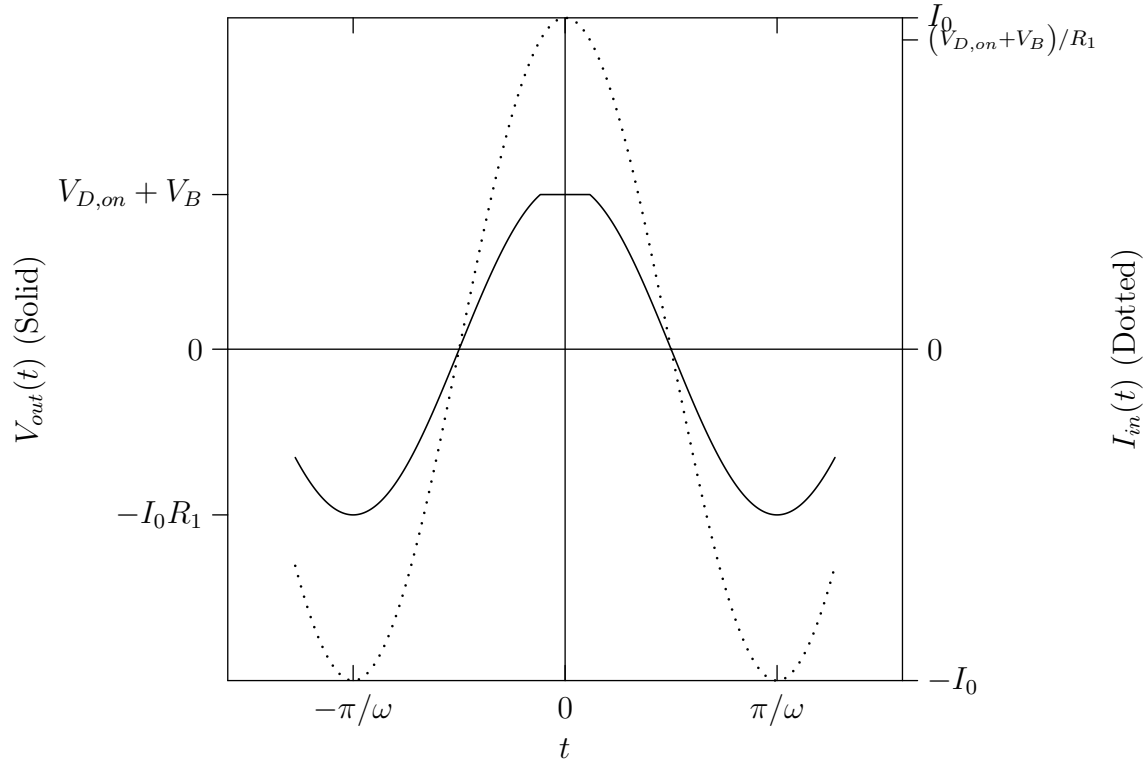
3.17 (a)

$$V_{out} = \begin{cases} I_{in}R_1 & I_{in} < \frac{V_{D,on}}{R_1} \\ V_{D,on} & I_{in} > \frac{V_{D,on}}{R_1} \end{cases}$$



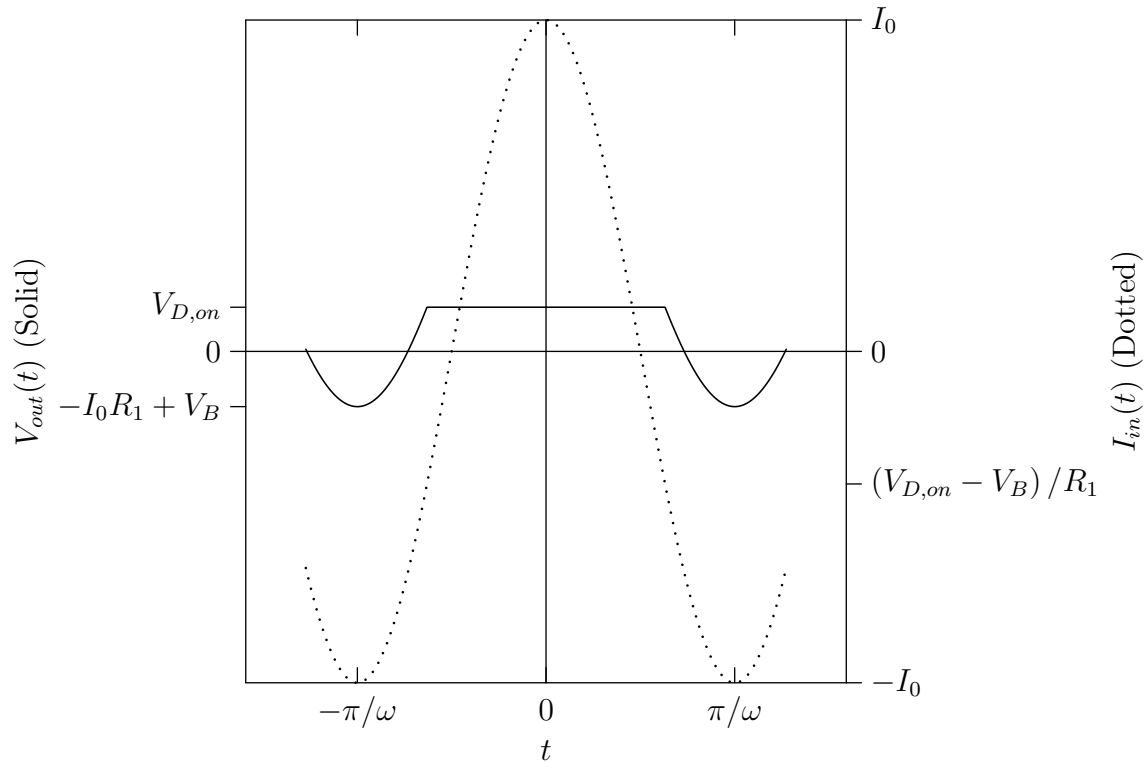

(b)

$$V_{out} = \begin{cases} I_{in}R_1 & I_{in} < \frac{V_{D,on}+V_B}{R_1} \\ V_{D,on} + V_B & I_{in} > \frac{V_{D,on}+V_B}{R_1} \end{cases}$$



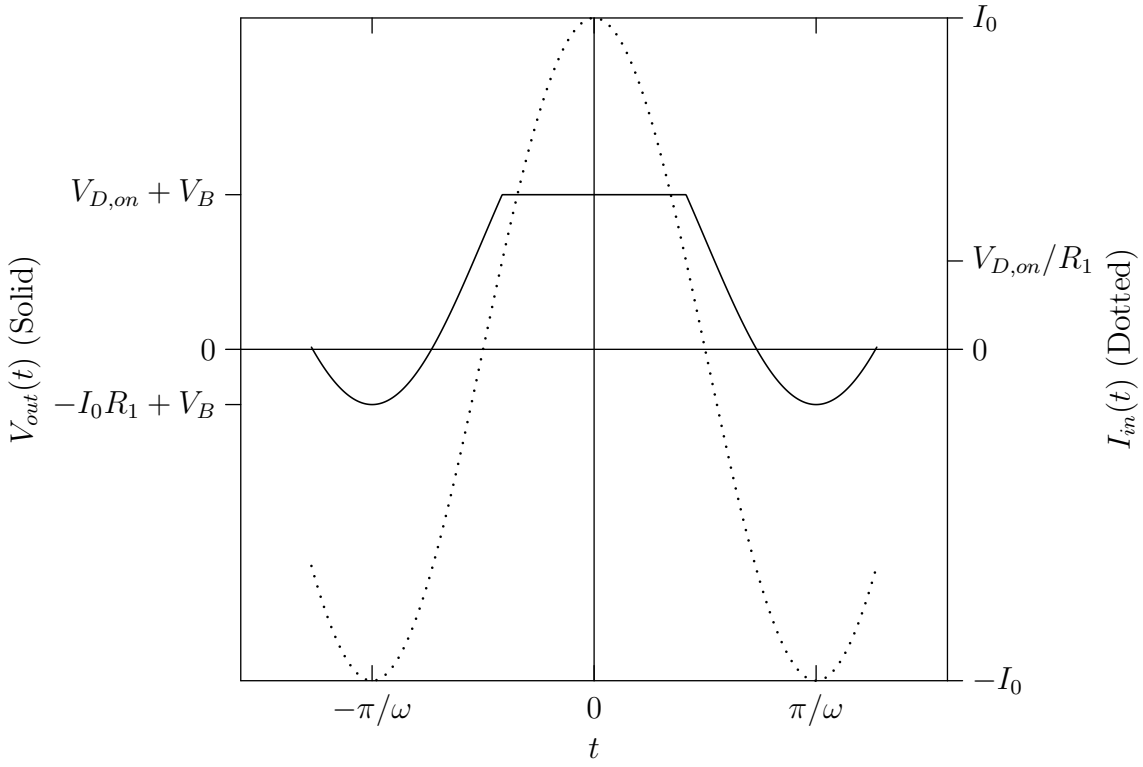

(c)

$$V_{out} = \begin{cases} I_{in}R_1 + V_B & I_{in} < \frac{V_{D,on}-V_B}{R_1} \\ V_{D,on} & I_{in} > \frac{V_{D,on}-V_B}{R_1} \end{cases}$$

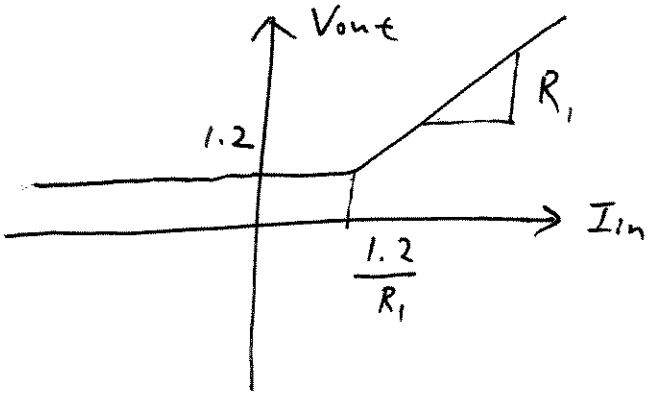


(d)

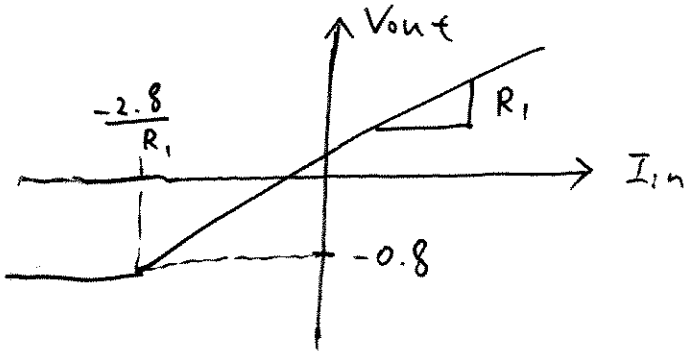
$$V_{out} = \begin{cases} I_{in}R_1 + V_B & I_{in} < \frac{V_{D,on}}{R_1} \\ V_{D,on} + V_B & I_{in} > \frac{V_{D,on}}{R_1} \end{cases}$$



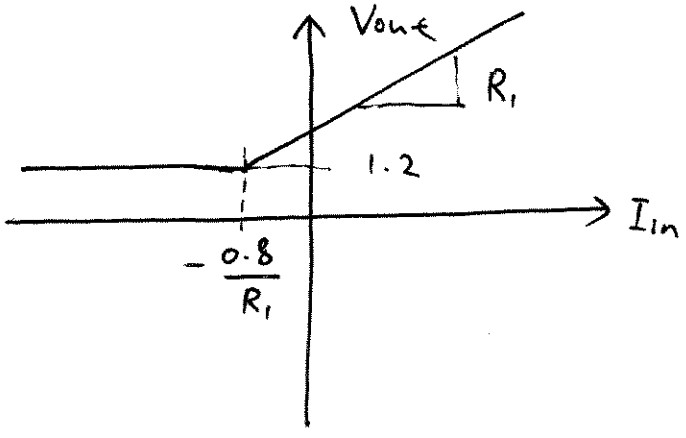
(18) a)



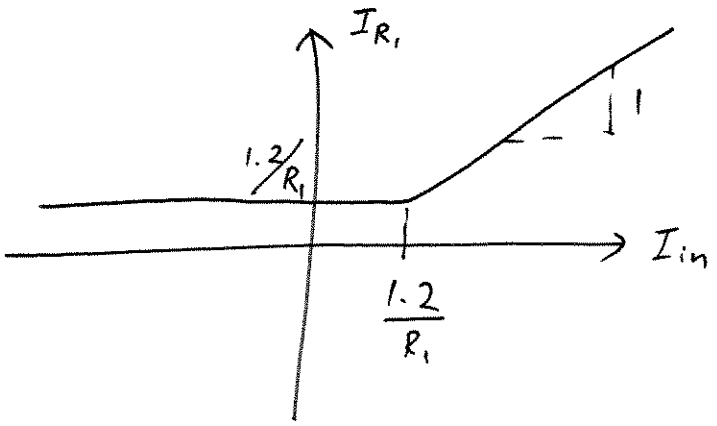

b)



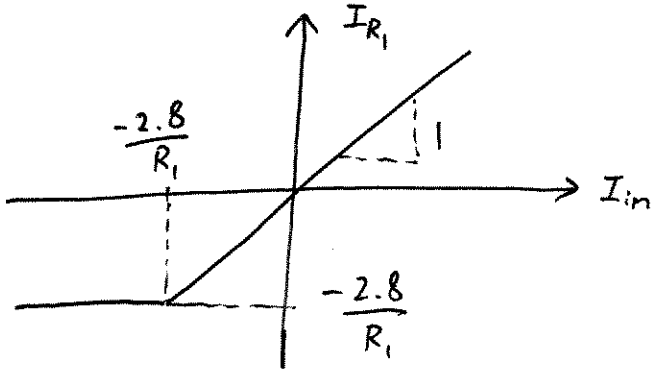

c)



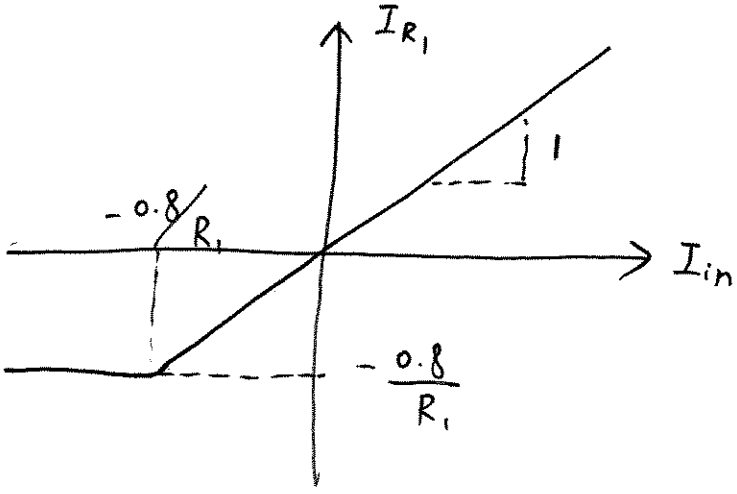
(19) a)


$I_{R_1}$
1
$1.2/R_1$
$I_{in}$
$\frac{1.2}{R_1}$

b)

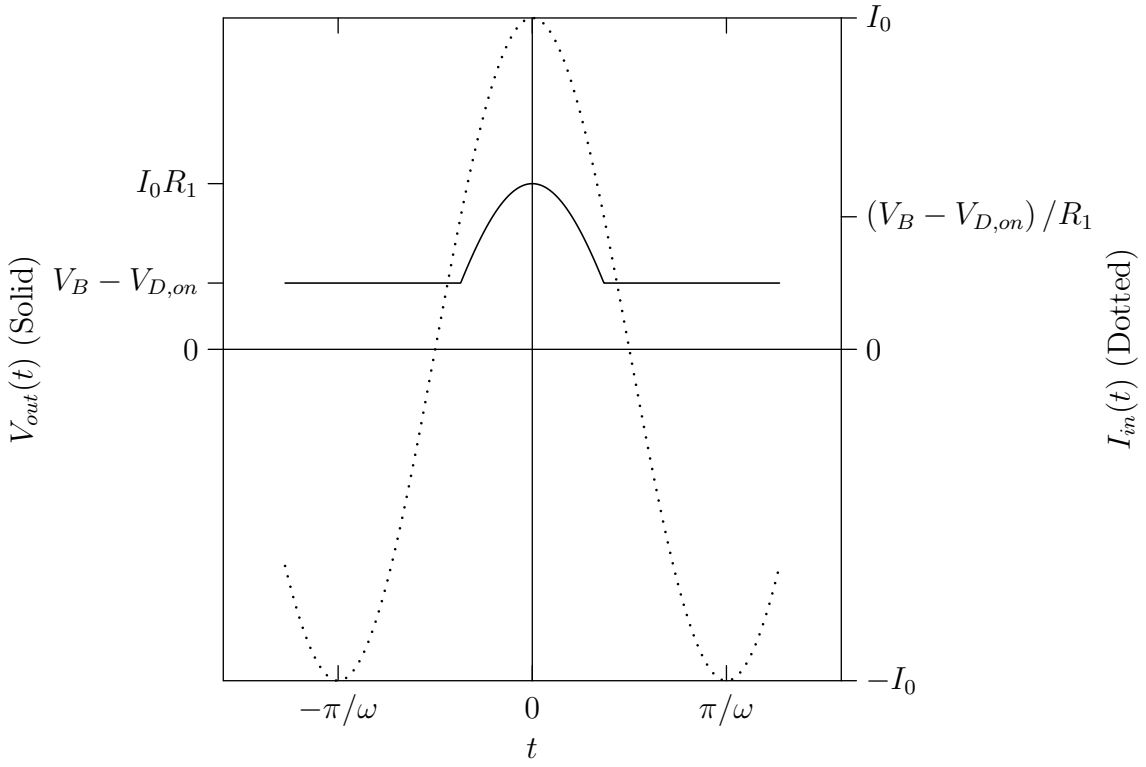

$I_{R_1}$
$\frac{-2.8}{R_1}$
1
$I_{in}$
$\frac{-2.8}{R_1}$

c)


$I_{R_1}$
1
$-0.8/R_1$
$I_{in}$
$\frac{-0.8}{R_1}$

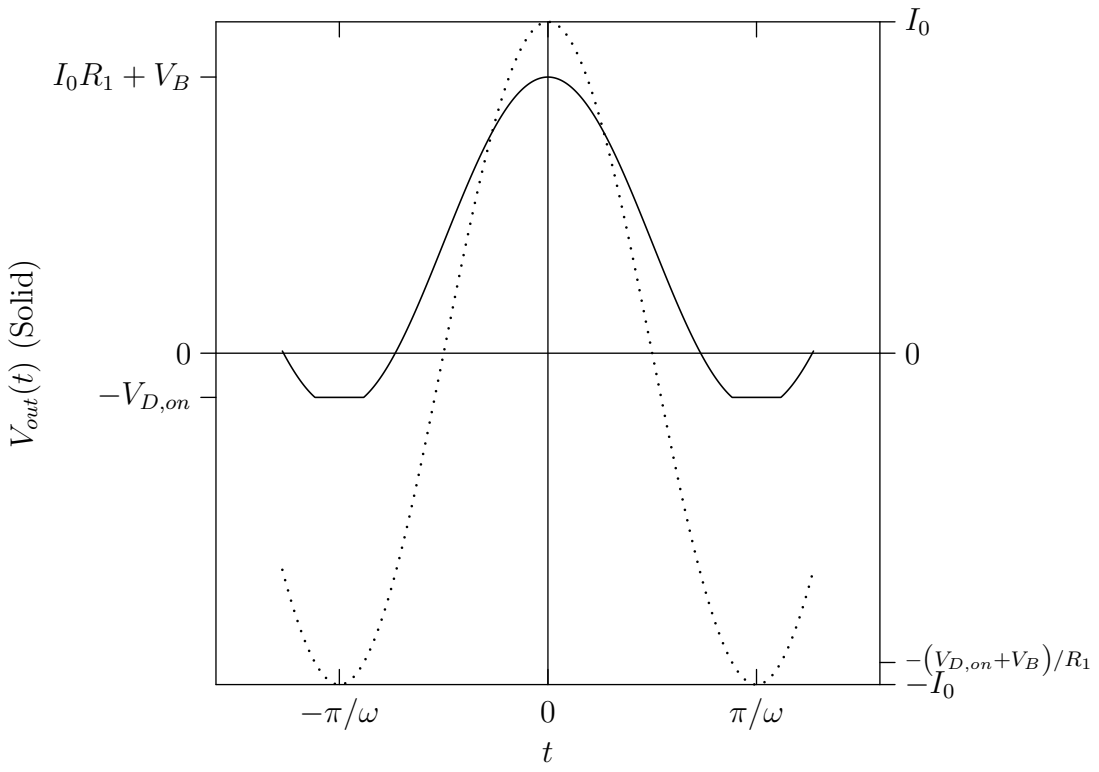
3.20 (a)

$$V_{out} = \begin{cases} I_{in}R_1 & I_{in} > \frac{V_B - V_{D,on}}{R_1} \\ V_B - V_{D,on} & I_{in} < \frac{V_B - V_{D,on}}{R_1} \end{cases}$$



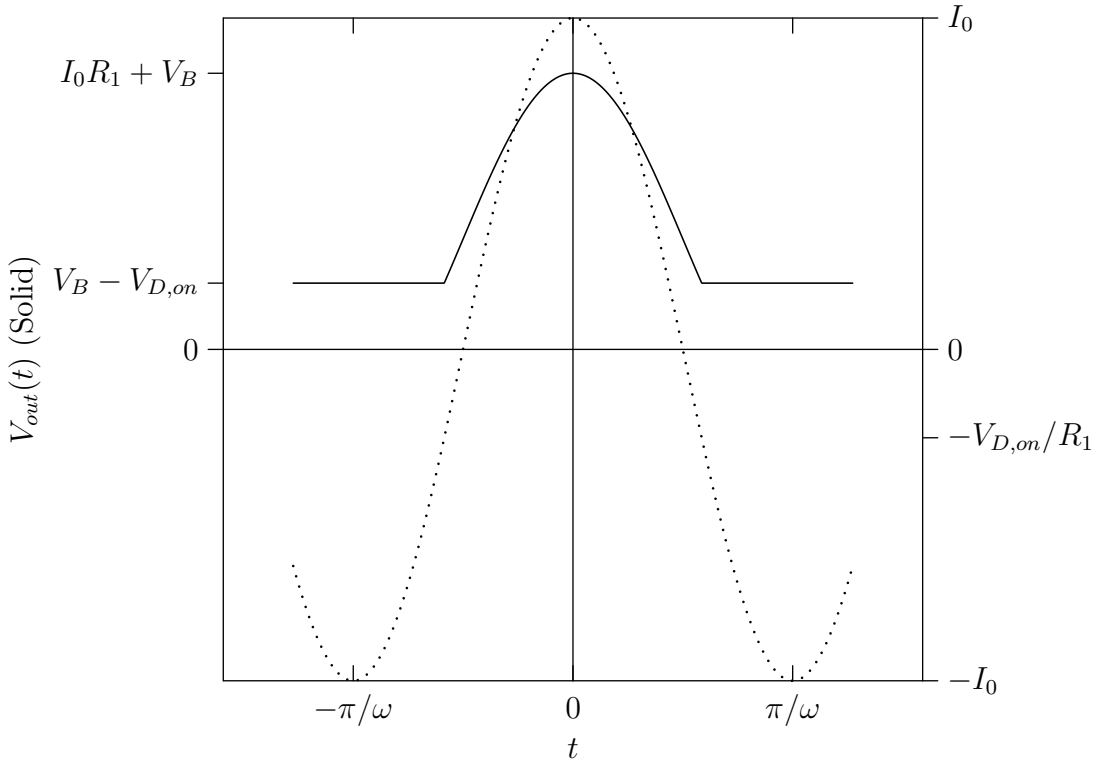

(b)

$$V_{out} = \begin{cases} I_{in}R_1 + V_B & I_{in} > -\frac{V_{D,on} + V_B}{R_1} \\ -V_{D,on} & I_{in} < -\frac{V_{D,on} + V_B}{R_1} \end{cases}$$

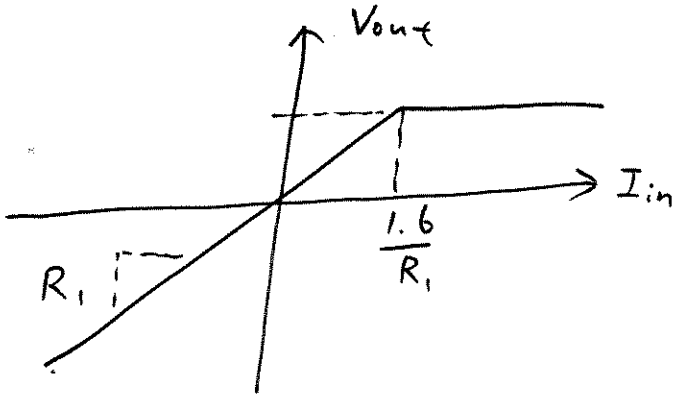



(c)

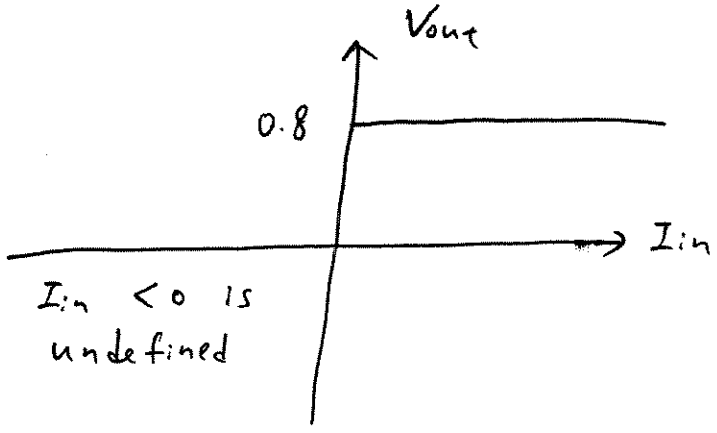
$$V_{out} = \begin{cases} I_{in}R_1 + V_B & I_{in} > -\frac{V_{D,on}}{R_1} \\ V_B - V_{D,on} & I_{in} < -\frac{V_{D,on}}{R_1} \end{cases}$$



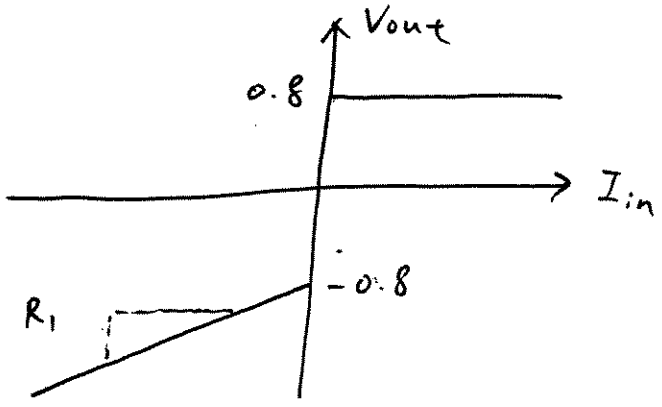
(21) a)


Vout
Iin
1.6
R1
R1

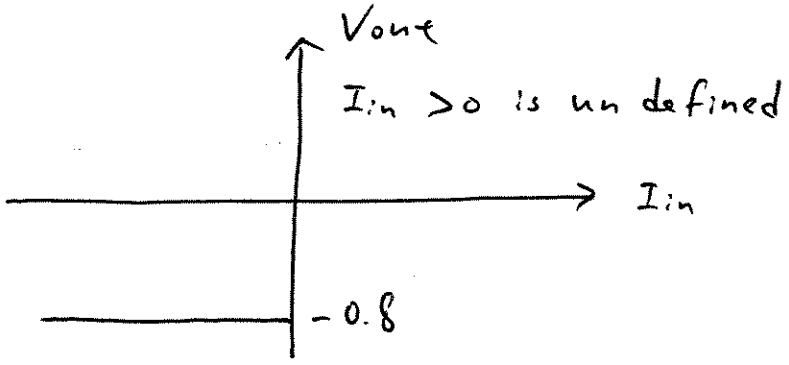
b)


Vout
0.8
Iin
Iin < 0 is
undefined

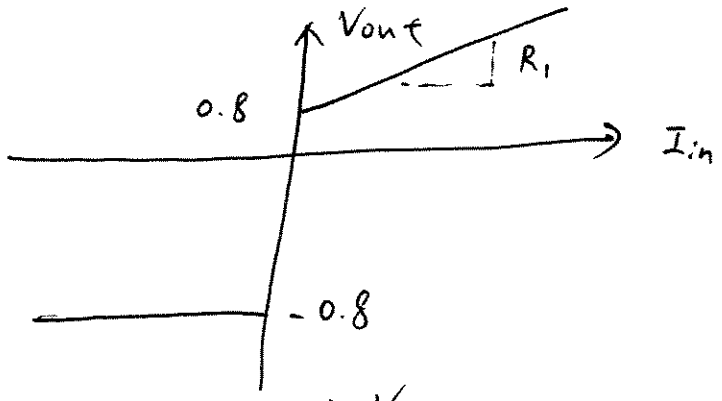
c)


Vout
0.8
Iin
-0.8
R1

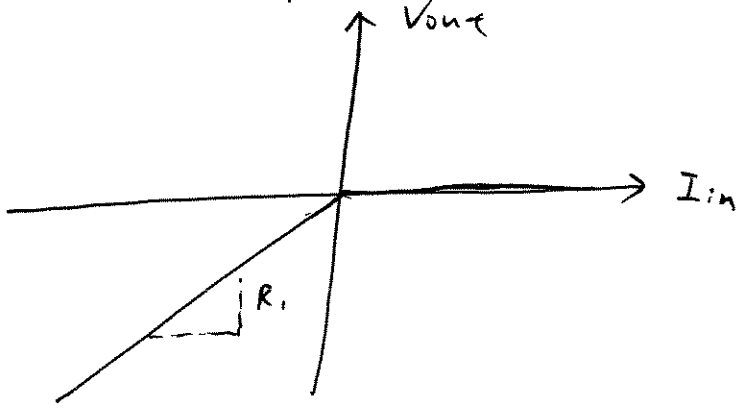
d)


Vout
Iin > 0 is un defined
Iin
-0.8

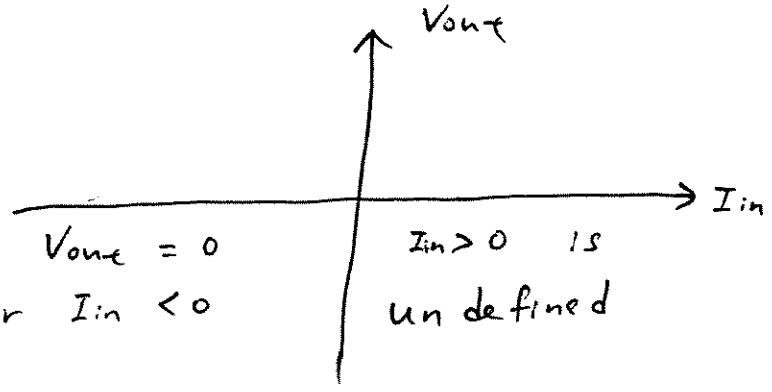
e)




f)



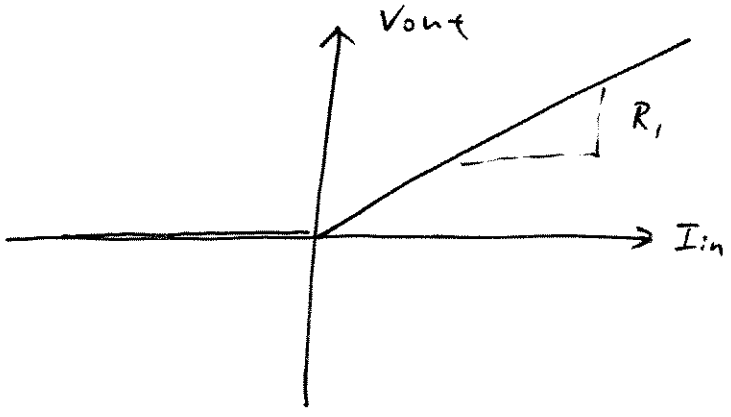

g)



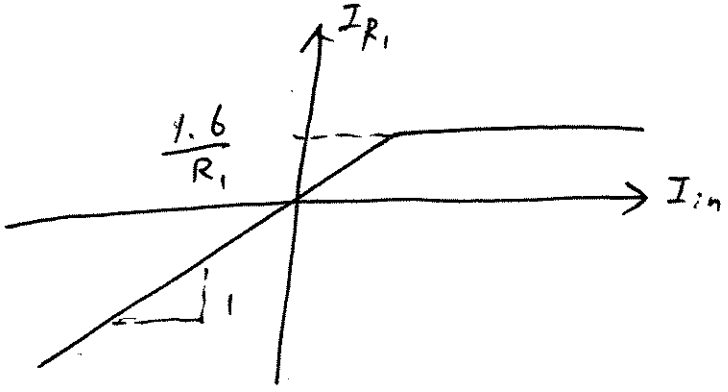

$V_{out} = 0$ for $I_{in} < 0$

$I_{in} > 0$ is undefined

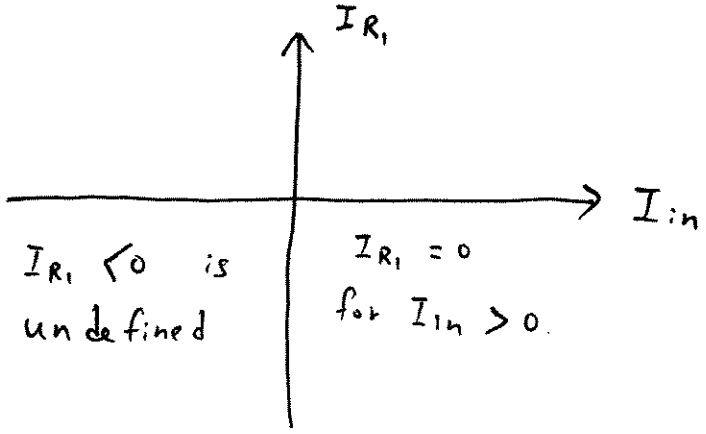
h)



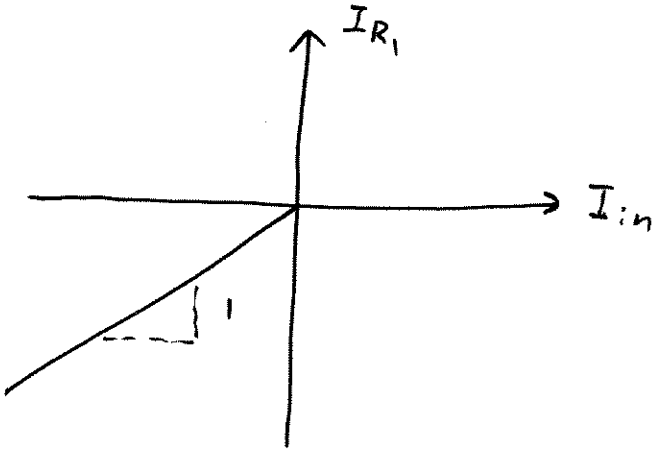
(22) a)


$I_{R_1}$
$\frac{1.6}{R_1}$
$I_{in}$
1

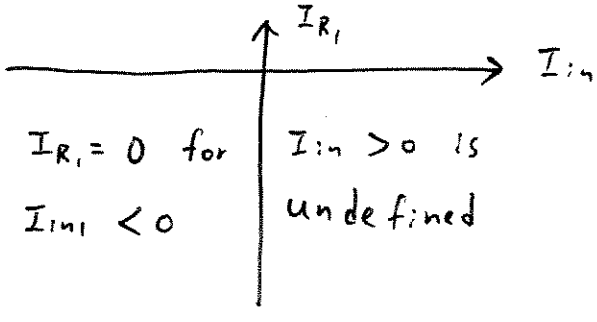
b)


$I_{R_1}$
$I_{in}$
$I_{R_1} < 0$ is undefined
$I_{R_1} = 0$ for $I_{in} > 0$.

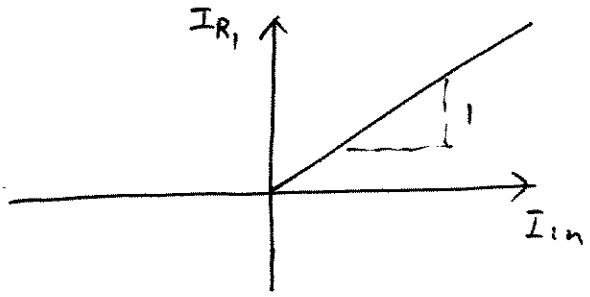
c)


$I_{R_1}$
$I_{in}$
1

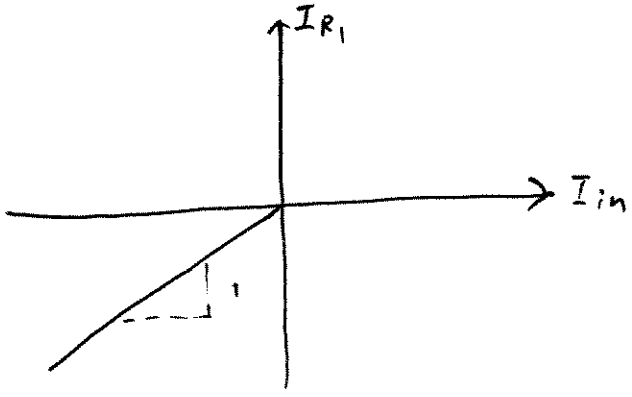
d)


$I_{R_1}$
$I_{in}$
$I_{R_1} = 0$ for $I_{in1} < 0$
$I_{in} > 0$ is undefined

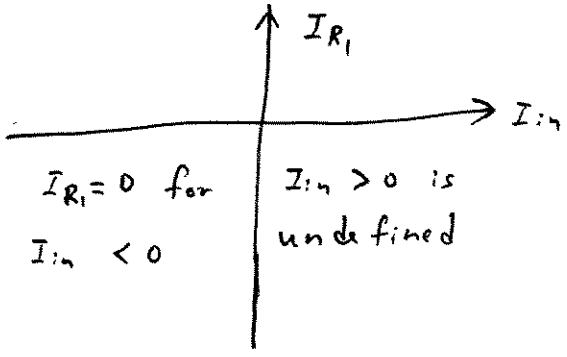
e)


$I_{R_1}$
1
$I_{in}$

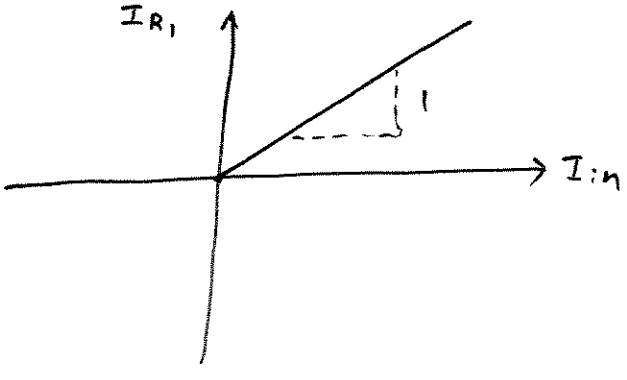
f)


$I_{R_1}$
$I_{in}$
1

g)

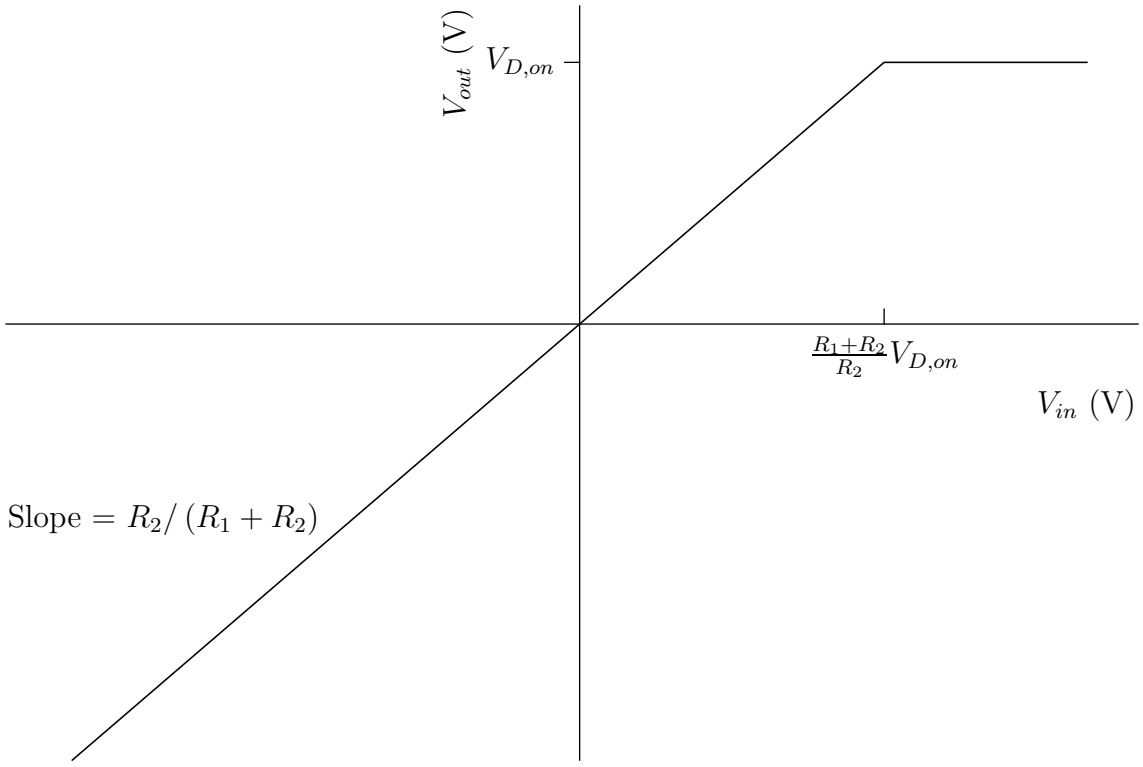

$I_{R_1}$
$I_{in}$
$I_{R_1} = 0$ for $I_{in} < 0$
$I_{in} > 0$ is undefined

h)


$I_{R_1}$
1
$I_{in}$

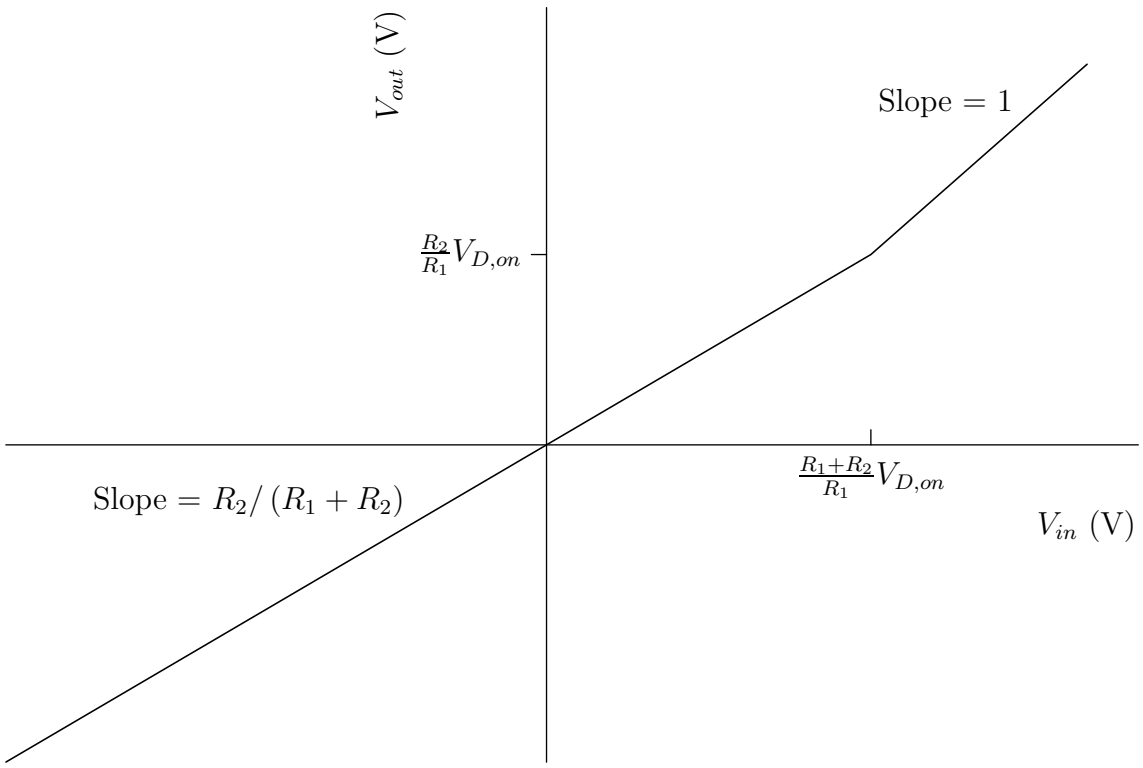
3.23 (a)

$$V_{out} = \begin{cases} \frac{R_2}{R_1+R_2} V_{in} & V_{in} < \frac{R_1+R_2}{R_2} V_{D,on} \\ V_{D,on} & V_{in} > \frac{R_1+R_2}{R_2} V_{D,on} \end{cases}$$




(b)

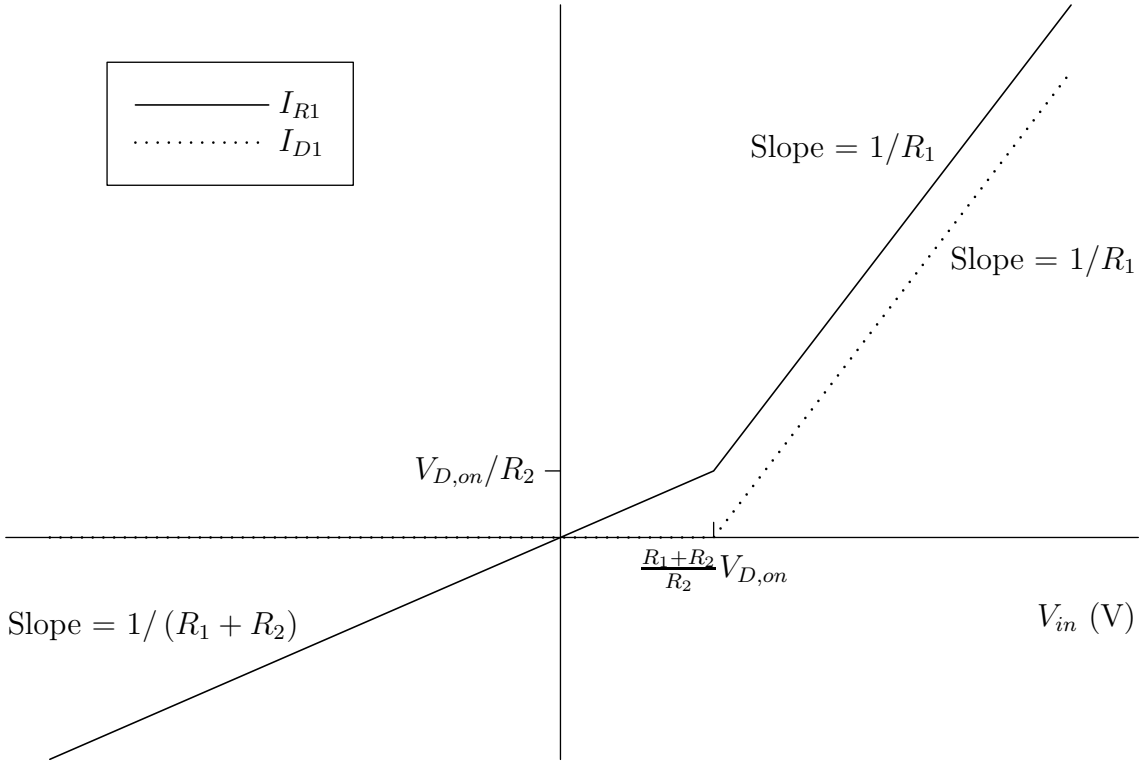
$$V_{out} = \begin{cases} \frac{R_2}{R_1+R_2} V_{in} & V_{in} < \frac{R_1+R_2}{R_1} V_{D,on} \\ V_{in} - V_{D,on} & V_{in} > \frac{R_1+R_2}{R_1} V_{D,on} \end{cases}$$


$V_{out}$ (V)
Slope = 1
$\frac{R_2}{R_1}V_{D,on}$
$\frac{R_1+R_2}{R_1}V_{D,on}$
Slope = $R_2/(R_1+R_2)$
$V_{in}$ (V)

3.24 (a)

$$I_{R1} = \begin{cases} \frac{V_{in}}{R_1+R_2} & V_{in} < \frac{R_1+R_2}{R_2} V_{D,on} \\ \frac{V_{in}-V_{D,on}}{R_1} & V_{in} > \frac{R_1+R_2}{R_2} V_{D,on} \end{cases}$$

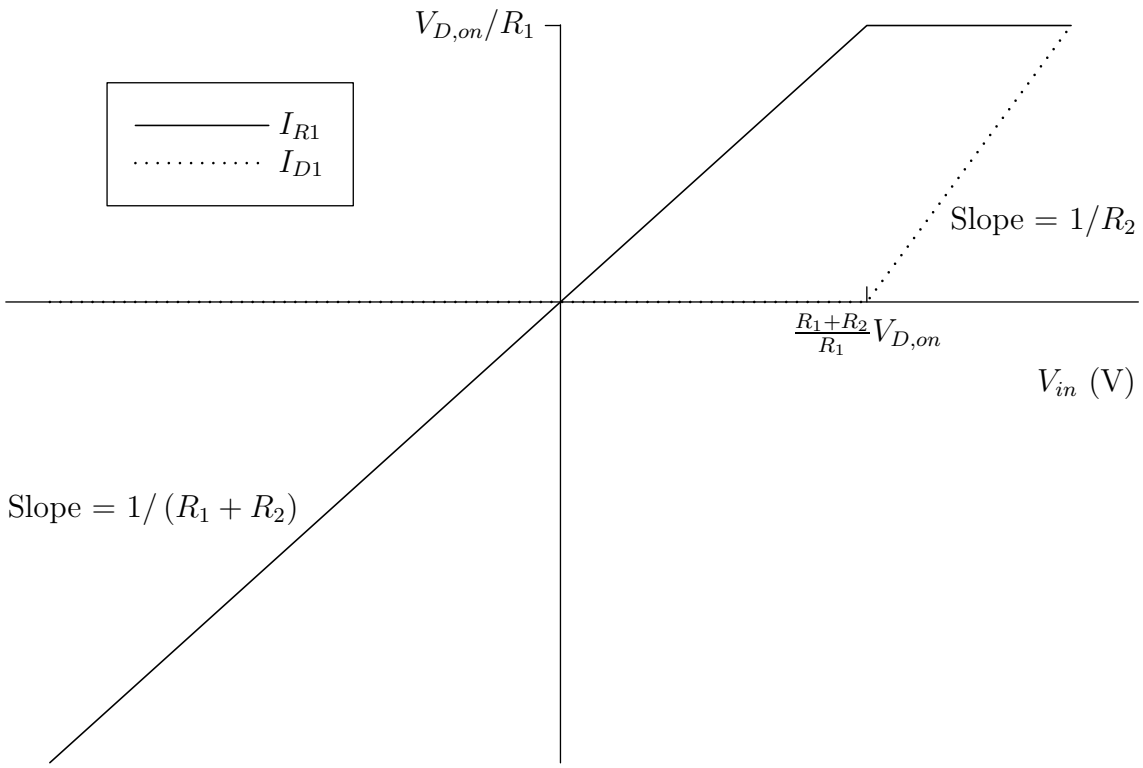
$$I_{D1} = \begin{cases} 0 & V_{in} < \frac{R_1+R_2}{R_2} V_{D,on} \\ \frac{V_{in}-V_{D,on}}{R_1} - \frac{V_{D,on}}{R_2} & V_{in} > \frac{R_1+R_2}{R_2} V_{D,on} \end{cases}$$




(b)

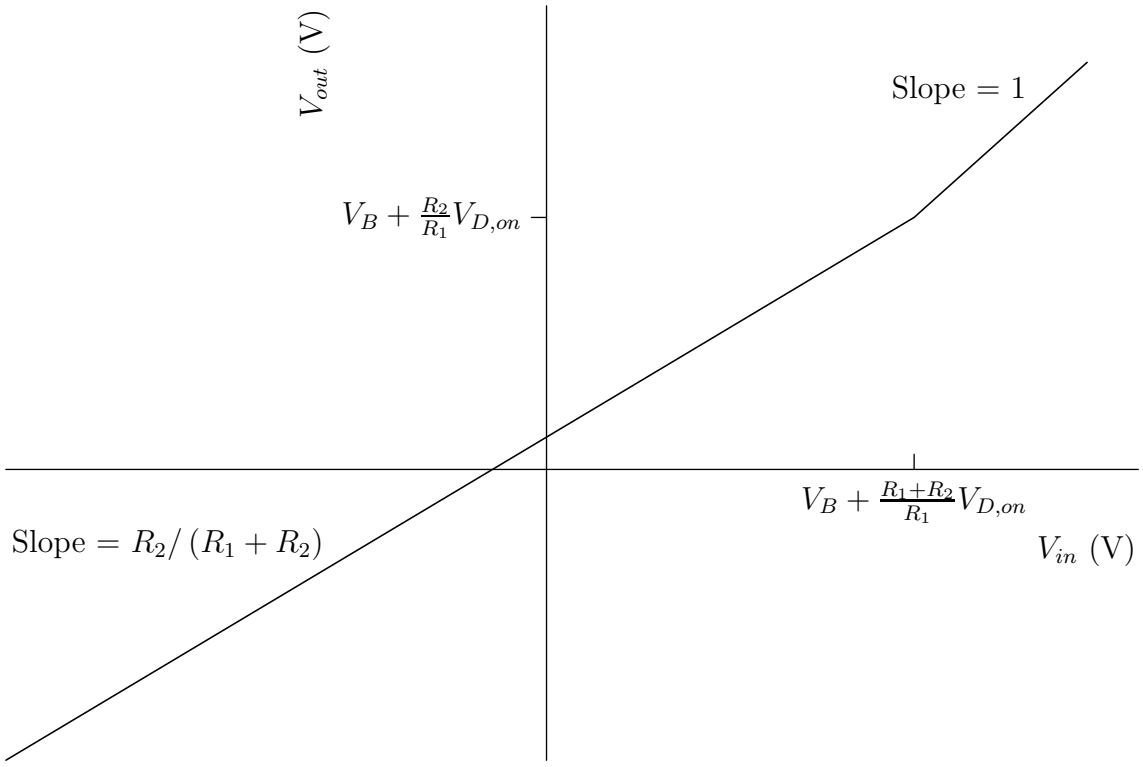
$$I_{R1} = \begin{cases} \frac{V_{in}}{R_1+R_2} & V_{in} < \frac{R_1+R_2}{R_1} V_{D,on} \\ \frac{V_{D,on}}{R_1} & V_{in} > \frac{R_1+R_2}{R_1} V_{D,on} \end{cases}$$

$$I_{D1} = \begin{cases} 0 & V_{in} < \frac{R_1+R_2}{R_1} V_{D,on} \\ \frac{V_{in}-V_{D,on}}{R_2} - \frac{V_{D,on}}{R_1} & V_{in} > \frac{R_1+R_2}{R_1} V_{D,on} \end{cases}$$


$V_{D,on}/R_1$
$I_{R1}$
$I_{D1}$
Slope $= 1/R_2$
$\frac{R_1+R_2}{R_1}V_{D,on}$
$V_{in}$ (V)
Slope $= 1/(R_1 + R_2)$

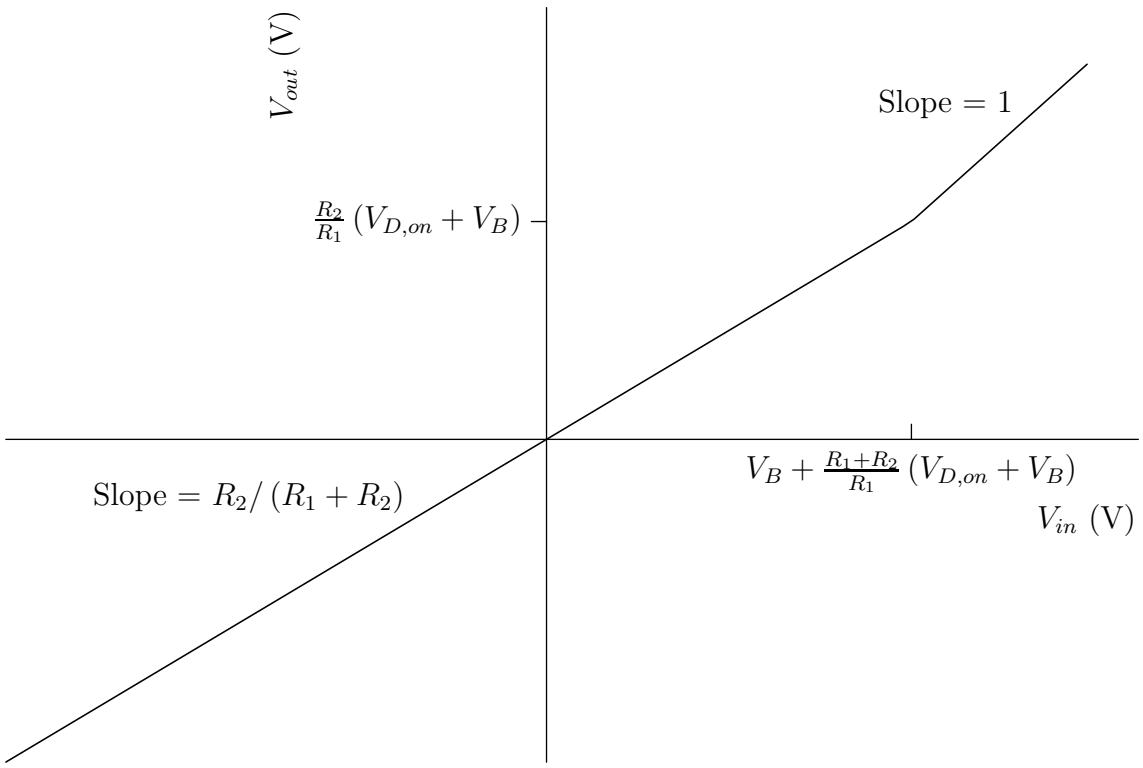
3.25 (a)

$$V_{out} = \begin{cases} V_B + \frac{R_2}{R_1+R_2}(V_{in} - V_B) & V_{in} < V_B + \frac{R_1+R_2}{R_1}V_{D,on} \\ V_{in} - V_{D,on} & V_{in} > V_B + \frac{R_1+R_2}{R_1}V_{D,on} \end{cases}$$



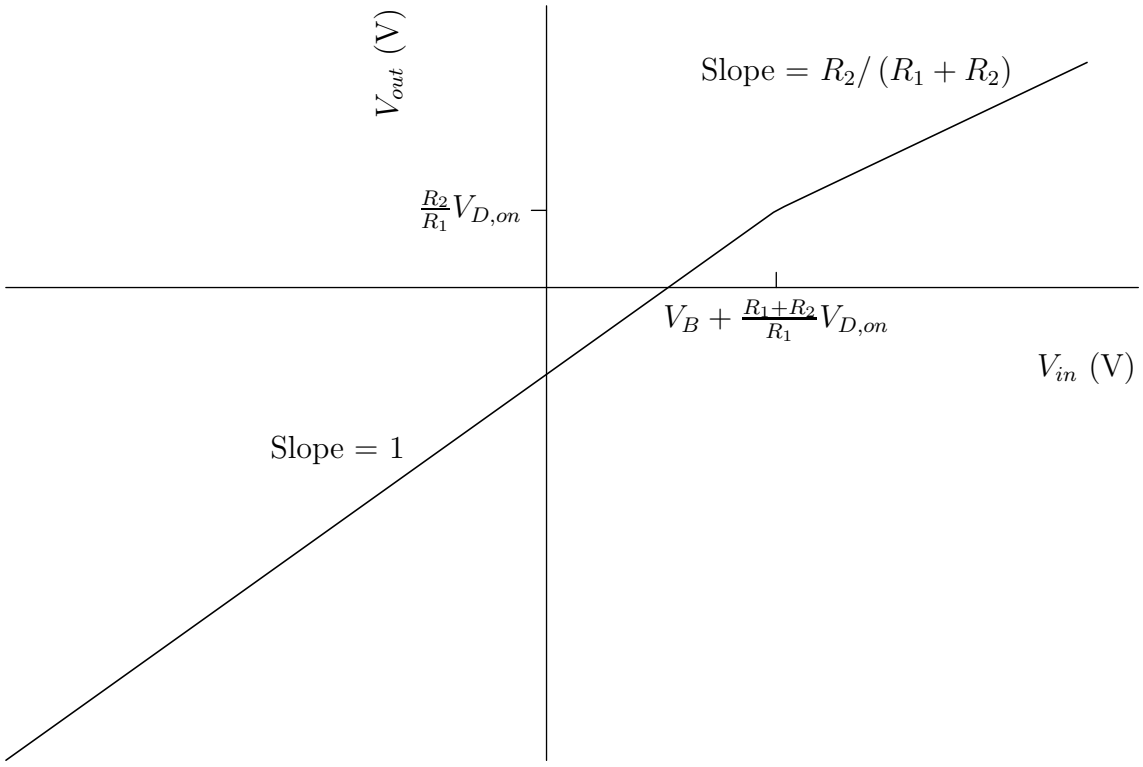

(b)

$$V_{out} = \begin{cases} \frac{R_2}{R_1+R_2}V_{in} & V_{in} < \frac{R_1+R_2}{R_1}(V_{D,on} + V_B) \\ V_{in} - V_{D,on} - V_B & V_{in} > \frac{R_1+R_2}{R_1}(V_{D,on} + V_B) \end{cases}$$



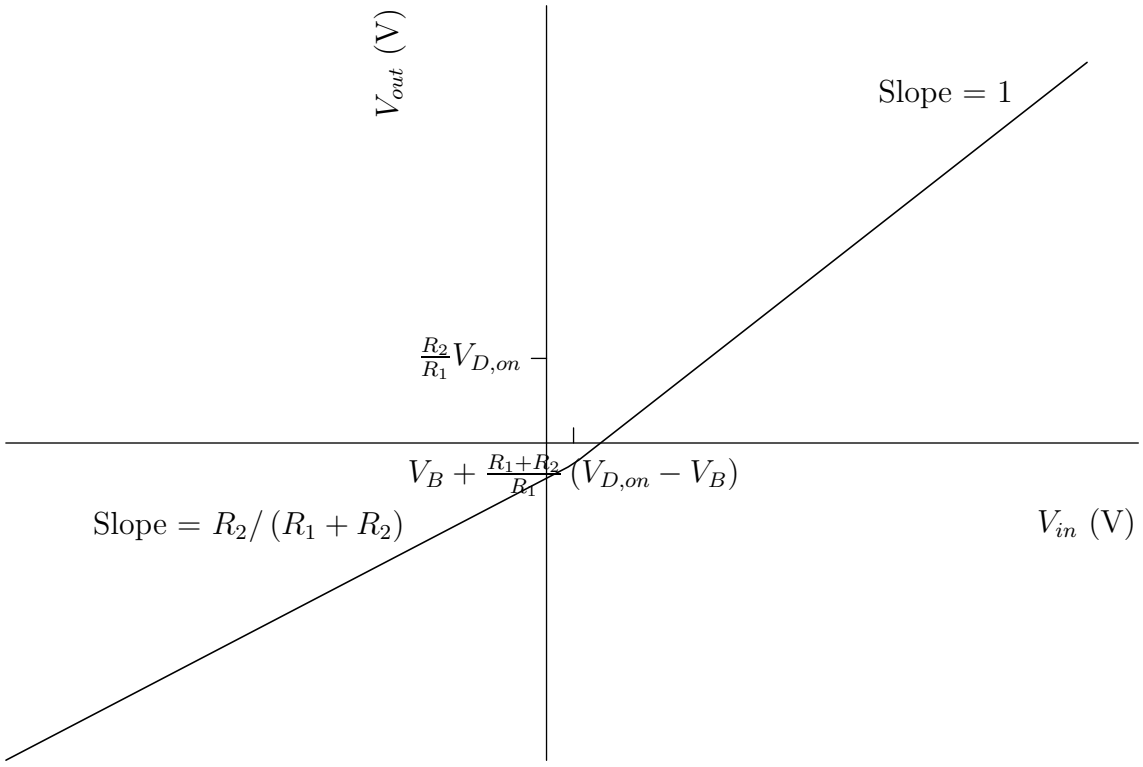

(c)

$$V_{out}=\begin{cases}\frac{R_2}{R_1+R_2}(V_{in}-V_B) & V_{in}>V_B+\frac{R_1+R_2}{R_1}V_{D,on}\\ V_{in}+V_{D,on}-V_B & V_{in}<V_B+\frac{R_1+R_2}{R_1}V_{D,on}\end{cases}$$



(d)

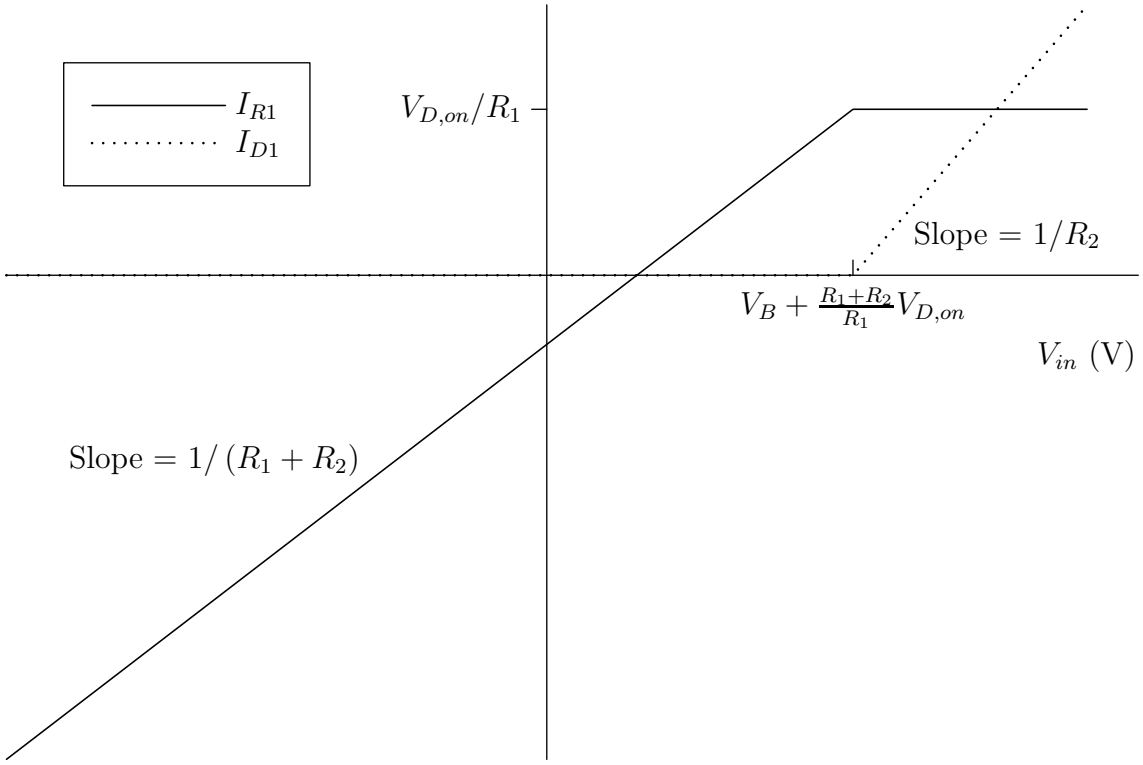
$$V_{out} = \begin{cases} \frac{R_2}{R_1+R_2}\left(V_{in} - V_B\right) & V_{in} < V_B + \frac{R_1+R_2}{R_1}\left(V_{D,on} - V_B\right) \\ V_{in} - V_{D,on} & V_{in} > V_B + \frac{R_1+R_2}{R_1}\left(V_{D,on} - V_B\right) \end{cases}$$



3.26 (a)

$$I_{R1} = \begin{cases} \frac{V_{in}-V_B}{R_1+R_2} & V_{in} < V_B + \frac{R_1+R_2}{R_1}V_{D,on} \\ \frac{V_{D,on}}{R_1} & V_{in} > V_B + \frac{R_1+R_2}{R_1}V_{D,on} \end{cases}$$

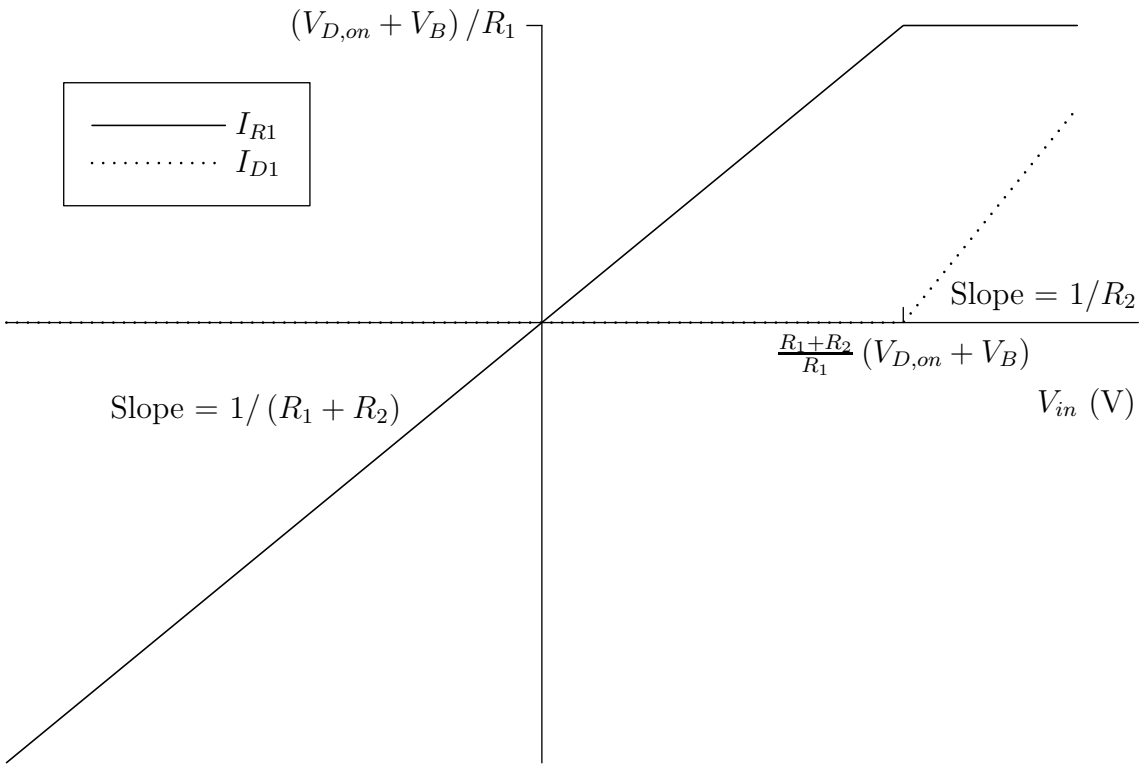
$$I_{D1} = \begin{cases} 0 & V_{in} < V_B + \frac{R_1+R_2}{R_1}V_{D,on} \\ \frac{V_{in}-V_{D,on}-V_B}{R_2} - \frac{V_{D,on}}{R_1} & V_{in} > V_B + \frac{R_1+R_2}{R_1}V_{D,on} \end{cases}$$




(b)

$$I_{R1} = \begin{cases} \frac{V_{in}}{R_1+R_2} & V_{in} < \frac{R_1+R_2}{R_1}(V_{D,on} + V_B) \\ \frac{V_{D,on}+V_B}{R_1} & V_{in} > \frac{R_1+R_2}{R_1}(V_{D,on} + V_B) \end{cases}$$

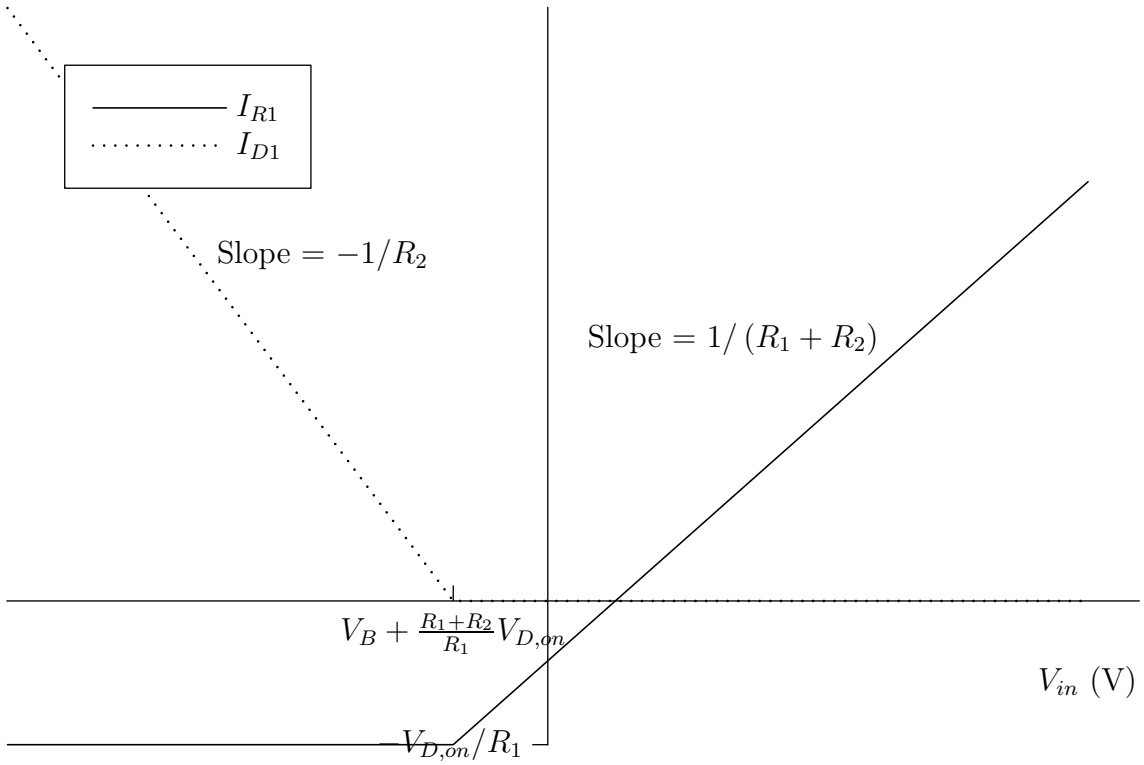
$$I_{D1} = \begin{cases} 0 & V_{in} < \frac{R_1+R_2}{R_1}(V_{D,on} + V_B) \\ \frac{V_{in}-V_{D,on}-V_B}{R_2} - \frac{V_{D,on}+V_B}{R_1} & V_{in} > \frac{R_1+R_2}{R_1}(V_{D,on} + V_B) \end{cases}$$




(c)

$$I_{R1} = \begin{cases} \frac{V_{in}-V_B}{R_1+R_2} & V_{in} > V_B - \frac{R_1+R_2}{R_1}V_{D,on} \\ -\frac{V_{D,on}}{R_1} & V_{in} < V_B - \frac{R_1+R_2}{R_1}V_{D,on} \end{cases}$$

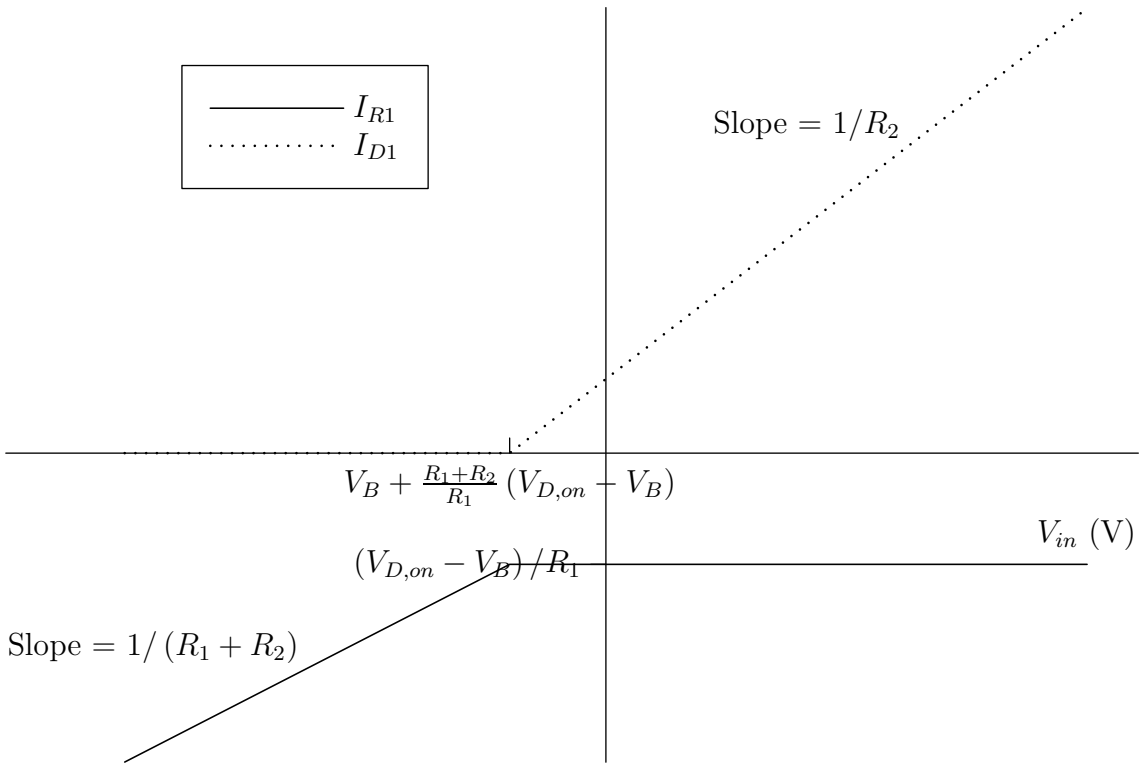
$$I_{D1} = \begin{cases} 0 & V_{in} > V_B - \frac{R_1+R_2}{R_1}V_{D,on} \\ -\frac{V_{in}+V_{D,on}+V_B}{R_2} - \frac{V_{D,on}}{R_1} & V_{in} < V_B - \frac{R_1+R_2}{R_1}V_{D,on} \end{cases}$$




(d)

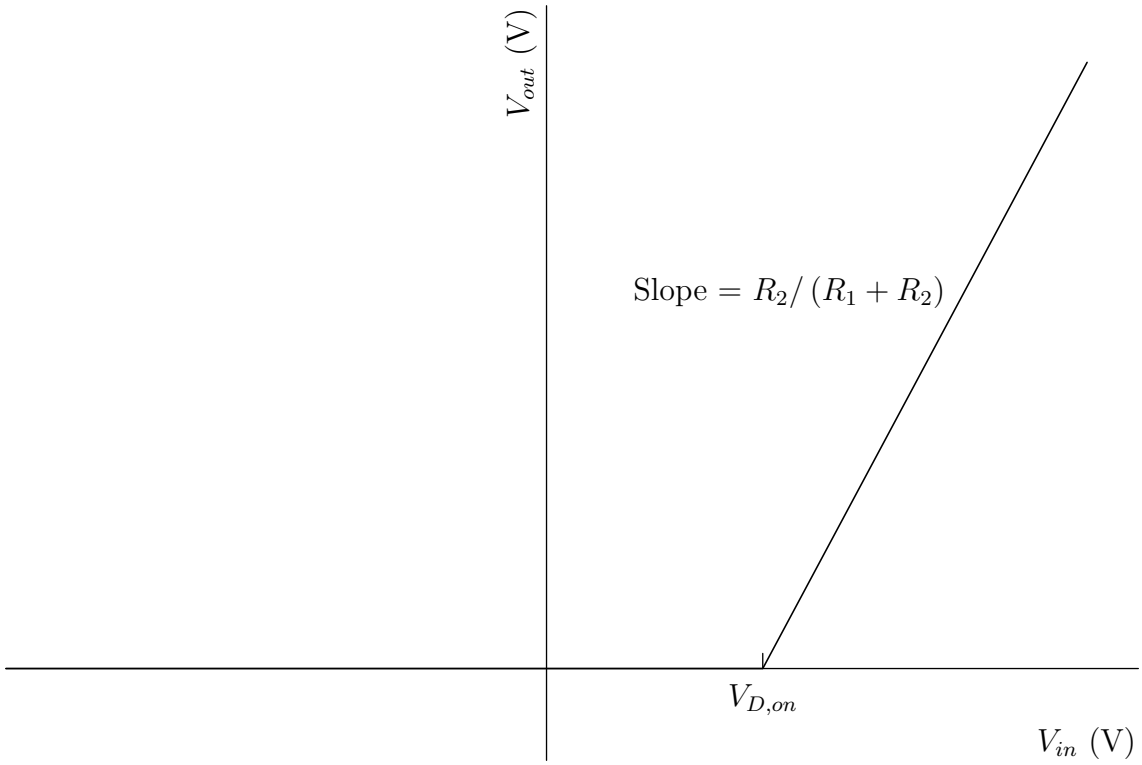
$$I_{R1} = \begin{cases} \frac{V_{in}-V_B}{R_1+R_2} & V_{in} < V_B + \frac{R_1+R_2}{R_1}\left(V_{D,on} - V_B\right) \\ \frac{V_{D,on}-V_B}{R_1} & V_{in} > V_B + \frac{R_1+R_2}{R_1}\left(V_{D,on} - V_B\right) \end{cases}$$

$$I_{D1} = \begin{cases} 0 & V_{in} < V_B + \frac{R_1+R_2}{R_1}\left(V_{D,on} - V_B\right) \\ \frac{V_{in}-V_{D,on}}{R_2} - \frac{V_{D,on}-V_B}{R_1} & V_{in} > V_B + \frac{R_1+R_2}{R_1}\left(V_{D,on} - V_B\right) \end{cases}$$


$I_{R1}$
$I_{D1}$
Slope $= 1/R_2$
$V_B + \frac{R_1+R_2}{R_1}(V_{D,on} - V_B)$
$V_{in}$ (V)
$(V_{D,on} - V_B)/R_1$
Slope $= 1/(R_1 + R_2)$

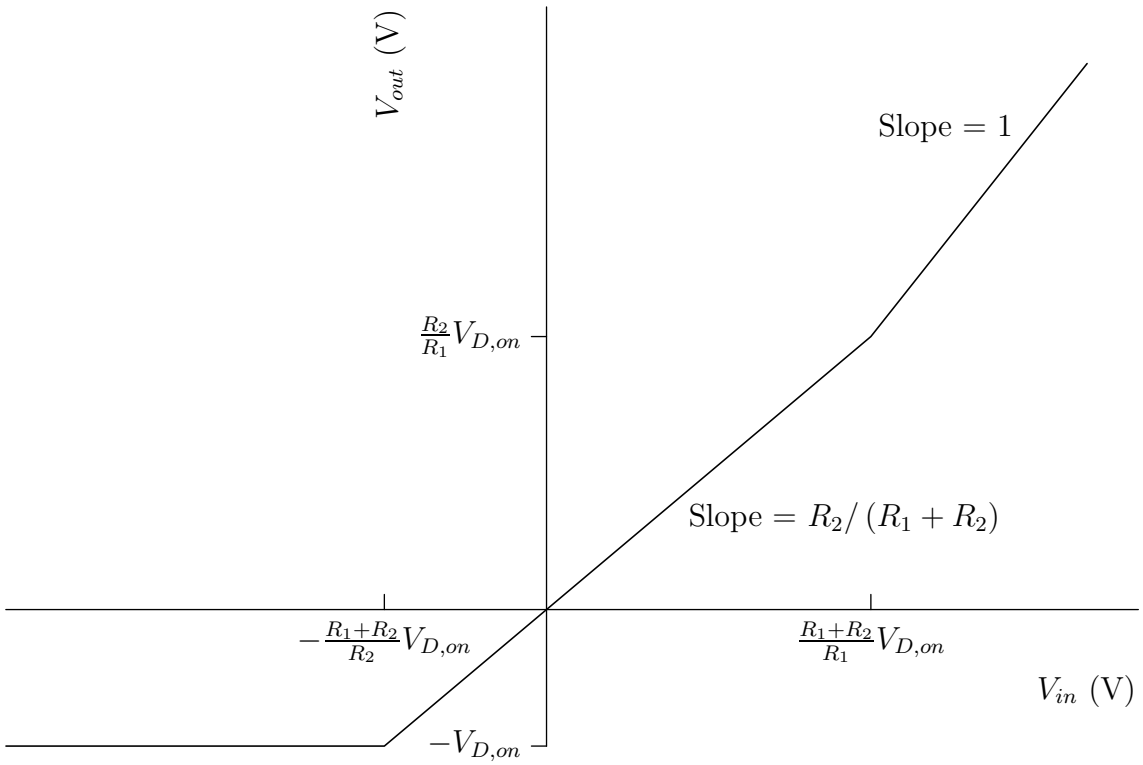
3.27 (a)

$$V_{out} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{R_2}{R_1+R_2}\left(V_{in} - V_{D,on}\right) & V_{in} > V_{D,on} \end{cases}$$



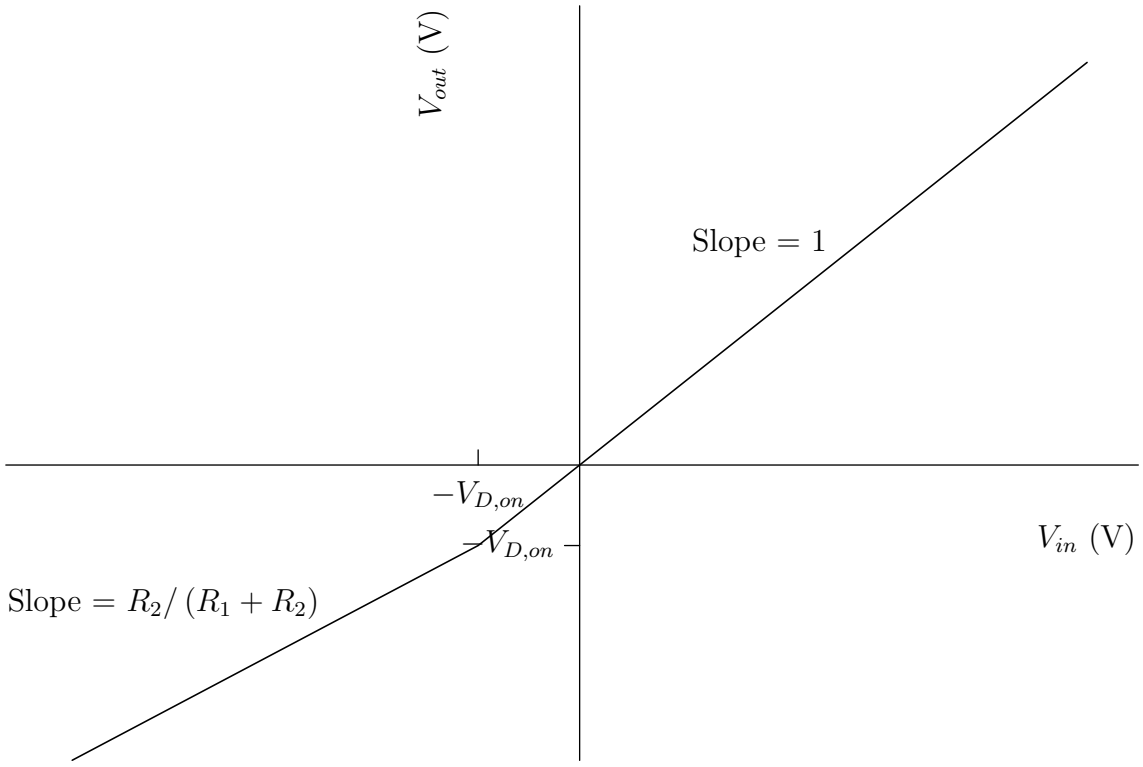

(b)

$$V_{out} = \begin{cases} -V_{D,on} & V_{in} < -\frac{R_1+R_2}{R_2}V_{D,on} \\ \frac{R_2}{R_1+R_2}V_{in} & -\frac{R_1+R_2}{R_2}V_{D,on} < V_{in} < \frac{R_1+R_2}{R_1}V_{D,on} \\ V_{in} - V_{D,on} & V_{in} > \frac{R_1+R_2}{R_1}V_{D,on} \end{cases}$$



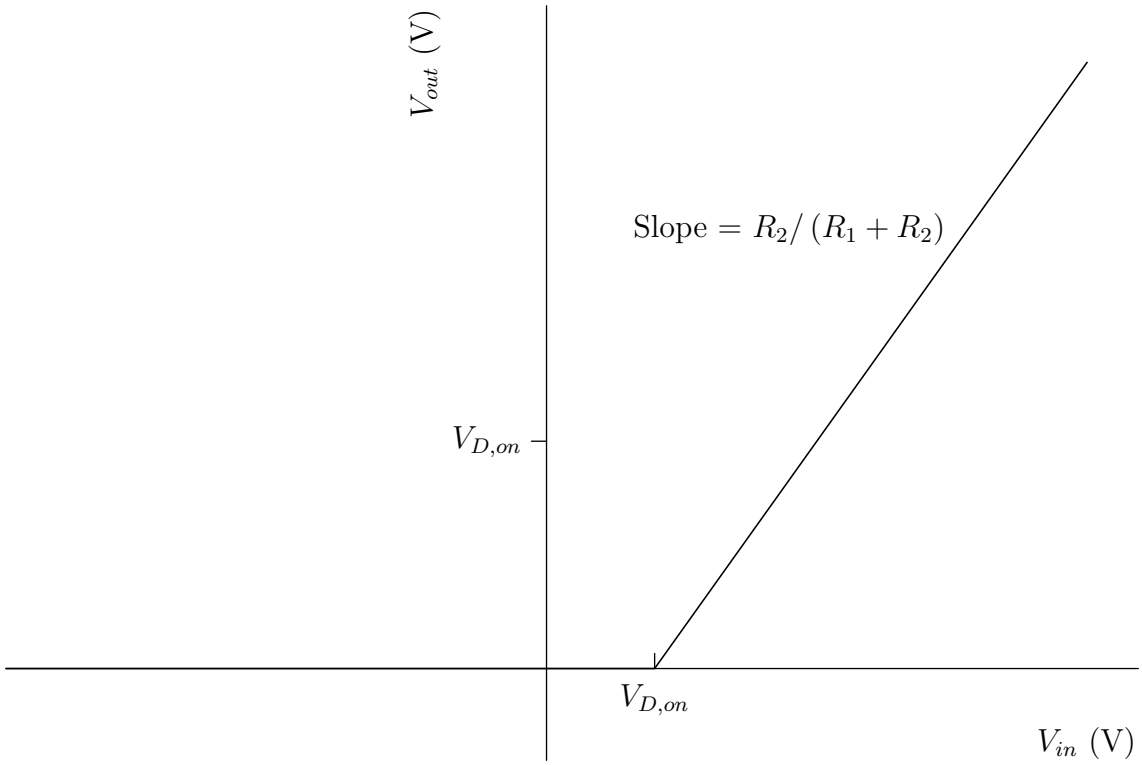

(c)

$$V_{out} = \begin{cases} \frac{R_2}{R_1+R_2}(V_{in}+V_{D,on}) - V_{D,on} & V_{in} < -V_{D,on} \\ V_{in} & V_{in} > -V_{D,on} \end{cases}$$



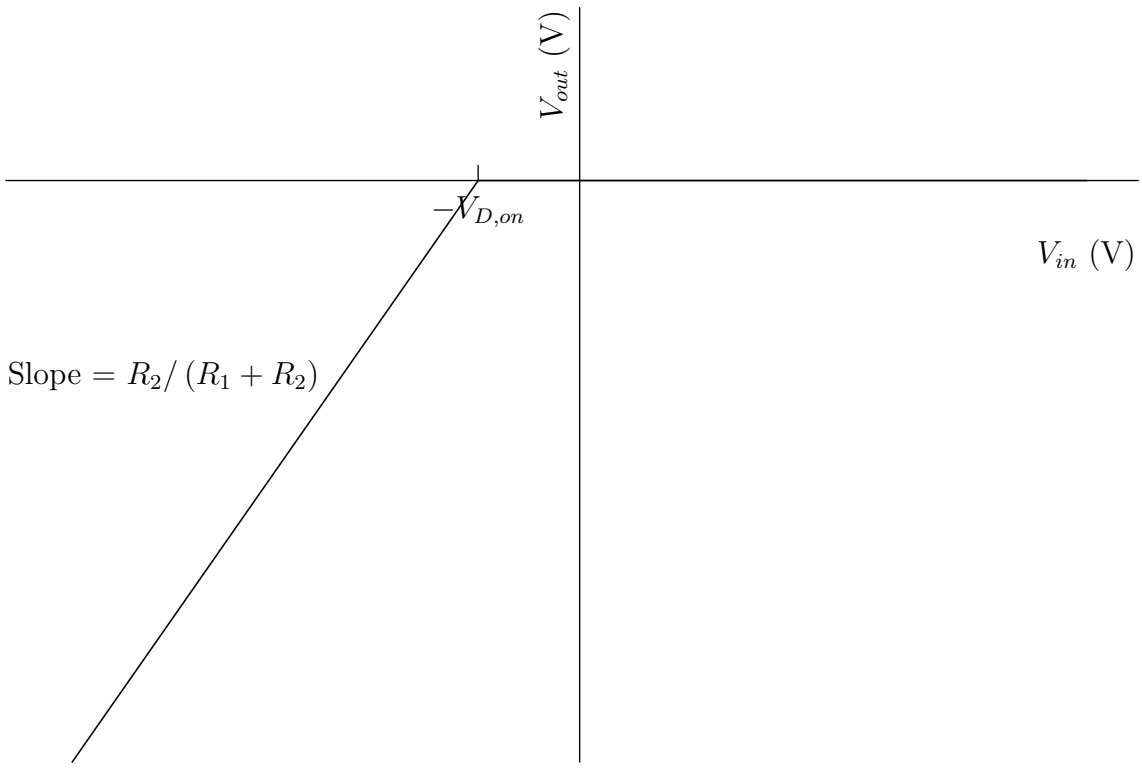
(d)

$$V_{out} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{R_2}{R_1+R_2}\left(V_{in} - V_{D,on}\right) & V_{in} > V_{D,on} \end{cases}$$




(e)

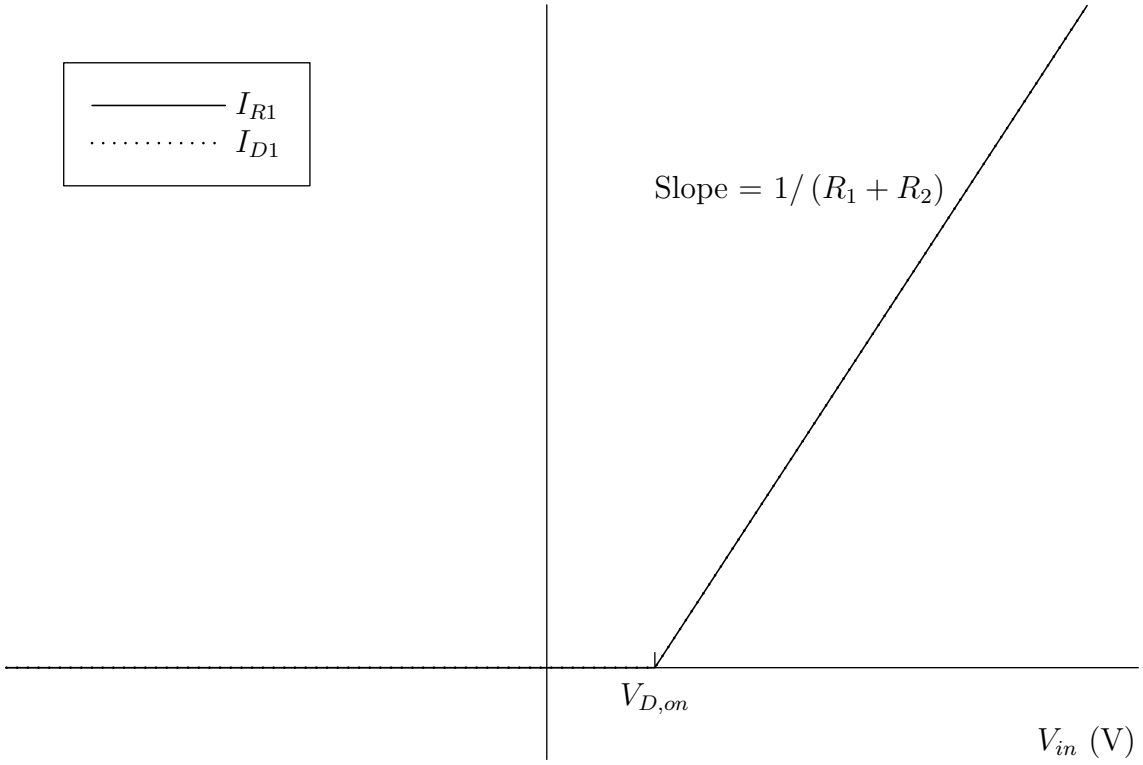
$$V_{out} = \begin{cases} \frac{R_2}{R_1+R_2}\left(V_{in} + V_{D,on}\right) & V_{in} < -V_{D,on} \\ 0 & V_{in} > -V_{D,on} \end{cases}$$


$V_{out}$ (V)
$-V_{D,on}$
$V_{in}$ (V)
Slope $= R_2/\left(R_1 + R_2\right)$

3.28 (a)

$$I_{R1} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{V_{in} - V_{D,on}}{R_1 + R_2} & V_{in} > V_{D,on} \end{cases}$$

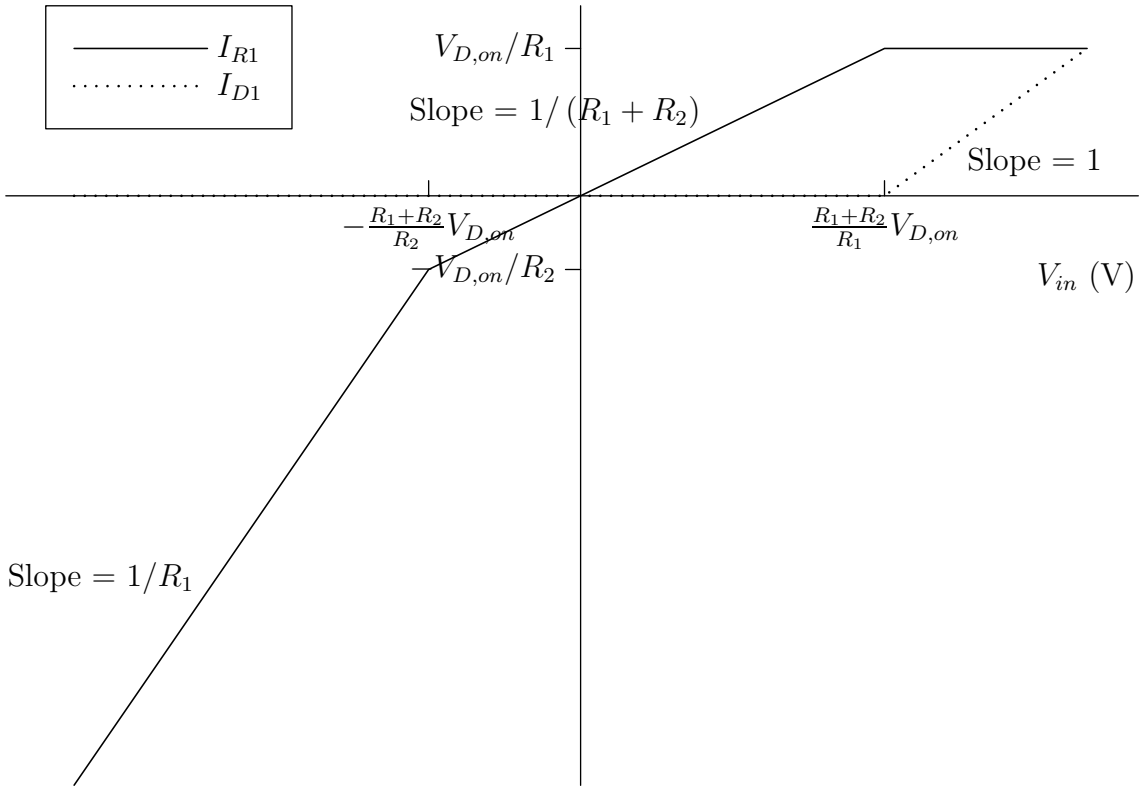
$$I_{D1} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{V_{in} - V_{D,on}}{R_1 + R_2} & V_{in} > V_{D,on} \end{cases}$$




(b)

$$I_{R1} = \begin{cases} \frac{V_{in} + V_{D,on}}{R_1} & V_{in} < -\frac{R_1 + R_2}{R_2} V_{D,on} \\ \frac{V_{in}}{R_1 + R_2} & -\frac{R_1 + R_2}{R_2} V_{D,on} < V_{in} < \frac{R_1 + R_2}{R_1} V_{D,on} \\ \frac{V_{D,on}}{R_1} & V_{in} > \frac{R_1 + R_2}{R_1} V_{D,on} \end{cases}$$

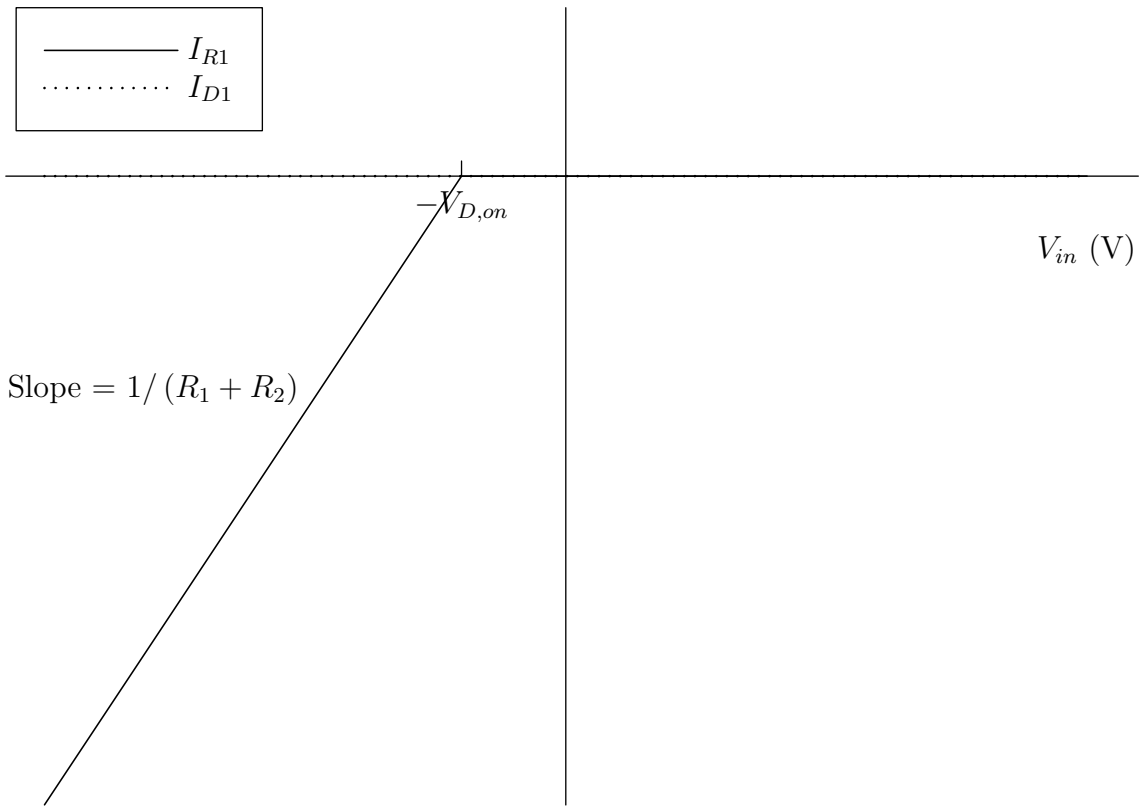
$$I_{D1} = \begin{cases} 0 & V_{in} < -\frac{R_1 + R_2}{R_2} V_{D,on} \\ 0 & -\frac{R_1 + R_2}{R_2} V_{D,on} < V_{in} < \frac{R_1 + R_2}{R_1} V_{D,on} \\ \frac{V_{in} - V_{D,on}}{R_2} - \frac{V_{D,on}}{R_1} & V_{in} > \frac{R_1 + R_2}{R_1} V_{D,on} \end{cases}$$




(c)

$$I_{R1} = \begin{cases} \frac{V_{in}+V_{D,on}}{R_1+R_2} & V_{in} < -V_{D,on} \\ 0 & V_{in} > -V_{D,on} \end{cases}$$

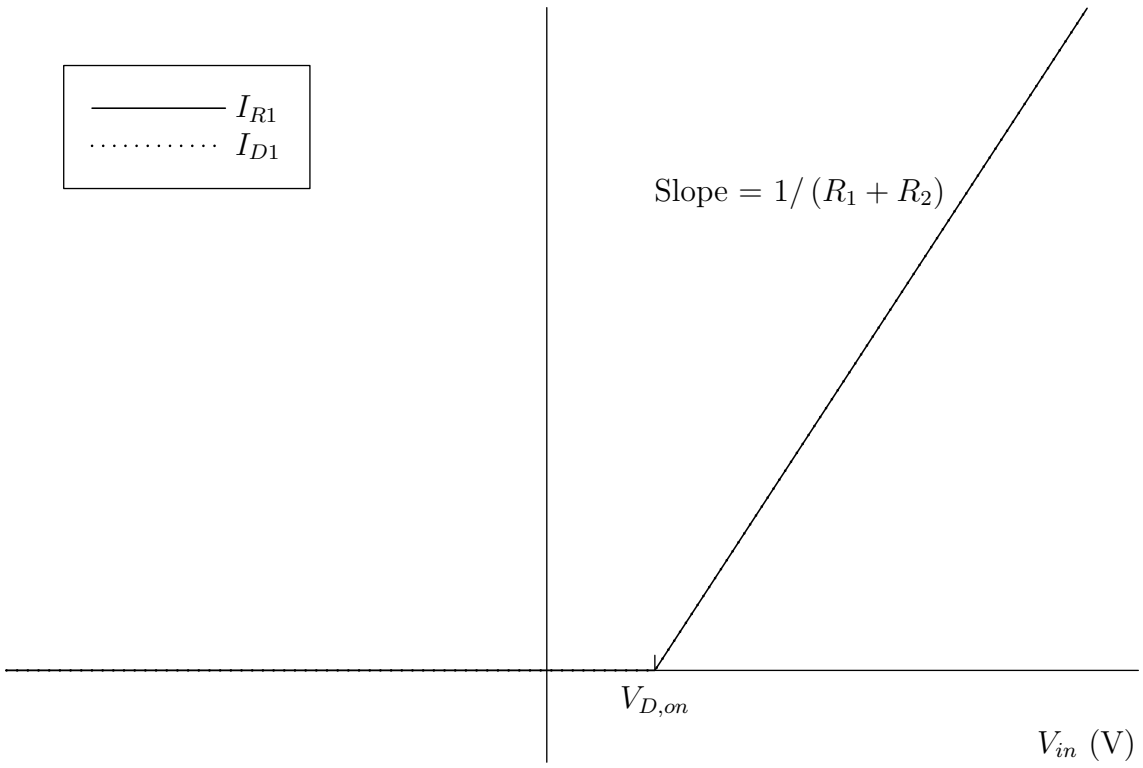
$$I_{D1} = \begin{cases} 0 & V_{in} < -V_{D,on} \\ 0 & V_{in} > -V_{D,on} \end{cases}$$




(d)

$$I_{R1} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{V_{in} - V_{D,on}}{R_1 + R_2} & V_{in} > V_{D,on} \end{cases}$$

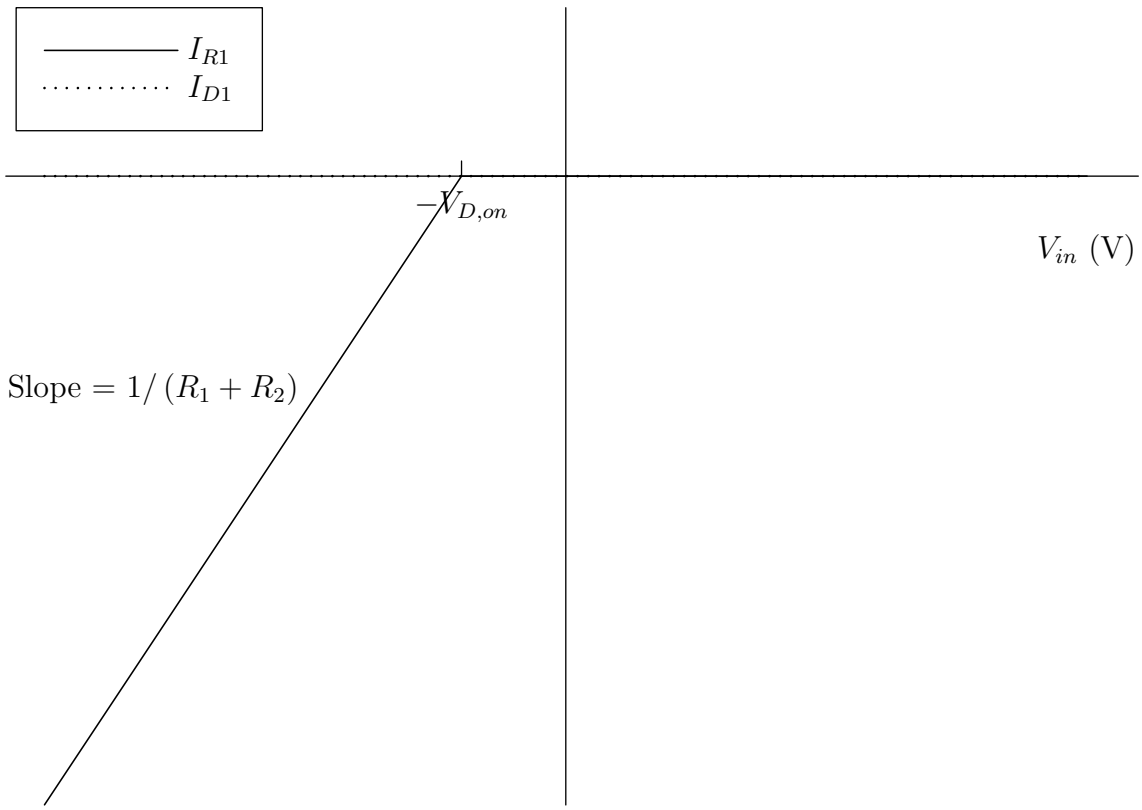
$$I_{D1} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{V_{in} - V_{D,on}}{R_1 + R_2} & V_{in} > V_{D,on} \end{cases}$$




(e)

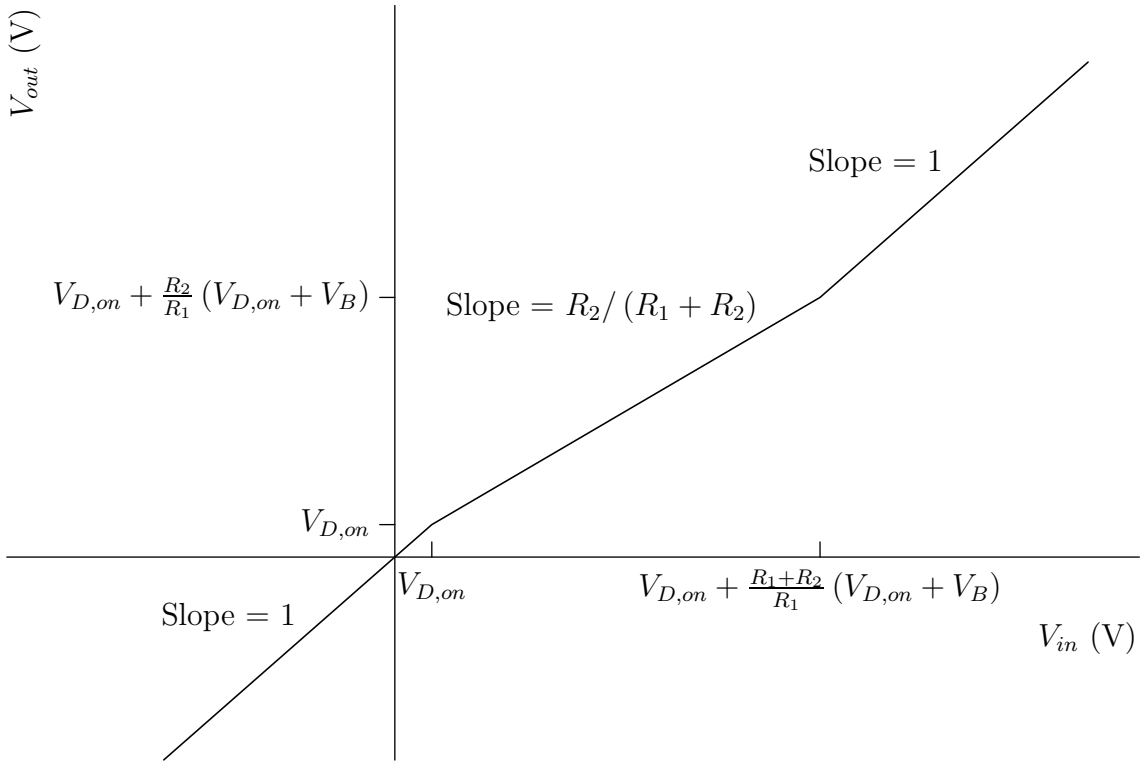
$$I_{R1} = \begin{cases} \frac{V_{in}+V_{D,on}}{R_1+R_2} & V_{in} < -V_{D,on} \\ 0 & V_{in} > -V_{D,on} \end{cases}$$

$$I_{D1} = \begin{cases} 0 & V_{in} < -V_{D,on} \\ 0 & V_{in} > -V_{D,on} \end{cases}$$


$I_{R1}$
$I_{D1}$
$-V_{D,on}$
$V_{in}$ (V)
Slope $= 1/\left(R_1 + R_2\right)$

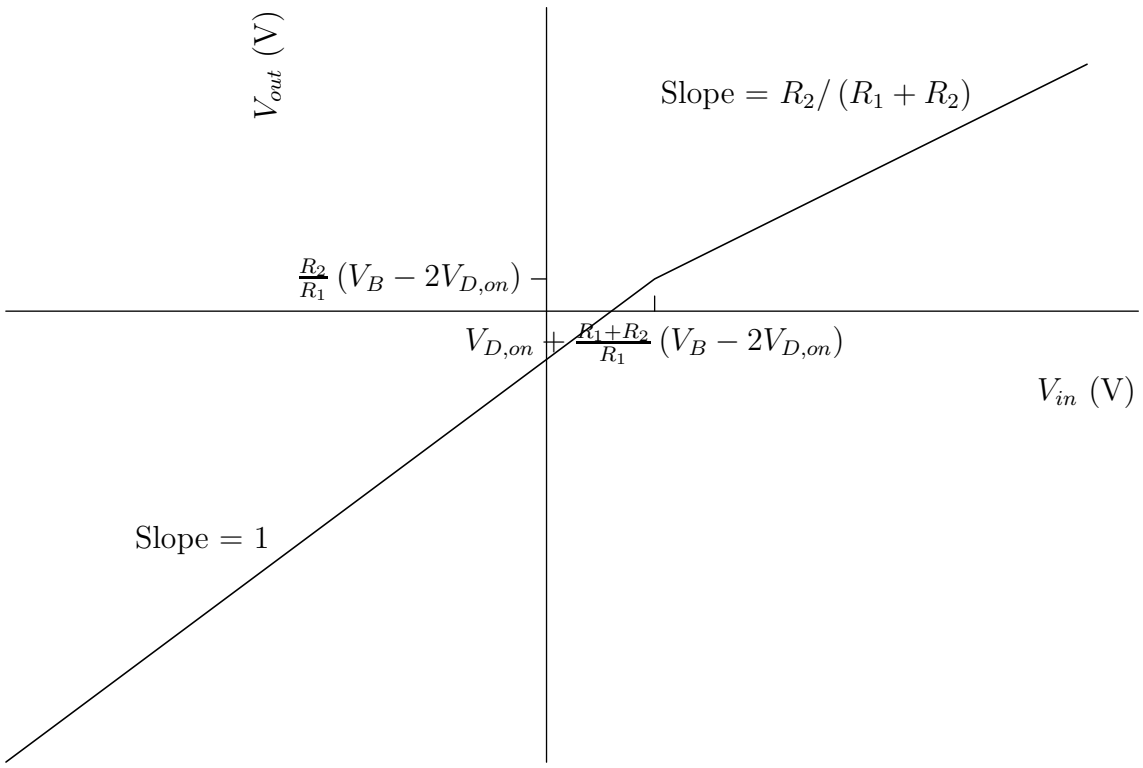
3.29 (a)

$$V_{out} = \begin{cases} V_{in} & V_{in} < V_{D,on} \\ V_{D,on} + \frac{R_2}{R_1+R_2}(V_{in} - V_{D,on}) & V_{D,on} < V_{in} < V_{D,on} + \frac{R_1+R_2}{R_1}(V_{D,on} + V_B) \\ V_{in} - V_{D,on} - V_B & V_{in} > V_{D,on} + \frac{R_1+R_2}{R_1}(V_{D,on} + V_B) \end{cases}$$



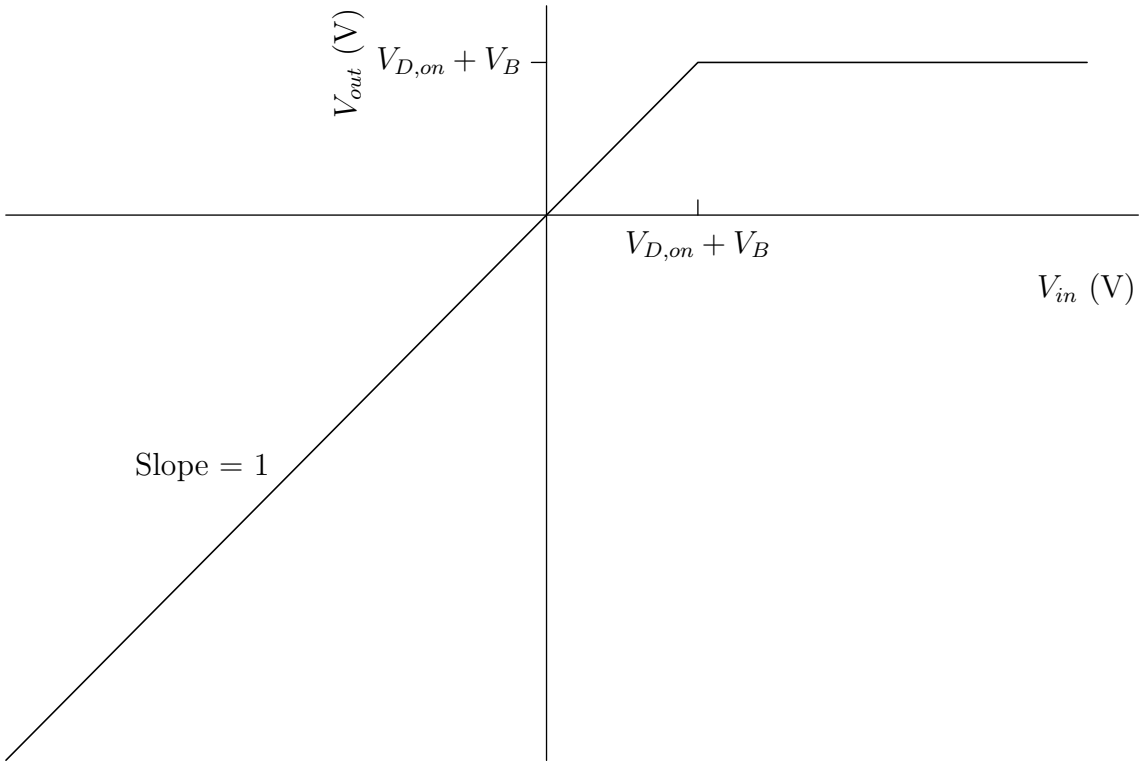

(b)

$$V_{out} = \begin{cases} V_{in} + V_{D,on} - V_B & V_{in} < V_{D,on} + \frac{R_1+R_2}{R_1}(V_B - 2V_{D,on}) \\ \frac{R_2}{R_1+R_2}(V_{in} - V_{D,on}) & V_{in} > V_{D,on} + \frac{R_1+R_2}{R_1}(V_B - 2V_{D,on}) \end{cases}$$



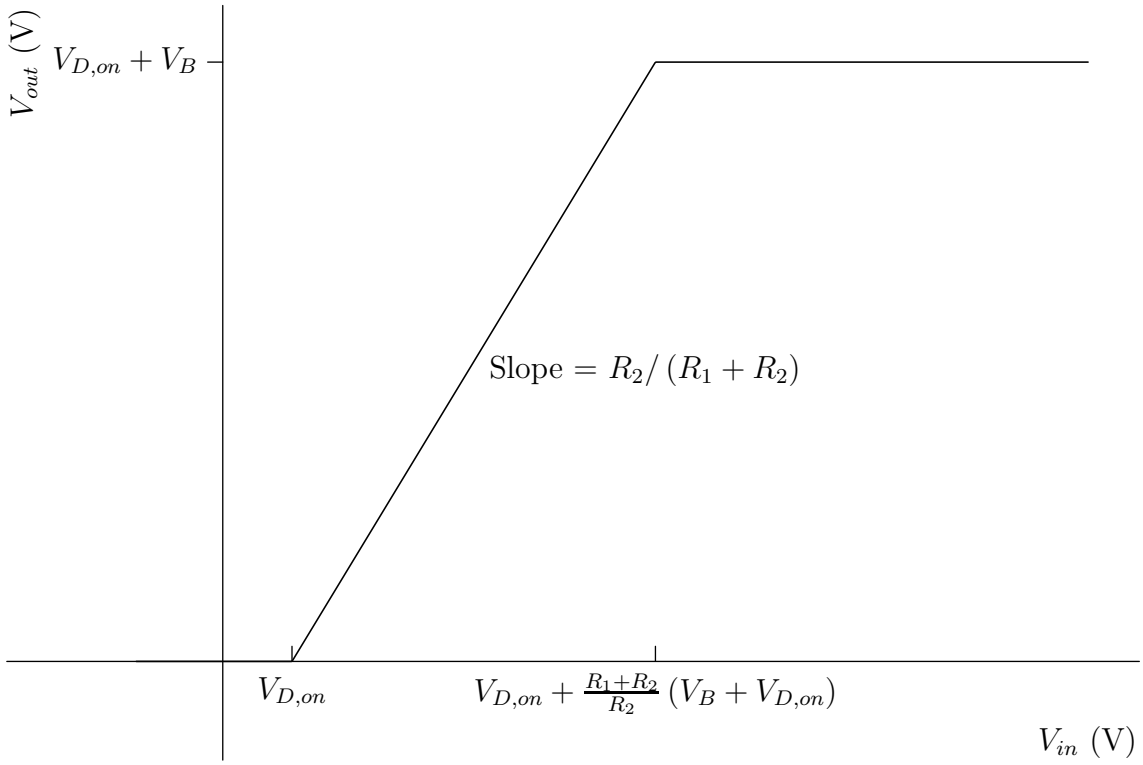

(c)

$$V_{out} = \begin{cases} V_{in} & V_{in} < V_{D,on} + V_B \\ V_{D,on} + V_B & V_{in} > V_{D,on} + V_B \end{cases}$$



(d)

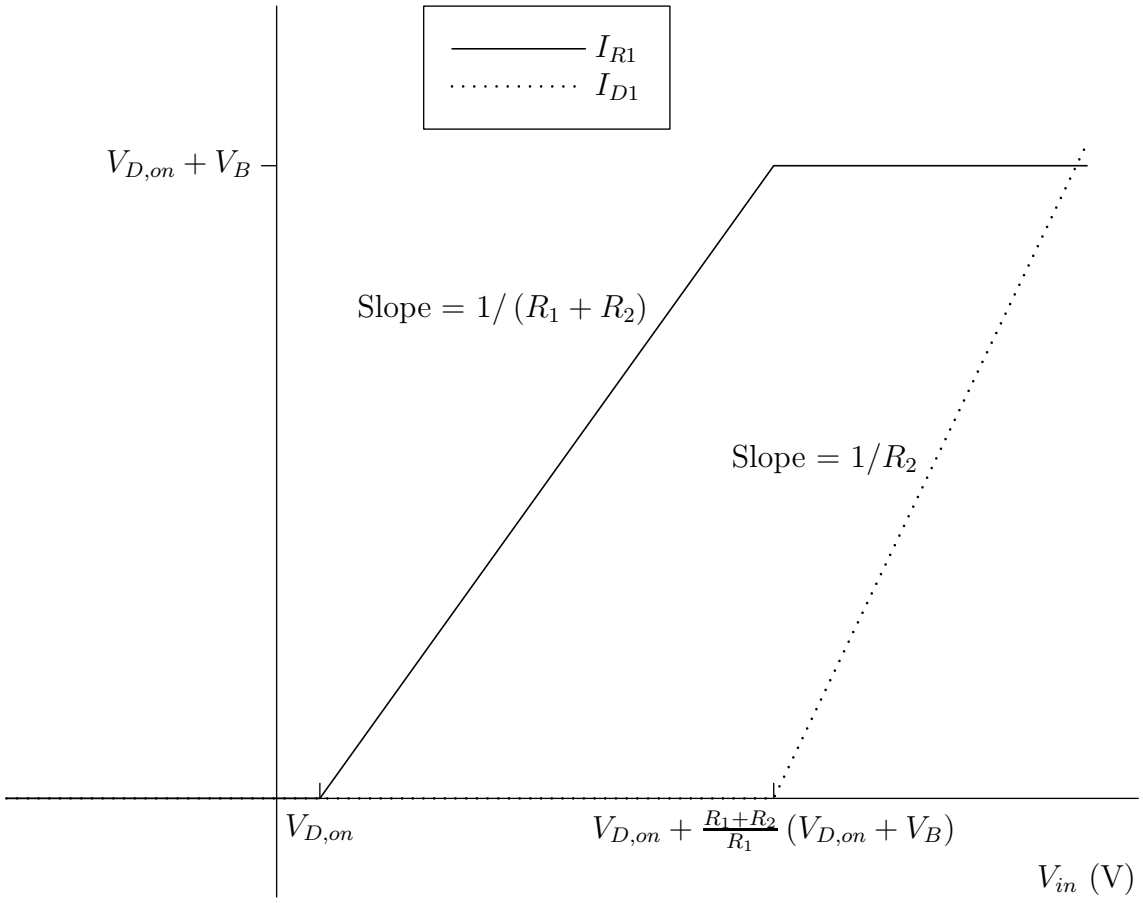
$$V_{out} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{R_2}{R_1+R_2}\left(V_{in} - V_{D,on}\right) & V_{D,on} < V_{in} < V_{D,on} + \frac{R_1+R_2}{R_2}\left(V_B + V_{D,on}\right) \\ V_{D,on} + V_B & V_{in} > V_{D,on} + \frac{R_1+R_2}{R_2}\left(V_B + V_{D,on}\right) \end{cases}$$



3.30 (a)

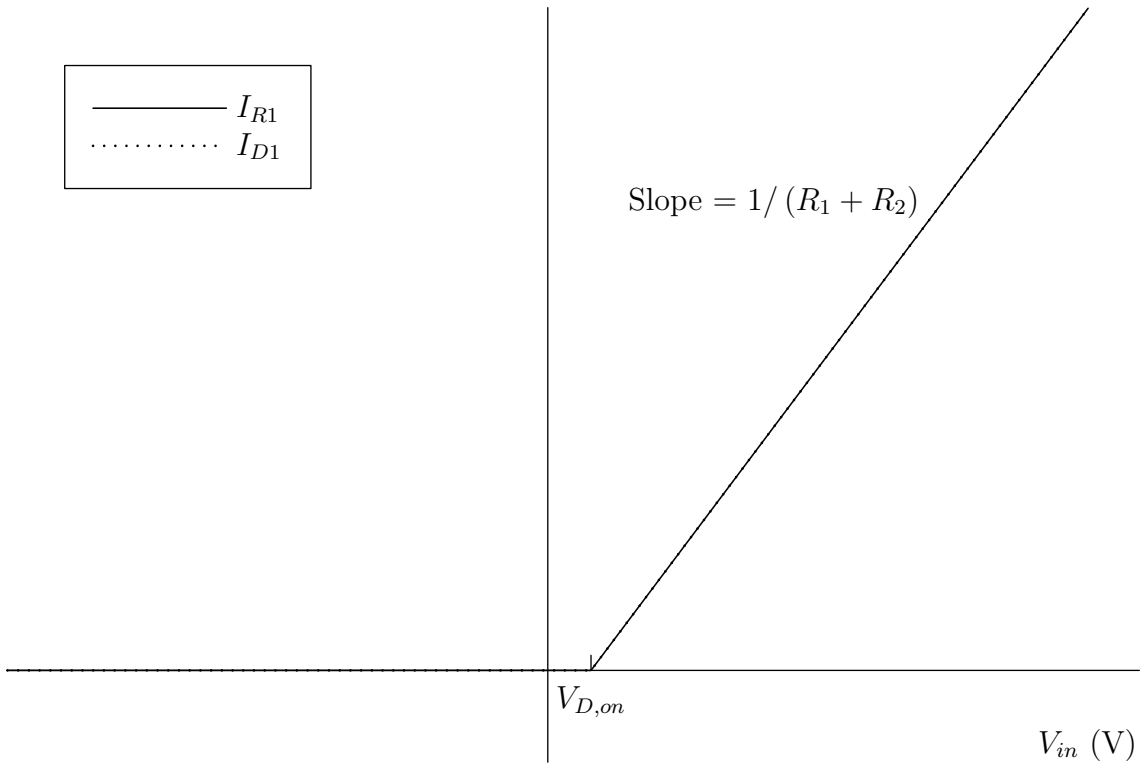
$$I_{R1} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{V_{in}-V_{D,on}}{R_1+R_2} & V_{D,on} < V_{in} < V_{D,on} + \frac{R_1+R_2}{R_1}(V_{D,on}+V_B) \\ \frac{V_{D,on}+V_B}{R_1} & V_{in} > V_{D,on} + \frac{R_1+R_2}{R_1}(V_{D,on}+V_B) \end{cases}$$

$$I_{D1} = \begin{cases} 0 & V_{in} < V_{D,on} + \frac{R_1+R_2}{R_1}(V_{D,on}+V_B) \\ \frac{V_{in}-2V_{D,on}-V_B}{R_2} - \frac{V_{D,on}+V_B}{R_1} & V_{in} > V_{D,on} + \frac{R_1+R_2}{R_1}(V_{D,on}+V_B) \end{cases}$$



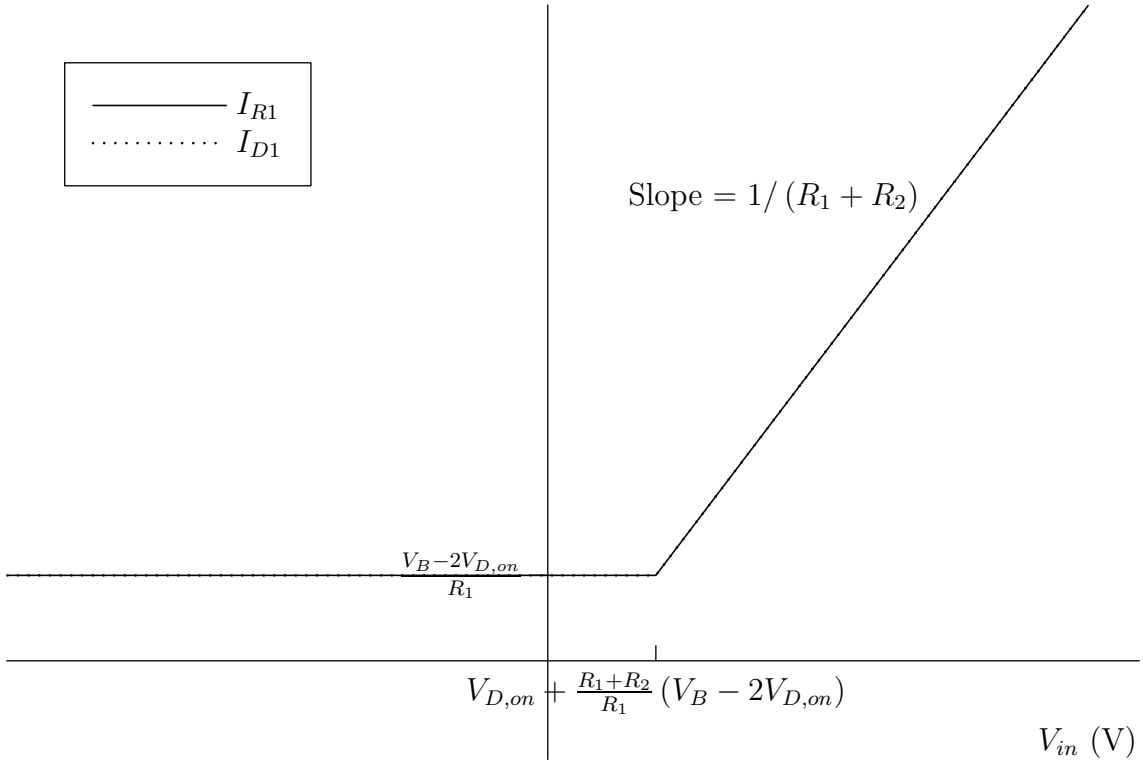

(b) If $V_B < 2V_{D,on}$:

$$I_{R1} = I_{D1} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{V_{in}-V_{D,on}}{R_1+R_2} & V_{in} > V_{D,on} \end{cases}$$




If $V_B > 2V_{D,on}$:

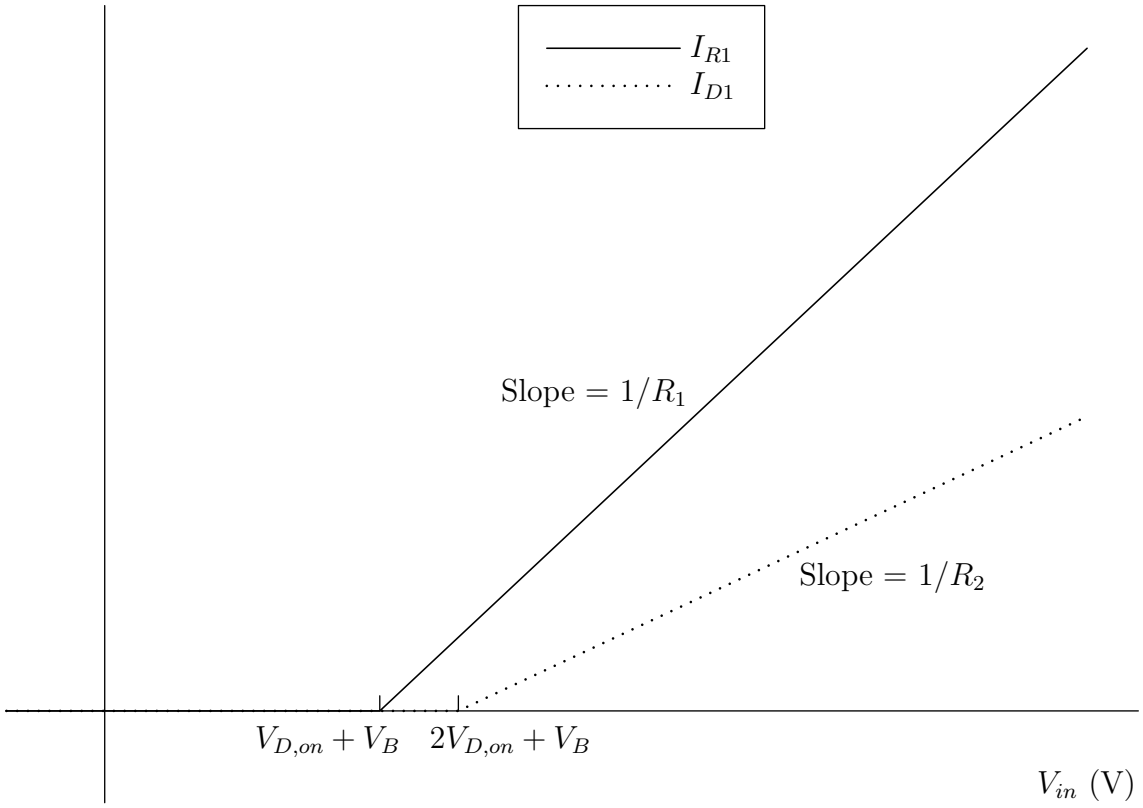
$$I_{R1} = I_{D1} = \begin{cases} \frac{V_B - 2V_{D,on}}{R_1} & V_{in} < V_{D,on} + \frac{R_1+R_2}{R_1}\left(V_B - 2V_{D,on}\right) \\ \frac{V_{in} - V_{D,on}}{R_1+R_2} & V_{in} > V_{D,on} + \frac{R_1+R_2}{R_1}\left(V_B - 2V_{D,on}\right) \end{cases}$$



(c)

$$I_{R1} = \begin{cases} 0 & V_{in} < V_{D,on} + V_B \\ \frac{V_{in} - V_{D,on} - V_B}{R_1} & V_{in} > V_{D,on} + V_B \end{cases}$$

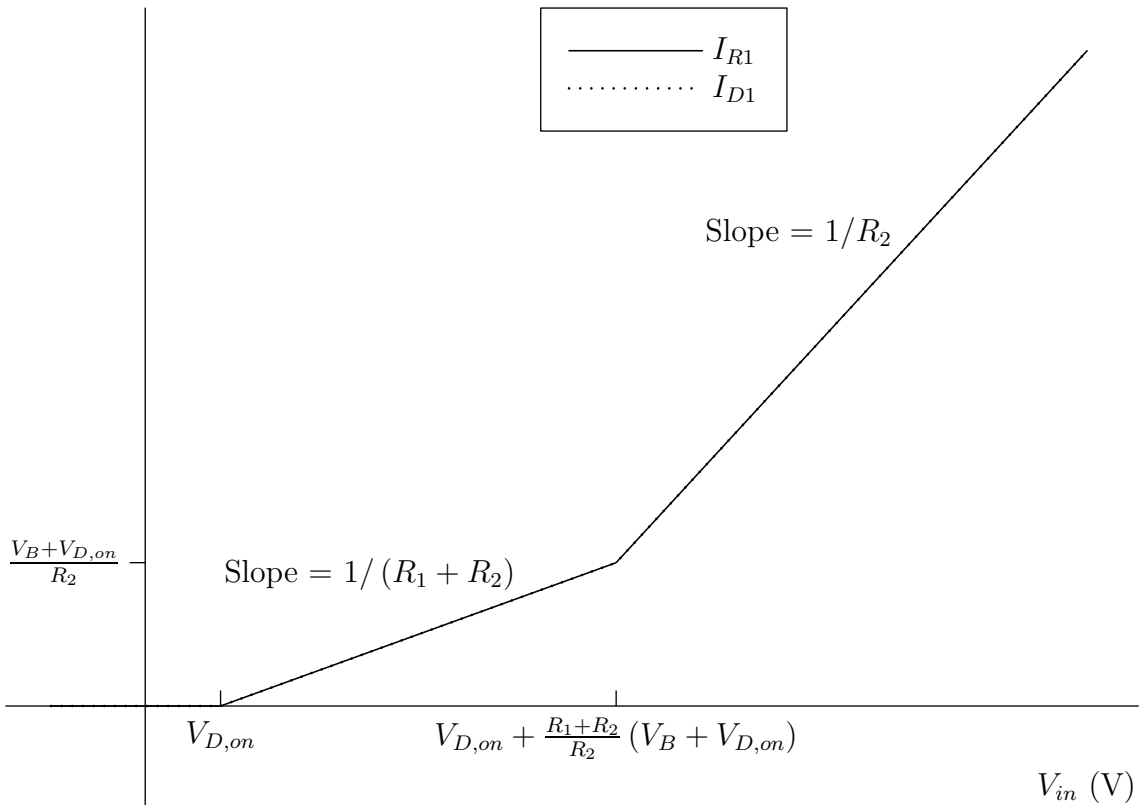
$$I_{D1} = \begin{cases} 0 & V_{in} < V_{D,on} + V_B \\ 0 & V_{D,on} + V_B < V_{in} < 2V_{D,on} + V_B \\ \frac{V_{in} - 2V_{D,on} - V_B}{R_2} & V_{in} > 2V_{D,on} + V_B \end{cases}$$




(d)

$$I_{R1} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{V_{in} - V_{D,on}}{R_1 + R_2} & V_{D,on} < V_{in} < V_{D,on} + \frac{R_1 + R_2}{R_2}(V_B + V_{D,on}) \\ \frac{V_{in} - 2V_{D,on} - V_B}{R_1} & V_{in} > V_{D,on} + \frac{R_1 + R_2}{R_2}(V_B + V_{D,on}) \end{cases}$$

$$I_{D1} = \begin{cases} 0 & V_{in} < V_{D,on} \\ \frac{V_{in} - V_{D,on}}{R_1 + R_2} & V_{D,on} < V_{in} < V_{D,on} + \frac{R_1 + R_2}{R_2}(V_B + V_{D,on}) \\ \frac{V_{in} - 2V_{D,on} - V_B}{R_1} & V_{in} > V_{D,on} + \frac{R_1 + R_2}{R_2}(V_B + V_{D,on}) \end{cases}$$


$I_{R1}$
$I_{D1}$
Slope $= 1/R_2$
$\frac{V_B+V_{D,on}}{R_2}$
Slope $= 1/\left(R_1+R_2\right)$
$V_{D,on}$
$V_{D,on}+\frac{R_1+R_2}{R_2}\left(V_B+V_{D,on}\right)$
$V_{in}$ (V)

3.31 (a)

$$I_{D1} = \frac{V_{in} - V_{D,on}}{R_1} = 1.6 \text{ mA}$$
$$r_{d1} = \frac{V_T}{I_{D1}} = 16.25\ \Omega$$
$$\Delta V_{out} = \frac{R_1}{r_d + R_1}\Delta V_{in} = \boxed{98.40 \text{ mV}}$$

(b)

$$I_{D1} = I_{D2} = \frac{V_{in} - 2V_{D,on}}{R_1} = 0.8 \text{ mA}$$
$$r_{d1} = r_{d2} = \frac{V_T}{I_{D1}} = 32.5\ \Omega$$
$$\Delta V_{out} = \frac{R_1 + r_{d2}}{R_1 + r_{d1} + r_{d2}}\Delta V_{in} = \boxed{96.95 \text{ mV}}$$

(c)

$$I_{D1} = I_{D2} = \frac{V_{in} - 2V_{D,on}}{R_1} = 0.8 \text{ mA}$$
$$r_{d1} = r_{d2} = \frac{V_T}{I_{D1}} = 32.5\ \Omega$$
$$\Delta V_{out} = \frac{r_{d2}}{r_{d1} + R_1 + r_{d2}}\Delta V_{in} = \boxed{3.05 \text{ mV}}$$

(d)

$$I_{D2} = \frac{V_{in} - V_{D,on}}{R_1} - \frac{V_{D,on}}{R_2} = 1.2 \text{ mA}$$
$$r_{d2} = \frac{V_T}{I_{D2}} = 21.67\ \Omega$$
$$\Delta V_{out} = \frac{R_2 \parallel r_{d2}}{R_1 + R_2 \parallel r_{d2}}\Delta V_{in} = \boxed{2.10 \text{ mV}}$$

3.32 (a)

$$\Delta V_{out} = \Delta I_{in} R_1 = \boxed{100 \text{ mV}}$$

(b)

$$I_{D1} = I_{D2} = I_{in} = 3 \text{ mA}$$
$$r_{d1} = r_{d2} = \frac{V_T}{I_{D1}} = 8.67 \ \Omega$$
$$\Delta V_{out} = \Delta I_{in} \left( R_1 + r_{d2} \right) = \boxed{100.867 \text{ mV}}$$

(c)

$$I_{D1} = I_{D2} = I_{in} = 3 \text{ mA}$$
$$r_{d1} = r_{d2} = \frac{V_T}{I_{D1}} = 8.67 \ \Omega$$
$$\Delta V_{out} = \Delta I_{in} r_{d2} = \boxed{0.867 \text{ mV}}$$

(d)

$$I_{D2} = I_{in} - \frac{V_{D,on}}{R_2} = 2.6 \text{ mA}$$
$$r_{d2} = \frac{V_T}{I_{D2}} = 10 \ \Omega$$
$$\Delta V_{out} = \Delta I_{in} \left( R_2 \parallel r_{d2} \right) = \boxed{0.995 \text{ mV}}$$

(33) a) $i_1 = i_{in}$

$= 0.1\ mA$

b) $i_{r_1} = i_{in}$

$= 0.1\ mA$

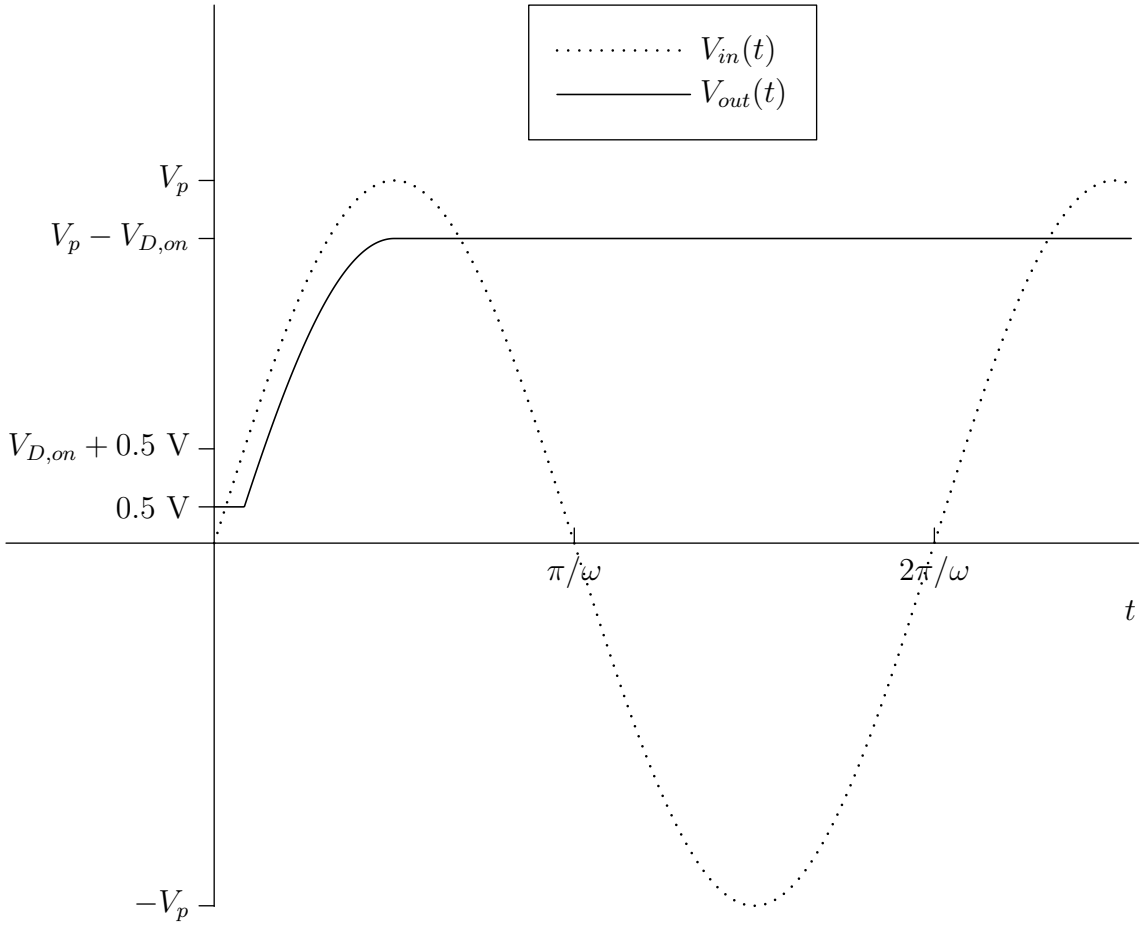
c) $i_{r_1} = i_{in}$

$= 0.1\ mA$

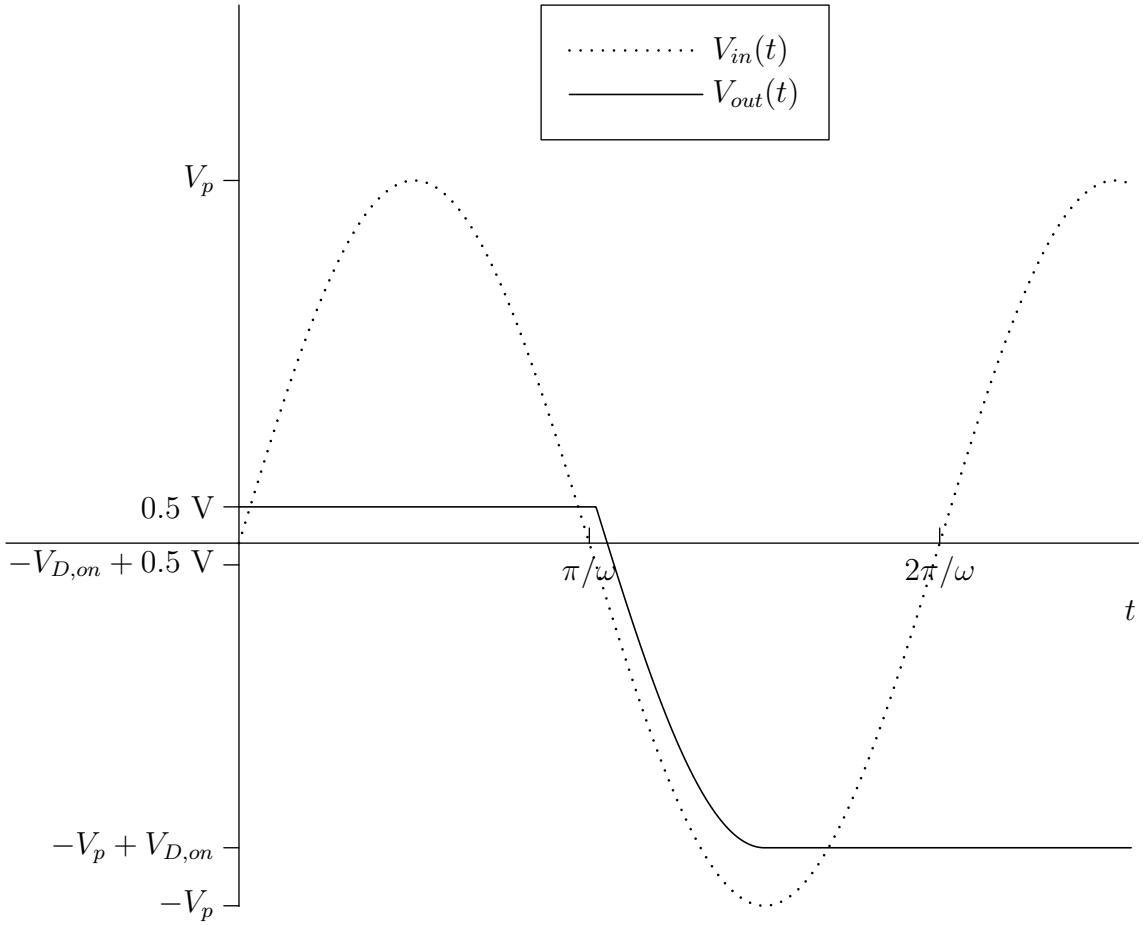
d) $i_{r_1} = i_{in}$

$= 0.1\ mA.$

3.34



3.35



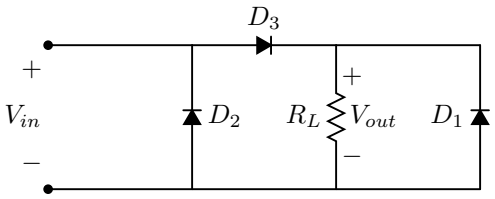
3.36

$$V_R \approx \frac{V_p - V_{D,on}}{R_L C_1 f_{in}}$$
$$V_p = 3.5 \text{ V}$$
$$R_L = 100 \ \Omega$$
$$C_1 = 1000 \ \mu\text{F}$$
$$f_{in} = 60 \text{ Hz}$$
$$V_R = \boxed{0.45 \text{ V}}$$

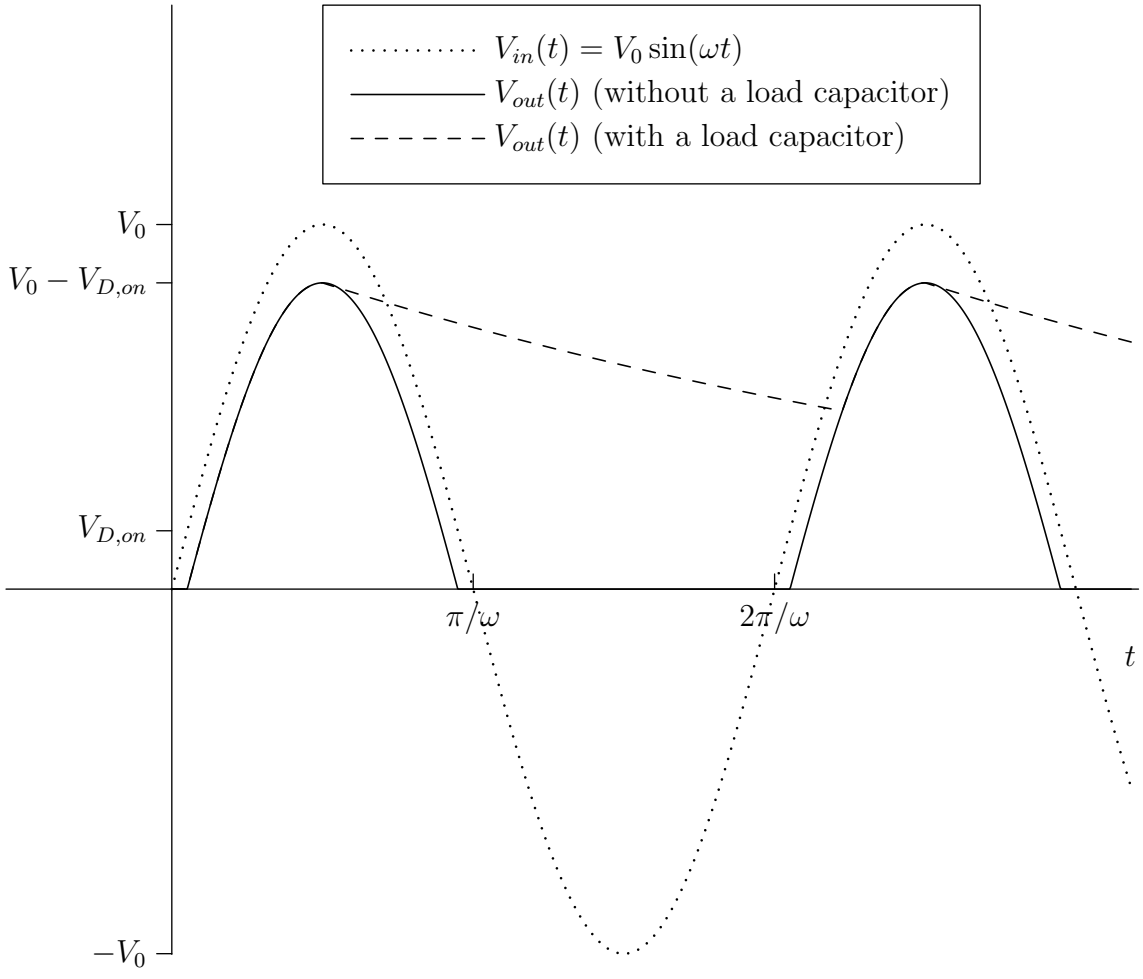
3.37

$$V_R = \frac{I_L}{C_1 f_{in}} \leq 300 \text{ mV}$$
$$f_{in} = 60 \text{ Hz}$$
$$I_L = 0.5 \text{ A}$$
$$C_1 \geq \frac{I_L}{(300 \text{ mV})\, f_{in}} = \boxed{27.78 \text{ mF}}$$

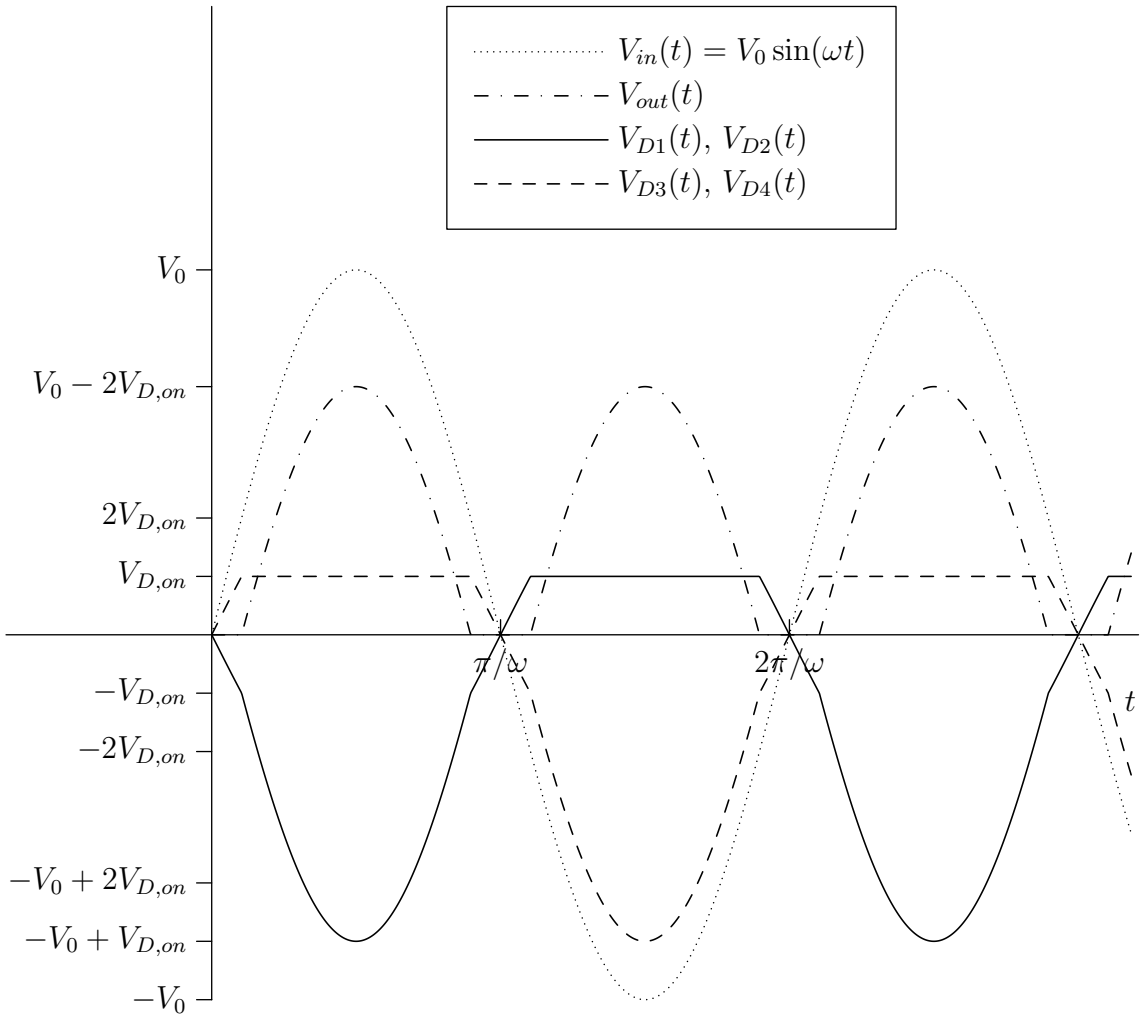
3.38 Shorting the input and output grounds of a full-wave rectifier shorts out the diode $D_4$ from Fig. 3.38(b). Redrawing the modified circuit, we have:




On the positive half-cycle, $D_3$ turns on and forms a half-wave rectifier along with $R_L$ (and $C_L$, if included). On the negative half-cycle, $D_2$ shorts the input (which could cause a dangerously large current to flow) and the output remains at zero. Thus, the circuit behaves like a half-wave recifier. The plots of $V_{out}(t)$ are shown below.



3.39 Note that the waveforms for $V_{D1}$ and $V_{D2}$ are identical, as are the waveforms for $V_{D3}$ and $V_{D4}$.



3.40 During the positive half-cycle, $D_2$ and $D_3$ will remain reverse-biased, causing $V_{out}$ to be zero as no current will flow through $R_L$. During the negative half-cycle, $D_1$ and $D_3$ will short the input (potentially causing damage to the devices), and once again, no current will flow through $R_L$ (even though $D_2$ will turn on, there will be no voltage drop across $R_L$). Thus, $V_{out}$ always remains at zero, and the circuit fails to act as a rectifier.

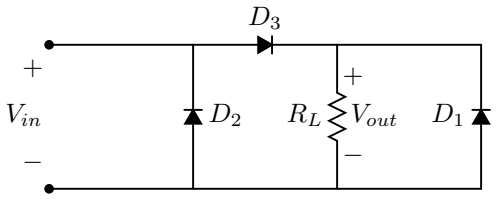
(41) Using Eq. (3.94),

$$V_R \approx \frac{1}{2} \cdot \frac{V_p - 2V_{p,on}}{R_L C_1 f_{in}}$$

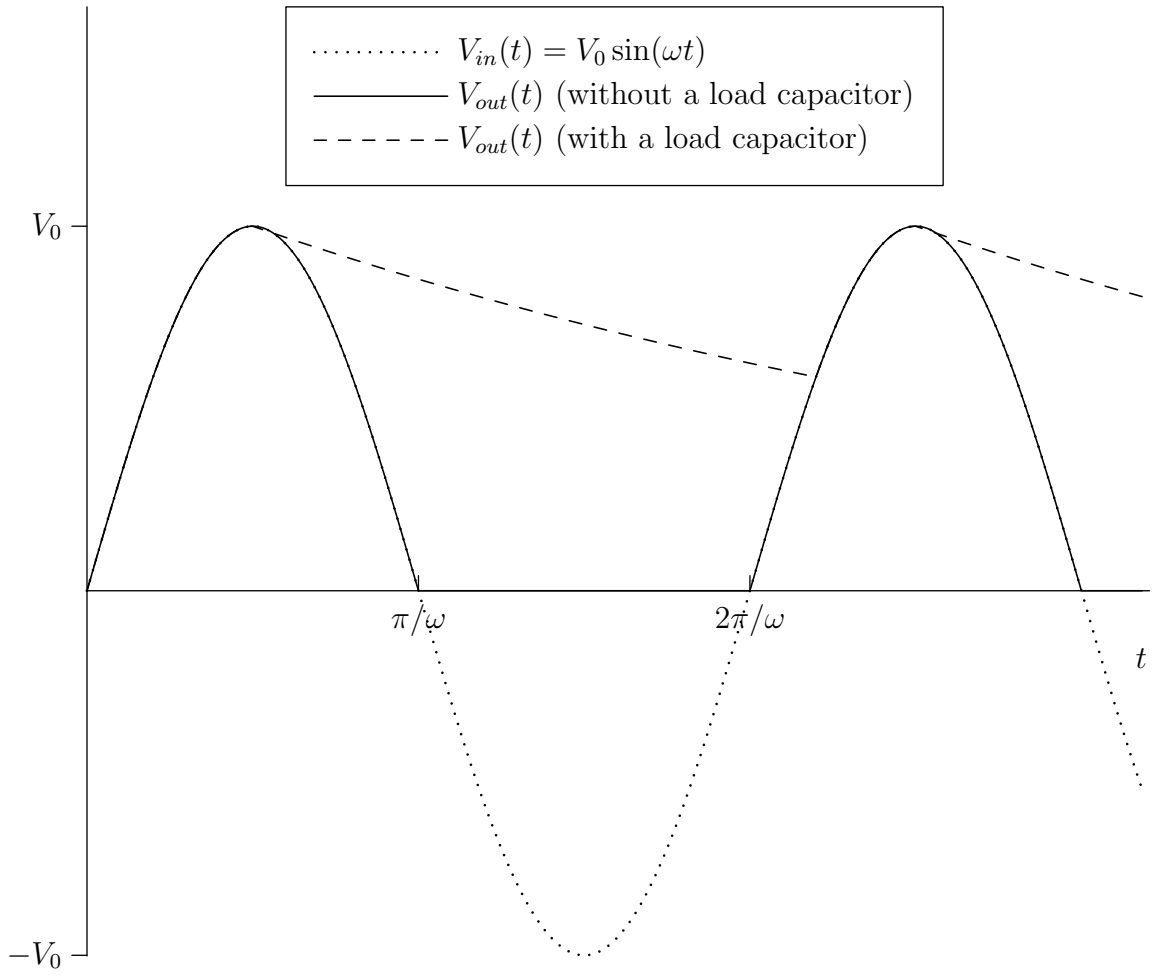
$$= \frac{1}{2} \cdot \frac{3 - 2 \times 0.8}{30 \times 1000 \times 10^{-6} \times 60}$$

$$= 0.389\,V$$

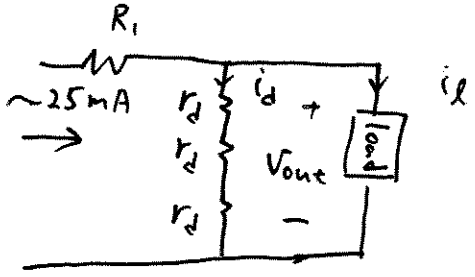
3.42 Shorting the negative terminals of $V_{in}$ and $V_{out}$ of a full-wave rectifier shorts out the diode $D_4$ from Fig. 3.38(b). Redrawing the modified circuit, we have:




On the positive half-cycle, $D_3$ turns on and forms a half-wave rectifier along with $R_L$ (and $C_L$, if included). On the negative half-cycle, $D_2$ shorts the input (which could cause a dangerously large current to flow) and the output remains at zero. Thus, the circuit behaves like a half-wave recifier. The plots of $V_{out}(t)$ are shown below.



(43) The circuit can be simplified as:




First, find $r_d$:

$$r_d = \frac{V_T}{I_D} \quad \text{(from eq. 3.60)}$$

$$= \frac{26\,mV}{5\,mA}$$

$$= 5.2\,\Omega$$

since $i_L = +1\,mA.$

$$i_d = -1\,mA.$$

∴ change in $V_{out}$,

ie. $V_{out} = (-1\,mA)(3 \times 5.2)$

$$= -15.6\,mV$$

3.44 (a) We know that when a capacitor is discharged by a constant current at a certain frequency, the ripple voltage is given by $\frac{I}{Cf_{in}}$, where $I$ is the constant current. In this case, we can calculate the current as approximately $\frac{V_p - 5V_{D,on}}{R_1}$ (since $V_p - 5V_{D,on}$ is the voltage drop across $R_1$, assuming $R_1$ carries a constant current). This gives us the following:

$$V_R \approx \frac{1}{2}\frac{V_p - 5V_{D,on}}{R_L C_1 f_{in}}$$
$$V_p = 5 \text{ V}$$
$$R_L = 1 \text{ k}\Omega$$
$$C_1 = 100 \text{ } \mu\text{F}$$
$$f_{in} = 60 \text{ Hz}$$
$$V_R = \boxed{166.67 \text{ mV}}$$

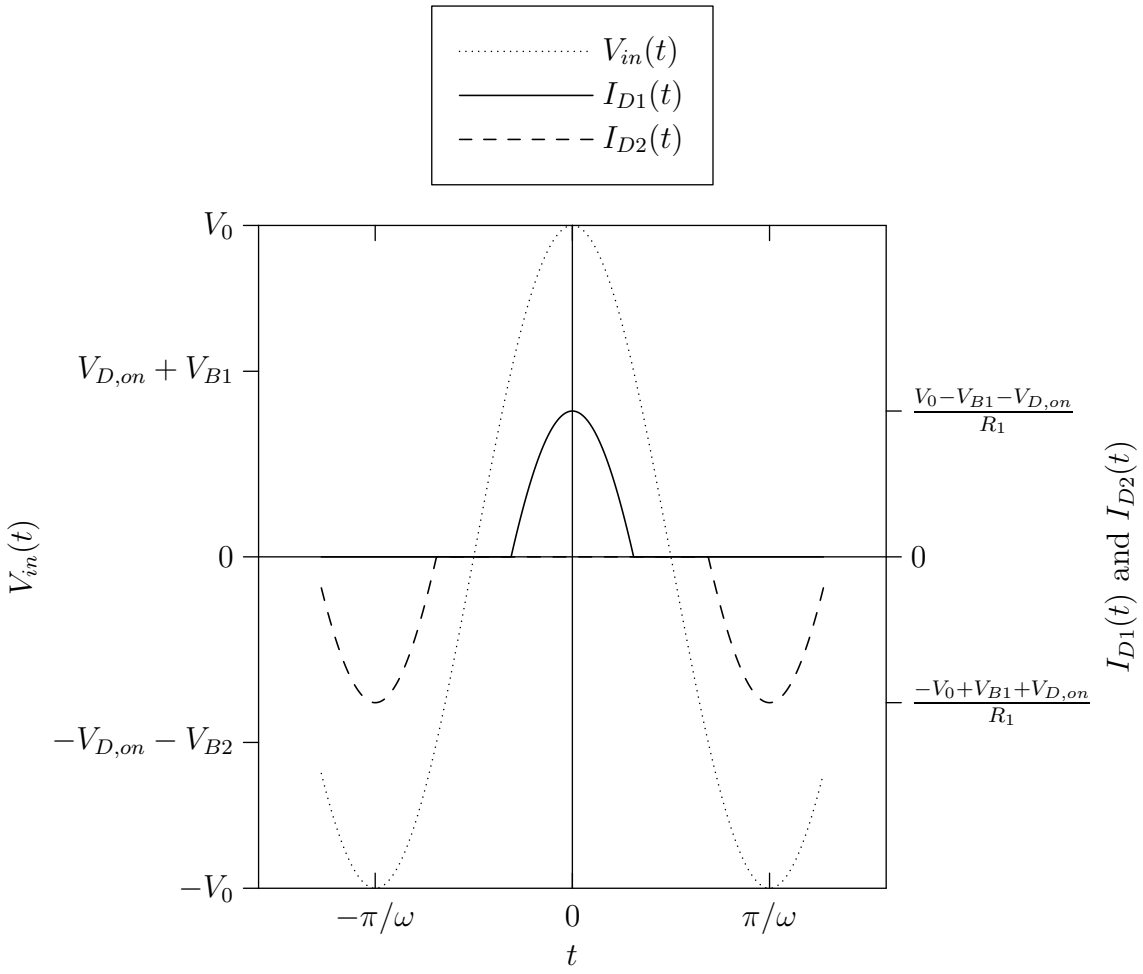
(b) The bias current through the diodes is the same as the bias current through $R_1$, which is $\frac{V_p - 5V_{D,on}}{R_1} = 1$ mA. Thus, we have:

$$r_d = \frac{V_T}{I_D} = 26 \ \Omega$$
$$V_{R,load} = \frac{3r_d}{R_1 + 3r_d} V_R = \boxed{12.06 \text{ mV}}$$

3.45

$$I_{D1} = \begin{cases} 0 & V_{in} < V_{D,on} + V_{B1} \\ \frac{V_{in} - V_{D,on} - V_{B1}}{R_1} & V_{in} > V_{D,on} + V_{B1} \end{cases}$$

$$I_{D2} = \begin{cases} \frac{V_{in} + V_{D,on} + V_{B2}}{R_1} & V_{in} < -V_{D,on} - V_{B2} \\ 0 & V_{in} > -V_{D,on} - V_{B2} \end{cases}$$



(46) With positive threshold = +2.2V,

$V_{B1} = 2.2 - 0.8$

$= +1.4V$ //

with negative threshold = −1.9V,

$-V_{B2} = -1.9 + 0.8$

$= -1.1V.$

$V_{B2} = 1.1V$ //

To meet the maximum current criterion,

Since $I_{R_1} = I_{D1}$ or $I_{D2}$,

$I_{D1}$ or $I_{D2}$ is at max when

$I_{R_1}$ is at max.

$I_{R_1}$ is at max when $|V_R|$ is max,

ie. $|V_R| = 5 - 1.9$

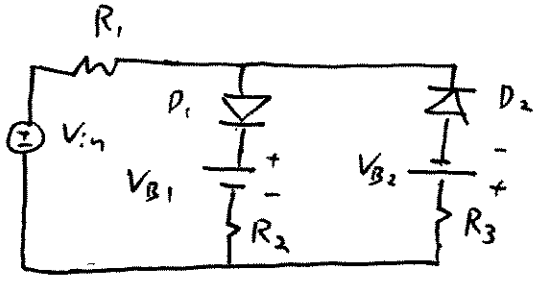
$= 3.1V.$

Since $I_{R_1} \leq 2\ mA.$

$R_1 \geq \frac{3.1}{2mA}$, ie. $R_1 \geq 1550\Omega$ //

(47)

The required circuit is:




Similar to Example 3.34,

$$V_{B1} = V_{B2} \simeq 2 - 0.8\ V$$

$$= 1.2\ V$$

To find $R_2$,

For $V_{in} > 2V$, $\frac{V_{out}}{V_{in}}$ has a slope of 0.5.

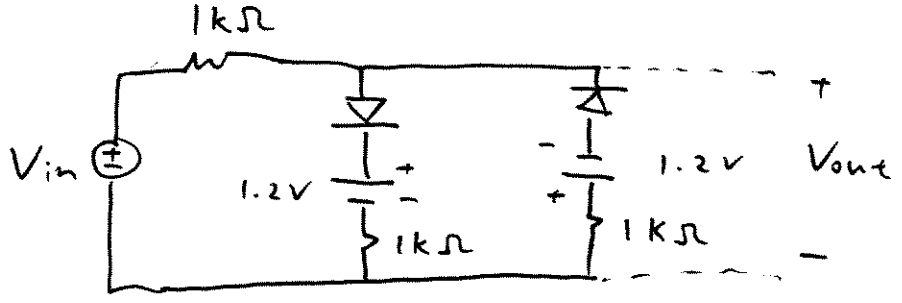
This implies $R_2 = R_1$

($R_1$ and $R_2$ forms a volt. divider).

Similarly, $R_3 = R_1$

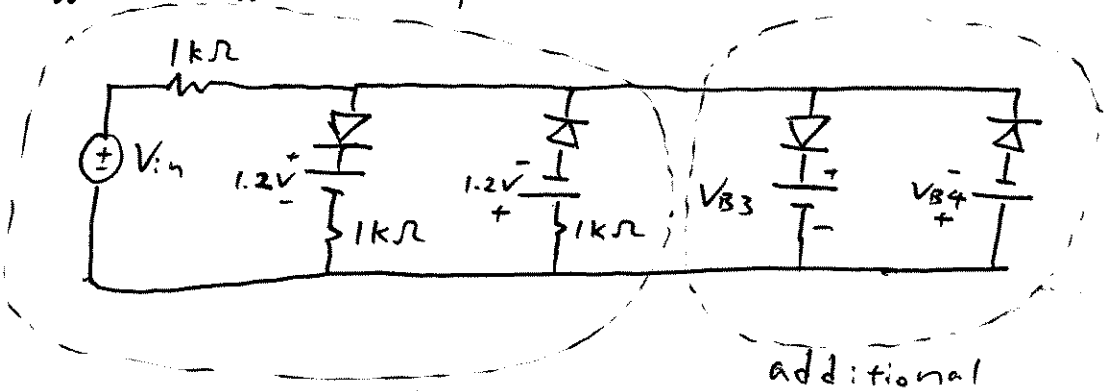
Thus, set $R_1 = R_2 = R_3 = 1\ k\Omega$.

The resulting circuit is:



(48) For $|V_{in}| < 4V$, the $V_{out} - V_{in}$ characteristic is similar to prob. (47).

To get voltage limiting characteristic for $V_{in} > 4V$, and $V_{in} < -4V$, we can shunt the circuit used in prob(47) with two antiparallel diodes as below:

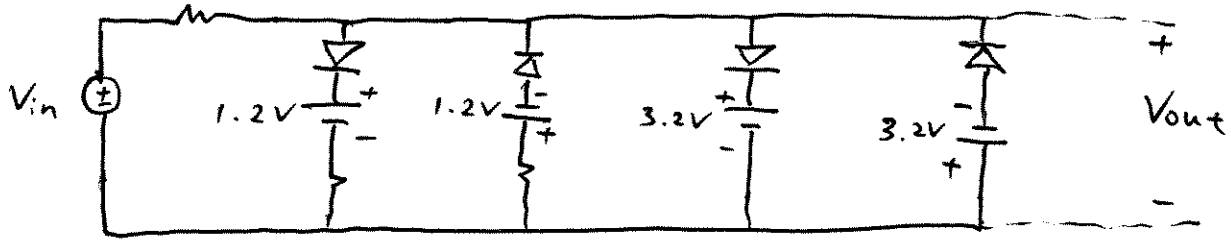



ckt used in prob. 47 — additional antiparallel diodes

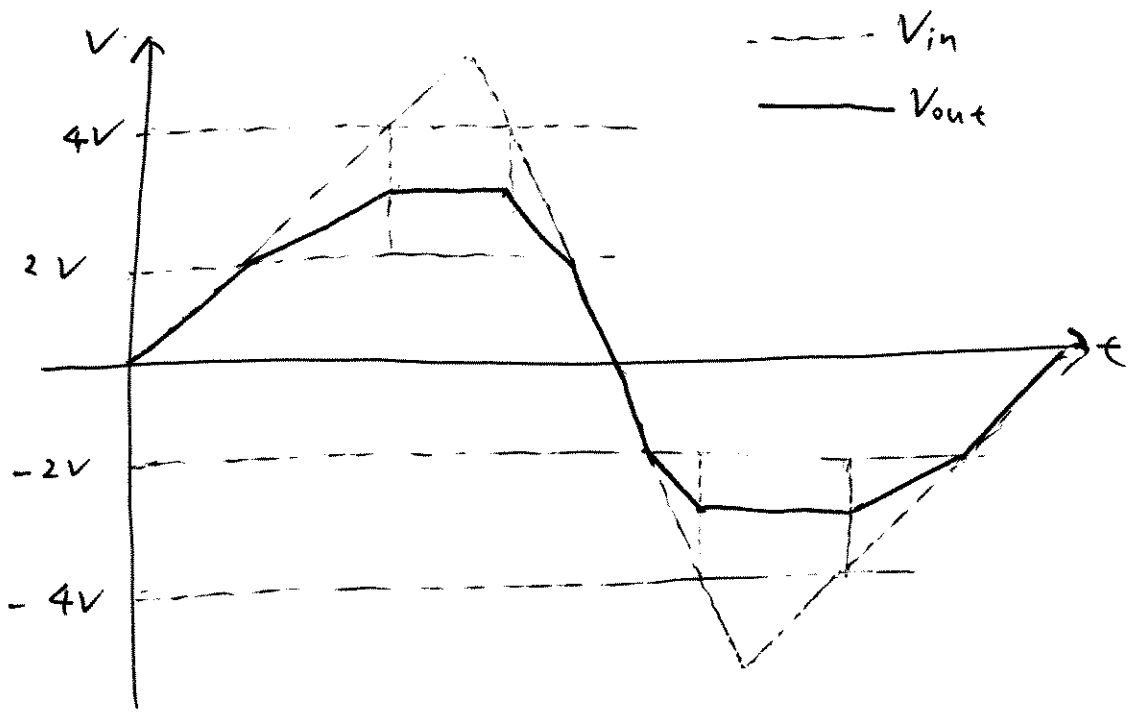
$$V_{B3} = V_{B4} = 4 - 0.8$$

$$= 3.2V$$

Resulting circuit is:

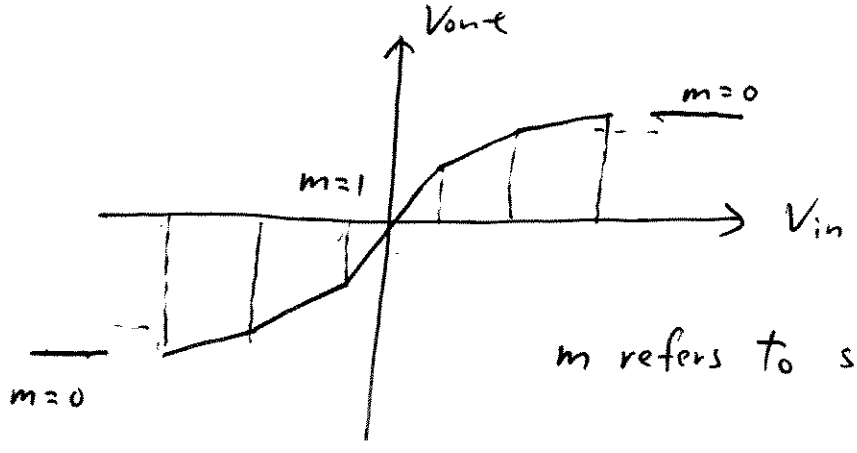


(49)




To get a better approximate of a sinusoid, the slope of the input-output characteristic should decrease more gradually from 1 to 0 through more sections.

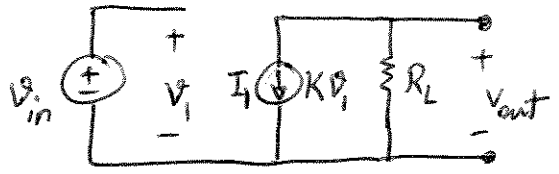
eg:




m refers to slope

# chapter 4

(4.1)




$K = 20\ mA/V$

$\left|\frac{V_{out}}{V_{in}}\right| = 15$ $\quad V_{in} = V_1$

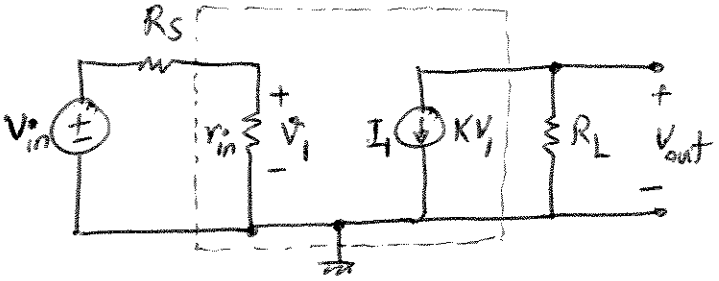
$$V_{out} = -I_1 R_L = -K R_L V_{in}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = -K R_L \Rightarrow \left|\frac{V_{out}}{V_{in}}\right| = K R_L$$

$$\Rightarrow K R_L = 15 \Rightarrow R_L = \frac{15}{20\ mA/V} = 750\ \Omega$$

$$\boxed{R_L = 750\ \Omega}$$

4.2




$\frac{V_{out}}{V_{in}} = ?$

$$V_1 = \frac{r_{in}}{r_{in} + R_S} V_{in}$$

$$I_1 = K V_1$$

$$V_{out} = -R_L I_1$$

$$\Rightarrow V_{out} = -K R_L V_1$$

$$\Rightarrow V_{out} = -K R_L \frac{r_{in}}{r_{in} + R_S} V_{in}$$

$$\Rightarrow \boxed{A_V = \frac{V_{out}}{V_{in}} = -K R_L \frac{r_{in}}{r_{in} + R_S}}$$

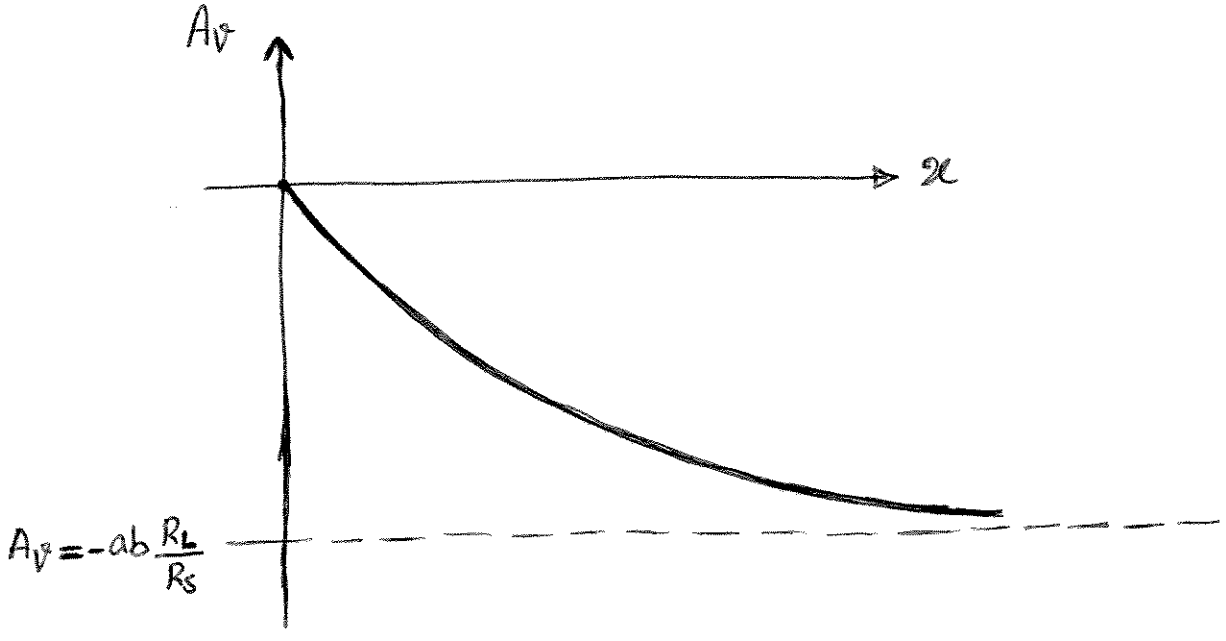
(4.3) From solution for problem 4.2,

$a>0$
$b>0$
$x \geq 0$

$$A_v = -K R_L \frac{r_{in}}{r_{in} + R_S}$$

$$\xrightarrow[K = bx]{r_{in} = a/x} A_v = -bx\, R_L \frac{a/x}{a/x + R_S} = -bR_L \frac{a}{\frac{a}{x} + R_S}$$

$$\Rightarrow \boxed{A_v = -bR_L \left( \frac{x}{1 + \frac{R_S}{a} x} \right)}$$

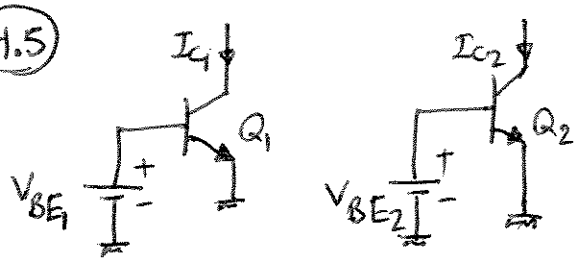


4.4 According to Equation (4.8), we have

$$\begin{aligned} I_C &= \frac{A_E q D_n n_i^2}{N_B W_B} \left( e^{V_{BE}/V_T} - 1 \right) \\ &\propto \frac{1}{W_B} \end{aligned}$$

We can see that if $W_B$ increases by a factor of two, then $\boxed{I_C \text{ decreases by a factor of two}}$.

(4.5)




$V_T = 26\,mV$

$I_{C_1} = I_{C_2}$

$V_{BE_1} - V_{BE_2} = 20\,mV$

$$I_C = \frac{A_E q D_n n_i^2}{N_E W_B}\left(e^{\frac{V_{BE}}{V_T}} - 1\right) \qquad \text{equation (4.8) page 136}$$

$$\Rightarrow I_C \simeq \frac{A_E q D_n n_i^2}{N_E W_B} e^{\frac{V_{BE}}{V_T}} \qquad A_E \equiv \text{Cross Section}$$

if $I_{C_1} = I_{C_2}$

$$\Rightarrow A_{E_1} \frac{q D_n n_i^2}{N_E W_B} e^{\frac{V_{BE_1}}{V_T}} = A_{E_2} \frac{q D_n n_i^2}{N_E W_B} e^{\frac{V_{BE_2}}{V_T}}$$

$$\Rightarrow \frac{A_{E_2}}{A_{E_1}} = \frac{e^{V_{BE_1}/V_T}}{e^{V_{BE_2}/V_T}}$$

$$\Rightarrow \frac{A_{E_2}}{A_{E_1}} = e^{(V_{BE_1} - V_{BE_2})/V_T} = e^{20\,mV/26\,mV}$$

$$\Rightarrow \boxed{\frac{A_{E_2}}{A_{E_1}} = e^{\frac{20}{26}} \simeq 2.16}$$

(6a) $I_X = 1^{mA} \Rightarrow I_{Q_1} = I_{Q_2} = 0.5^{mA}$

$$I_{Q_1} = I_{S_1} e^{\frac{V_{BE_1}}{V_T}} \Rightarrow 5\times10^{-4} = 3\times10^{-16}\, e^{\frac{V_B}{26^{mV}}}$$

$$\Rightarrow V_B = 26^{mV} \ln\left(\tfrac{5}{3}\times10^{12}\right) \Rightarrow \boxed{V_B \simeq 731.7^{mV}}$$

(6b) $I_Y = I_{S_3} e^{\frac{V_B}{V_T}}$

$$\Rightarrow I_{S_3} = I_Y e^{-\frac{V_B}{V_T}} = 2.5\times10^{-3} \times e^{-\frac{V_B}{26^{mV}}} = 2.5\times10^{-3} \times \frac{1}{\frac{5}{3}\times10^{12}}$$

$$\Rightarrow \boxed{I_{S_3} = 1.5\times10^{-15}\, A}$$

(7a) $I_X = I_1 + I_2$

$$\Rightarrow I_X = I_{S_1} e^{\frac{V_B}{V_T}} + I_{S_2} e^{\frac{V_B}{V_T}} \Rightarrow I_X = (I_{S_1} + I_{S_2}) e^{\frac{V_B}{V_T}}$$

$$\Rightarrow V_B = V_T \ln\left(\frac{I_X}{I_{S_1} + I_{S_2}}\right) \xrightarrow{I_{S_1} = 2I_{S_2}} \boxed{V_B = V_T \ln\left(\frac{I_X}{\frac{3}{2} I_{S_1}}\right)}$$

$$V_B = 26\times10^{-3} \ln\left(\frac{1.2\times10^{-3}}{\frac{3}{2}\times 5\times10^{-16}}\right) \Rightarrow \boxed{V_B \simeq 730.6\ mV}$$

(7b) Transistors at the edge of the active mode $\Rightarrow V_C = V_B$

applying KVL, we have:

$$V_{CC} = R_C I_X + V_B \Rightarrow \boxed{R_C = \frac{V_{CC} - V_B}{I_X}}$$

$$\Rightarrow R_C = \frac{2.5 - 0.73}{1.2\times10^{-3}}$$

$$\Rightarrow \boxed{R_C \simeq 1475\ \Omega}$$

(8a) Same as 7a,

$$\boxed{V_B \simeq 730.6\,mV}$$

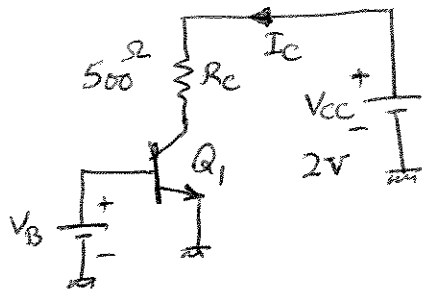
(8b) According to 7b,

$$R_C = \frac{V_{CC} - V_B}{I_X} = \frac{1.5 - 0.73}{1.2 \times 10^{-3}}$$

$$\Rightarrow \boxed{R_C \simeq 642\ \Omega}$$

⑨ $Q_1$ is at the edge of the active region $\Rightarrow V_C = V_B$

applying KVL, we have:




$$V_{CC} = R_C I_C + V_C$$

$$\xrightarrow{V_C = V_B} V_{CC} = R_C I_C + V_B$$

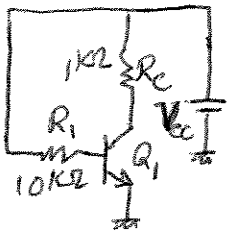
$$\Rightarrow V_{CC} = R_C I_S e^{V_B / V_T} + V_B$$

$$\Rightarrow 500\,\Omega \times 5 \times 10^{-16}\, e^{\frac{V_B}{26\,mV}} + V_B = 2V$$

Using numerical methods or simply, trial & error:

$$\boxed{V_B \simeq 760\ mV}$$

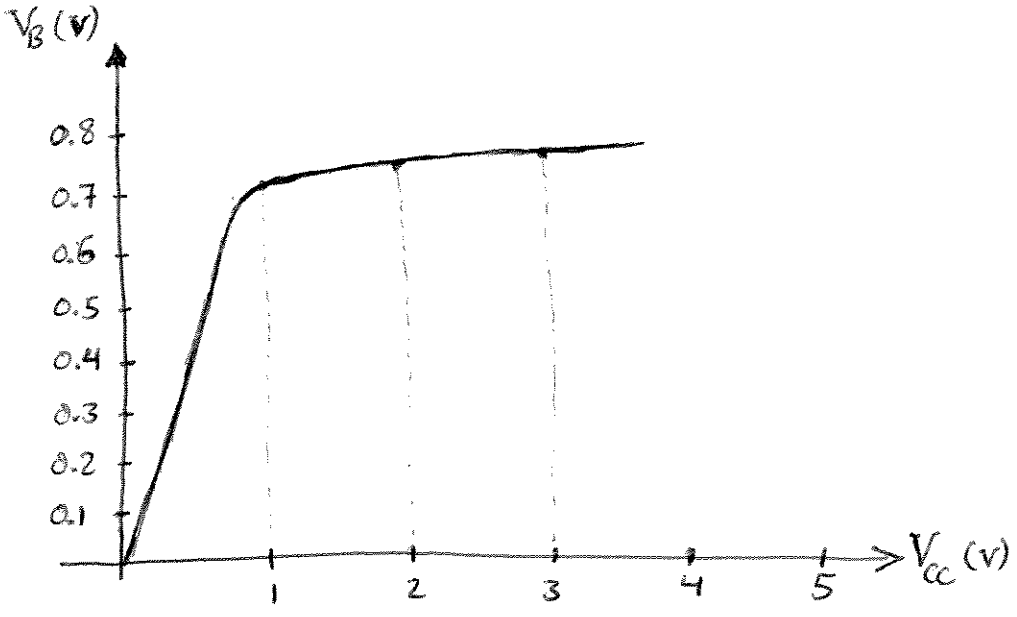
(10) $Q_1$ at the edge of saturation $\Rightarrow V_C = V_B$




Hence: $V_{CC} = R_C I_C + V_B$

$$\Rightarrow V_{CC} = R_C I_S e^{V_B/V_T} + V_B$$

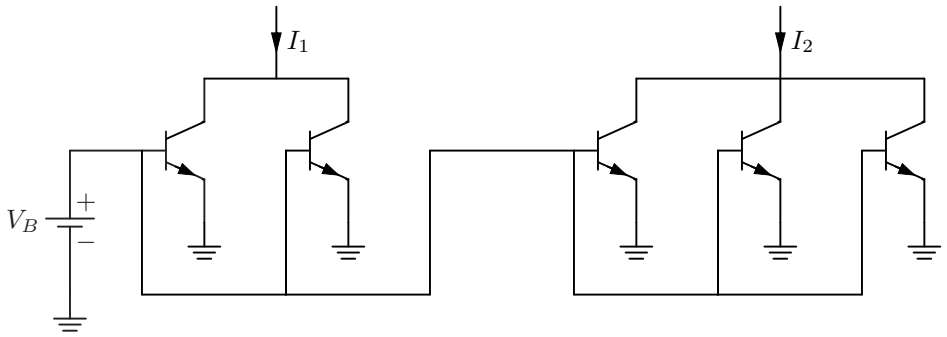
$I_S = 3\times10^{-16}$ A $\rightarrow$ $V_{CC} = 3\times10^{-13} e^{V_B/V_T} + V_B$ with $V_{CC} = 2$ V $\rightarrow$ $V_B \simeq 755$ mV



4.11

$$\begin{aligned}
V_{BE} &= 1.5\text{ V} - I_E(1\text{ k}\Omega) \\
&\approx 1.5\text{ V} - I_C(1\text{ k}\Omega)\text{ (assuming }\beta \gg 1) \\
&= V_T \ln\left(\frac{I_C}{I_S}\right) \\
I_C &= 775\ \mu\text{A} \\
V_X &\approx I_C(1\text{ k}\Omega) \\
&= \boxed{775\text{ mV}}
\end{aligned}$$

4.12 Since we have only integer multiples of a unit transistor, we need to find the largest number that divides both $I_1$ and $I_2$ evenly (i.e., we need to find the largest $x$ such that $I_1/x$ and $I_2/x$ are integers). This will ensure that we use the fewest transistors possible. In this case, it's easy to see that we should pick $x = 0.5$ mA, meaning each transistor should have 0.5 mA flowing through it. Therefore, $I_1$ should be made up of 1 mA/0.5 mA = 2 parallel transistors, and $I_2$ should be made up of 1.5 mA/0.5 mA = 3 parallel transistors. This is shown in the following circuit diagram.




Now we have to pick $V_B$ so that $I_C = 0.5$ mA for each transistor.

$$\begin{aligned} V_B &= V_T \ln\left(\frac{I_C}{I_S}\right) \\ &= (26\text{ mV}) \ln\left(\frac{5 \times 10^{-4}\text{ A}}{3 \times 10^{-16}\text{ A}}\right) \\ &= \boxed{732\text{ mV}} \end{aligned}$$

(13) Using the same technique as in problem 12, we have:

$$\frac{n_1}{I_1} = \frac{n_2}{I_2} = \frac{n_3}{I_3}$$

$$\Rightarrow \frac{n_1}{0.2} = \frac{n_2}{0.3} = \frac{n_3}{0.45} \Rightarrow \boxed{\frac{n_1}{4} = \frac{n_2}{6} = \frac{n_3}{9}}$$

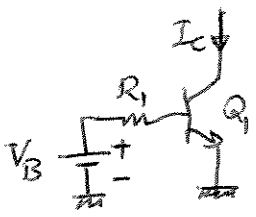
So lets choose $\boxed{\begin{cases} n_1 = 4 \\ n_2 = 6 \\ n_3 = 9 \end{cases}}$

Hence,

$$I_1 = n_1 I_S e^{V_B/V_T} \Rightarrow 0.2 \times 10^{-3} = 4 \times 3 \times 10^{-16} e^{\frac{V_B}{26mV}}$$

$$\Rightarrow \boxed{V_B \simeq 672 \text{ mV}}$$

(19) From KVL,




$$V_B = R_1 I_B + V_{BE_{Q_1}}$$

$$I_B = \frac{I_C}{\beta} = \frac{1\,mA}{100} \Rightarrow \boxed{I_B = 10^{-5}\,A}$$

$$V_{BE_{Q_1}} = V_T \ln\left(\frac{I_C}{I_S}\right) = 26\times10^{-3} \ln\left(\frac{10^{-3}}{7\times10^{-16}}\right)$$

$$\Rightarrow \boxed{V_{BE_{Q_1}} \simeq 727.7\,mV}$$

Therefore,

$$V_B = R_1 I_B + V_{BE_{Q_1}}$$

$$\simeq 10\,k\Omega \times 10^{-5}\,A + 728\times10^{-3}$$

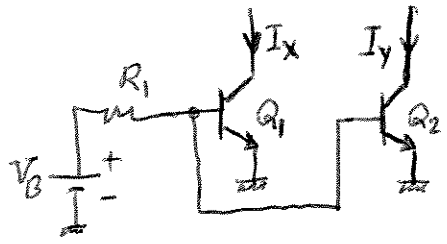
$$\Rightarrow V_B \simeq 0.1 + 0.728 \Rightarrow \boxed{V_B \simeq 0.828\ V}$$

4.15

$$\begin{aligned}\frac{V_B - V_{BE}}{R_1} &= I_B \\ &= \frac{I_C}{\beta} \\ I_C &= \frac{\beta}{R_1}\left[V_B - V_T \ln(I_C/I_S)\right] \\ I_C &= \boxed{786\ \mu\text{A}}\end{aligned}$$

(16)

$$\begin{cases} I_X = I_{S_1} \exp\left(\dfrac{V_{BE_1}}{V_T}\right) \\ I_Y = I_{S_2} \exp\left(\dfrac{V_{BE_2}}{V_T}\right) \\ V_{BE_1} = V_{BE_2} = V_{BE} \end{cases}$$




$$\Rightarrow \frac{I_X}{I_Y} = \frac{I_{S_1}}{I_{S_2}} = \frac{2\cancel{I_{S_2}}}{\cancel{I_{S_2}}} \Rightarrow \boxed{\frac{I_X}{I_Y} = 2}$$

$$\begin{cases} I_X = \beta_1 I_{B_1} \\ I_Y = \beta_2 I_{B_2} \\ \beta_1 = \beta_2 \end{cases}$$

$$\Rightarrow \boxed{\frac{I_{B_1}}{I_{B_2}} = \frac{I_X}{I_Y} = 2}$$

Applying KVL:

$$V_B = R_1 \left( I_{B_1} + I_{B_2} \right) + V_{BE}$$

$$V_{BE} = V_{BE_1} = V_T \ln\left(\frac{I_X}{I_{S_1}}\right) = 26^{mV} \ln\left(\frac{1^{mA}}{4\times10^{-16}}\right) \simeq 742\,mV$$

$$I_{B_1} = \frac{I_X}{\beta} \xrightarrow{\beta=100} I_{B_1} = \frac{1^{mA}}{100} = 10\mu A$$

$$\frac{I_{B_1}}{I_{B_2}} = 2 \longrightarrow I_{B_2} = \frac{I_{B_1}}{2} = \frac{10.\mu A}{2} \Rightarrow I_{B_2} = 5\mu A$$

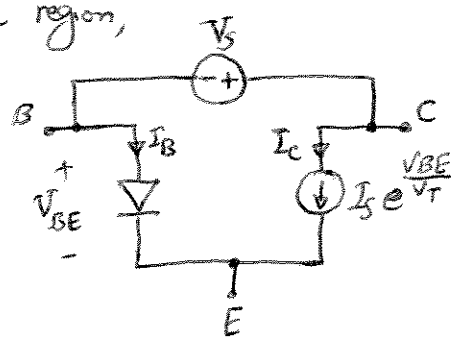
Hence: $V_B = 5\times10^{3}\,\Omega\times(10\mu A + 5\mu A) + 0.742^{V}$

$$= 0.075 + 0.742 \Longrightarrow \boxed{V_B \simeq 0.817^{V}}$$

4.17 First, note that $V_{BE1} = V_{BE2} = V_{BE}$.

$$\begin{aligned}
V_B &= (I_{B1} + I_{B2})R_1 + V_{BE} \\
&= \frac{R_1}{\beta}(I_X + I_Y) + V_T \ln(I_X/I_{S1}) \\
I_{S2} &= \frac{5}{3} I_{S1} \\
\Rightarrow I_Y &= \frac{5}{3} I_X \\
V_B &= \frac{8R_1}{3\beta} I_X + V_T \ln(I_X/I_{S1}) \\
I_X &= \boxed{509\ \mu\text{A}} \\
I_Y &= \boxed{848\ \mu\text{A}}
\end{aligned}$$

(18) Since Transistor is in Forward active region,




No change across $V_{BE}$

$\Downarrow$

No change in $I_B$

$\Downarrow$

No change in $I_C$

(19) $g_m = \frac{I_C}{V_T}$

$$\Rightarrow g_m = \frac{I_S \exp\left(\frac{V_{BE}}{V_T}\right)}{V_T} \Rightarrow \boxed{V_{BE} = V_T \ln\left(\frac{g_m V_T}{I_S}\right)}$$

$$\xrightarrow[g_m = \frac{1}{13\Omega}]{I_S = 6\times10^{-16}\,A} V_{BE} = 26\,mV \cdot \ln\left(\frac{\frac{1}{13\Omega} \times 26\times10^{-3}}{6\times10^{-16}}\right)$$

$$\Rightarrow \boxed{V_{BE} \simeq 750\ mV}$$

(20) $g_m = \frac{I_C}{V_T}$

$$\Delta g_m = \frac{\Delta I_C}{V_T} = \frac{1}{V_T}\Delta\left(I_S e^{V_{BE}/V_T}\right) \simeq \frac{I_S}{V_T^2} e^{\frac{V_{BE}}{V_T}} \Delta V_{BE}$$

$$\Rightarrow \boxed{\Delta g_m \simeq \frac{I_C}{V_T^2} \Delta V_{BE}}$$

$$\Rightarrow \Delta g_m \simeq \frac{g_m}{V_T} \Delta V_{BE}$$

$$\Rightarrow \boxed{\frac{\Delta g_m}{g_m} \simeq \frac{1}{V_T} \Delta V_{BE}}$$

$$\left.\frac{\Delta g_m}{g_m}\right|_{I_C = 1mA} \overset{max}{=\!=} 0.1 \quad \Rightarrow \Delta V_{BE_{max}} = 0.1\, V_T$$

$$\Rightarrow \boxed{\Delta V_{BE} \leqslant 2.6\ mV}$$

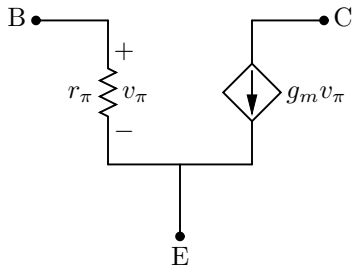
4.21 (a)

$$V_{BE} = \boxed{0.8\text{ V}}$$
$$I_C = I_S e^{V_{BE}/V_T}$$
$$= \boxed{18.5\text{ mA}}$$
$$V_{CE} = V_{CC} - I_C R_C$$
$$= \boxed{1.58\text{ V}}$$

$Q_1$ is operating in forward active. Its small-signal parameters are

$$g_m = I_C/V_T = \boxed{710\text{ mS}}$$
$$r_\pi = \beta/g_m = \boxed{141\ \Omega}$$
$$r_o = \boxed{\infty}$$

The small-signal model is shown below.



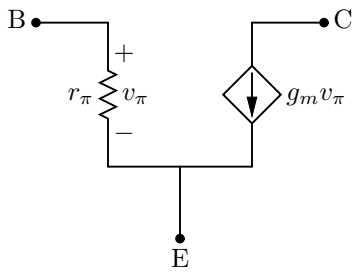

(b)

$$I_B = 10\ \mu\text{A}$$
$$I_C = \beta I_B = \boxed{1\text{ mA}}$$
$$V_{BE} = V_T \ln(I_C/I_S) = \boxed{724\text{ mV}}$$
$$V_{CE} = V_{CC} - I_C R_C$$
$$= \boxed{1.5\text{ V}}$$

$Q_1$ is operating in forward active. Its small-signal parameters are

$$g_m = I_C/V_T = \boxed{38.5\text{ mS}}$$
$$r_\pi = \beta/g_m = \boxed{2.6\text{ k}\Omega}$$
$$r_o = \boxed{\infty}$$

The small-signal model is shown below.



(c)

$$I_E = \frac{V_{CC} - V_{BE}}{R_C} = \frac{1+\beta}{\beta} I_C$$

$$I_C = \frac{\beta}{1+\beta} \frac{V_{CC} - V_T \ln(I_C/I_S)}{R_C}$$

$$I_C = \boxed{1.74 \text{ mA}}$$

$$V_{BE} = V_T \ln(I_C/I_S) = \boxed{739 \text{ mV}}$$

$$V_{CE} = V_{BE} = \boxed{739 \text{ mV}}$$

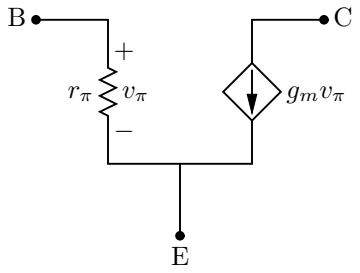
$Q_1$ is operating in forward active. Its small-signal parameters are

$$g_m = I_C/V_T = \boxed{38.5 \text{ mS}}$$

$$r_\pi = \beta/g_m = \boxed{2.6 \text{ k}\Omega}$$

$$r_o = \boxed{\infty}$$

The small-signal model is shown below.



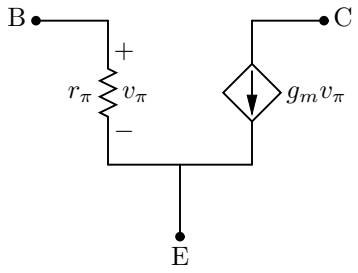
4.22 (a)

$$I_B = 10\ \mu\text{A}$$
$$I_C = \beta I_B = \boxed{1\ \text{mA}}$$
$$V_{BE} = V_T \ln(I_C/I_S) = \boxed{739\ \text{mV}}$$
$$V_{CE} = V_{CC} - I_E(1\ \text{k}\Omega)$$
$$= V_{CC} - \frac{1+\beta}{\beta}(1\ \text{k}\Omega)$$
$$= \boxed{0.99\ \text{V}}$$

$Q_1$ is operating in forward active. Its small-signal parameters are

$$g_m = I_C/V_T = \boxed{38.5\ \text{mS}}$$
$$r_\pi = \beta/g_m = \boxed{2.6\ \text{k}\Omega}$$
$$r_o = \boxed{\infty}$$

The small-signal model is shown below.



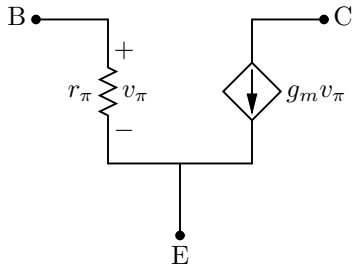

(b)

$$I_E = \frac{V_{CC} - V_{BE}}{1\ \text{k}\Omega} = \frac{1+\beta}{\beta} I_C$$
$$I_C = \frac{\beta}{1+\beta}\frac{V_{CC} - V_T \ln(I_C/I_S)}{1\ \text{k}\Omega}$$
$$I_C = \boxed{1.26\ \text{mA}}$$
$$V_{BE} = V_T \ln(I_C/I_S) = \boxed{730\ \text{mV}}$$
$$V_{CE} = V_{BE} = \boxed{730\ \text{mV}}$$

$Q_1$ is operating in forward active. Its small-signal parameters are

$$g_m = I_C/V_T = \boxed{48.3\ \text{mS}}$$
$$r_\pi = \beta/g_m = \boxed{2.07\ \text{k}\Omega}$$
$$r_o = \boxed{\infty}$$

The small-signal model is shown below.



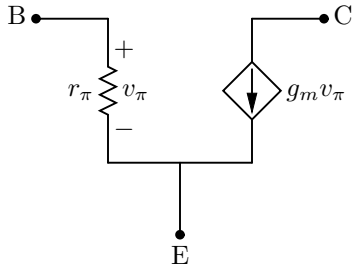

(c)

$$I_E = 1 \text{ mA}$$
$$I_C = \frac{\beta}{1+\beta} I_E = \boxed{0.99 \text{ mA}}$$
$$V_{BE} = V_T \ln(I_C/I_S) = \boxed{724 \text{ mV}}$$
$$V_{CE} = V_{BE} = \boxed{724 \text{ mV}}$$

$Q_1$ is operating in forward active. Its small-signal parameters are

$$g_m = I_C/V_T = \boxed{38.1 \text{ mS}}$$
$$r_\pi = \beta/g_m = \boxed{2.63 \text{ k}\Omega}$$
$$r_o = \boxed{\infty}$$

The small-signal model is shown below.



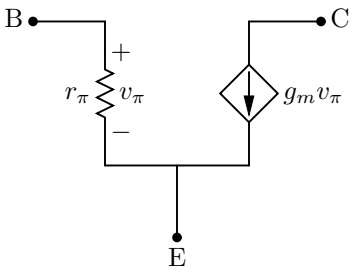

(d)

$$I_E = 1 \text{ mA}$$
$$I_C = \frac{\beta}{1+\beta} I_E = \boxed{0.99 \text{ mA}}$$
$$V_{BE} = V_T \ln(I_C/I_S) = \boxed{724 \text{ mV}}$$
$$V_{CE} = V_{BE} = \boxed{724 \text{ mV}}$$

$Q_1$ is operating in forward active. Its small-signal parameters are

$$g_m = I_C/V_T = \boxed{38.1 \text{ mS}}$$
$$r_\pi = \beta/g_m = \boxed{2.63 \text{ k}\Omega}$$
$$r_o = \boxed{\infty}$$

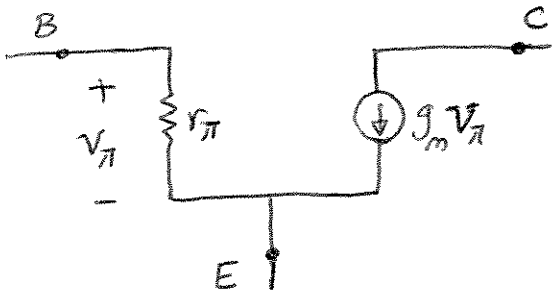
The small-signal model is shown below.


B
C
+
$r_\pi$
$v_\pi$
−
$g_m v_\pi$
E

(23) $I_C = I_S \exp\left(\frac{V_{BE}}{nV_T}\right)$ $\qquad I_C = \beta I_B$

$$g_m = \frac{\partial I_C}{\partial V_{BE}} = \frac{1}{nV_T} I_S \exp\left(\frac{V_{BE}}{nV_T}\right) = \frac{I_C}{nV_T}$$

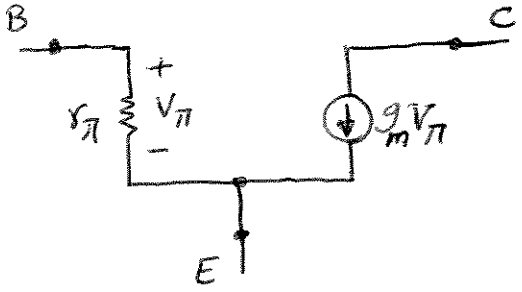
$$r_\pi = \frac{\partial V_{BE}}{\partial I_B} = \frac{\partial V_{BE}}{\frac{1}{\beta}\partial I_C} = \frac{\beta}{g_m} = \frac{n\beta V_T}{I_C}$$



(24) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)$ , $I_C = \alpha I_B^2 \Rightarrow \frac{\partial I_B}{\partial I_C} = \frac{1}{2\sqrt{\alpha I_C}}$

$$g_m = \frac{\partial I_C}{\partial V_{BE}} = \frac{I_S}{V_T} \exp\left(\frac{V_{BE}}{V_T}\right) = \boxed{\frac{I_C}{V_T}}$$

$$r_\pi = \frac{\partial V_{BE}}{\partial I_B} = \frac{\partial V_{BE}}{\frac{1}{2\sqrt{\alpha I_C}} \partial I_C} = \frac{2\sqrt{\alpha I_C}}{g_m} = \frac{2\sqrt{\alpha I_C}}{I_C / V_T} = \boxed{2V_T \sqrt{\frac{\alpha}{I_C}}}$$



(25) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]$ $V_{BE}$ is Constant

$$\Delta I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\frac{1}{V_A}\Delta V_{CE}$$

$$\Rightarrow \frac{\Delta I_C}{I_C} = \frac{I_S \exp\left(\frac{V_{BE}}{V_T}\right)\frac{1}{V_A}\cdot \Delta V_{CE}}{I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]} = \frac{\Delta V_{CE}}{V_A + V_{CE}}$$

$$\frac{\Delta I_C}{I_{C_{min}}} < 0.05 \Rightarrow \frac{\Delta V_{CE}}{V_A + V_{CE_{min}}} < 0.05$$

$$\Rightarrow 20\,\Delta V_{CE} < V_A + V_{CE_{min}}$$

$$\left.\begin{array}{l} \Delta V_{CE} = 2\,V \\ V_{CE_{min}} = 1\,V \end{array}\right\} \Rightarrow 40 < V_A + 1 \Rightarrow \boxed{V_A > 39\,V}$$

(26)

a) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right) = 5\times10^{-17} \exp\left(\frac{800\,mV}{26\,mV}\right) \simeq \boxed{1.15\,mA}$

$V_X = V_{CC} - R_C I_C = 2.5\,V - 1\,k\Omega \times 1.15\,mA$

$\boxed{V_X = 1.35\,V}$ Transistor is in Forward Active Region

b) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]$

$\Rightarrow I_C = 5\times10^{-17} \exp\left(\frac{800}{26}\right)\left[1 + \frac{V_X}{5\,V}\right]$ equation 1

Also we know: $V_X = V_{CC} - R_C I_C \Rightarrow I_C = \frac{V_{CC} - V_X}{R_C}$ equation 2

equations 1, 2 $\Rightarrow \frac{V_{CC} - V_X}{R_C} = 5\times10^{-17} \exp\left(\frac{800}{26}\right)\left[1 + \frac{V_X}{5}\right]$

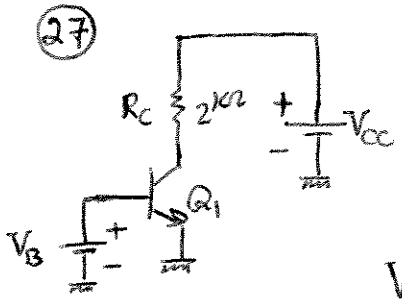
$\Rightarrow V_X + 5\times10^{-14} \exp\left(\frac{800}{26}\right)\left[1 + \frac{V_X}{5}\right] = 2.5$

$\Rightarrow 1.2306\, V_X \simeq 1.347$

$\Rightarrow \boxed{V_X \simeq 1.095\ V} \xRightarrow{\text{equation 1}} \boxed{I_C \simeq 1.406\,mA}$

Transistor is in Forward Active Region

(27)




$I_S = 8 \times 10^{-17}\,A \qquad V_A = 5^V$

Applying KVL:

$$V_{CC} = R_C I_C + V_{CE}$$

$$\Rightarrow V_{CC} = R_C I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right] + V_{CE}$$

$V_{BE}$ Constant $\Rightarrow$

$$\boxed{\Delta V_{CC} = \left[R_C I_S \exp\left(\frac{V_{BE}}{V_T}\right)\frac{1}{V_A} + 1\right].\Delta V_{CE}} \qquad \text{equation } \underline{1}$$

$$I_C = I_S e^{\frac{V_{BE}}{V_T}}\left[1 + \frac{V_{CE}}{V_A}\right] \Rightarrow \Delta I_C = I_S e^{\frac{V_{BE}}{V_T}} \times \frac{1}{V_A}\,\Delta V_{CE}$$

$$\Rightarrow \boxed{\Delta V_{CE} = \frac{1}{I_S e^{\frac{V_{BE}}{V_T}} \times \frac{1}{V_A}} . \Delta I_C} \qquad \text{equation } \underline{2}$$

equations 1, 2 $\Rightarrow$

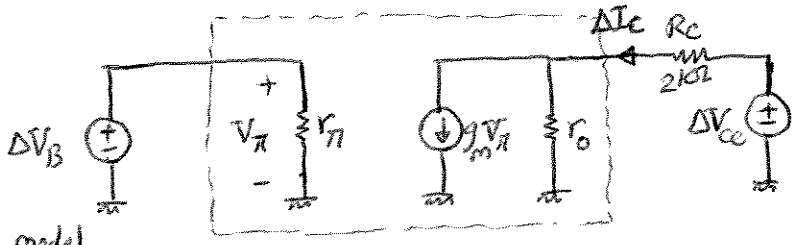
$$\Delta I_C = \frac{I_S e^{\frac{V_{BE}}{V_T}} \times \frac{1}{V_A}}{1 + R_C I_S e^{\frac{V_{BE}}{V_T}} \times \frac{1}{V_A}} . \Delta V_{CC}$$

$$\Rightarrow \boxed{\Delta I_C = \frac{I_S \exp\left(\frac{V_{BE}}{V_T}\right)}{V_A + R_C I_S \exp\left(\frac{V_{BE}}{V_T}\right)} . \Delta V_{CC}} = \frac{1}{r_o + R_C} . \Delta V_{CC}$$

Could also be obtained using small signal model

$$\Rightarrow \Delta I_C = \frac{2.31 \times 10^{-4}}{5 + 0.4613} \times 0.5 \Rightarrow \Delta I_C \simeq 0.021\,mA$$

(28)




We use small signal model,
Assuming that the required $\Delta V_B$ is small enough.

Applying Superposition,

$$\Delta I_C = \left(\frac{1}{r_o + R_C}\right)\Delta V_{CC} + \left(\frac{g_m r_o}{r_o + R_C}\right)\Delta V_B$$

$$\Delta I_C = 0 \Rightarrow \boxed{\Delta V_B = -\frac{1}{g_m r_o}\Delta V_{CC}}$$

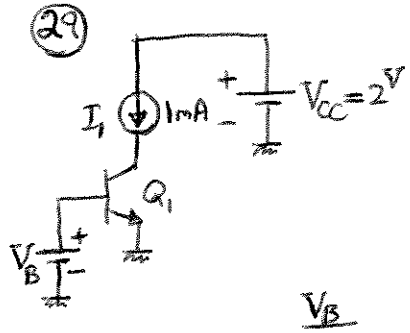
$$\Delta V_B = -\frac{1}{\frac{I_C}{V_T}\cdot\frac{V_A}{I_C}}\Delta V_{CC} \Rightarrow \Delta V_B = -\frac{V_T}{V_A}\Delta V_{CC}$$

$$\Rightarrow \Delta V_B = -\frac{26\times10^{-3}}{5}\times(3-2.5)$$

$$\Rightarrow \boxed{\Delta V_B = -2.6\ mV}$$

which is small enough for small signal model

(29)




$I_S = 3\times10^{-17}\,A$

a) $I_C = I_S e^{\frac{V_B}{V_T}} \Rightarrow V_B = V_T \ln\left(\frac{I_C}{I_S}\right) = 26^{mV} \ln\left(\frac{10^{-3}}{3\times10^{-17}}\right)$

$\Rightarrow \boxed{V_B \simeq 809.6\ mV}$

b) $I_C = I_S e^{\frac{V_B}{V_T}}\left(1+\frac{V_{CE}}{V_A}\right)$

$10^{-3} = 3\times10^{-17} e^{\frac{V_B}{V_T}}\left(1+\frac{1.5}{5}\right) \Rightarrow e^{\frac{V_B}{V_T}} = \frac{10^{14}}{3.9}$

$\Rightarrow V_B = 26^{mV} \ln\left(\frac{10^{14}}{3.9}\right) \Rightarrow \boxed{V_B \simeq 802.8\ mV}$

(30) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]$

$$r_o^{-1} = \frac{dI_C}{dV_{CE}} = I_S \exp\left(\frac{V_{BE}}{V_T}\right) \cdot \frac{1}{V_A} \simeq \frac{I_C}{V_A} \Rightarrow r_o \simeq \frac{V_A}{I_C}$$

$$r_o > 10^{k\Omega} \Rightarrow \frac{V_A}{I_C} > 10^{k\Omega}$$

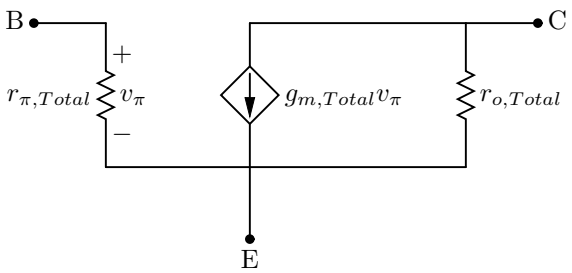
$$\Rightarrow V_A > 10^{k\Omega} \times 2^{mA}$$

$$\Rightarrow \boxed{V_A > 20\,V}$$

4.31

$$
\begin{aligned}
I_C &= I_S e^{V_{BE}/V_T}\left(1+\frac{V_{CE}}{V_A}\right)\\
I_{C,Total} &= nI_C\\
&= nI_S e^{V_{BE}/V_T}\left(1+\frac{V_{CE}}{V_A}\right)\\
g_{m,Total} &= \frac{\partial I_C}{\partial V_{BE}}\\
&= n\frac{I_S}{V_T}e^{V_{BE}/V_T}\\
&\approx n\frac{I_C}{V_T}\\
&= ng_m\\
&= \boxed{n\times 0.4435\ \text{S}}\\
I_{B,Total} &= \frac{1}{\beta}I_{C,Total}\\
r_{\pi,Total} &= \left(\frac{\partial I_{B,Total}}{\partial V_{BE}}\right)^{-1}\\
&\approx \left(\frac{I_{C,Total}}{\beta V_T}\right)^{-1}\\
&= \left(\frac{nI_C}{\beta V_T}\right)^{-1}\\
&= \frac{r_\pi}{n}\\
&= \boxed{\frac{225.5\ \Omega}{n}}\ (\text{assuming } \beta = 100)\\
r_{o,Total} &= \left(\frac{\partial I_{C,Total}}{\partial V_{CE}}\right)^{-1}\\
&\approx \left(\frac{I_{C,Total}}{V_A}\right)^{-1}\\
&= \frac{V_A}{nI_C}\\
&= \frac{r_o}{n}\\
&= \boxed{\frac{693.8\ \Omega}{n}}
\end{aligned}
$$

The small-signal model is shown below.



4.32 (a)

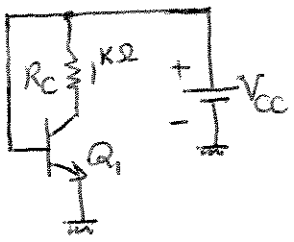
$$\begin{aligned} V_{BE} &= V_{CE} \text{ (for } Q_1 \text{ to operate at the edge of saturation)} \\ V_T \ln(I_C/I_S) &= V_{CC} - I_C R_C \\ I_C &= 885.7\ \mu\text{A} \\ V_B = V_{BE} &= \boxed{728.5\ \text{mV}} \end{aligned}$$

(b) Let $I_C'$, $V_B'$, $V_{BE}'$, and $V_{CE}'$ correspond to the values where the collector-base junction is forward biased by 200 mV.

$$\begin{aligned} V_{BE}' &= V_{CE}' + 200\ \text{mV} \\ V_T \ln(I_C'/I_S) &= V_{CC} - I_C' R_C + 200\ \text{mV} \\ I_C' &= 984.4\ \mu\text{A} \\ V_B' &= 731.3\ \text{mV} \end{aligned}$$

Thus, $V_B$ can increase by $V_B' - V_B = \boxed{2.8\ \text{mV}}$ if we allow soft saturation.

(33)




$I_S = 7\times10^{-16}A$ , $V_A = \infty$ $\Rightarrow$ $r_o = \infty$

Applying KVL,

$$V_{CC} = R_C I_C + V_{CE} \xrightarrow{V_{CE} = V_{BE} - 0.2^V} R_C I_C + V_{BE} - 0.2^V = V_{CC}$$

$$\Rightarrow R_C I_S e^{\frac{V_{BE}}{V_T}} + V_{BE} - 0.2^V = V_{CC}$$

$$\xRightarrow{V_{BE} = V_{CC}} R_C I_S e^{\frac{V_{CC}}{V_T}} + V_{CC} - 0.2 = V_{CC}$$

$$\Rightarrow R_C I_S e^{V_{CC}/V_T} = 0.2^V$$

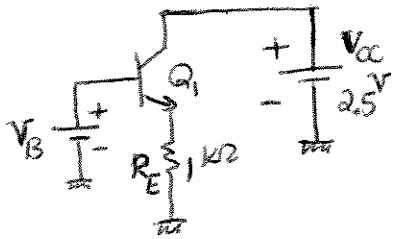
$$\Rightarrow 1^{K\Omega} \times 7\times10^{-16}\, e^{V_{CC}/26^{mV}} = 0.2^V$$

$$\Rightarrow \boxed{V_{CC} \simeq 686\ mV}$$

4.34

$$
\begin{aligned}
V_{BE} &= V_{CC} - I_B R_B \\
V_T \ln(I_C/I_S) &= V_{CC} - I_C R_B/\beta \\
I_C &= 1.67 \text{ mA} \\
V_{BC} &= V_{CC} - I_B R_B - (V_{CC} - I_C R_C) \\
&< 200 \text{ mV} \\
I_C R_C - I_B R_B &< 200 \text{ mV} \\
R_C &< \frac{200 \text{ mV} + I_B R_B}{I_C} \\
&= \frac{200 \text{ mV} + I_C R_B/\beta}{I_C} \\
R_C &< \boxed{1.12 \text{ k}\Omega}
\end{aligned}
$$

(35) $I_S = 5\times10^{-16}\,A$, $V_A = \infty \Rightarrow r_o = \infty$




Soft Saturation $\Rightarrow V_{BC} = 200\,mV$

$\Rightarrow V_B = V_C + 0.2^{V} \Rightarrow \boxed{V_B = 2.7\,V}$

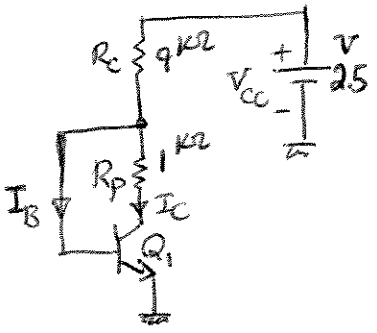
Applying KVL $\Rightarrow V_B = V_{BE} + R_E I_E \xrightarrow{I_E \simeq I_C} V_B = V_{BE} + R_E I_C$

$\Rightarrow V_{BE} + 1^{k\Omega}\times I_S e^{\frac{V_{BE}}{V_T}} = 2.7^{V}$

$\Rightarrow V_{BE} + 5\times10^{-13} e^{\frac{V_{BE}}{V_T}} = 2.7^{V} \Rightarrow \boxed{V_{BE} \simeq 754\,mV}$

$I_C = I_S e^{V_{BE}/V_T} = 5\times10^{-16} e^{\frac{0.754}{0.026}} \Rightarrow \boxed{I_C \simeq 2\,mA}$

(36) $\beta = 100, \quad V_A = \infty \Rightarrow r_o = \infty$




$$V_{BC} = 0.2\,V \Rightarrow R_P I_C = 0.2\,V$$

$$\Rightarrow \boxed{I_C = \frac{0.2\,V}{R_P}}$$

$$V_{BE} = V_{CC} - R_C(I_B + I_C)$$

$$\overset{\beta=100}{\Rightarrow} V_{BE} = V_{CC} - \frac{\beta+1}{\beta} R_C I_C \Rightarrow \boxed{V_{BE} = V_{CC} - \frac{\beta+1}{\beta}\frac{R_C}{R_P} \times 0.2}$$

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right) \Rightarrow I_S = I_C \exp\left(-\frac{V_{BE}}{V_T}\right)$$

$$\Rightarrow \boxed{I_S = \frac{0.2}{R_P} \exp\left[\frac{0.2}{V_T}\cdot\frac{\beta+1}{\beta}\cdot\frac{R_C}{R_P} - \frac{V_{CC}}{V_T}\right]}$$

$$\overset{\beta=100}{\Rightarrow} \boxed{I_S \simeq \frac{0.2}{R_P} \exp\left[\frac{0.2}{V_T}\frac{R_C}{R_P} - \frac{V_{CC}}{V_T}\right]}$$

$$\Rightarrow \boxed{I_S \simeq 4.06 \times 10^{-16}\,A}$$

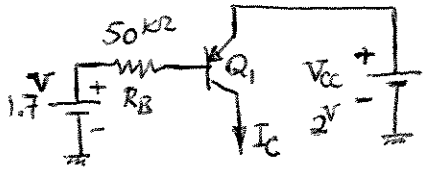
(37) $I_{S_1} = 3I_{S_2} = 6\times10^{-16}\,A$

$$I_1 = I_{S_1}\exp\left(\frac{V_{EB_1}}{V_T}\right) = 6\times10^{-16}\exp\left(\frac{300}{26}\right) \Rightarrow \boxed{I_1 \simeq 6.155\times10^{-11}\,A}$$

$$I_2 = I_{S_2}\exp\left(\frac{V_{EB_2}}{V_T}\right) = 2\times10^{-16}\exp\left(\frac{820}{26}\right) \Rightarrow \boxed{I_2 \simeq 10\,mA}$$

$$I_X = I_1 + I_2 \Rightarrow \boxed{I_X \simeq 10\,mA}$$

(38) $I_S = 2\times10^{-17}\,A$ $\qquad \beta = 100$




Applying KVL,

$$V_{CC} = V_{EB} + R_B I_B + 1.7\,V$$

$$\Rightarrow 2\,V = V_{EB} + R_B \frac{I_C}{\beta} + 1.7\,V$$

$$\Rightarrow 0.3\,V = V_{EB} + \frac{50\,k\Omega}{100} I_C$$

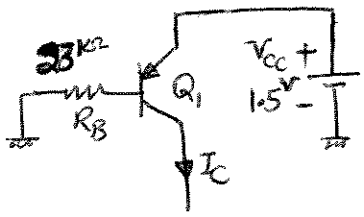
$$\Rightarrow 0.3\,V = V_{EB} + 500 \times I_S\, e^{\frac{V_{EB}}{V_T}}$$

$$\Rightarrow 0.3\,V = V_{EB} + 10^{-14}\, e^{\frac{V_{EB}}{26\,mV}} \Rightarrow \boxed{V_{EB} \simeq 0.3\ V}$$

$$I_C = I_S\, e^{\frac{V_{EB}}{V_T}} \Rightarrow I_C = 2\times10^{-17}\, e^{\frac{300}{26}}$$

$$\Rightarrow \boxed{I_C \simeq 2.05 \times 10^{-12}\ A}$$

(39) $I_C = 3mA$ , $\beta = 100$ , $R_B = 23\,k\Omega$




Applying KVL,

$$V_{CC} = V_{EB} + R_B I_B \Rightarrow V_{CC} = V_{EB} + R_B \frac{I_C}{\beta}$$

$$\Rightarrow -I_C \frac{R_B}{\beta} + V_{CC} = V_{EB}$$

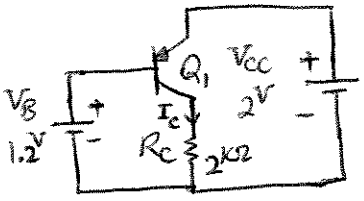
$$I_C = I_S e^{V_{EB}/V_T}$$

$$\Rightarrow I_S = I_C e^{-\frac{V_{EB}}{V_T}}$$

$$\Rightarrow \boxed{I_S = I_C e^{\frac{1}{V_T}\left(\frac{R_B I_C}{\beta} - V_{CC}\right)}}$$

$$\Rightarrow \boxed{I_S \simeq 8.85 \times 10^{-17}\ A}$$

(40) At the edge of active $\Rightarrow V_{BC} = 0$




$$I_C = \frac{V_B - V_{BC}}{R_C} = \frac{V_B}{R_C}$$

$$\Rightarrow I_C = \frac{1.2^V}{2^{K\Omega}} \Rightarrow \boxed{I_C \simeq 0.6\ mA}$$

$$I_C = I_S \exp\left(\frac{V_{EB}}{V_T}\right) \Rightarrow I_S = I_C \exp\left(-\frac{V_{EB}}{V_T}\right)$$

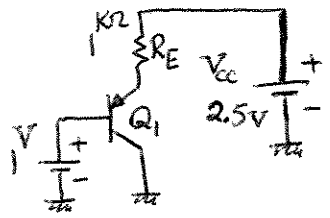
$$\Rightarrow I_S = 0.6 \times 10^{-3} \exp\left(-\frac{800}{26}\right)$$

$$\Rightarrow \boxed{I_S \simeq 2.6 \times 10^{-17}\ A}$$

4.41

$$
\begin{aligned}
V_{EB} &= V_{EC} \text{ (for } Q_1 \text{ to operate at the edge of saturation)} \\
V_{CC} - I_B R_B &= V_{CC} - I_C R_C \\
I_C R_B / \beta &= I_C R_C \\
R_B / \beta &= R_C \\
\beta &= R_B / R_C \\
&= \boxed{100}
\end{aligned}
$$

(42) $I_S = 3\times10^{-17}\,A$




Applying KVL,

$$V_{CC} = R_E I_E + V_{EB} + 1\,V \xrightarrow{I_E \simeq I_C} V_{CC} = R_E I_C + V_{EB} + 1\,V$$

$$\Rightarrow 2.5 = 1\,K\Omega \times 3\times10^{-17}\, e^{V_{EB}/26\,mV} + V_{EB} + 1\,V$$

$$\Rightarrow V_{EB} + 3\times10^{-14}\, e^{V_{EB}/26\,mV} = 1.5\,V$$

$$\Rightarrow \boxed{V_{EB} \simeq 800.5\ mV}$$

$$I_C = I_S e^{\frac{V_{EB}}{V_T}} = 3\times10^{-17}\, e^{\frac{800.5}{26}} \Rightarrow \boxed{I_C \simeq 0.705\ mA}$$

(43) $I_S = 3\times10^{-17}\,A$, $\beta = 100$, $V_A = \infty \Rightarrow \boxed{r_o = \infty}$

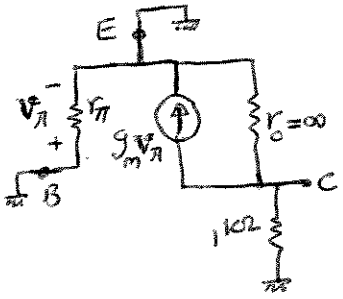
a) $V_{EB} = 2.5 - 1.7 = 0.8\ V$

$$I_C = I_S \exp\left(\frac{V_{EB}}{V_T}\right) = 3\times10^{-17}\exp\left(\frac{800}{26}\right) \Rightarrow \boxed{I_C \simeq 0.692\ mA}$$

$$V_{EC} = V_{CC} - R_C I_C = 2.5 - 1\,k\Omega \times 0.692\,mA \Rightarrow \boxed{V_{EC} \simeq 1.808\ V}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.692\,mA}{26\,mV} \Rightarrow \boxed{g_m \simeq 26.6\,mS}$$

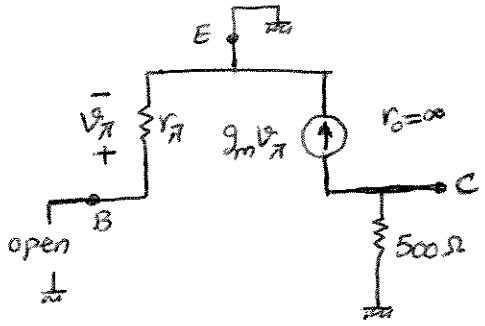
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{26.6\times10^{-3}} \Rightarrow \boxed{r_\pi \simeq 3.76\ k\Omega}$$




b) $V_{EB} = V_T \ln\left(\frac{I_C}{I_S}\right) \Rightarrow V_{EB} = V_T \ln\left(\frac{\beta I_B}{I_S}\right)$

$$\Rightarrow V_{EB} = 26\,mV \times \ln\left(\frac{100\times20\times10^{-6}}{3\times10^{-17}}\right)$$

$$\Rightarrow \boxed{V_{EB} \simeq 827.6\ mV}$$




$$I_C = \beta I_B \Rightarrow \boxed{I_C = 2\,mA}$$

$$V_{EC} = V_{CC} - R_C I_C = 2.5 - 0.5\,k\Omega \times 2\,mA \Rightarrow \boxed{V_{EC} = 1.5\ V}$$

$$g_m = \frac{I_C}{V_T} = \frac{2\,mA}{26\,mV} \Rightarrow \boxed{g_m \simeq 77\,mS} \qquad r_\pi = \frac{\beta}{g_m} \Rightarrow \boxed{r_\pi \simeq 1.3\,k\Omega}$$

(43) Continued .....

c) Applying KVL,

$$V_{CC} = V_{EB} + (I_C + I_B)\times 2^{k\Omega} \simeq V_{EB} + 2^{k\Omega} \times I_C$$

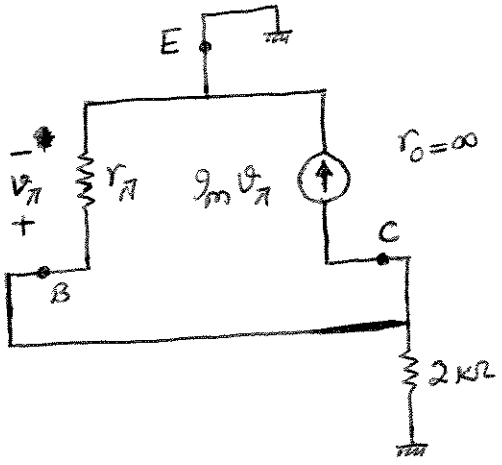
$$\Rightarrow V_{EB} + 2^{k\Omega} \times I_S e^{\frac{V_{EB}}{V_T}} = V_{CC}$$

$$\Rightarrow V_{EB} + 6\times 10^{-14} e^{\frac{V_{EB}}{26mV}} = 2.5^{V} \Rightarrow \boxed{V_{EB} \simeq 805 mV}$$

$$I_C = \frac{V_{CC} - V_{EB}}{R} = \frac{2.5 - 0.805}{2^{k\Omega}} \Rightarrow \boxed{I_C \simeq 847.5 \mu A}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.8475\times 10^{-3}}{0.026} \Rightarrow \boxed{g_m \simeq 32.6\ mS}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{32.6\times 10^{-3}} \Rightarrow \boxed{r_\pi \simeq 3068\ \Omega}$$



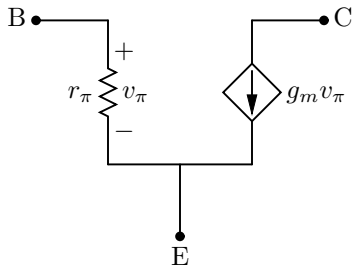
4.44 (a)

$$\begin{aligned} I_B &= 2\ \mu\text{A} \\ I_C &= \beta I_B \\ &= \boxed{200\ \mu\text{A}} \\ V_{EB} &= V_T \ln(I_C/I_S) \\ &= \boxed{768\ \text{mV}} \\ V_{EC} &= V_{CC} - I_E(2\ \text{k}\Omega) \\ &= V_{CC} - \frac{1+\beta}{\beta} I_C(2\ \text{k}\Omega) \\ &= \boxed{2.1\ \text{V}} \end{aligned}$$

$Q_1$ is operating in forward active. Its small-signal parameters are

$$\begin{aligned} g_m &= I_C/V_T = \boxed{7.69\ \text{mS}} \\ r_\pi &= \beta/g_m = \boxed{13\ \text{k}\Omega} \\ r_o &= \boxed{\infty} \end{aligned}$$

The small-signal model is shown below.



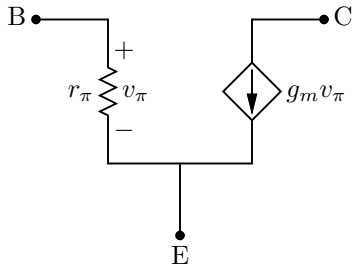

(b)

$$\begin{aligned} I_E &= \frac{V_{CC} - V_{EB}}{5\ \text{k}\Omega} \\ \frac{1+\beta}{\beta} I_C &= \frac{V_{CC} - V_T \ln(I_C/I_S)}{5\ \text{k}\Omega} \\ I_C &= \boxed{340\ \mu\text{A}} \\ V_{EB} &= \boxed{782\ \text{mV}} \\ V_{EC} &= V_{EB} = \boxed{782\ \text{mV}} \end{aligned}$$

$Q_1$ is operating in forward active. Its small-signal parameters are

$$\begin{aligned} g_m &= I_C/V_T = \boxed{13.1\ \text{mS}} \\ r_\pi &= \beta/g_m = \boxed{7.64\ \text{k}\Omega} \\ r_o &= \boxed{\infty} \end{aligned}$$

The small-signal model is shown below.




(c)

$$I_E = \frac{1+\beta}{\beta} I_C = 0.5 \text{ mA}$$

$$I_C = \boxed{495 \text{ μA}}$$

$$V_{EB} = \boxed{971 \text{ mV}}$$

$$V_{EC} = V_{EB} = \boxed{971 \text{ mV}}$$

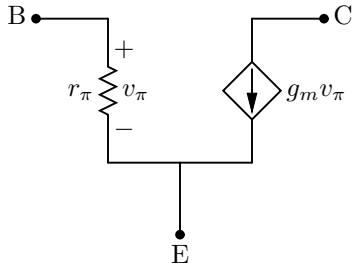
$Q_1$ is operating in forward active. Its small-signal parameters are

$$g_m = I_C/V_T = \boxed{19.0 \text{ mS}}$$

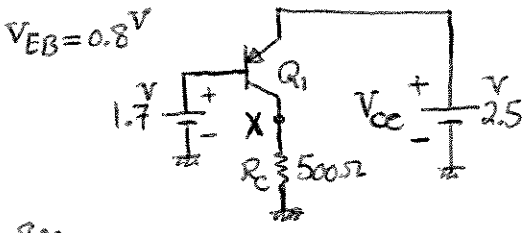
$$r_\pi = \beta/g_m = \boxed{5.25 \text{ kΩ}}$$

$$r_o = \boxed{\infty}$$

The small-signal model is shown below.



(45) $I_S = 5 \times 10^{-17}\ A$




a) $V_A = \infty \Rightarrow r_o = \infty$

$$I_C = I_S e^{V_{EB}/V_T} \Rightarrow I_C = 5 \times 10^{-17} e^{\frac{800}{26}} \Rightarrow \boxed{I_C \simeq 1.15\ mA}$$

$$V_X = R_C I_C = 0.5\ k\Omega \times 1.15\ mA \Rightarrow \boxed{V_X \simeq 0.58\ V}$$

b) $V_A = 6\ V$

$$I_C = I_S e^{\frac{V_{EB}}{V_T}} \left(1 + \frac{V_{EC}}{V_A}\right) \quad , \quad V_{EC} = V_{CC} - R_C I_C$$

$$\Rightarrow I_C = I_S e^{V_{EB}/V_T} \left(1 + \frac{V_{CC} - R_C I_C}{V_A}\right)$$

$$\Rightarrow I_C = I_S e^{V_{EB}/V_T} \left(1 + \frac{V_{CC}}{V_A}\right) - \frac{I_S R_C}{V_A} e^{\frac{V_{EB}}{V_T}} I_C$$

$$\Rightarrow \boxed{I_C = \frac{I_S e^{V_{EB}/V_T} \left(1 + \frac{V_{CC}}{V_A}\right)}{1 + \frac{I_S R_C}{V_A} e^{V_{EB}/V_T}}} = \frac{5 \times 10^{-17} e^{\frac{800}{26}} \left(1 + \frac{2.5}{6}\right)}{1 + \frac{5 \times 10^{-17} \times 0.5\ k\Omega}{6} e^{\frac{800}{26}}}$$

$$\Rightarrow \boxed{I_C = 1.49\ mA} \qquad V_X = R_C I_C = 500 \times 1.49 \times 10^{-3} \Rightarrow \boxed{V_X = 0.745\ V}$$

(46) $r_o = 60\,K\Omega$ , $I_C = 2\,mA$

$$r_o = \frac{V_A}{I_C} \Rightarrow 60 \times 10^3\,\Omega = \frac{V_A}{2 \times 10^{-3}\,A} \Rightarrow \boxed{V_A = 120\,V}$$

(47) $r_o = 60\ k\Omega$, $I_C = 1\ mA$

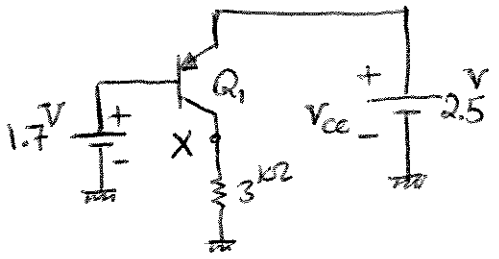
$$r_o = \frac{V_A}{I_C} \Rightarrow \boxed{V_A = r_o \cdot I_C} \Rightarrow \underline{\boxed{V_A \propto I_C}}$$

$$\Rightarrow V_A = 60\ k\Omega \times 1\ mA$$

$$\Rightarrow \boxed{V_A = 60\ V}$$

$V_A$ is half the value in problem 46 as $V_A$ is proportional to $I_C$.

(48) $V_A = 5\,V$




a) At the edge of active mode

$$\Rightarrow \boxed{V_X = V_B = 1.7\,V}$$

$$I_C = \frac{V_X}{R_C} = \frac{1.7\,V}{3\,k\Omega} \Rightarrow \boxed{I_C \simeq 0.567\,mA}$$

$$I_C = I_S\, e^{\frac{V_{BE}}{V_T}}\left(1 + \frac{V_{EC}}{V_A}\right) \Rightarrow \boxed{I_S = \frac{I_C\, e^{-\frac{V_{BE}}{V_T}}}{1 + \frac{V_{EC}}{V_A}}}$$

$$I_S = \frac{0.567 \times 10^{-3} \times e^{-\frac{800}{26}}}{1 + \frac{2.5 - 1.7}{5}} \Rightarrow \boxed{I_S \simeq 2.118 \times 10^{-17}\,A}$$

b) $V_A = \infty$

$$I_C = I_S\, e^{\frac{V_{EB}}{V_T}} \Rightarrow \boxed{I_S = I_C e^{-\frac{V_{EB}}{V_T}}}$$

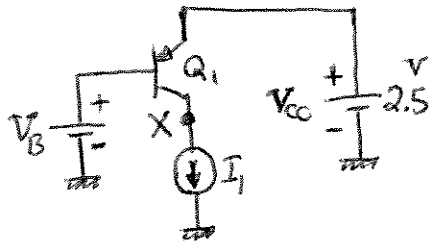
$$I_S = 0.567 \times 10^{-3}\, e^{-\frac{800}{26}} \Rightarrow \boxed{I_S \simeq 2.457 \times 10^{-17}\,A}$$

$I_S$ increases

4.49 The direction of current flow in the large-signal model (Fig. 4.40) indicates the direction of positive current flow when the transistor is properly biased.

The direction of current flow in the small-signal model (Fig. 4.43) indicates the direction of positive change in current flow when the base-emitter voltage $v_{be}$ increases. For example, when $v_{be}$ increases, the current flowing into the collector increases, which is why $i_c$ is shown flowing into the collector in Fig. 4.43. Similar reasoning can be applied to the direction of flow of $i_b$ and $i_e$ in Fig. 4.43.

(50) $I_S = 6\times10^{-16}\,A$, $V_A = 5\,V$, $I_1 = 2mA$




a) $I_C = I_S e^{\frac{V_{EB}}{V_T}}\left(1+\frac{V_{EC}}{V_A}\right)$

$$\Rightarrow V_{EB} = V_T \ln\left(\frac{I_C}{I_S\left(1+\frac{V_{EC}}{V_A}\right)}\right)$$

$V_{EC} = V_{CC} - V_X$, $V_{EB} = V_{CC} - V_B$

$$\boxed{V_B = V_{CC} - V_T \ln\left(\frac{I_C}{I_S\left(1+\frac{V_{CC}-V_X}{V_A}\right)}\right)}$$

$$\Rightarrow V_B = 2.5 - 0.026\ln\left(\frac{2\times10^{-3}}{6\times10^{-16}\left(1+\frac{2.5-1}{5}\right)}\right) \Rightarrow \boxed{V_B \simeq 1.757\,V}$$

b) $I_C = I_S e^{\frac{V_{EB}}{V_T}}\left(1+\frac{V_{EC}}{V_A}\right) \Rightarrow 1+\frac{V_{EC}}{V_A} = \frac{I_C}{I_S} e^{-\frac{V_{EB}}{V_T}}$

$V_{EC} = V_{CC} - V_X$, $V_{EB} = V_{CC} - V_B$

$$\boxed{V_X = V_{CC} - V_A\left(\frac{I_C}{I_S} e^{-\frac{V_{EB}}{V_T}} - 1\right)}$$

$$\Delta V_X \simeq \frac{dV_X}{dV_{EB}}\Delta V_{EB} \Rightarrow \Delta V_X \simeq \frac{V_A}{V_T}\cdot\frac{I_C}{I_S} e^{-\frac{V_{EB}}{V_T}} \Delta V_{EB}$$

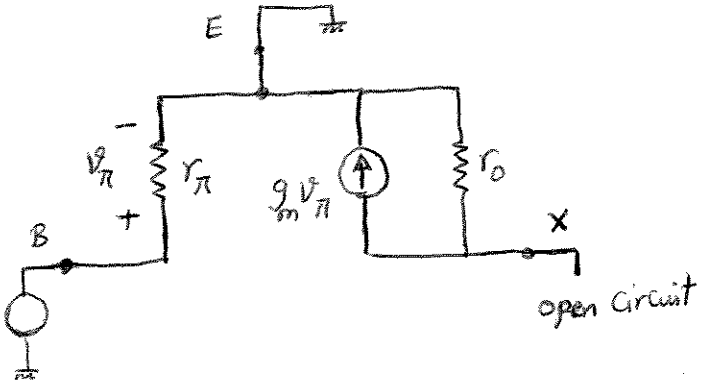
$\Delta V_{EB} = -\Delta V_B$

$$\boxed{\Delta V_X \simeq -\frac{V_A}{V_T}\cdot\frac{I_C}{I_S} e^{-\frac{V_{EB}}{V_T}} \Delta V_B}$$

$$\Rightarrow \Delta V_X \simeq -\frac{5}{0.026}\times\frac{2\times10^{-3}}{6\times10^{-16}}\exp\left(-\frac{2.5-1.757}{0.026}\right)\times 0.1\times10^{-3} \Rightarrow \boxed{\Delta V_X \simeq -24.9\,mV}$$

(50) Continued .....

c)



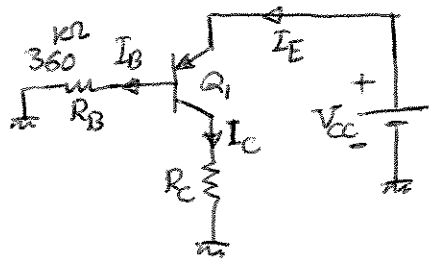

$$r_o = \frac{V_A}{I_C} = \frac{5\,V}{2\,mA} \Rightarrow \boxed{r_o \simeq 2.5\ k\Omega}$$

$$g_m = \frac{I_C}{V_T} = \frac{2\,mA}{0.026\,V} \Rightarrow \boxed{g_m \simeq 76.9\ mS}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{\frac{2}{26}} \Rightarrow \boxed{r_\pi = 1.3\ k\Omega}$$

(51) $\beta = 100, \quad V_A = \infty \Rightarrow r_o = \infty$

$R_B = 360\ k\Omega$




a) given: $V_C = V_B + 0.2^V$

$$\Rightarrow R_C I_C = R_B I_B + 0.2^V$$

$$\Rightarrow R_C I_C = R_B \frac{I_C}{\beta} + 0.2^V \Rightarrow \boxed{I_C = \frac{0.2^V}{R_C - \frac{R_B}{\beta}}} \Rightarrow \boxed{I_C = 0.5^{mA}}$$

$$I_C = I_S e^{+\frac{V_{EB}}{V_T}} \Rightarrow I_S = I_C e^{-\frac{V_{EB}}{V_T}} \Rightarrow I_S = I_C e^{-\left(\frac{V_{CC} - R_B I_B}{V_T}\right)}$$

$$\Rightarrow \boxed{I_S = \left(\frac{0.2}{R_C - \frac{R_B}{\beta}}\right) \exp\left[-\frac{1}{V_T}\left(V_{CC} - R_B \times \frac{0.2^V}{\beta\left(R_C - \frac{R_B}{\beta}\right)}\right)\right]}$$

$$\Rightarrow \boxed{I_S \simeq 10^{-15}\ A = 1\ FA}$$

b) $g_m = \frac{I_C}{V_T}$

$$\Rightarrow \boxed{g_m = \frac{0.2^V}{V_T\left(R_C - \frac{R_B}{\beta}\right)}} \Rightarrow \boxed{g_m \simeq 19.23\ mS}$$

(5.2) $I_S = 5\times10^{-16}A$, $\beta = 100$, $V_A = \infty \Rightarrow r_o = \infty$

a) $V_{EB} = 0 \Rightarrow Q_1$ is off $\quad I_C \simeq 0$

b) $I_B = 0 \Rightarrow Q_1$ is off

c) Applying KVL: $V_{CC} \simeq V_{EB} + 1^{k\Omega} \times I_C$

$\Rightarrow V_{EB} + 1^{k\Omega} \times I_S e^{\frac{V_{EB}}{V_T}} \simeq V_{CC} \Rightarrow V_{EB} + 5\times10^{-13} e^{\frac{V_{EB}}{26^{mV}}} \simeq 2.5^{V}$

$\Rightarrow \boxed{V_{EB} \simeq 751\,mV} \qquad I_C = 5\times10^{-16} e^{\frac{0.751}{0.026}} \Rightarrow \boxed{I_C \simeq 1.8mA}$

with this current, Transistor is saturated. Note $V_B < V_C$ Always

(d) $V_{BC} = 0 \Rightarrow$ Transistor is at the edge of saturation

e) $I_C \simeq 0.5mA \Rightarrow V_{EB} = V_T \ln\left(\frac{I_C}{I_S}\right) = 26^{mV} \ln\left(\frac{0.5mA}{5\times10^{-16}}\right)$

$\Rightarrow \boxed{V_{EB} \simeq 718\,mV}$

$V_{collector} = 500^{\Omega} \times I_C \Rightarrow \boxed{V_C = 0.25^{V}}$

As $V_B = 0$, $V_C = 0.25^{V} \Rightarrow$ Transistor is soft saturated

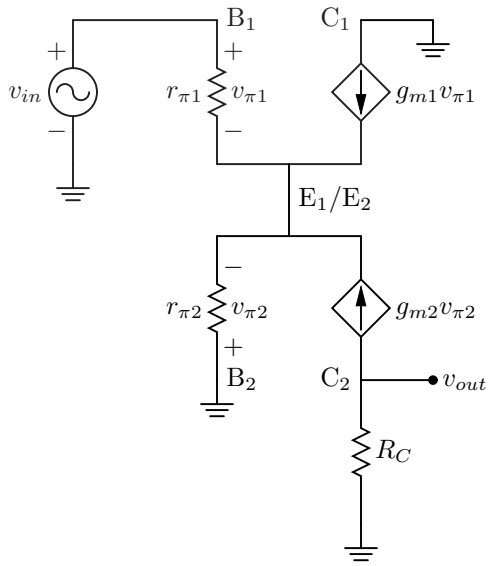
4.53 (a)

$$\begin{aligned}
V_{CB2} &< 200 \text{ mV} \\
I_{C2}R_C &< 200 \text{ mV} \\
I_{C2} &< 400 \text{ μA} \\
V_{EB2} &= V_{E2} \\
&= V_T \ln(I_{C2}/I_{S2}) \\
&< 741 \text{ mV} \\
\frac{\beta_2}{1+\beta_2} I_{E2}R_C &< 200 \text{ mV} \\
\frac{\beta_2}{1+\beta_2}\frac{1+\beta_1}{\beta_1} I_{C1}R_C &< 200 \text{ mV} \\
I_{C1} &< 396 \text{ μA} \\
V_{BE1} &= V_T \ln(I_{C1}/I_{S1}) \\
&< 712 \text{ mV} \\
V_{in} &= V_{BE1} + V_{EB2} \\
&< \boxed{1.453 \text{ V}}
\end{aligned}$$

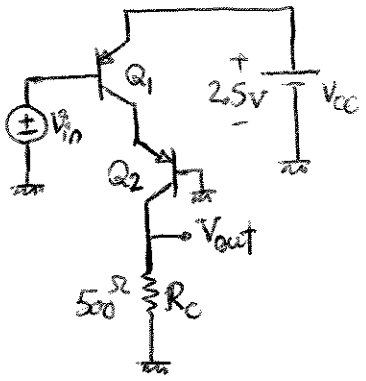
(b)

$$\begin{aligned}
I_{C1} &= 396 \text{ μA} \\
I_{C2} &= 400 \text{ μA} \\
g_{m1} &= \boxed{15.2 \text{ mS}} \\
r_{\pi 1} &= \boxed{6.56 \text{ kΩ}} \\
r_{o1} &= \boxed{\infty} \\
g_{m2} &= \boxed{15.4 \text{ mS}} \\
r_{\pi 2} &= \boxed{3.25 \text{ kΩ}} \\
r_{o2} &= \boxed{\infty}
\end{aligned}$$

The small-signal model is shown below.



(54) $I_{S_1} = 3I_{S_2} = 5\times10^{-16}\,A$, $\beta_1 = 100$, $\beta_2 = 50$, $V_A = \infty$




a) $V_{B_2} = 0 \xrightarrow[\text{Forward biased by } 200mV]{Q_2 \text{ Base-Collector}} V_{C_2} = 0.2\,V$

$$\Rightarrow I_{C_2 max} = \frac{V_{C_2 max}}{R_C} = \frac{0.2\,V}{500\,\Omega} \Rightarrow \boxed{I_{C_2 max} = 0.4\,mA}$$

As shown: $I_{C_1} \cong I_{C_2}$ (Note: $I_{C_1} = I_{E_2} = \frac{\beta_2+1}{\beta_2} I_{C_2}$ precisely)

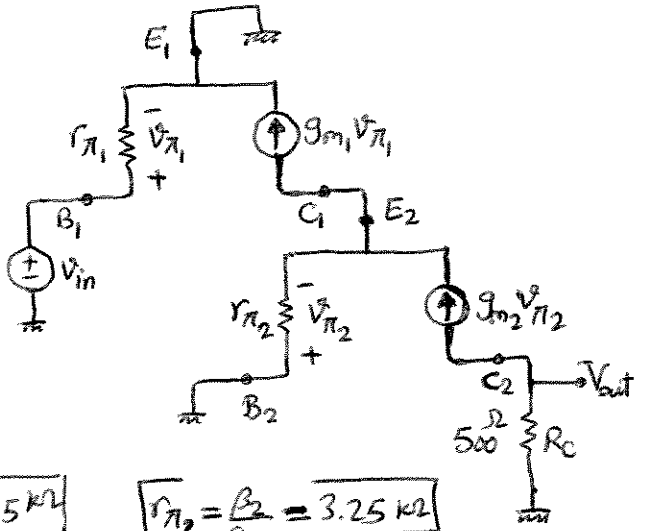
$$I_{C_1} \cong I_{S_1} e^{V_{EB_1}/V_T} \Rightarrow V_{EB_1} = V_T \ln\left(\frac{I_{C_1}}{I_{S_1}}\right) \Rightarrow V_{CC} - V_{in} = V_T \ln\left(\frac{I_{C_1}}{I_{S_1}}\right)$$

$$\Rightarrow \boxed{V_{in} = V_{CC} - V_T \ln\left(\frac{I_{C_1}}{I_{S_1}}\right)} \Rightarrow V_{in} = 2.5 - 0.026 \ln\left(\frac{4\times10^{-4}}{5\times10^{-16}}\right)$$

$$\Rightarrow \boxed{V_{in} \simeq 1.787\,V}$$ This is minimum acceptable $V_{in}$

b) $$g_{m_1} = \frac{I_{C_1}}{V_T} \simeq \frac{0.4\,mA}{26\,mV}$$

$$g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{0.4\,mA}{26\,mV}$$




$$\Rightarrow \boxed{g_{m_1} = g_{m_2} \simeq 15.4\,mS}$$

$$r_{\pi_1} = \frac{\beta_1}{g_{m_1}} = \frac{100}{\frac{0.4}{26}} \Rightarrow \boxed{r_{\pi_1} = 6.5\,k\Omega} \qquad \boxed{r_{\pi_2} = \frac{\beta_2}{g_{m_2}} = 3.25\,k\Omega}$$

$$V_{EB_2} = V_T \ln\left(\frac{I_{C_2}}{I_{S_2}}\right) = 26\,mV \ln\left(\frac{0.4\times10^{-3}}{5\times10^{-16}}\right) \Rightarrow \boxed{V_{EB_2} \simeq 741\,mV} \Rightarrow \boxed{V_{EC_1} \simeq 1.759\,V}$$

$Q_1$ in active mode

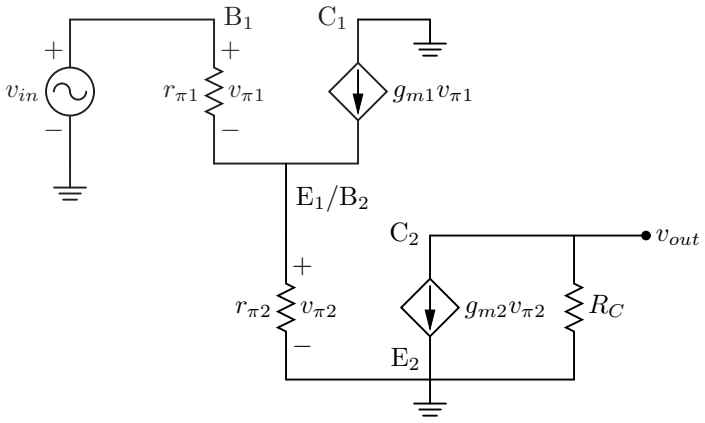
4.55 (a)

$$\begin{aligned}
V_{BC2} &< 200 \text{ mV} \\
V_{BE2} - (V_{CC} - I_{C2}R_C) &< 200 \text{ mV} \\
V_T \ln(I_{C2}/I_{S2}) + I_{C2}R_C - V_{CC} &< 200 \text{ mV} \\
I_{C2} &< 3.80 \text{ mA} \\
V_{BE2} &< 799.7 \text{ mV} \\
I_{E1} = \frac{1+\beta_1}{\beta_1} I_{C1} &= I_{B2} = I_{C2}/\beta_2 \\
I_{C1} &< 75.3 \text{ μA} \\
V_{BE1} &< 669.2 \text{ mV} \\
V_{in} &= V_{BE1} + V_{BE2} \\
&< \boxed{1.469 \text{ V}}
\end{aligned}$$

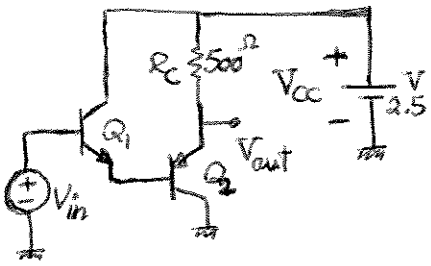
(b)

$$\begin{aligned}
I_{C1} &= 75.3 \text{ μA} \\
I_{C2} &= 3.80 \text{ mA} \\
g_{m1} &= \boxed{2.90 \text{ mS}} \\
r_{\pi 1} &= \boxed{34.5 \text{ kΩ}} \\
r_{o1} &= \boxed{\infty} \\
g_{m2} &= \boxed{146.2 \text{ mS}} \\
r_{\pi 2} &= \boxed{342 \text{ Ω}} \\
r_{o2} &= \boxed{\infty}
\end{aligned}$$

The small-signal model is shown below.



(56) $I_{S_1} = 2I_{S_2} = 6\times10^{-17}\,A, \quad \beta_1 = 80, \quad \beta_2 = 100$




a) $I_{C_2} = 2^{mA}$

$$V_{EB_2} = V_T \ln\frac{I_{C_2}}{I_{S_2}} = 26^{mV}\ln\left(\frac{2\times10^{-3}}{3\times10^{-17}}\right) \simeq 827.6^{mV}$$

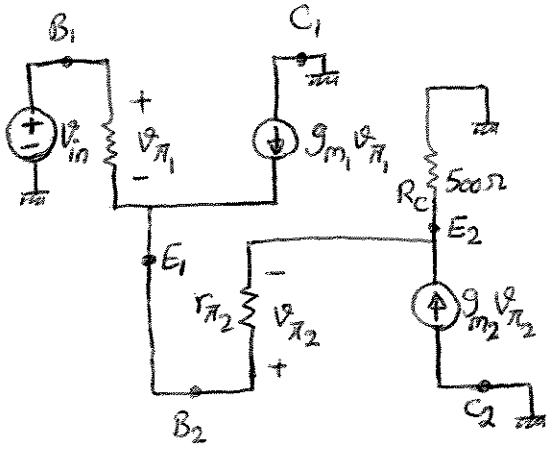
$$V_{BE_1} = V_T \ln\frac{I_{C_1}}{I_{S_1}} = 26^{mV}\ln\left(\frac{\frac{2\times10^{-3}}{100}}{6\times10^{-17}}\right) \simeq 689.9\ mV$$

$$\boxed{V_{in} = V_{CC} - R_C I_{C_2} - V_{EB_2} + V_{BE_1}} = 2.5 - 0.5^{k\Omega}\times 2^{mA} - 0.8276 + 0.6899$$

$$\Rightarrow \boxed{V_{in} \simeq 1.362\ V}$$

b) $g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{2^{mA}}{26^{mV}} \Rightarrow \boxed{g_{m_2} \simeq 76.9\ mS}$

$$g_{m_1} = \frac{I_{C_1}}{V_T} = \frac{\frac{2^{mA}}{100}}{26^{mV}} \Rightarrow \boxed{g_{m_1} \simeq 769\,\mu S}$$



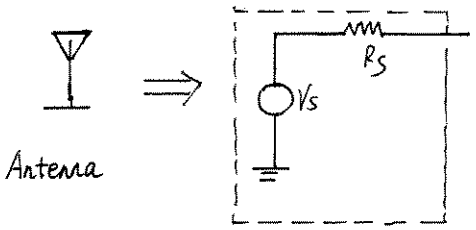

$$r_{\pi_1} = \frac{\beta_1}{g_{m_1}} = \frac{80}{\frac{1}{1300}}$$

$$\Rightarrow \boxed{r_{\pi_1} = 104\ k\Omega}$$

$$r_{\pi_2} = \frac{\beta_2}{g_{m_2}} = \frac{100}{\frac{2}{26}} \Rightarrow \boxed{r_{\pi_2} = 1300\,\Omega}$$

$V_A = \infty \Rightarrow r_o = \infty$

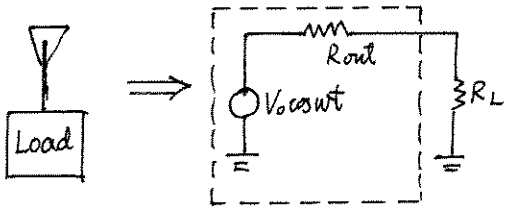
17)




Thevenin Equivalent:

$V_S = V_0 \cos \omega t$

$R_S = R_{out}$




Average power delivered to load = $(I_{RMS})^2 R_L$,

$I_{RMS} = \frac{V_{RMS}}{R_{out} + R_L}$, $V_{RMS} = \frac{V_0}{\sqrt{2}} \Rightarrow I_{RMS} = \frac{V_0}{\sqrt{2}(R_{out} + R_L)}$

Average power $= (I_{RMS})^2 R_L = \frac{V_0^2 R_L}{2(R_{out} + R_L)^2}$ (Eq. 1)

Plot of Average Power

When $R_L$ is small, Eq. 1 is small.

When $R_L$ is large, Eq. 1 is also small.

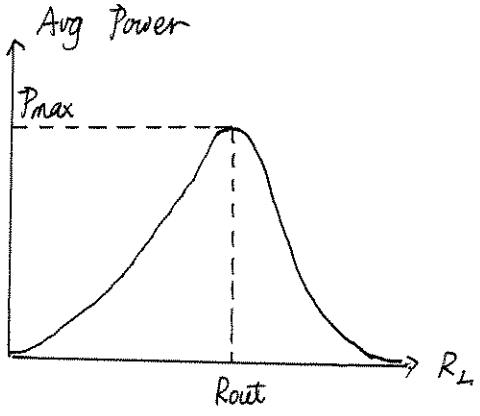
So for some $R_L$ between zero and infinity, the average power will reach its peak. Lets take the derivative of Eq. 1 with respect to $R_L$ to find the optimum $R_L$.

$$\frac{\partial}{\partial R_L}\left[\frac{V_0^2 R_L}{2(R_{out}+R_L)^2}\right] = \frac{V_0^2}{2(R_{out}+R_L)^2} - \frac{V_0^2 R_L}{(R_{out}+R_L)^3}$$

Setting it to zero and solve for $R_L$

$$\frac{V_0^2}{2(R_{out}+R_L)^2} = \frac{V_0^2 R_L}{(R_{out}+R_L)^3} \Rightarrow \frac{(R_{out}+R_L)}{2} = R_L$$

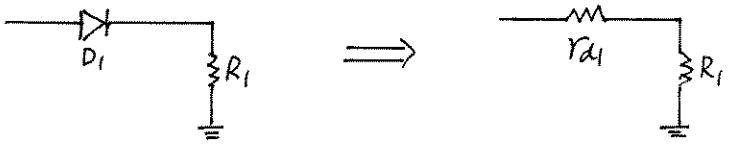
$$\Rightarrow R_{out} + R_L = 2R_L \Rightarrow R_L = R_{out}$$



2) In small signal operation, a diode can be replaced by a linear resistor if changes are small.

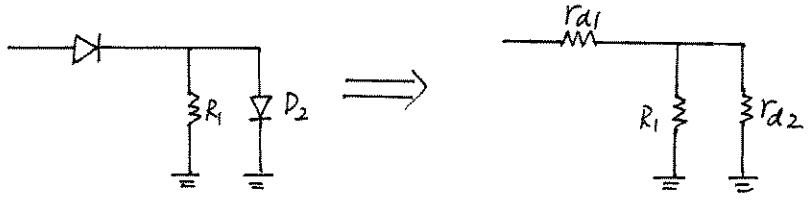



a).



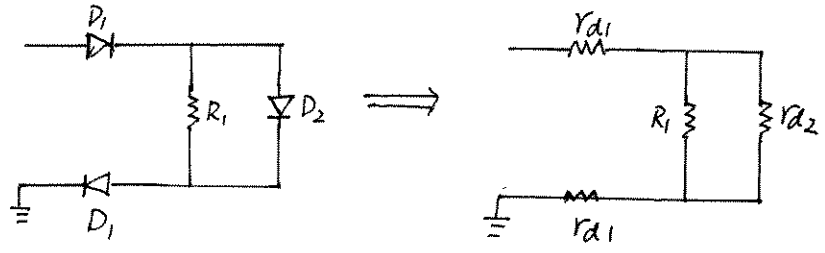

$R_{in} = r_{d1} + R_1$

b).



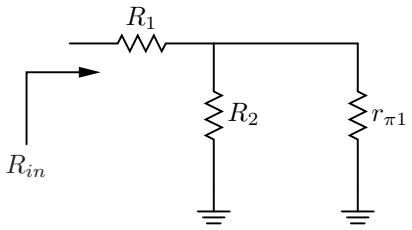

$R_{in} = r_{d1} + R_1 // r_{d2}$ ( // means in parallel )

c).



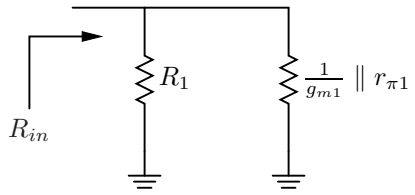

$R_{in} = 2r_{d1} + R_1 // r_{d2}$

5.3 (a) Looking into the base of $Q_1$ we see an equivalent resistance of $r_{\pi 1}$, so we can draw the following equivalent circuit for finding $R_{in}$:



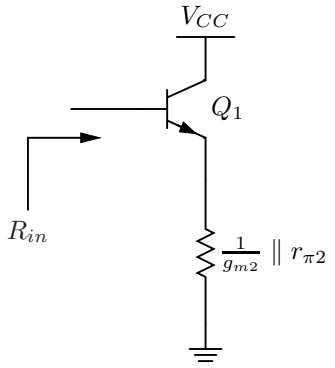

$$R_{in} = \boxed{R_1 + R_2 \parallel r_{\pi 1}}$$

(b) Looking into the emitter of $Q_1$ we see an equivalent resistance of $\frac{1}{g_{m1}} \parallel r_{\pi 1}$, so we can draw the following equivalent circuit for finding $R_{in}$:



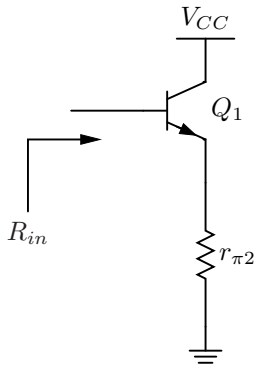

$$R_{in} = \boxed{R_1 \parallel \frac{1}{g_{m1}} \parallel r_{\pi 1}}$$

(c) Looking down from the emitter of $Q_1$ we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding $R_{in}$:



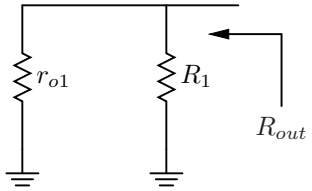

$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

(d) Looking into the base of $Q_2$ we see an equivalent resistance of $r_{\pi 2}$, so we can draw the following equivalent circuit for finding $R_{in}$:



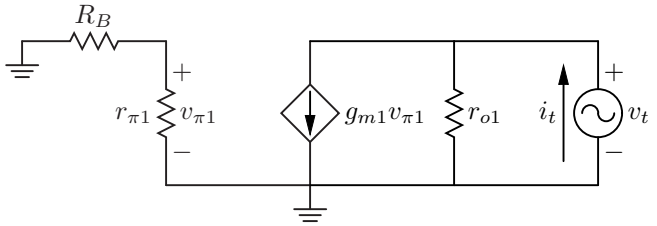

$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1) r_{\pi 2}}$$

5.4 (a) Looking into the collector of $Q_1$ we see an equivalent resistance of $r_{o1}$, so we can draw the following equivalent circuit for finding $R_{out}$:



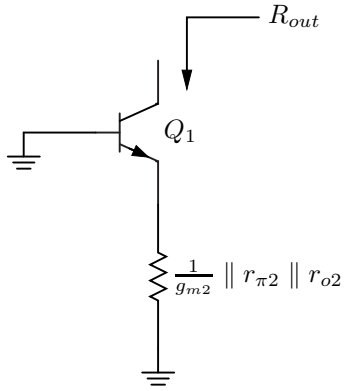

$$R_{out} = \boxed{r_{o1} \parallel R_1}$$

(b) Let's draw the small-signal model and apply a test source at the output.



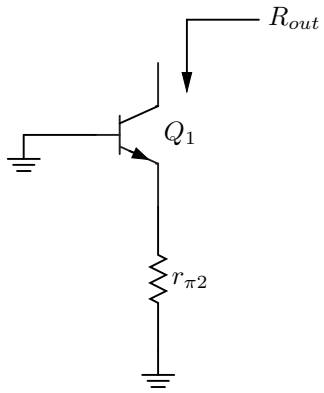

$$i_t = g_{m1}v_{\pi 1} + \frac{v_t}{r_{o1}}$$
$$v_{\pi 1} = 0$$
$$i_t = \frac{v_t}{r_{o1}}$$
$$R_{out} = \frac{v_t}{i_t} = \boxed{r_{o1}}$$

(c) Looking down from the emitter of $Q_1$ we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}$, so we can draw the following equivalent circuit for finding $R_{out}$:



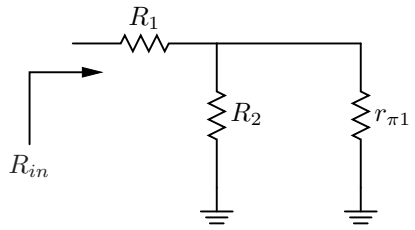

$$R_{out} = \boxed{r_{o1} + (1 + g_{m1}r_{o1})\left(r_{\pi 1} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}\right)}$$

(d) Looking into the base of $Q_2$ we see an equivalent resistance of $r_{\pi 2}$, so we can draw the following equivalent circuit for finding $R_{out}$:



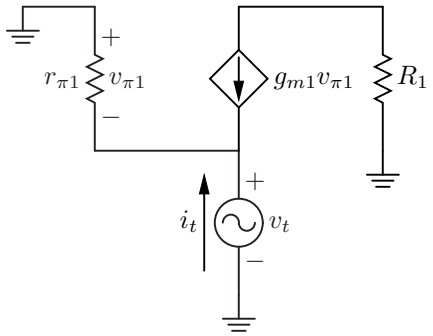

$$R_{out} = \boxed{r_{o1} + (1 + g_{m1} r_{o1}) (r_{\pi 1} \parallel r_{\pi 2})}$$

5.5 (a) Looking into the base of $Q_1$ we see an equivalent resistance of $r_{\pi 1}$, so we can draw the following equivalent circuit for finding $R_{in}$:



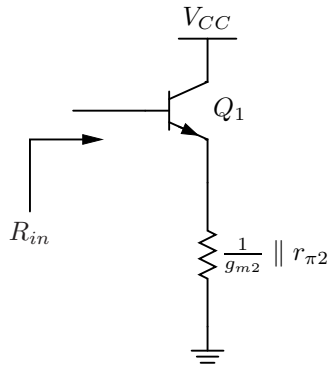

$$R_{in} = \boxed{R_1 + R_2 \parallel r_{\pi 1}}$$

(b) Let's draw the small-signal model and apply a test source at the input.



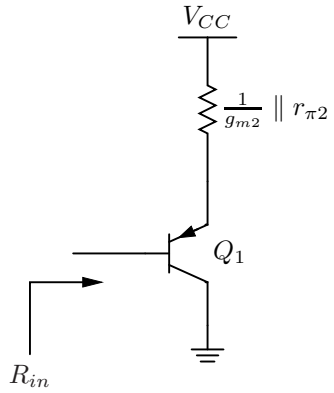

$$\begin{aligned} i_t &= -\frac{v_{\pi 1}}{r_{\pi 1}} - g_{m1} v_{\pi 1} \\ v_{\pi 1} &= -v_t \\ i_t &= \frac{v_t}{r_{\pi 1}} + g_{m1} v_t \\ i_t &= v_t \left( g_{m1} + \frac{1}{r_{\pi 1}} \right) \\ R_{in} &= \frac{v_t}{i_t} = \boxed{\frac{1}{g_{m1}} \parallel r_{\pi 1}} \end{aligned}$$

(c) From our analysis in part (b), we know that looking into the emitter we see a resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$. Thus, we can draw the following equivalent circuit for finding $R_{in}$:




$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

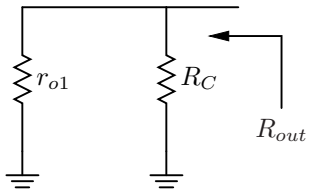
(d) Looking up from the emitter of $Q_1$ we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding $R_{in}$:




$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

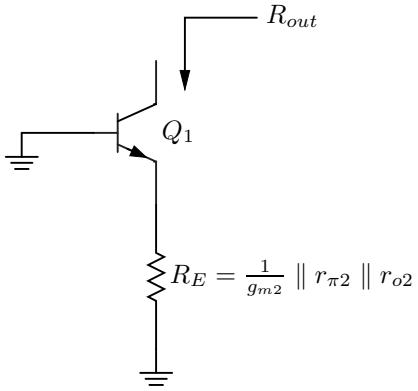
(e) We know that looking into the base of $Q_2$ we see $R_{in} = \boxed{r_{\pi 2}}$ if the emitter is grounded. Thus, transistor $Q_1$ does not affect the input impedance of this circuit.

5.6 (a) Looking into the collector of $Q_1$ we see an equivalent resistance of $r_{o1}$, so we can draw the following equivalent circuit for finding $R_{out}$:




$$R_{out} = \boxed{R_C \parallel r_{o1}}$$

(b) Looking into the emitter of $Q_2$ we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}$, so we can draw the following equivalent circuit for finding $R_{out}$:




$$R_{out} = \boxed{r_{o1} + (1 + g_{m1} r_{o1}) \left( r_{\pi 1} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2} \right)}$$

5.7 (a)

$$V_{CC} - I_B(100\ \text{k}\Omega) = V_{BE} = V_T \ln(I_C/I_S)$$
$$V_{CC} - \frac{1}{\beta} I_C(100\ \text{k}\Omega) = V_T \ln(I_C/I_S)$$
$$I_C = \boxed{1.754\ \text{mA}}$$
$$V_{BE} = V_T \ln(I_C/I_S) = \boxed{746\ \text{mV}}$$
$$V_{CE} = V_{CC} - I_C(500\ \Omega) = \boxed{1.62\ \text{V}}$$

$Q_1$ is operating in forward active.

(b)

$$I_{E1} = I_{E2} \Rightarrow V_{BE1} = V_{BE2}$$
$$V_{CC} - I_{B1}(100\ \text{k}\Omega) = 2V_{BE1}$$
$$V_{CC} - \frac{1}{\beta} I_{C1}(100\ \text{k}\Omega) = 2V_T \ln(I_{C1}/I_S)$$
$$I_{C1} = I_{C2} = \boxed{1.035\ \text{mA}}$$
$$V_{BE1} = V_{BE2} = \boxed{733\ \text{mV}}$$
$$V_{CE2} = V_{BE2} = \boxed{733\ \text{mV}}$$
$$V_{CE1} = V_{CC} - I_C(1\ \text{k}\Omega) - V_{CE2}$$
$$= \boxed{733\ \text{mV}}$$

Both $Q_1$ and $Q_2$ are at the edge of saturation.

(c)

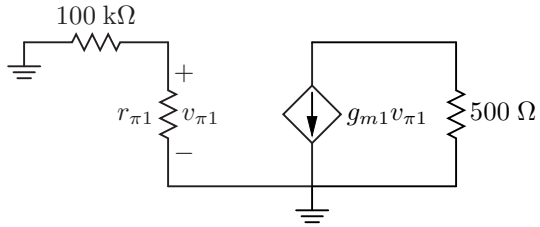
$$V_{CC} - I_B(100\ \text{k}\Omega) = V_{BE} + 0.5\ \text{V}$$
$$V_{CC} - \frac{1}{\beta} I_C(100\ \text{k}\Omega) = V_T \ln(I_C/I_S) + 0.5\ \text{V}$$
$$I_C = \boxed{1.262\ \text{mA}}$$
$$V_{BE} = \boxed{738\ \text{mV}}$$
$$V_{CE} = V_{CC} - I_C(1\ \text{k}\Omega) - 0.5\ \text{V}$$
$$= \boxed{738\ \text{mV}}$$

$Q_1$ is operating at the edge of saturation.

5.8 See Problem 7 for the derivation of $I_C$ for each part of this problem.

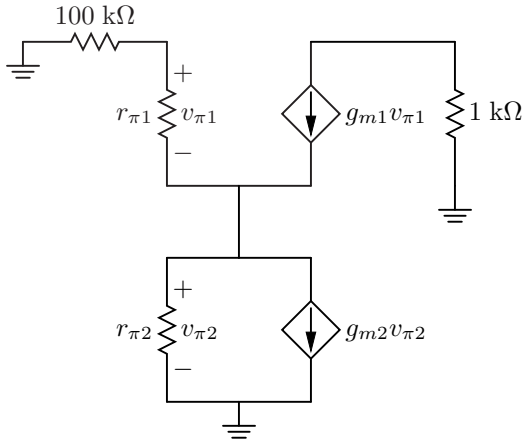
(a)

$$I_{C1} = 1.754 \text{ mA}$$
$$g_{m1} = I_{C1}/V_T = \boxed{67.5 \text{ mS}}$$
$$r_{\pi 1} = \beta/g_{m1} = \boxed{1.482 \text{ k}\Omega}$$



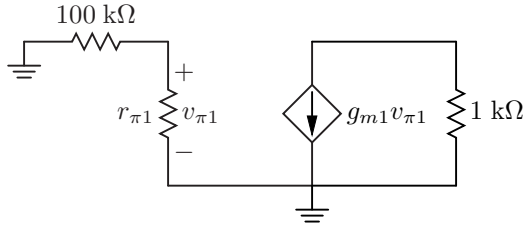

(b)

$$I_{C1} = I_{C2} = 1.034 \text{ mA}$$
$$g_{m1} = g_{m2} = I_{C1}/V_T = \boxed{39.8 \text{ mS}}$$
$$r_{\pi 1} = r_{\pi 2} = \beta/g_{m1} = \boxed{2.515 \text{ k}\Omega}$$




(c)

$$I_{C1} = 1.26 \text{ mA}$$
$$g_{m1} = I_{C1}/V_T = \boxed{48.5 \text{ mS}}$$
$$r_{\pi 1} = \beta/g_{m1} = \boxed{2.063 \text{ k}\Omega}$$



5.9 (a)

$$\frac{V_{CC} - V_{BE}}{34\ \text{k}\Omega} - \frac{V_{BE}}{16\ \text{k}\Omega} = I_B = \frac{I_C}{\beta}$$

$$I_C = \beta\frac{V_{CC} - V_T \ln(I_C/I_S)}{34\ \text{k}\Omega} - \beta\frac{V_T \ln(I_C/I_S)}{16\ \text{k}\Omega}$$

$$I_C = \boxed{677\ \mu\text{A}}$$

$$V_{BE} = \boxed{726\ \text{mV}}$$

$$V_{CE} = V_{CC} - I_C(3\ \text{k}\Omega) = \boxed{468\ \text{mV}}$$

$Q_1$ is in soft saturation.

(b)

$$I_{E1} = I_{E2}$$

$$\Rightarrow I_{C1} = I_{C2}$$

$$\Rightarrow V_{BE1} = V_{BE2} = V_{BE}$$

$$\frac{V_{CC} - 2V_{BE}}{9\ \text{k}\Omega} - \frac{2V_{BE}}{16\ \text{k}\Omega} = I_{B1} = \frac{I_{C1}}{\beta}$$

$$I_{C1} = \beta\frac{V_{CC} - 2V_T \ln(I_{C1}/I_S)}{9\ \text{k}\Omega} - \beta\frac{2V_T \ln(I_{C1}/I_S)}{16\ \text{k}\Omega}$$

$$I_{C1} = I_{C2} = \boxed{1.72\ \text{mA}}$$

$$V_{BE1} = V_{BE2} = V_{CE2} = \boxed{751\ \text{mV}}$$

$$V_{CE1} = V_{CC} - I_{C1}(500\ \Omega) - V_{CE2} = \boxed{890\ \text{mV}}$$

$Q_1$ is in forward active and $Q_2$ is on the edge of saturation.

(c)

$$\frac{V_{CC} - V_{BE} - 0.5\ \text{V}}{12\ \text{k}\Omega} - \frac{V_{BE} + 0.5\ \text{V}}{13\ \text{k}\Omega} = I_B = \frac{I_C}{\beta}$$

$$I_C = \beta\frac{V_{CC} - V_T \ln(I_C/I_S) - 0.5\ \text{V}}{12\ \text{k}\Omega} - \beta\frac{V_T \ln(I_C/I_S) + 0.5\ \text{V}}{13\ \text{k}\Omega}$$

$$I_C = \boxed{1.01\ \text{mA}}$$

$$V_{BE} = \boxed{737\ \text{mV}}$$

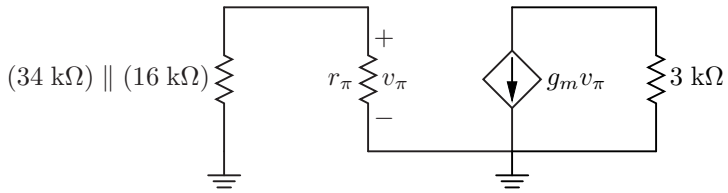
$$V_{CE} = V_{CC} - I_C(1\ \text{k}\Omega) - 0.5\ \text{V} = \boxed{987\ \text{mV}}$$

$Q_1$ is in forward active.

5.10 See Problem 9 for the derivation of $I_C$ for each part of this problem.

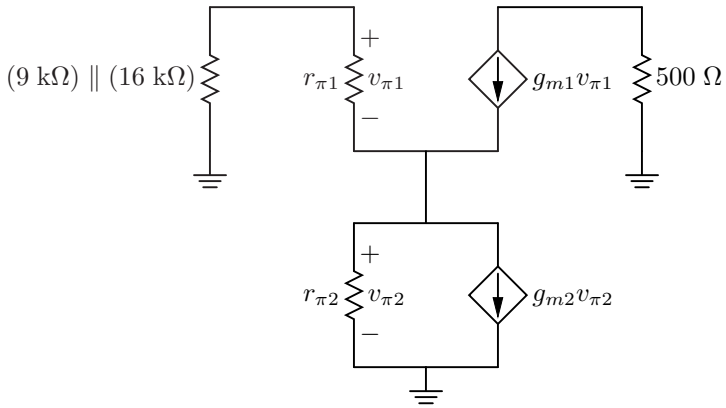
(a)

$$I_C = 677\ \mu\text{A}$$
$$g_m = I_C/V_T = \boxed{26.0\ \text{mS}}$$
$$r_\pi = \beta/g_m = \boxed{3.84\ \text{k}\Omega}$$



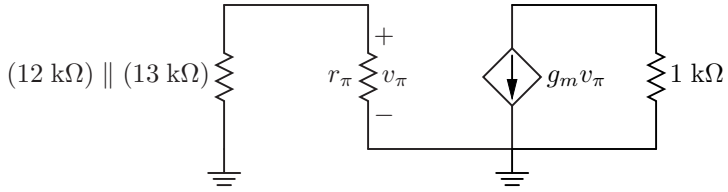

(b)

$$I_{C1} = I_{C2} = 1.72\ \text{mA}$$
$$g_{m1} = g_{m2} = I_{C1}/V_T = \boxed{66.2\ \text{mS}}$$
$$r_{\pi 1} = r_{\pi 2} = \beta/g_{m1} = \boxed{1.51\ \text{k}\Omega}$$




(c)

$$I_C = 1.01\ \text{mA}$$
$$g_m = I_C/V_T = \boxed{38.8\ \text{mS}}$$
$$r_\pi = \beta/g_m = \boxed{2.57\ \text{k}\Omega}$$



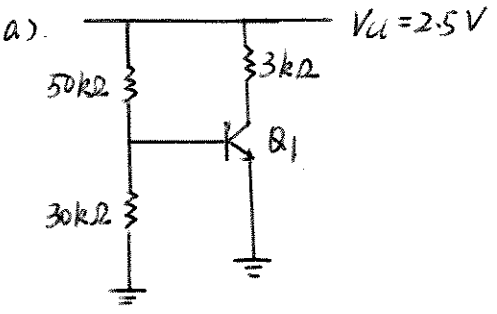
5.11 (a)

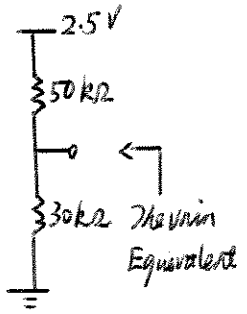
$$
\begin{aligned}
V_{CE} &\geq V_{BE} \text{ (in order to guarantee operation in the active mode)} \\
V_{CC} - I_C(2\ \text{k}\Omega) &\geq V_{BE} \\
V_{CC} - I_C(2\ \text{k}\Omega) &\geq V_T \ln(I_C/I_S) \\
I_C &\leq 886\ \mu\text{A} \\
\frac{V_{CC} - V_{BE}}{R_B} - \frac{V_{BE}}{3\ \text{k}\Omega} &= I_B = \frac{I_C}{\beta} \\
\frac{V_{CC} - V_T \ln(I_C/I_S)}{R_B} - \frac{V_T \ln(I_C/I_S)}{3\ \text{k}\Omega} &= \frac{I_C}{\beta} \\
R_B \left( \frac{I_C}{\beta} + \frac{V_T \ln(I_C/I_S)}{3\ \text{k}\Omega} \right) &= V_{CC} - V_T \ln(I_C/I_S) \\
R_B &= \frac{V_{CC} - V_T \ln(I_C/I_S)}{\frac{I_C}{\beta} + \frac{V_T \ln(I_C/I_S)}{3\ \text{k}\Omega}} \\
R_B &\geq \boxed{7.04\ \text{k}\Omega}
\end{aligned}
$$

(b)

$$
\begin{aligned}
\frac{V_{CC} - V_{BE}}{R_B} - \frac{V_{BE}}{3\ \text{k}\Omega} &= I_B = \frac{I_C}{\beta} \\
I_C &= \beta \frac{V_{CC} - V_T \ln(I_C/I_S)}{R_B} - \beta \frac{V_T \ln(I_C/I_S)}{3\ \text{k}\Omega} \\
I_C &= 1.14\ \text{mA} \\
V_{BE} &= 735\ \text{mV} \\
V_{CE} &= V_{CC} - I_C(2\ \text{k}\Omega) = 215\ \text{mV} \\
V_{BC} &= V_{BE} - V_{CE} = \boxed{520\ \text{mV}}
\end{aligned}
$$

12). a).




$V_{th} = 2.5 \times \frac{30}{50+30}$

$= 0.9375$

$R_{th} = \frac{30 \times 50}{30+50} k\Omega$

$= 18.75\ k$

Since $I_C = 0.5mA$, $I_B = \frac{I_C}{\beta} = 0.005mA$.

$I_B = \frac{V_{th} - V_{BE}}{R_{th}} \Rightarrow V_{BE} = V_{th} - I_B \cdot R_{th} = 0.84375$

$I_C = I_S e^{(\frac{V_{BE}}{V_T})} \Rightarrow I_S = \frac{I_C}{e^{(V_{BE}/V_T)}} = 4.03 \times 10^{-15} (mA)$

b). At the edge of saturation means $V_{BE} - V_{CE} = 0$.

(soft saturation not allowed)

$V_{CE} = 2.5 - I_C \cdot (3k)$, in which $I_C = \beta I_B = \beta (\frac{V_{th} - V_{BE}}{R_{th}})$

so $V_{BE} = 2.5 - \beta (\frac{V_{th} - V_{BE}}{R_{th}}) \cdot (3k)$

Solve this equation:

$V_{BE} = 0.83$.

$I_S = \frac{I_C}{e^{(V_{BE}/V_T)}} = \frac{\beta (\frac{V_{th} - V_{BE}}{R_{th}})}{e^{(V_{BE}/V_T)}} = 7.84 \times 10^{-15} (mA)$

5.13 We know the input resistance is $R_{in} = R_1 \parallel R_2 \parallel r_\pi$. Since we want the minimum values of $R_1$ and $R_2$ such that $R_{in} > 10\ \text{k}\Omega$, we should pick the maximum value allowable for $r_\pi$, which means picking the minimum value allowable for $g_m$ (since $r_\pi \propto 1/g_m$), which is $g_m = 1/260$ S.

$$
\begin{aligned}
g_m &= \frac{1}{260}\ \text{S} \\
I_C &= g_m V_T = 100\ \mu\text{A} \\
V_{BE} &= V_T \ln(I_C/I_S) = 760\ \text{mV} \\
I_B &= \frac{I_C}{\beta} = 1\ \mu\text{A} \\
\frac{V_{CC} - V_{BE}}{R_1} - \frac{V_{BE}}{R_2} &= I_B \\
R_1 &= \frac{V_{CC} - V_{BE}}{I_B + \frac{V_{BE}}{R_2}} \\
r_\pi &= \frac{\beta}{g_m} = 26\ \text{k}\Omega \\
R_{in} &= R_1 \parallel R_2 \parallel r_\pi \\
&= \left( \frac{V_{CC} - V_{BE}}{I_B + \frac{V_{BE}}{R_2}} \right) \parallel R_2 \parallel r_\pi \\
&> 10\ \text{k}\Omega \\
R_2 &> \boxed{23.57\ \text{k}\Omega} \\
R_1 &> \boxed{52.32\ \text{k}\Omega}
\end{aligned}
$$

5.14

$$
\begin{aligned}
g_m &= \frac{I_C}{V_T} \geq \frac{1}{26}\ \text{S} \\
r_\pi &= \frac{\beta}{g_m} = 2.6\ \text{k}\Omega \\
R_{in} &= R_1 \parallel R_2 \parallel r_\pi \\
&\leq r_\pi
\end{aligned}
$$

According to the above analysis, $R_{in}$ cannot be greater than 2.6 k$\Omega$. This means that the requirement that $R_{in} \geq 10$ k$\Omega$ cannot be met. Qualitatively, the requirement for $g_m$ to be large forces $r_\pi$ to be small, and since $R_{in}$ is bounded by $r_\pi$, it puts an upper bound on $R_{in}$ that, in this case, is below the required 10 k$\Omega$.

5.15

$$
\begin{aligned}
R_{out} &= R_C = R_0 \\
A_v &= -g_m R_C = -g_m R_0 = -\frac{I_C}{V_T} R_0 = -A_0 \\
I_C &= \frac{A_0}{R_0} V_T \\
r_\pi &= \beta \frac{V_T}{I_C} = \beta \frac{R_0}{A_0} \\
V_{BE} &= V_T \ln(I_C / I_S) = V_T \ln\left(\frac{A_0 V_T}{R_0 I_S}\right) \\
\frac{V_{CC} - V_{BE}}{R_1} - \frac{V_{BE}}{R_2} &= I_B = \frac{I_C}{\beta} \\
R_1 &= \frac{V_{CC} - V_{BE}}{\frac{I_C}{\beta} + \frac{V_{BE}}{R_2}} \\
R_{in} &= R_1 \parallel R_2 \parallel r_\pi \\
&= \left( \frac{V_{CC} - V_T \ln\left(\frac{A_0 V_T}{R_0 I_S}\right)}{\frac{I_C}{\beta} + \frac{V_T}{R_2} \ln\left(\frac{A_0 V_T}{R_0 I_S}\right)} \right) \parallel R_2 \parallel \beta \frac{R_0}{A_0}
\end{aligned}
$$

In order to maximize $R_{in}$, we can let $R_2 \to \infty$. This gives us

$$
R_{in,max} = \boxed{\left( \beta \frac{V_{CC} - V_T \ln\left(\frac{A_0 V_T}{R_0 I_S}\right)}{I_C} \right) \parallel \beta \frac{R_0}{A_0}}
$$

5.16 (a)

$$\begin{aligned} I_C &= 0.25 \text{ mA} \\ V_{BE} &= 696 \text{ mV} \\ \frac{V_{CC} - V_{BE} - I_E R_E}{R_1} - \frac{V_{BE} + I_E R_E}{R_2} &= I_B = \frac{I_C}{\beta} \\ R_1 &= \frac{V_{CC} - V_{BE} - \frac{1+\beta}{\beta} I_C R_E}{\frac{I_C}{\beta} + \frac{V_{BE} + \frac{1+\beta}{\beta} I_C R_E}{R_2}} \\ &= \boxed{22.74 \text{ k}\Omega} \end{aligned}$$

(b) First, consider a 5 % increase in $R_E$.

$$\begin{aligned} R_E &= 210 \ \Omega \\ \frac{V_{CC} - V_{BE} - I_E R_E}{R_1} - \frac{V_{BE} + I_E R_E}{R_2} &= I_B = \frac{I_C}{\beta} \\ \frac{V_{CC} - V_T \ln(I_C/I_S) - \frac{1+\beta}{\beta} I_C R_E}{R_1} - \frac{V_T \ln(I_C/I_S) + \frac{1+\beta}{\beta} I_C R_E}{R_2} &= I_B = \frac{I_C}{\beta} \\ I_C &= 243 \ \mu\text{A} \\ \frac{I_C - I_{C,nom}}{I_{C,nom}} \times 100 &= \boxed{-2.6 \ \%} \end{aligned}$$

Now, consider a 5 % decrease in $R_E$.

$$\begin{aligned} R_E &= 190 \ \Omega \\ I_C &= 257 \ \mu\text{A} \\ \frac{I_C - I_{C,nom}}{I_{C,nom}} \times 100 &= \boxed{+2.8 \ \%} \end{aligned}$$

5.17

$$
\begin{aligned}
V_{CE} &\geq V_{BE} \text{ (in order to guarantee operation in the active mode)} \\
V_{CC} - I_C R_C &\geq V_T \ln(I_C/I_S) \\
I_C &\leq 833\ \mu\text{A} \\
\frac{V_{CC} - V_{BE} - I_E R_E}{30\ \text{k}\Omega} - \frac{V_{BE} + I_E R_E}{R_2} &= I_B = \frac{I_C}{\beta} \\
R_2 &= \frac{V_{BE} + I_E R_E}{\frac{V_{CC} - V_{BE} - I_E R_E}{30\ \text{k}\Omega} - \frac{I_C}{\beta}} \\
&= \frac{V_T \ln(I_C/I_S) + \frac{1+\beta}{\beta} I_C R_E}{\frac{V_{CC} - V_T \ln(I_C/I_S) - \frac{1+\beta}{\beta} I_C R_E}{30\ \text{k}\Omega} - \frac{I_C}{\beta}} \\
R_2 &\leq \boxed{20.66\ \text{k}\Omega}
\end{aligned}
$$

5.18 (a) First, note that $V_{BE1} = V_{BE2} = V_{BE}$, but since $I_{S1} = 2I_{S2}$, $I_{C1} = 2I_{C2}$. Also note that $\beta_1 = \beta_2 = \beta = 100$.

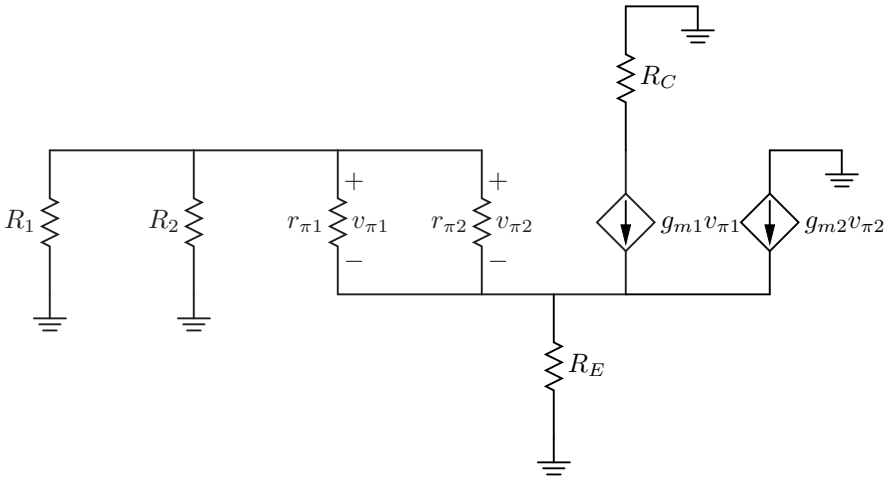
$$I_{B1} = \frac{I_{C1}}{\beta} = \frac{V_{CC} - V_{BE} - (I_{E1} + I_{E2})R_E}{R_1} - \frac{V_{BE} + (I_{E1} + I_{E2})R_E}{R_2}$$

$$I_{C1} = \beta \frac{V_{CC} - V_T \ln(I_{C1}/I_{S1}) - \frac{3}{2}\frac{1+\beta}{\beta} I_{C1} R_E}{R_1} - \frac{V_T \ln(I_{C1}/I_{S1}) + \frac{3}{2}\frac{1+\beta}{\beta} I_{C1} R_E}{R_2}$$

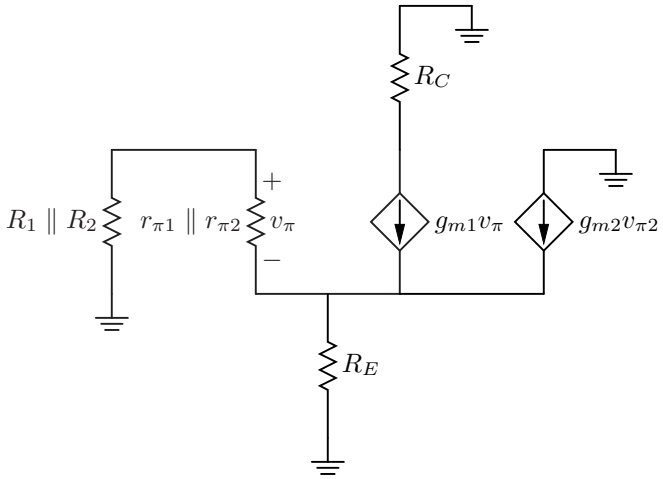
$$I_{C1} = \boxed{707\ \mu\text{A}}$$

$$I_{C2} = \frac{I_{C1}}{2} = \boxed{354\ \mu\text{A}}$$

(b) The small-signal model is shown below.




We can simplify the small-signal model as follows:



$$g_{m1} = I_{C1}/V_T = \boxed{27.2\ \text{mS}}$$
$$r_{\pi 1} = \beta_1/g_{m1} = \boxed{3.677\ \text{k}\Omega}$$
$$g_{m2} = I_{C2}/V_T = \boxed{13.6\ \text{mS}}$$
$$r_{\pi 2} = \beta_2/g_{m2} = \boxed{7.355\ \text{k}\Omega}$$

5.19 (a)

$$I_{E1} = I_{E2} \Rightarrow V_{BE1} = V_{BE2}$$

$$\frac{V_{CC} - 2V_{BE1}}{9\ \text{k}\Omega} - \frac{2V_{BE1}}{16\ \text{k}\Omega} = I_{B1} = \frac{I_{C1}}{\beta_1}$$

$$I_{C1} = \beta_1 \frac{V_{CC} - 2V_T \ln(I_{C1}/I_{S1})}{9\ \text{k}\Omega} - \beta_1 \frac{2V_T \ln(I_{C1}/I_{S1})}{16\ \text{k}\Omega}$$

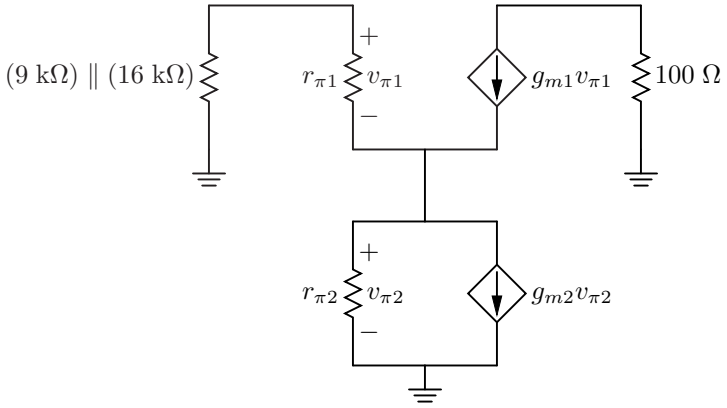
$$I_{C1} = I_{C2} = \boxed{1.588\ \text{mA}}$$

$$V_{BE1} = V_{BE2} = V_T \ln(I_{C1}/I_{S1}) = \boxed{754\ \text{mV}}$$

$$V_{CE2} = V_{BE2} = \boxed{754\ \text{mV}}$$

$$V_{CE1} = V_{CC} - I_{C1}(100\ \Omega) - V_{CE2} = \boxed{1.587\ \text{V}}$$

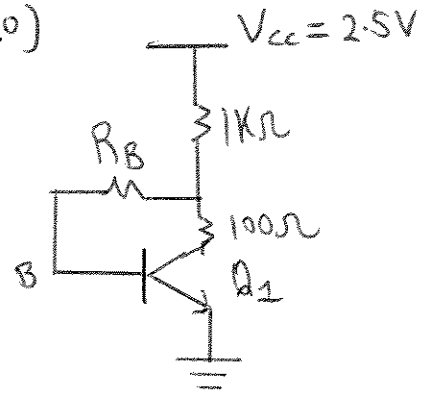
(b) The small-signal model is shown below.




$$g_{m1} = g_{m2} = \frac{I_{C1}}{V_T} = \boxed{61.1\ \text{mS}}$$

$$r_{\pi 1} = r_{\pi 2} = \frac{\beta_1}{g_{m1}} = \boxed{1.637\ \text{k}\Omega}$$

20)




$I_c = 1mA$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.750V$

$V_B = 2.5 - (I_E(1K\Omega) + I_B R_B) = 0.750V$

$I_E = 1.01mA$

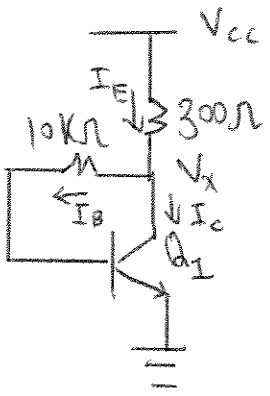
$I_B = 0.01mA$

$V_B = 2.5 - 1.01 - 0.01 R_B = 0.750$

$0.74 = 0.01 R_B$

$R_B = 74K\Omega$

21)




$V_x = 1.1V$

$\beta = 100$

$I_s = ?$

$$I_E = I_B + I_c$$

$$I_E = \frac{2.5 - 1.1}{300\Omega} = 4.67\,mA$$

$$I_B = \frac{I_c}{\beta}$$

$$I_E = \frac{I_c}{\beta} + I_c = 4.67\,mA$$

$$I_c = 4.624\,mA$$

$$I_s = \frac{I_c}{e^{\left(\frac{V_{BE}}{V_T}\right)}}, \quad V_{BE} = 1.1 - \frac{4.624}{100}(10K) = 0.6376V$$

$$I_s = 1.035 \times 10^{-10}\,mA$$

$$I_s = 1.035 \times 10^{-13}\,A$$

5.22

$$V_{CC} - I_E(500\ \Omega) - I_B(20\ \text{k}\Omega) - I_E(400\ \Omega) = V_{BE}$$

$$V_{CC} - \frac{1+\beta}{\beta} I_C(500\ \Omega + 400\ \Omega) - \frac{1}{\beta} I_C(20\ \text{k}\Omega) = V_T \ln(I_C/I_S)$$

$$I_C = \boxed{1.584\ \text{mA}}$$

$$V_{BE} = V_T \ln(I_C/I_S) = \boxed{754\ \text{mV}}$$

$$V_{CE} = V_{CC} - I_E(500\ \Omega) - I_E(400\ \Omega)$$

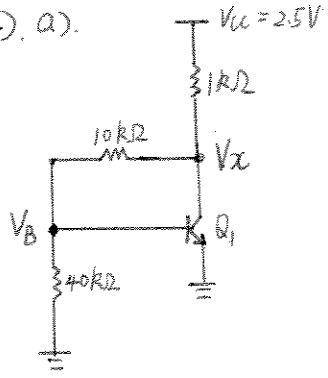
$$= V_{CC} - \frac{1+\beta}{\beta} I_C(500\ \Omega + 400\ \Omega) = \boxed{1.060\ \text{V}}$$

$Q_1$ is operating in forward active.

5.23

$$
\begin{aligned}
V_{BC} &\leq 200\text{ mV} \\
V_{CC} - I_E(1\text{ k}\Omega) - I_B R_B - (V_{CC} - I_E(1\text{ k}\Omega) - I_C(500\ \Omega)) &\leq 200\text{ mV} \\
I_C(500\ \Omega) - I_B R_B &\leq 200\text{ mV} \\
I_B R_B &\geq I_C(500\ \Omega) - 200\text{ mV} \\
V_{CC} - I_E(1\text{ k}\Omega) - I_B R_B &= V_{BE} = V_T \ln(I_C/I_S) \\
V_{CC} - \frac{1+\beta}{\beta} I_C(1\text{ k}\Omega) - I_C(500\ \Omega) + 200\text{ mV} &\leq V_T \ln(I_C/I_S) \\
I_C &\geq 1.29\text{ mA} \\
R_B &\geq \frac{I_C(500\ \Omega) - 200\text{ mV}}{\frac{I_C}{\beta}} \\
&\geq \boxed{34.46\text{ k}\Omega}
\end{aligned}
$$

24). a).




$I_S = 8 \times 10^{-16} A$

$\beta = 100$

$V_A = \infty$

$V_x = 2.5 - \left(\frac{I_C}{\alpha} + \frac{V_B}{40k}\right) \cdot 1k$

$V_x = \left(\frac{V_B}{40k} + I_B\right) 10k + V_B = \left(\frac{V_B}{40k} + \frac{I_C}{\beta}\right) 10k + V_B$

Equating $V_x \Rightarrow 2.5 - \left(V_B + \frac{V_B \cdot 1k}{40k} + \frac{V_B \cdot 10k}{40k}\right) = \frac{I_C}{\alpha} \cdot 1k + \frac{I_C}{\beta} \cdot 10k$

$\Rightarrow I_C = \frac{2.5 - 1.275 V_B}{\frac{1k}{\alpha} + \frac{10k}{\beta}}$

Guess $V_B = 0.8$

$I_C = \frac{1.48}{\frac{1k}{0.99} + \frac{10k}{100}} = 1.33\,m(A)$

Then

$V_B = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.732$, not 0.8.

Reiterate

$I_C = \frac{1.5667}{1.11} = 1.4113\,(mA)$

$V_B = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.733$

So $V_B$ converges to 0.73V

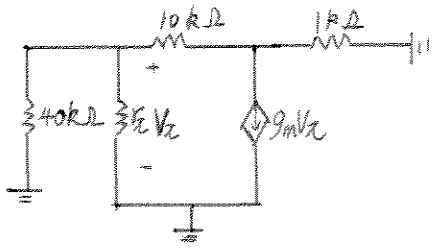
$I_C = 1.41\,mA$

$I_B = 14.1\,\mu A$

$V_{CE} = 2.5\,V - \left(\frac{1.41}{0.99} + \frac{0.73}{40}\right) \times 1V = 1.06V$

$V_{BE} = 0.73V$

24 b) Small Signal



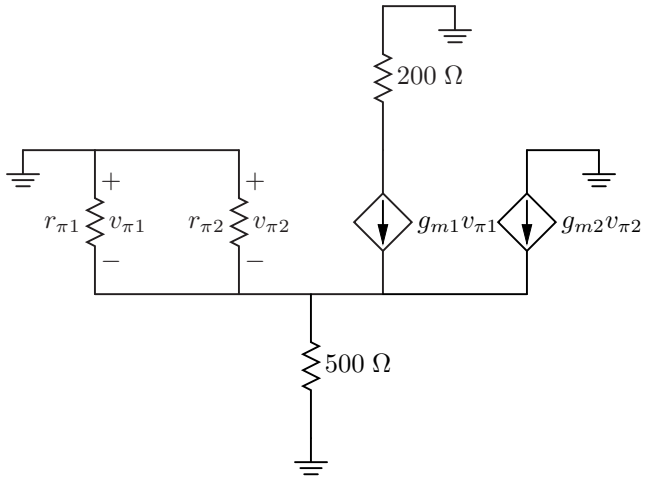

$$g_m = \frac{I_C}{V_T} = 0.054\ S$$

$$r_\pi = \frac{\beta}{g_m} = 1844\Omega$$

5.25 (a)

$$\begin{aligned} I_{C1} &= 1 \text{ mA} \\ V_{CC} - (I_{E1} + I_{E2})(500\ \Omega) &= V_T \ln(I_{C2}/I_{S2}) \\ V_{CC} - \left(\frac{1+\beta}{\beta} I_{C1} + \frac{1+\beta}{\beta} I_{C2}\right)(500\ \Omega) &= V_T \ln(I_{C2}/I_{S2}) \\ I_{C2} &= 2.42 \text{ mA} \\ V_B - (I_{E1} + I_{E2})(500\ \Omega) &= V_T \ln(I_{C1}/I_{S1}) \\ V_B - \left(\frac{1+\beta}{\beta} I_{C1} + \frac{1+\beta}{\beta} I_{C2}\right)(500\ \Omega) &= V_T \ln(I_{C1}/I_{S1}) \\ V_B &= \boxed{2.68 \text{ V}} \end{aligned}$$

(b) The small-signal model is shown below.




$$\begin{aligned} g_{m1} &= I_{C1}/V_T = \boxed{38.5 \text{ mS}} \\ r_{\pi 1} &= \beta_1/g_{m1} = \boxed{2.6 \text{ k}\Omega} \\ g_{m2} &= I_{C2}/V_T = \boxed{93.1 \text{ mS}} \\ r_{\pi 2} &= \beta_2/g_{m2} = \boxed{1.074 \text{ k}\Omega} \end{aligned}$$

5.26 (a)

$$V_{CC} - I_B(60\ \text{k}\Omega) = V_{EB}$$
$$V_{CC} - \frac{1}{\beta_{pnp}} I_C(60\ \text{k}\Omega) = V_T \ln(I_C/I_S)$$
$$I_C = \boxed{1.474\ \text{mA}}$$
$$V_{EB} = V_T \ln(I_C/I_S) = \boxed{731\ \text{mV}}$$
$$V_{EC} = V_{CC} - I_C(200\ \Omega) = \boxed{2.205\ \text{V}}$$

$Q_1$ is operating in forward active.

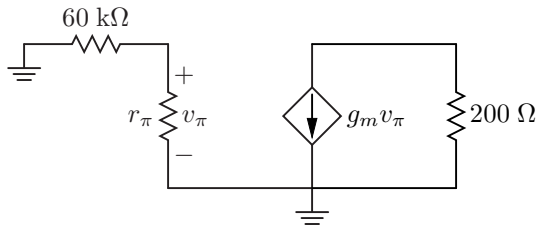
(b)

$$V_{CC} - V_{BE1} - I_{B2}(80\ \text{k}\Omega) = V_{EB2}$$
$$V_{CC} - V_T \ln(I_{C1}/I_S) - I_{B2}(80\ \text{k}\Omega) = V_T \ln(I_{C2}/I_S)$$
$$I_{C1} = \frac{\beta_{npn}}{1+\beta_{npn}} I_{E1}$$
$$= \frac{\beta_{npn}}{1+\beta_{npn}} I_{E2}$$
$$= \frac{\beta_{npn}}{1+\beta_{npn}} \cdot \frac{1+\beta_{pnp}}{\beta_{pnp}} I_{C2}$$
$$V_{CC} - V_T \ln\left(\frac{\beta_{npn}}{1+\beta_{npn}} \cdot \frac{1+\beta_{pnp}}{\beta_{pnp}} \cdot \frac{I_{C2}}{I_S}\right) - \frac{1}{\beta_{pnp}} I_{C2}(80\ \text{k}\Omega) = V_T \ln(I_{C2}/I_S)$$
$$I_{C2} = \boxed{674\ \mu\text{A}}$$
$$V_{BE2} = V_T \ln(I_{C2}/I_S) = \boxed{711\ \text{mV}}$$
$$I_{C1} = \boxed{680\ \mu\text{A}}$$
$$V_{BE1} = V_T \ln(I_{C1}/I_S) = \boxed{711\ \text{mV}}$$
$$V_{CE1} = V_{BE1} = \boxed{711\ \text{mV}}$$
$$V_{CE2} = V_{CC} - V_{CE1} - I_{C2}(300\ \Omega)$$
$$= \boxed{1.585\ \text{V}}$$

$Q_1$ is operating on the edge of saturation. $Q_2$ is operating in forward active.

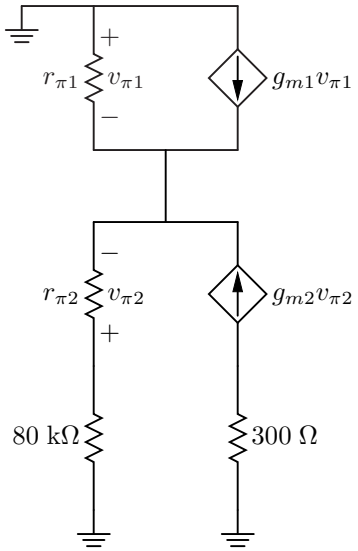
5.27 See Problem 26 for the derivation of $I_C$ for each part of this problem.

(a) The small-signal model is shown below.




$$I_C = 1.474 \text{ mA}$$
$$g_m = \frac{I_C}{V_T} = \boxed{56.7 \text{ mS}}$$
$$r_\pi = \frac{\beta}{g_m} = \boxed{1.764 \text{ k}\Omega}$$

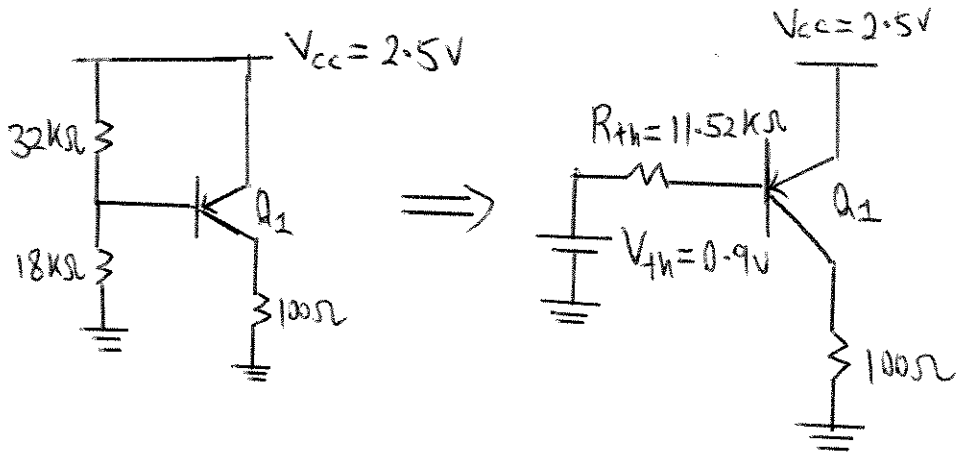
(b) The small-signal model is shown below.




$$I_{C1} = 680 \text{ μA}$$
$$g_{m1} = \frac{I_{C1}}{V_T} = \boxed{26.2 \text{ mS}}$$
$$r_{\pi 1} = \frac{\beta_{npn}}{g_{m1}} = \boxed{3.824 \text{ k}\Omega}$$
$$I_{C2} = 674 \text{ μA}$$
$$g_{m2} = \frac{I_{C2}}{V_T} = \boxed{25.9 \text{ mS}}$$
$$r_{\pi 2} = \frac{\beta_{pnp}}{g_{m2}} = \boxed{1.929 \text{ k}\Omega}$$

28)

a)




$$I_c = \beta_{pnp}\left(\frac{2.5 - |V_{BE}| - V_{th}}{R_{th}}\right)$$

Guess $|V_{BE}| = 0.7\,V$, $I_c = 3.91mA$

$$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.757V$$

Reiterate, $|V_{BE}| = 0.757V$, $I_c = 3.66mA$

$$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.755V$$

Reiterate, $|V_{BE}| = 0.755V$, $I_c = 3.67mA$

$$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.755V, \text{ converged!!}$$

$V_c = (3.67mA)(0.1k\Omega) = 0.367V$, $V_B = 2.5 - 0.755 = 1.745V$

$Q_1$ in forward active.

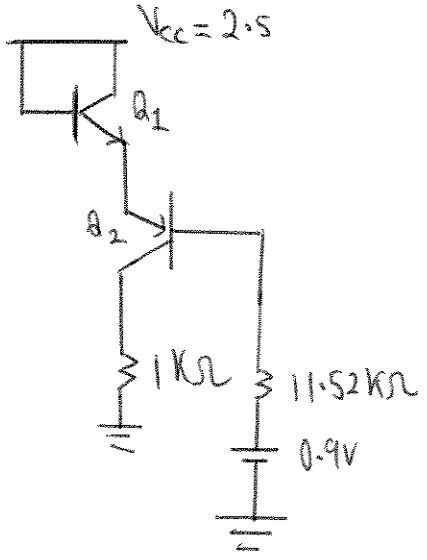
Bias point:

$I_c = 3.67mA$ $\quad |V_{BE}| = 0.755$

$I_B = 73.4\mu A$ $\quad |V_{CE}| = 2.5 - 0.367 = 2.133V$

28)

b)




$$I_{c_2} = \left(\frac{2.5 - (V_{BE_1} + V_{BE_2}) - 0.9}{11.52\,K}\right) 50$$

$I_{c_1} = I_{c_2}(1.0099)$
(From $\beta$ relation)

$$V_{BE_1} = V_T \ln\left(\frac{I_{c_1}}{I_s}\right)$$

$$|V_{BE_2}| = V_T \ln\left(\frac{I_{c_2}}{I_s}\right)$$

Guess, $V_{BE_1} = V_{BE_2} = 0.7V$

$I_{c_2} \approx 0.868\,mA$, $I_{c_1} = 0.877\,mA$

$V_{BE_1} = V_T \ln\left(\frac{I_{c_1}}{I_s}\right) = 0.718V$, $|V_{BE_2}| = V_T \ln\left(\frac{I_{c_2}}{I_s}\right) = 0.717$

Reiterate, $V_{BE_1} = 0.718V$, $|V_{BE_2}| = 0.717V$

$I_{c_2} = 0.716\,mA$, $I_{c_1} = 0.723\,mA$

$V_{BE_1} = V_T \ln\left(\frac{I_{c_1}}{I_s}\right) = 0.713V$, $|V_{BE_2}| = V_T \ln\left(\frac{I_{c_2}}{I_s}\right) = 0.712V$

Reiterate, $V_{BE_1} = 0.713V$, $|V_{BE_2}| = 0.712V$

$I_{c_2} = 0.760\,mA$, $I_{c_1} = 0.767\,mA$

$V_{BE_1} = 0.714V$, $|V_{BE_2}| = 0.714V$

28)

b)

Reiterate, $V_{BE_1} = 0.714\,V$, $|V_{BE_2}| = 0.714V$

$$I_{C_2} = 0.747mA, \quad I_{C_1} = 0.754mA$$

$$V_{BE_1} = V_t \ln\left(\frac{I_C}{I_S}\right) = 0.714V,$$

$$|V_{BE_2}| = 0.714\,V$$

$$V_{B_2} = \frac{(0.747mA)}{50}(11.52K\Omega) + 0.9 = 1.07\,V$$

$$V_{C_2} = (0.747mA)(1K\Omega) = 0.747\,V$$

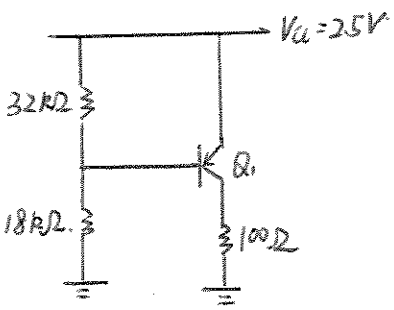
$Q_2$ is in forward-active region. $Q_1$ is always in forward-active region.

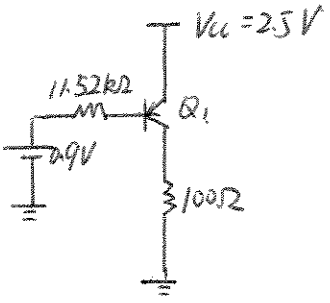
Bias point:

| | |
|---|---|
| $V_{BE_1} = 0.714V$ | $\|V_{BE_2}\| = 0.714V$ |
| $I_{C_1} = 0.754\,mA$ | $I_{C_2} = 0.747mA$ |
| $I_{B_1} = 7.54\mu A$ | $I_{B_2} = 14.94\,\mu A$ |
| $V_{CE_1} = 0.714\,V$ | $\|V_{CE_2}\| = 2.5 - 0.714 - 0.747 = 1.039\,V$ |

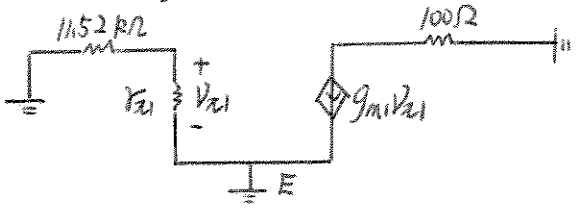
29)

a).



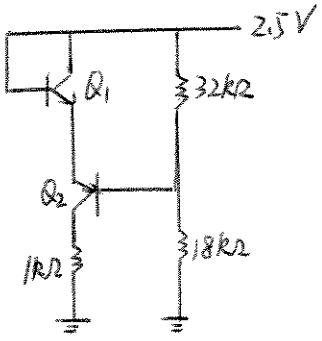

Small signal:

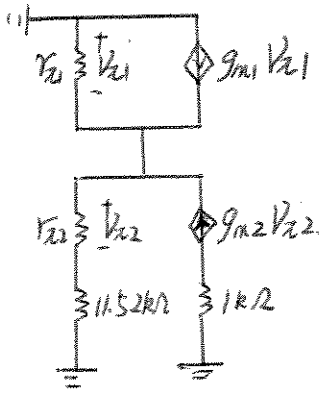



$$g_{m1} = \frac{3.67mA}{26mV} = 0.141\ S$$

$$r_\pi = \frac{50}{0.141}\ \Omega = 354.2\ \Omega$$

b).




$g_{m1} = 0.029\ S$

$r_{\pi 1} = 3448.3\ \Omega$

$g_{m2} = 0.0287\ S$

$r_{\pi 2} = 17403\ \Omega$

5.30

$$\begin{aligned}
V_{CC} - I_C(1\text{ k}\Omega) &= V_{EC} = V_{EB} \text{ (in order for } Q_1 \text{ to operate at the edge of saturation)} \\
&= V_T \ln(I_C/I_S) \\
I_C &= 1.761\text{ mA} \\
V_{EB} &= 739\text{ mV} \\
\frac{V_{CC} - V_{EB}}{R_B} - \frac{V_{EB}}{5\text{ k}\Omega} &= I_B = \frac{I_C}{\beta} \\
R_B &= 9.623\text{ k}\Omega
\end{aligned}$$

First, let's consider when $R_B$ is 5 % larger than its nominal value.

$$\begin{aligned}
R_B &= 10.104\text{ k}\Omega \\
\frac{V_{CC} - V_T \ln(I_C/I_S)}{R_B} - \frac{V_T \ln(I_C/I_S)}{5\text{ k}\Omega} &= \frac{I_C}{\beta} \\
I_C &= 1.411\text{ mA} \\
V_{EB} &= 733\text{ mV} \\
V_{EC} &= V_{CC} - I_C(1\text{ k}\Omega) = 1.089\text{ V} \\
V_{CB} &= \boxed{-355\text{ mV}} \text{ (the collector-base junction is reverse biased)}
\end{aligned}$$

Now, let's consider when $R_B$ is 5 % smaller than its nominal value.

$$\begin{aligned}
R_B &= 9.142\text{ k}\Omega \\
\frac{V_{CC} - V_T \ln(I_C/I_S)}{R_B} - \frac{V_T \ln(I_C/I_S)}{5\text{ k}\Omega} &= \frac{I_C}{\beta} \\
I_C &= 2.160\text{ mA} \\
V_{EB} &= 744\text{ mV} \\
V_{EC} &= V_{CC} - I_C(1\text{ k}\Omega) = 340\text{ mV} \\
V_{CB} &= \boxed{405\text{ mV}} \text{ (the collector-base junction is forward biased)}
\end{aligned}$$

5.31

$$\frac{V_{BC} + I_C(5\text{ k}\Omega)}{10\text{ k}\Omega} - \frac{V_{CC} - V_{BC} - I_C(5\text{ k}\Omega)}{10\text{ k}\Omega} = I_B = \frac{I_C}{\beta}$$

$$V_{BC} = 300\text{ mV}$$

$$I_C = 194\ \mu\text{A}$$

$$V_{EB} = V_T \ln(I_C/I_S) = 682\text{ mV}$$

$$V_{CC} - I_E R_E - I_C(5\text{ k}\Omega) = V_{EC} = V_{EB} + 300\text{ mV}$$

$$V_{CC} - \frac{1+\beta}{\beta} I_C R_E - I_C(5\text{ k}\Omega) = V_{EB} + 300\text{ mV}$$

$$R_E = \boxed{2.776\text{ k}\Omega}$$

Let's look at what happens when $R_E$ is halved.

$$R_E = 1.388\text{ k}\Omega$$

$$\frac{V_{CC} - I_E R_E - V_{EB}}{10\text{ k}\Omega} - \frac{V_{CC} - (V_{CC} - I_E R_E - V_{EB})}{10\text{ k}\Omega} = I_B = \frac{I_C}{\beta}$$

$$\beta\frac{V_{CC} - \frac{1+\beta}{\beta} I_C R_E - V_T \ln(I_C/I_S)}{10\text{ k}\Omega} - \beta\frac{V_{CC} - \left(V_{CC} - \frac{1+\beta}{\beta} I_C R_E - V_T \ln(I_C/I_S)\right)}{10\text{ k}\Omega} = I_C$$

$$I_C = 364\ \mu\text{A}$$

$$V_{EB} = 698\ \mu\text{V}$$

$$V_{EC} = 164\ \mu\text{V}$$

Thus, when $R_E$ is halved, $Q_1$ operates in deep saturation.

5.32

$$V_{CC} - I_B(20\ \text{k}\Omega) - I_E(1.6\ \text{k}\Omega) = V_{BE} = V_T \ln(I_C/I_S)$$

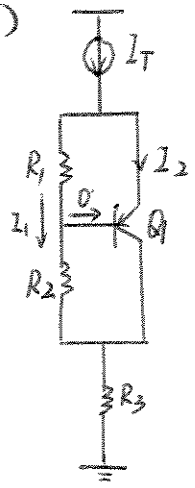
$$V_{CC} - \frac{I_C}{\beta}(20\ \text{k}\Omega) - \frac{1+\beta}{\beta} I_C(1.6\ \text{k}\Omega) = V_{BE} = V_T \ln(I_C/I_S)$$

$$I_S = \frac{I_C}{e^{\left[V_{CC} - \frac{I_C}{\beta}(20\ \text{k}\Omega) - \frac{1+\beta}{\beta} I_C(1.6\ \text{k}\Omega)\right]/V_T}}$$

$$I_C = 1\ \text{mA}$$

$$I_S = \boxed{3 \times 10^{-14}\ \text{A}}$$

33)




If Base current is neglected, $I_C = I_E$

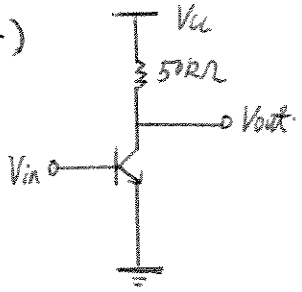
$$I_1 = \frac{V_E - V_C}{R_1 + R_2}$$

$$|V_{BE}| = I_1 R_1 = \frac{V_E - V_C}{R_1 + R_2} R_1 = \frac{|V_{CE}|}{R_1 + R_2} R_1$$

$$\text{So} \quad \frac{|V_{CE}|}{|V_{BE}|} = \frac{R_1 + R_2}{R_1}$$

Let $A = \frac{R_1 + R_2}{R_1}$, $|V_{CE}| = A\,|V_{BE}|$, thus $|V_{BE}|$ is multiplied.

34)




$A_V = g_m R_C = 20$

$\frac{I_C R_C}{V_T} = 20 \Rightarrow I_C = \frac{20 V_T}{R_C}$

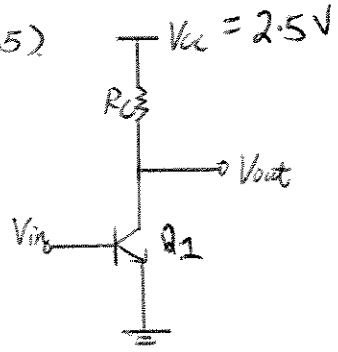
$I_C = 0.0104 mA$

$V_{CC} - (50k\Omega)(0.0104mA) = V_{BE}$

$\Rightarrow V_{CC} - 50 \times 0.0104\ V = 0.8\ V$

$\Rightarrow V_{CC} = 1.32\ V$

35)




$$V_A = 10V \;,\; r_o = \frac{V_A}{I_C} \;,\; g_m = \frac{I_c}{V_T}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = g_m (R_C // r_o) = g_m \left(\frac{R_C r_o}{R_C + r_o}\right) = \frac{R_C V_A}{V_T (R_C + \frac{V_A}{I_C})}$$

As the equation above shows, a large gain means a large $I_c$. However, a large $I_c$ will drive $Q_1$ into saturation. So a tradeoff must be made. The maximum limit for $I_c$ is when it drives $Q_1$ into the edge of saturation, namely, $V_{BE} = V_{CE}$.

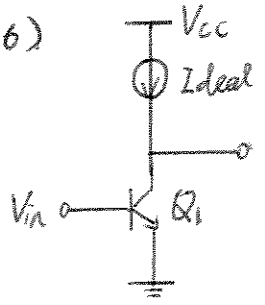
$$V_{CE} = V_{cc} - I_c (1K)$$

$$V_{BE} = 0.8V \;,\; V_{cc} = 2.5V$$

$$0.8 = 2.5 - I_c 1K$$

$$I_c = 1.7mA$$

36)



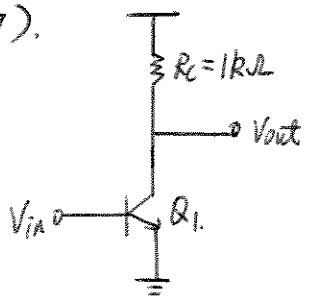

$A_v = 50$

$R_{out} = r_o = 10K\Omega$

$$A_v = g_m R_{out} = \frac{I_c}{V_T} R_{out} = 50$$

$$I_c = 50 \left( \frac{V_T}{R_{out}} \right) = 0.13mA$$

37).




$$I_c = I_s \exp\left(\frac{V_{BE}}{2V_T}\right)$$

$$g_m = \frac{\partial I_c}{\partial V_{BE}} = \frac{I_c}{2V_T}$$

$$R_{out} = R_c$$

$$\left|\frac{V_{out}}{V_{in}}\right| = g_m R_{out} = \frac{I_c R_c}{2V_T} = \frac{(1mA)(1K\Omega)}{(2)(0.026V)} = 19.23$$

5.38 (a)

$$A_v = \boxed{-g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{in} = \boxed{r_{\pi 1}}$$

$$R_{out} = \boxed{\frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(b)

$$A_v = \boxed{-g_{m1}\left(R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{in} = \boxed{r_{\pi 1}}$$

$$R_{out} = \boxed{R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

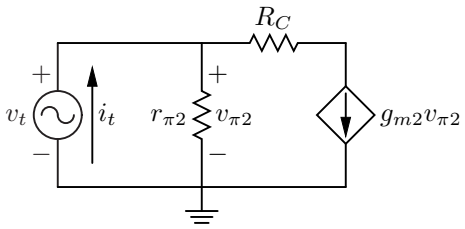
(c)

$$A_v = \boxed{-g_{m1}\left(R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{in} = \boxed{r_{\pi 1}}$$

$$R_{out} = \boxed{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(d) Let's determine the equivalent resistance seen looking up from the output by drawing a small-signal model and applying a test source.




$$i_t = \frac{v_{\pi 2}}{r_{\pi 2}} + g_{m2}v_{\pi 2}$$

$$v_{\pi 2} = v_t$$

$$i_t = v_t\left(\frac{1}{r_{\pi 2}} + g_{m2}\right)$$

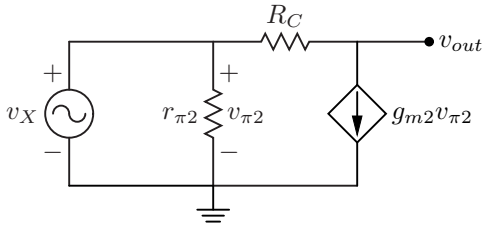
$$\frac{v_t}{i_t} = \frac{1}{g_{m2}} \parallel r_{\pi 2}$$

$$A_v = \boxed{-g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{in} = \boxed{r_{\pi 1}}$$

$$R_{out} = \boxed{\frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(e) From (d), we know the gain from the input to the collector of $Q_1$ is $-g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$. If we find the gain from the collector of $Q_1$ to $v_{out}$, we can multiply these expressions to find the overall gain. Let's draw the small-signal model to find the gain from the collector of $Q_1$ to $v_{out}$. I'll refer to the collector of $Q_1$ as node $X$ in the following derivation.



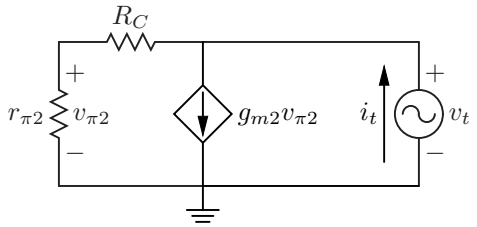

$$\begin{aligned}
\frac{v_X - v_{out}}{R_C} &= g_{m2}v_{\pi 2} \\
v_{\pi 2} &= v_X \\
\frac{v_X - v_{out}}{R_C} &= g_{m2}v_X \\
v_X\left(\frac{1}{R_C} - g_{m2}\right) &= \frac{v_{out}}{R_C} \\
\frac{v_{out}}{v_X} &= 1 - g_{m2}R_C
\end{aligned}$$

Thus, we have

$$A_v = \boxed{-g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)(1 - g_{m2}R_C)}$$

$$R_{in} = \boxed{r_{\pi 1}}$$

To find the output resistance, let's draw the small-signal model and apply a test source at the output. Note that looking into the collector of $Q_1$ we see infinite resistance, so we can exclude it from the small-signal model.



$$
\begin{aligned}
i_t &= g_{m2} v_{\pi 2} + \frac{v_{\pi 2}}{r_{\pi 2}} \\
v_{\pi 2} &= \frac{r_{\pi 2}}{r_{\pi 2} + R_C} v_t \\
i_t &= \left( g_{m2} + \frac{1}{r_{\pi 2}} \right) \frac{r_{\pi 2}}{r_{\pi 2} + R_C} v_t \\
R_{out} &= \frac{v_t}{i_t} \\
&= \boxed{\left( \frac{1}{g_{m2}} \parallel r_{\pi 2} \right) \frac{r_{\pi 2} + R_C}{r_{\pi 2}}}
\end{aligned}
$$

5.39 (a)

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}\right)}$$

$$R_{in} = \boxed{r_{\pi 1}}$$

$$R_{out} = \boxed{r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}}$$

(b)

$$A_v = \boxed{-g_{m1}\left[r_{o1} \parallel \left(R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}\right)\right]}$$

$$R_{in} = \boxed{r_{\pi 1}}$$

$$R_{out} = \boxed{r_{o1} \parallel \left(R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}\right)}$$

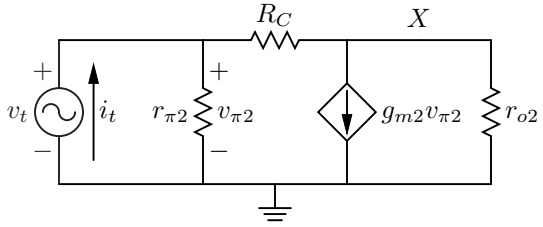
(c)

$$A_v = \boxed{-g_{m1}\left[r_{o1} \parallel \left(R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}\right)\right]}$$

$$R_{in} = \boxed{r_{\pi 1}}$$

$$R_{out} = \boxed{r_{o1} \parallel \left(R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}\right)}$$

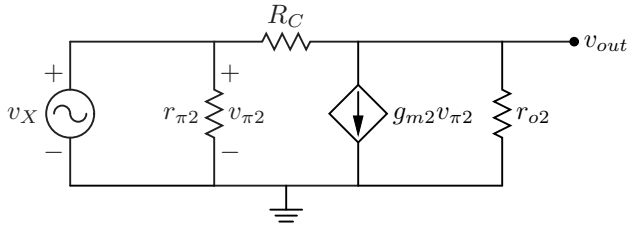
(d) Let's determine the equivalent resistance seen looking up from the output by drawing a small-signal model and applying a test source.



$$
\begin{aligned}
i_t &= \frac{v_{\pi 2}}{r_{\pi 2}} + \frac{v_t - v_X}{R_C} \\
\frac{v_X - v_t}{R_C} + g_{m2} v_{\pi 2} + \frac{v_X}{r_{o2}} &= 0 \\
v_{\pi 2} &= v_t \\
v_X \left( \frac{1}{R_C} + \frac{1}{r_{o2}} \right) &= v_t \left( \frac{1}{R_C} - g_{m2} \right) \\
v_X &= v_t \left( \frac{1}{R_C} - g_{m2} \right) (r_{o2} \parallel R_C) \\
i_t &= \frac{v_t}{r_{\pi 2}} + \frac{v_t}{R_C} - \frac{1}{R_C} v_t \left( \frac{1}{R_C} - g_{m2} \right) (r_{o2} \parallel R_C) \\
&= v_t \left[ \frac{1}{r_{\pi 2}} + \frac{1}{R_C} - \frac{1}{R_C} \left( \frac{1}{R_C} - g_{m2} \right) (r_{o2} \parallel R_C) \right] \\
&= v_t \left[ \frac{1}{r_{\pi 2}} + \frac{1}{R_C} + \left( g_{m2} - \frac{1}{R_C} \right) \frac{r_{o2}}{r_{o2} + R_C} \right] \\
\frac{v_t}{i_t} &= r_{\pi 2} \parallel R_C \parallel \left[ \frac{r_{o2} + R_C}{r_{o2}} \frac{1}{g_{m2} - \frac{1}{R_C}} \right]
\end{aligned}
$$

$$
\begin{aligned}
A_v &= \boxed{-g_{m1} \left( r_{o1} \parallel r_{\pi 2} \parallel R_C \parallel \left[ \frac{r_{o2} + R_C}{r_{o2}} \frac{1}{g_{m2} - \frac{1}{R_C}} \right] \right)} \\
R_{in} &= \boxed{r_{\pi 1}} \\
R_{out} &= \boxed{r_{o1} \parallel r_{\pi 2} \parallel R_C \parallel \left[ \frac{r_{o2} + R_C}{r_{o2}} \frac{1}{g_{m2} - \frac{1}{R_C}} \right]}
\end{aligned}
$$

(e) From (d), we know the gain from the input to the collector of $Q_1$ is $-g_{m1} \left( r_{o1} \parallel r_{\pi 2} \parallel R_C \parallel \left[ \frac{r_{o2}+R_C}{r_{o2}} \frac{1}{g_{m2} - \frac{1}{R_C}} \right] \right)$. If we find the gain from the collector of $Q_1$ to $v_{out}$, we can multiply these expressions to find the overall gain. Let's draw the small-signal model to find the gain from the collector of $Q_1$ to $v_{out}$. I'll refer to the collector of $Q_1$ as node $X$ in the following derivation.



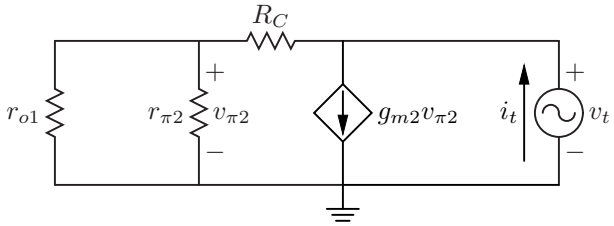
$$\begin{aligned}
\frac{v_{out}-v_X}{R_C}+g_{m2}v_{\pi 2}+\frac{v_{out}}{r_{o2}}&=0\\
v_{\pi 2}&=v_X\\
\frac{v_{out}-v_X}{R_C}+g_{m2}v_X+\frac{v_{out}}{r_{o2}}&=0\\
v_{out}\left(\frac{1}{R_C}+\frac{1}{r_{o2}}\right)&=v_X\left(\frac{1}{R_C}-g_{m2}\right)\\
\frac{v_{out}}{v_X}&=\left(\frac{1}{R_C}-g_{m2}\right)(R_C\parallel r_{o2})
\end{aligned}$$

Thus, we have

$$A_v=\boxed{-g_{m1}\left(r_{o1}\parallel r_{\pi 2}\parallel R_C\parallel\left[\frac{r_{o2}+R_C}{r_{o2}}\frac{1}{g_{m2}-\frac{1}{R_C}}\right]\right)\left(\frac{1}{R_C}-g_{m2}\right)(R_C\parallel r_{o2})}$$

$$R_{in}=\boxed{r_{\pi 1}}$$

To find the output resistance, let's draw the small-signal model and apply a test source at the output. Note that looking into the collector of $Q_1$ we see $r_{o1}$, so we replace $Q_1$ in the small-signal model with this equivalent resistance. Also note that $r_{o2}$ appears from the output to ground, so we can remove it from this analysis and add it in parallel at the end to find $R_{out}$.




$$\begin{aligned}
i_t&=g_{m2}v_{\pi 2}+\frac{v_{\pi 2}}{r_{\pi 2}\parallel r_{o1}}\\
v_{\pi 2}&=\frac{r_{\pi 2}\parallel r_{o1}}{r_{\pi 2}\parallel r_{o1}+R_C}v_t\\
i_t&=\left(g_{m2}+\frac{1}{r_{\pi 2}\parallel r_{o1}}\right)\frac{r_{\pi 2}\parallel r_{o1}}{r_{\pi 2}\parallel r_{o1}+R_C}v_t\\
R_{out}&=r_{o2}\parallel\frac{v_t}{i_t}\\
&=\boxed{r_{o2}\parallel\left[\left(\frac{1}{g_{m2}}\parallel r_{\pi 2}\parallel r_{o1}\right)\frac{r_{\pi 2}\parallel r_{o1}+R_C}{r_{\pi 2}\parallel r_{o1}}\right]}
\end{aligned}$$

40)

Gain of a degenerated CE stage ($V_A = \infty$)

$$A_V = \frac{-R_C}{\frac{1}{g_m} + R_E} = \frac{-R_C g_m}{1 + R_E g_m}$$

$$\frac{\partial A_V}{\partial I_C} = R_C \left( \frac{g_m R_E}{(1 + R_E g_m)^2} \frac{\partial g_m}{\partial I_C} - \frac{\partial g_m / \partial I_C}{1 + g_m R_E} \right)$$

$$\frac{\partial g_m}{\partial I_C} = \frac{1}{V_T} = \frac{1}{26mV} = 38.46 \left(\frac{1}{V}\right)$$

a) $g_m R_E = 3$

$$\frac{\partial A_V}{\partial I_C} = R_C (-2.404) \quad , \quad \partial I_C = 0.1 I_C$$

$$\partial A_V = -R_C I_C (0.24)$$

$$\text{Relative Change in gain} = \frac{\partial A_V}{A_V} = \frac{-0.24 (R_C I_C)}{\frac{-R_C I_C}{V_T (1 + R_E g_m)}} = 2.5\%$$

40)

b) $g_m R_E = 7$

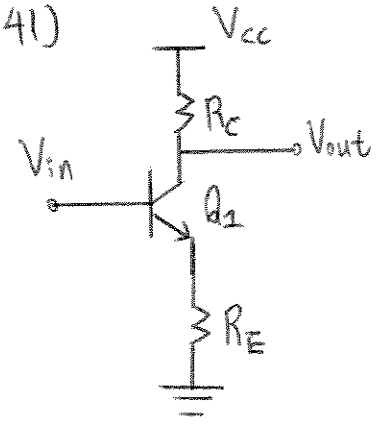
$$\frac{\partial A_v}{\partial I_c} = -R_c \, 0.6$$

$$\partial A_v = -R_c I_c (0.06)$$

Relative change in gain

$$\frac{\partial A_v}{A_v} = \frac{-0.06 (R_c I_c)}{\frac{-R_c I_c}{V_T (1 + R_E g_m)}} = 1.25\%$$

41)




$V_A = \infty$

$R_C I_C = 20 V_T$

$R_E I_C = 5 V_T$

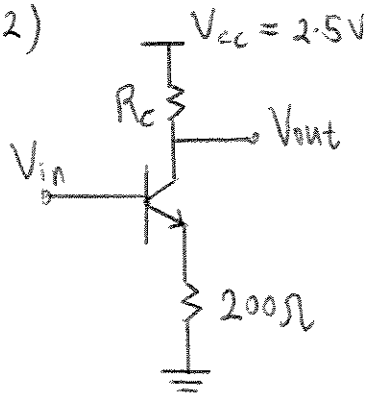
$$|A_v| = \frac{R_C}{R_E + \frac{1}{g_m}} = \frac{R_C}{R_E + \frac{V_T}{I_C}} = \frac{R_C I_C}{R_E I_C + V_T}$$

Assume $\beta$ is large, so $I_C = I_E$.

$R_C I_C = 20 V_T$, $R_E I_C = 5 V_T$

$$|A_v| = \frac{20 V_T}{5 V_T + V_T} = \frac{20 V_T}{6 V_T} = 3.33$$

42)




$$|A_v| = \frac{R_c I_c}{R_E I_c + V_T} = 10$$

Edge of Saturation

$$V_{CE} = V_{BE} = 2.5 - I_c(R_c + R_E)$$

$V_{BE} = 0.8V \Rightarrow I_c R_c = 1.7 - I_c 0.2$ (operating point)

$|A_v| = 10 \Rightarrow R_c I_c = 10(R_E I_c + V_T)$ (Gain Equation)

Equating the two equations above $\Rightarrow$

$1.7 - 0.2 I_c = 2 I_c + 0.26 \Rightarrow I_c = 0.655mA$

Check for $V_{BE} \Rightarrow V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.725$, not 0.8, Reiterate

$I_c R_c = 1.775 - I_c 0.2$ (operating point)

$I_c R_c = 2 I_c + 0.26$ (Gain equation)

Equating the two equations $\Rightarrow I_c = 0.689mA$

Check for $V_{BE} \Rightarrow V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.727V$, iterate 1 more time

$I_c R_c = 1.773 - I_c 0.2$ (operating point)

$I_c R_c = 2 I_c + 0.26$ (Gain equation)

42)

Equating the two equations $\Rightarrow$ $I_c = 0.688\,mA$

Check for $V_{BE}$ $\Rightarrow$ $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.727V$, converged

$I_c = 0.688mA$

$$R_c = \frac{2I_c + 0.26}{I_c} = \frac{(2)(0.688) + 0.26}{0.688}$$

$R_c = 2.38K\Omega$

$R_{in} = r_\pi + (1+\beta) R_E$

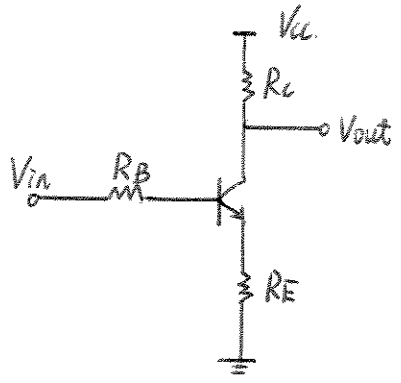
$$R_{in} = \frac{\beta}{g_m} + (101)(0.2) = 24.0K\Omega$$

5.43

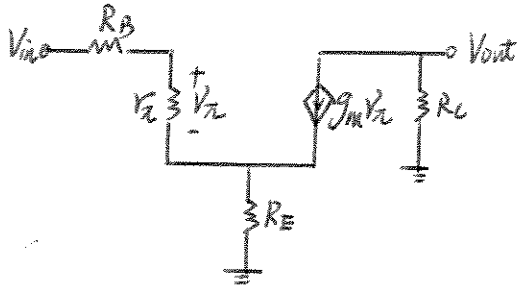
$$\begin{aligned}
A_v &= -\frac{R_C}{\frac{1}{g_m} + (200\ \Omega)} \\
&= -\frac{R_C}{\frac{V_T}{I_C} + (200\ \Omega)} \\
&= -100 \\
R_C &= 100\frac{V_T}{I_C} + 100(200\ \Omega) \\
I_C R_C - I_E(200\ \Omega) &= V_{CE} = V_{BE} = V_T \ln(I_C/I_S) \\
I_C\left(100\frac{V_T}{I_C} + 100(200\ \Omega)\right) - \frac{1+\beta}{\beta} I_C(200\ \Omega) &= V_T \ln(I_C/I_S)
\end{aligned}$$

We can see that this equation has no solution. For example, if we let $I_C = 0$, we see that according to the left side, we should have $V_{BE} = 2.6$ V, which is clearly an infeasible value. Qualitatively, we know that in order to achieve a large gain, we need a large value for $R_C$. However, increasing $R_C$ will result in a smaller value of $V_{CE}$, eventually driving the transistor into saturation. When $A_v = -100$, there is no value of $R_C$ that will provide such a large gain without driving the transistor into saturation.

44) $V_A = \infty$




Small Signal →



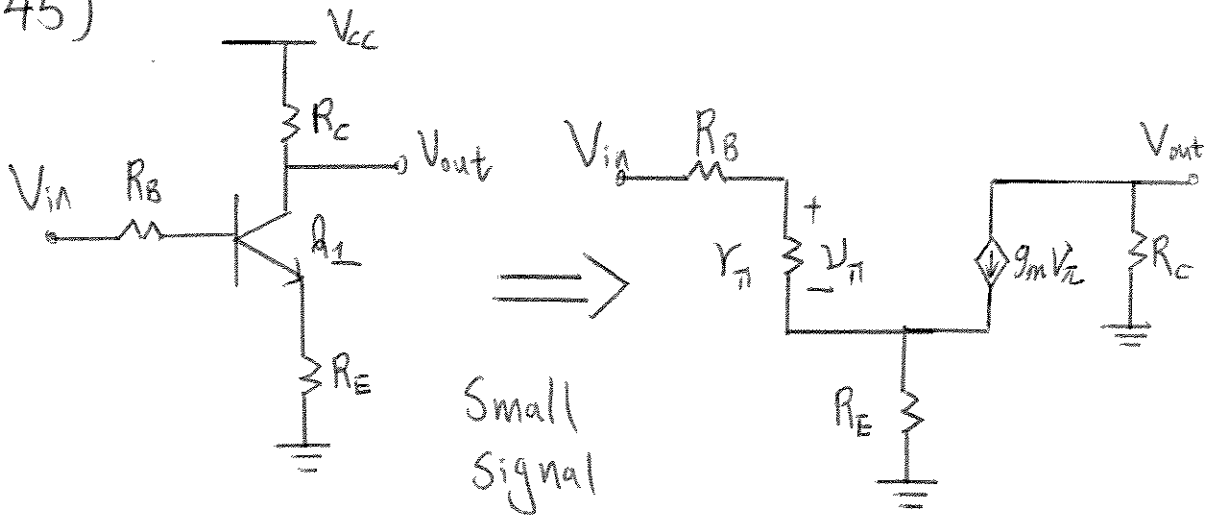

$$V_{out} = -g_m V_\pi R_C$$

$$V_\pi = \frac{V_{in} r_\pi}{R_B + r_\pi + (\beta+1) R_E}$$

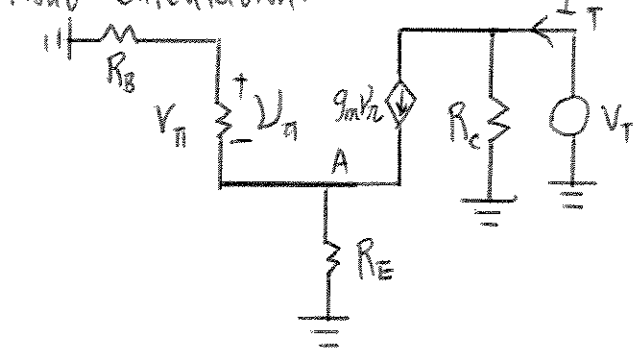
$$V_{out} = \frac{-g_m r_\pi R_C V_{in}}{R_B + r_\pi + (\beta+1) R_E} = \frac{-\beta R_C V_{in}}{R_B + r_\pi + (\beta+1) R_E} = \frac{-R_C V_{in}}{\frac{R_B}{\beta} + \frac{1}{g_m} + \frac{\beta+1}{\beta} R_E}$$

$$\frac{V_{out}}{V_{in}} \approx \frac{-R_C}{\frac{R_B}{\beta+1} + \frac{1}{g_m} + R_E}$$

45)




$R_{out}$ Calculation:




$$V_A = g_m v_\pi (R_E // R_B + r_\pi) \quad (1)$$

$$v_\pi = \frac{-V_A r_\pi}{r_\pi + R_B} \Rightarrow V_A = \frac{-v_\pi (r_\pi + R_B)}{r_\pi} \quad (2)$$

The only possible solution for 1) and 2) is $v_\pi = V_A = 0$, since 1) is positive and 2) is negative.

$$v_\pi = 0 \Rightarrow g_m v_\pi \Rightarrow 0 \Rightarrow \frac{V_T}{I_T} = R_C$$

Therefore, $R_{out} = R_C$

5.46 (a)

$$A_v = \boxed{-\frac{R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1) R_E}$$

$$R_{out} = \boxed{R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(b)

$$A_v = \boxed{-\frac{R_C}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}} \parallel r_{\pi 2}}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{out} = \boxed{R_C}$$

(c)

$$A_v = \boxed{-\frac{R_C}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}} \parallel r_{\pi 2}}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{out} = \boxed{R_C}$$

(d)

$$A_v = \boxed{-\frac{R_C}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}} \parallel r_{\pi 2} + \frac{R_B}{1+\beta_1}}}$$

$$R_{in} = \boxed{R_B + r_{\pi 1} + (1+\beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{out} = \boxed{R_C}$$

(e)

$$A_v = \boxed{-\frac{R_C}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}} \parallel r_{\pi 2} + \frac{R_B}{1+\beta_1}}}$$

$$R_{in} = \boxed{R_B + r_{\pi 1} + (1+\beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$R_{out} = \boxed{R_C}$$

5.47 (a)

$$A_v = \boxed{-\frac{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1) R_E}$$

$$R_{out} = \boxed{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(b)

$$A_v = -\frac{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E} \cdot \frac{\frac{1}{g_{m2}} \parallel r_{\pi 2}}{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

$$= \boxed{-\frac{\frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1) R_E}$$

$$R_{out} = \boxed{\frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(c)

$$A_v = \boxed{-\frac{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + \frac{1}{g_{m3} \parallel r_{\pi 3}}}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1) \left( \frac{1}{g_{m3} \parallel r_{\pi 3}} \right)}$$

$$R_{out} = \boxed{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

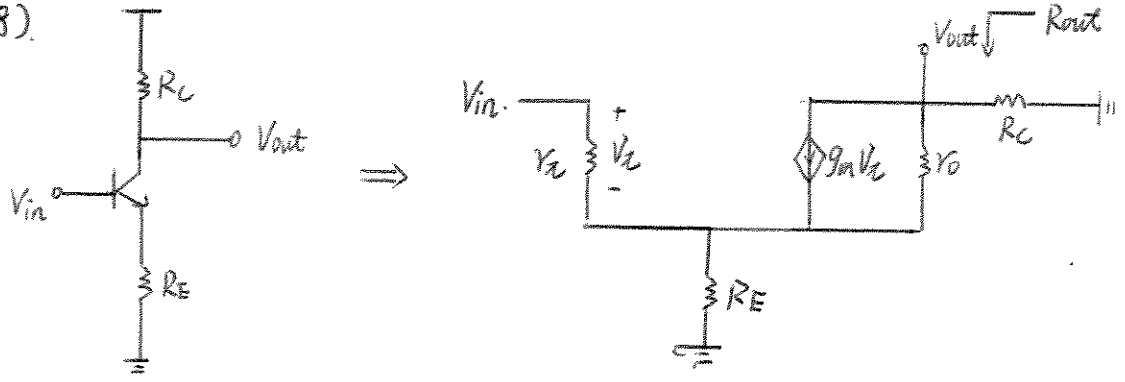
(d)

$$A_v = \boxed{-\frac{R_C \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1) R_E}$$

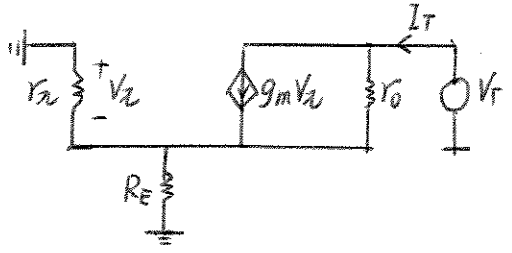
$$R_{out} = \boxed{R_C \parallel r_{\pi 2}}$$

48)




$R_{out} = R_C \,//\, R_{eq}$

Solve for $R_{eq}$.




$$I_T = g_m v_\pi + \frac{(V_T + v_\pi)}{r_o}$$

$$v_\pi = -I_T (r_\pi \,//\, R_E)$$

$$I_T = -g_m I_T (r_\pi \,//\, R_E) + \frac{(V_T - I_T (r_\pi \,//\, R_E))}{r_o}$$

$$\frac{V_T}{I_T} = r_o \left(1 + \frac{(r_\pi \,//\, R_E)}{r_o}\right) + g_m (r_\pi \,//\, R_E)\Big)$$

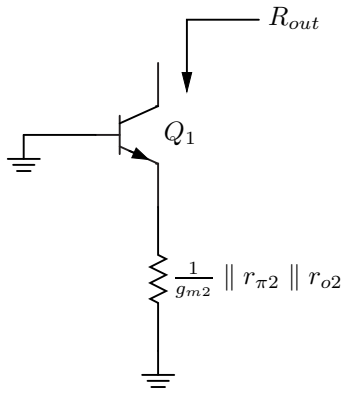
$$\frac{V_T}{I_T} = r_o + (1 + g_m r_o)(r_\pi \,//\, R_E)$$

$$R_{eq} = r_o + (1 + g_m r_o)(r_\pi \,//\, R_E)$$

$$R_{out} = R_C \,//\, r_o + (1 + g_m r_o)(r_\pi \,//\, R_E)$$

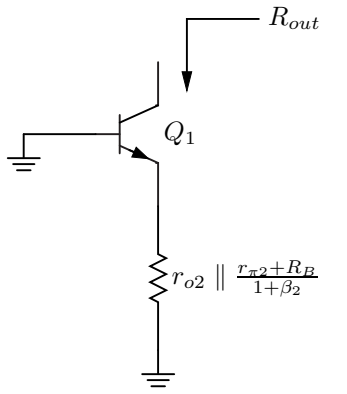
$$R_{out} \approx R_C \,//\, r_o (1 + g_m (r_\pi \,//\, R_E)) \quad \text{since } g_m r_o \gg 1$$

5.49 (a) Looking into the emitter of $Q_2$ we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2}$, so we can draw the following equivalent circuit for finding $R_{out}$:



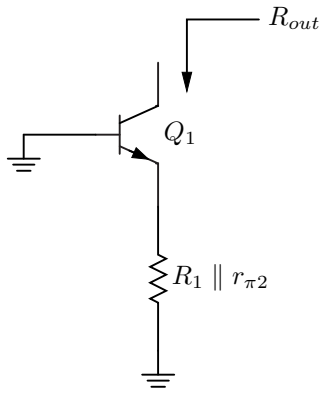

$$R_{out} = \boxed{r_{o1} + (1 + g_{m1} r_{o1}) \left( r_{\pi 1} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2} \parallel r_{o2} \right)}$$

(b) Looking into the emitter of $Q_2$ we see an equivalent resistance of $r_{o2} \parallel \frac{r_{\pi 2} + R_B}{1+\beta_2}$ ($r_{o2}$ simply appears in parallel with the resistance seen when $V_A = \infty$), so we can draw the following equivalent circuit for finding $R_{out}$:



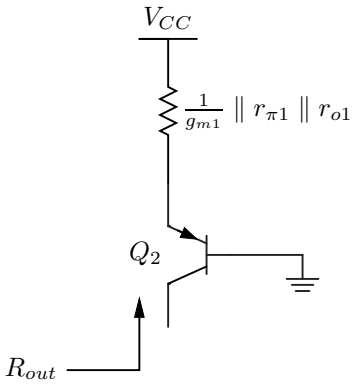

$$R_{out} = \boxed{r_{o1} + (1 + g_{m1} r_{o1}) \left( r_{\pi 1} \parallel r_{o2} \parallel \frac{r_{\pi 2} + R_B}{1 + \beta_2} \right)}$$

(c) Looking down from the emitter of $Q_1$ we see an equivalent resistance of $R_1 \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding $R_{out}$:



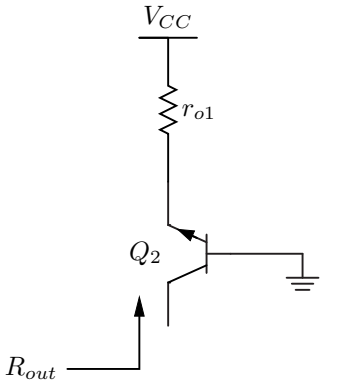

$$R_{out} = \boxed{r_{o1} + (1 + g_{m1} r_{o1}) \left( r_{\pi 1} \parallel R_1 \parallel r_{\pi 2} \right)}$$

5.50 (a) Looking into the emitter of $Q_1$ we see an equivalent resistance of $\frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel r_{o1}$, so we can draw the following equivalent circuit for finding $R_{out}$:




$$R_{out} = \boxed{r_{o2} + (1 + g_{m2} r_{o2}) \left( r_{\pi 2} \parallel \frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel r_{o1} \right)}$$

(b) Looking into the emitter of $Q_1$ we see an equivalent resistance of $r_{o1}$, so we can draw the following equivalent circuit for finding $R_{out}$:




$$R_{out} = \boxed{r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel r_{o1})}$$

Comparing this to the solution to part (a), we can see that the output resistance is larger because instead of a factor of $1/g_{m1}$ dominating the parallel resistors in the expression, $r_{\pi 2}$ dominates (assuming $r_{o1} \gg r_{\pi 2}$).

51). $r_\pi = \beta V_T / I_C$.

$$R_{in} = r_\pi \,//\, R_B = \frac{\frac{\beta V_T}{I_C} R_B}{\frac{\beta V_T}{I_C} + R_B} = \frac{V_T R_B}{V_T + \frac{I_C}{\beta} R_B} = \frac{V_T R_B}{V_T + I_B R_B}$$

Since $I_B R_B >> V_T \Rightarrow R_{in} \approx \frac{V_T R_B}{I_B R_B} = \frac{V_T}{I_B} = \frac{V_T}{\frac{I_C}{\beta}} = \frac{\beta V_T}{I_C} \approx r_\pi$

So $R_{in} = r_\pi \,//\, R_B \approx r_\pi$.

5.52 (a)

$$V_{CC} - I_B(100\ \text{k}\Omega) - I_E(100\ \Omega) = V_{BE} = V_T \ln(I_C/I_S)$$
$$V_{CC} - \frac{1}{\beta} I_C(100\ \text{k}\Omega) - \frac{1+\beta}{\beta} I_C(100\ \Omega) = V_T \ln(I_C/I_S)$$
$$I_C = 1.6\ \text{mA}$$
$$A_v = -\frac{1\ \text{k}\Omega}{\frac{1}{g_m} + 100\ \Omega}$$
$$g_m = 61.6\ \text{mS}$$
$$A_v = \boxed{-8.60}$$

(b)

$$V_{CC} - I_B(50\ \text{k}\Omega) - I_E(2\ \text{k}\Omega) = V_T \ln(I_C/I_S)$$
$$I_C = 708\ \mu\text{A}$$
$$A_v = -\frac{1\ \text{k}\Omega}{\frac{1}{g_m} + \frac{(1\ \text{k}\Omega)\|(50\ \text{k}\Omega)}{1+\beta}}$$
$$g_m = 27.2\ \text{mS}$$
$$A_v = \boxed{-21.54}$$

(c)

$$I_B = \frac{I_C}{\beta} = \frac{V_{CC} - V_{BE} - I_E(2.5\ \text{k}\Omega)}{14\ \text{k}\Omega} - \frac{V_{BE} + I_E(2.5\ \text{k}\Omega)}{11\ \text{k}\Omega}$$
$$I_C = \beta \frac{V_{CC} - V_T \ln(I_C/I_S) - \frac{1+\beta}{\beta} I_C(2.5\ \text{k}\Omega)}{14\ \text{k}\Omega} - \beta \frac{V_T \ln(I_C/I_S) + \frac{1+\beta}{\beta} I_C(2.5\ \text{k}\Omega)}{11\ \text{k}\Omega}$$
$$I_C = 163\ \mu\text{A}$$
$$A_v = -\frac{10\ \text{k}\Omega}{\frac{1}{g_m} + 500\ \Omega + \frac{(1\ \text{k}\Omega)\|(14\ \text{k}\Omega)\|(11\ \text{k}\Omega)}{1+\beta}}$$
$$g_m = 6.29\ \text{mS}$$
$$A_v = \boxed{-14.98}$$

5.53 (a)

$$\begin{aligned} I_C &= \frac{V_{CC} - 1.5\text{ V}}{R_C} \\ &= 4\text{ mA} \\ V_{BE} &= V_T \ln(I_C/I_S) = 832\text{ mV} \\ I_B &= \frac{V_{CC} - V_{BE}}{R_B} = 66.7\ \mu\text{A} \\ \beta &= \frac{I_C}{I_B} = \boxed{60} \end{aligned}$$

(b) Assuming the speaker has an impedance of 8 Ω, the gain of the amplifier is

$$\begin{aligned} A_v &= -g_m\,(R_C \parallel 8\ \Omega) \\ &= -\frac{I_C}{V_T}\,(R_C \parallel 8\ \Omega) \\ &= \boxed{-1.19} \end{aligned}$$

Thus, the circuit provides greater than unity gain.

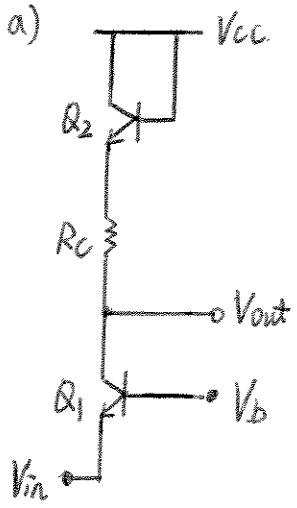
5.54 (a)

$$A_v = g_m R_C$$
$$g_m = \frac{I_C}{V_T} = 76.9 \text{ mS}$$
$$A_v = \boxed{38.46}$$
$$R_{in} = \frac{1}{g_m} \parallel r_\pi$$
$$r_\pi = \frac{\beta}{g_m} = 1.3 \text{ k}\Omega$$
$$R_{in} = \boxed{12.87 \ \Omega}$$
$$R_{out} = R_C = \boxed{500 \ \Omega}$$

(b) Since $A_v = g_m R_C$ and $g_m$ is fixed for a given value of $I_C$, $R_C$ should be chosen as large as possible to maximize the gain of the amplifier. $V_b$ should be chosen as small as possible to maximize the headroom of the amplifier (since in order for $Q_1$ to remain in forward active, we require $V_b < V_{CC} - I_C R_C$).

55) $V_A = \infty$

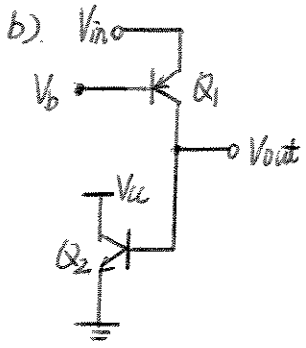
a)




$$|A_V| = \frac{R_C + \frac{1}{g_{m2}} // r_{\pi 2}}{\frac{1}{g_{m1}}}$$

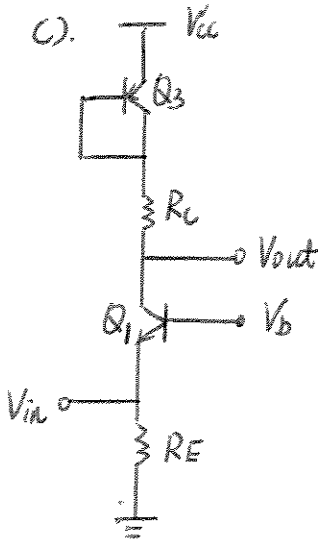
$$= g_{m1}\left(R_C + \frac{1}{g_{m2}} // r_{\pi 2}\right)$$

b).




$$|A_V| = \frac{r_{\pi 2}}{\frac{1}{g_{m1}}} = g_{m1} r_{\pi 2}$$

c).

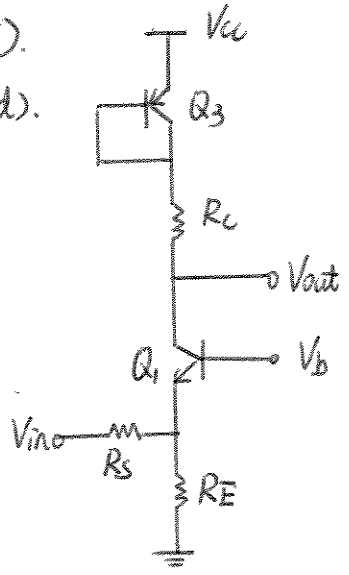



$$|A_V| = \frac{R_C + \frac{1}{g_{m3}} // r_{\pi 3}}{\frac{1}{g_{m1}}}$$

$$= g_{m1}\left(R_C + \frac{1}{g_{m3}} // r_{\pi 3}\right)$$

55).

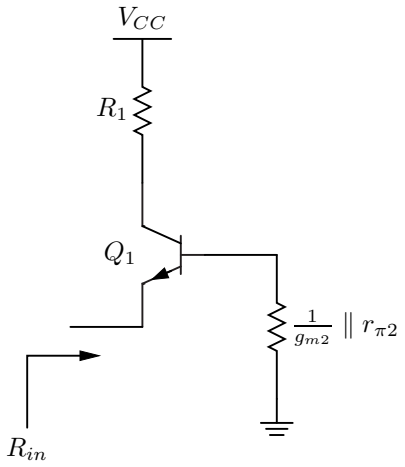
d).




$$|A_V| = \left| \frac{V_{out}}{V_A} \right| \left| \frac{V_A}{V_{in}} \right|$$

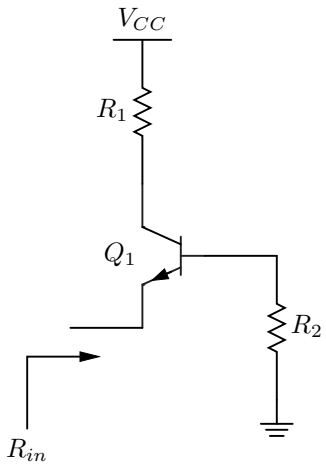
$$= \left[ g_{m1} \left( R_C + \frac{1}{g_{m3}} // r_{z3} \right) \right] \left( \frac{R_E // \frac{1}{g_{m1}}}{R_E // \frac{1}{g_{m1}} + R_S} \right)$$

5.56 (a) Looking into the emitter of $Q_2$ we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding $R_{in}$:



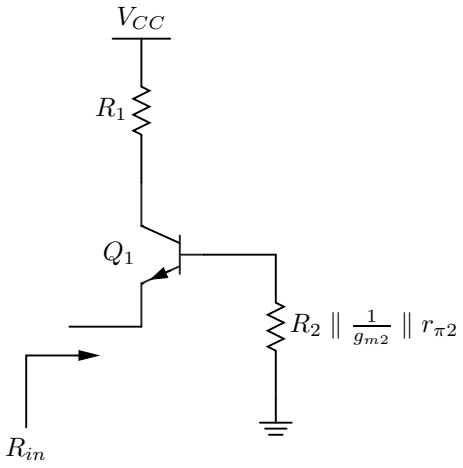

$$R_{in} = \boxed{\frac{r_{\pi 1} + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{1 + \beta_1}}$$

(b) Looking right from the base of $Q_1$ we see an equivalent resistance of $R_2$, so we can draw the following equivalent circuit for finding $R_{in}$:



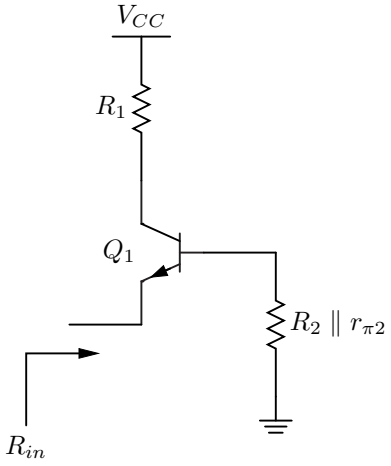

$$R_{in} = \boxed{\frac{r_{\pi 1} + R_2}{1 + \beta_1}}$$

(c) Looking right from the base of $Q_1$ we see an equivalent resistance of $R_2 \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding $R_{in}$:



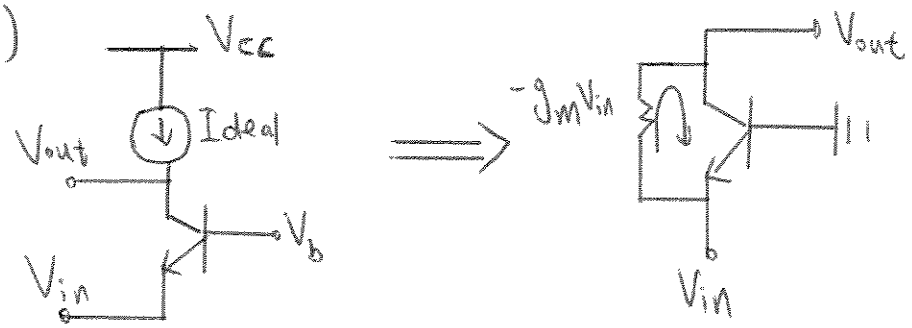

$$R_{in} = \boxed{\frac{r_{\pi 1} + R_2 \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}{1 + \beta_1}}$$

(d) Looking right from the base of $Q_1$ we see an equivalent resistance of $R_2 \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding $R_{in}$:




$$R_{in} = \boxed{\frac{r_{\pi 1} + R_2 \parallel r_{\pi 2}}{1 + \beta_1}}$$

57)




Since an ideal current source is an open circuit, the signal current produced by the transistor has no where to go but $r_o$.

So $V_{out} = -(g_m(0 - V_{in}))r_o + V_{in}$

$V_{out} = g_m r_o V_{in} + V_{in}$

$V_{out} = V_{in}(g_m r_o + 1)$

$\frac{V_{out}}{V_{in}} = 1 + g_m r_o$

5.58 (a)

$$I_B = \frac{I_C}{\beta} = \frac{V_{CC} - V_{BE} - I_E(400\ \Omega)}{13\ \text{k}\Omega} - \frac{V_{BE} + I_E(400\ \Omega)}{12\ \text{k}\Omega}$$

$$I_C = \beta\frac{V_{CC} - V_T \ln(I_C/I_S) - \frac{1+\beta}{\beta}I_C(400\ \Omega)}{13\ \text{k}\Omega} - \beta\frac{V_T \ln(I_C/I_S) + \frac{1+\beta}{\beta}I_C(400\ \Omega)}{12\ \text{k}\Omega}$$

$$I_C = \boxed{1.02\ \text{mA}}$$

$$V_{BE} = V_T \ln(I_C/I_S) = \boxed{725\ \text{mV}}$$

$$V_{CE} = V_{CC} - I_C(1\ \text{k}\Omega) - I_E(400\ \Omega) = \boxed{1.07\ \text{V}}$$

$Q_1$ is operating in forward active.

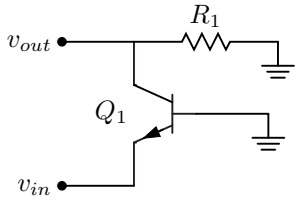
(b)

$$A_v = g_m(1\ \text{k}\Omega)$$

$$g_m = 39.2\ \text{mS}$$

$$A_v = \boxed{39.2}$$

5.61 For small-signal analysis, we can draw the following equivalent circuit.




$$A_v = \boxed{g_m R_1}$$

$$R_{in} = \boxed{\frac{1}{g_m} \parallel r_\pi}$$

$$R_{out} = \boxed{R_1}$$

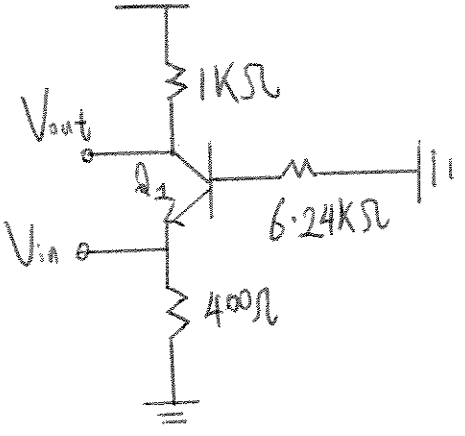
59)

$C_B = 0$

a) Since $C_B$ was not considered during DC analysis, it has no effect on operating point analysis. So it is still the same as 58).

$V_{BE} = 0.725\,V$
$I_c = 1.0163\,mA$
$I_B = 10.163\,\mu A$
$V_{CE} = 1.07\,V$

b) Since capacitor is frequency dependent, the circuit's AC analysis will be different.




$$|A_v| = \frac{1k}{\frac{1}{g_m} + \frac{6.24K\Omega}{\beta+1}} = 11.4$$

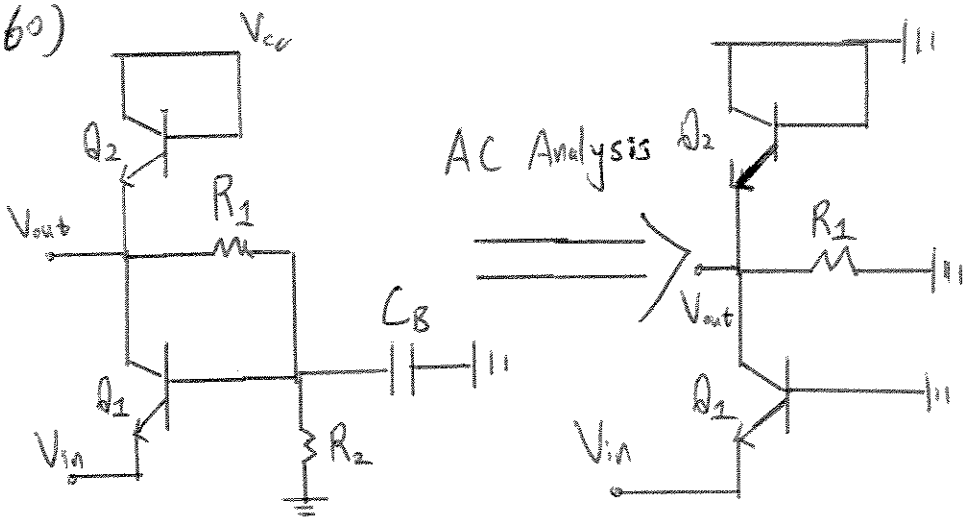
$$R_{in} = 400\Omega \,//\, \left(\frac{1}{g_m} + \frac{6.24K\Omega}{\beta+1}\right)$$

$$R_{in} = 71.7\Omega$$

$$R_{out} = 1K\Omega$$

Note: 6.24 KΩ is $R_{THEV}$ of 13 KΩ and 12 KΩ Combination.

60)

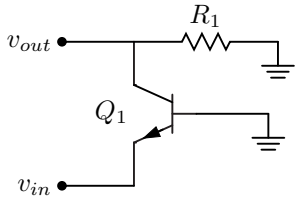



$$R_{out} = \frac{1}{g_{m_2}} // r_{\pi_2} // R_1 \approx \frac{1}{g_{m_2}} // R_1$$

$$A_v = \left|\frac{V_{out}}{V_{in}}\right| = g_{m_1}\left(\frac{1}{g_{m_2}} // r_{\pi_2} // R_1\right) \approx g_{m_1}\left(\frac{1}{g_{m_2}} // R_1\right)$$

$$R_{in} = \frac{1}{g_m} // r_{\pi_1} \approx \frac{1}{g_{m_1}}$$

5.61 For small-signal analysis, we can draw the following equivalent circuit.

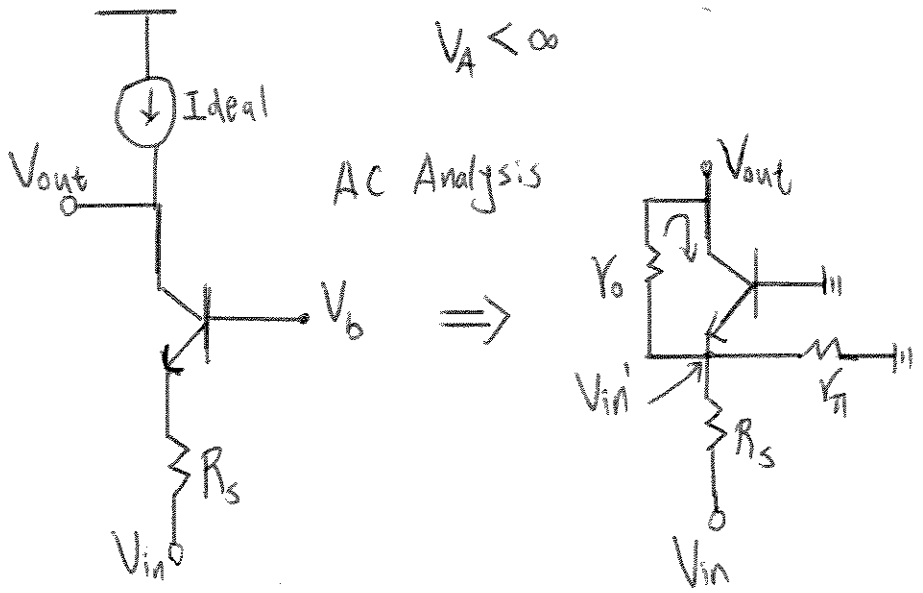



$$A_v = \boxed{g_m R_1}$$

$$R_{in} = \boxed{\frac{1}{g_m} \parallel r_\pi}$$

$$R_{out} = \boxed{R_1}$$

62)




$$A_v = \frac{V_{out}}{V_{in}} = \left(\frac{V_{in}'}{V_{in}}\right)\left(\frac{V_{out}}{V_{in}'}\right), \quad \left(\frac{V_{in}'}{V_{in}}\right) = \frac{r_\pi}{r_\pi + R_s}$$

Since $V_{out}$ is float, so looking at emitter and $r_o$, We will see an infinite impedance.

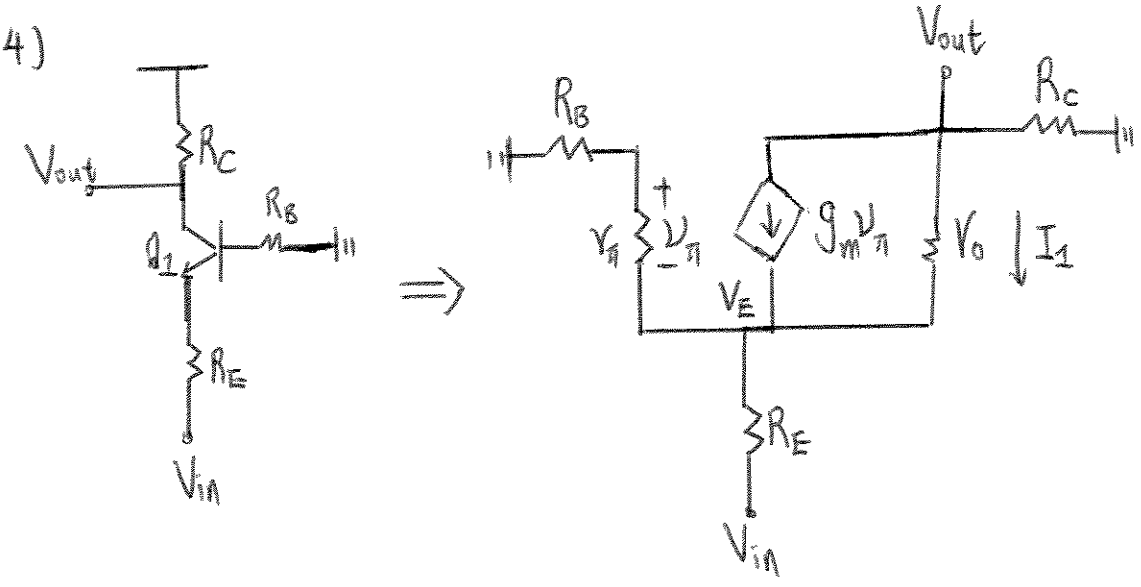
$$\frac{V_{out}}{V_{in}'} \Rightarrow -g_m(-V_{in}')r_o + V_{in}' = V_{out} \Rightarrow \frac{V_{out}}{V_{in}'} = (g_m r_o + 1)$$

$$A_v = (g_m r_o + 1)\left(\frac{r_\pi}{r_\pi + R_s}\right).$$

5.63 Since $I_{S1} = 2I_{S2}$ and they're biased identically, we know that $I_{C1} = 2I_{C2}$, which means $g_{m1} = 2g_{m2}$.

$$\frac{v_{out1}}{v_{in}} = g_{m1}R_C = 2g_{m2}R_C$$
$$\frac{v_{out2}}{v_{in}} = g_{m2}R_C$$
$$\Rightarrow \boxed{\frac{v_{out1}}{v_{in}} = 2\frac{v_{out2}}{v_{in}}}$$

64)




$$V_{out} = -(I_1 + g_m v_\pi) R_c \quad , \quad I_1 = \frac{V_{out} - V_E}{r_o}$$

$$V_{out} = -\left(\frac{V_{out} - V_E}{r_o} + g_m v_\pi\right) R_c \quad , \quad V_E = -\frac{g_m v_\pi}{\beta}(r_\pi + R_B)$$

$$V_{out} = -\left(\frac{V_{out} + \frac{g_m v_\pi}{\beta}(r_\pi + R_B)}{r_o} + g_m v_\pi\right) R_c$$

Rearranging

$$v_\pi = \frac{-\left(1 + \frac{R_c}{r_o}\right)}{\frac{g_m (r_\pi + R_B) R_c}{\beta r_o} + g_m R_c} V_{out} = A V_{out}$$

Summing the voltage at node E.

$$V_E - \left(\left(1 + \frac{1}{\beta}\right) g_m v_\pi + \frac{(V_{out} - V_E)}{r_o}\right) R_E = V_{in} \qquad (1)$$

64)

Writing $V_E$ in terms of $V_\pi$, and $V_\pi$ in terms of $V_{out}$

1) becomes

$$-\frac{g_m A V_{out}}{\beta}(r_\pi + R_B)\left(1+\frac{R_E}{r_o}\right) - \left(1+\frac{1}{\beta}\right) g_m A V_{out} R_E - \frac{V_{out}}{r_o} R_E = V_{in}$$

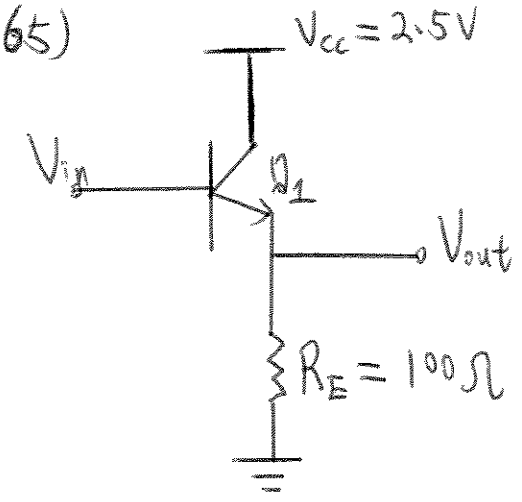
Solving $V_{out}/V_{in} \Rightarrow$

$$\frac{V_{out}}{V_{in}} = \frac{1}{-\frac{g_m A}{\beta}(r_\pi + R_B)\left(1+\frac{R_E}{r_o}\right) - \left(1+\frac{1}{\beta}\right) g_m A R_E - \frac{R_E}{r_o}}$$

Substituting $A$ into equation

$$\frac{V_{out}}{V_{in}} = \frac{\frac{g_m (r_\pi + R_B) R_C}{\beta r_o} + g_m R_C}{g_m\left(1+\frac{R_C}{r_o}\right)(r_\pi + R_B)\left(1+\frac{R_E}{r_o}\right) + \left(1+\frac{1}{\beta}\right) g_m \left(1+\frac{R_C}{r_o}\right) R_E - \frac{R_E}{r_o}\left(\frac{g_m (r_\pi + R_B) R_C}{\beta r_o} + g_m R_C\right)}$$

65)




$R_E = 100\Omega$

$V_A = \infty$

$|A_v| = 0.8$

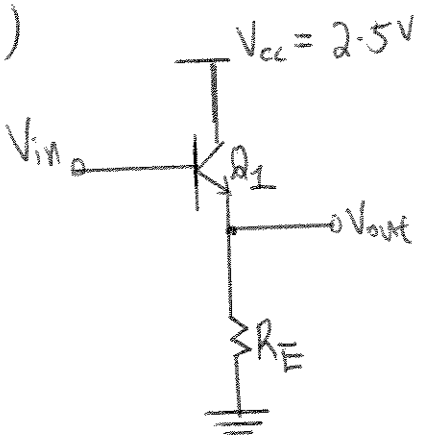
$$|A_v| = \frac{R_E}{R_E + \frac{1}{g_m}} = \frac{R_E I_c}{R_E I_c + V_T} = 0.8$$

$$\Rightarrow R_E I_c = 0.8(R_E I_c + V_T), \quad R_E = 100\Omega$$

$$\Rightarrow 0.1 I_c = 0.08 I_c + 0.0208 \Rightarrow 0.02 I_c = 0.0208$$

$$\Rightarrow I_c = 1.04\,mA$$

66)




$|A_v| > 0.9$
$R_{in} > 10K\Omega$

$$|A_v| = \frac{R_E I_c}{R_E I_c + V_T} > 0.9 \Rightarrow R_E I_c > 0.9[R_E I_c + V_T]$$

$\Rightarrow R_E I_c > 9V_T = 234\,mV$, Let $R_E I_c = 240\,mV$

$R_{in} = r_\pi + (1+\beta)R_E > 10K \Rightarrow 100V_T + (101)R_E I_c > 10K\Omega I_c$
Substituting $R_E I_c = 240\,mV \Rightarrow I_c < 2.684\,mA$

Choose $I_c$ to be $2.5\,mA \Rightarrow R_E = 96\Omega$

To Verify:

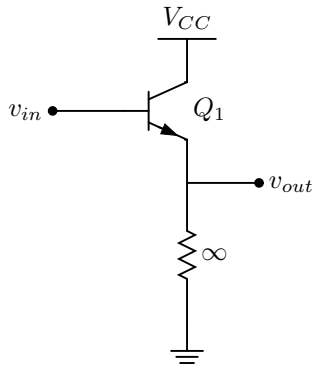
$$R_{in} = 100\frac{(0.026)}{2.5} + (101)0.096 = 10.74K\Omega$$

$$|A_v| = \frac{(0.096)(2.5)}{(0.096)(2.5) + 0.026} = 0.902$$

5.67

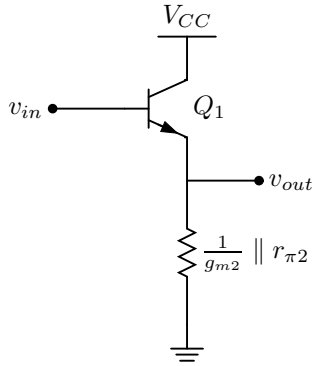
$$
\begin{aligned}
R_{out} &= \frac{r_\pi + R_S}{1+\beta} \\
&= \frac{\beta V_T / I_C + R_S}{1+\beta} \\
&\leq 5\ \Omega \\
I_C &= \frac{\beta}{1+\beta} I_E = \frac{\beta}{1+\beta} I_1 \\
\frac{\frac{\beta(1+\beta)V_T}{\beta I_1} + R_S}{1+\beta} &= \frac{\frac{(1+\beta)V_T}{I_1} + R_S}{1+\beta} \\
&\leq 5\ \Omega \\
I_1 &\geq \boxed{8.61\ \text{mA}}
\end{aligned}
$$

5.68 (a) Looking into the collector of $Q_2$ we see an equivalent resistance of $r_{o2} = \infty$, so we can draw the following equivalent circuit:



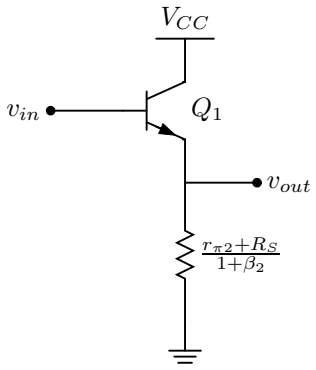

$$A_v = \boxed{1}$$
$$R_{in} = \boxed{\infty}$$
$$R_{out} = \boxed{\frac{1}{g_{m1}} \parallel r_{\pi 1}}$$

(b) Looking down from the emitter of $Q_1$ we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit:




$$A_v = \boxed{\frac{\frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}} \parallel r_{\pi 2}}}$$
$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$
$$R_{out} = \boxed{\frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

(c) Looking into the emitter of $Q_2$ we see an equivalent resistance of $\frac{r_{\pi 2} + R_S}{1 + \beta_2}$, so we can draw the following equivalent circuit:

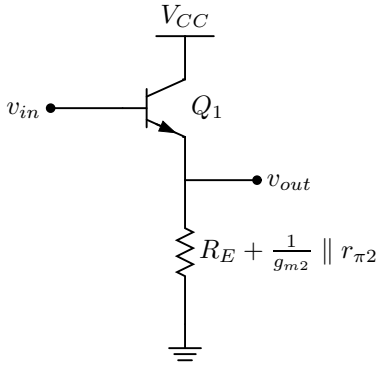



$$A_v = \boxed{\frac{\frac{r_{\pi 2}+R_S}{1+\beta_2}}{\frac{1}{g_{m1}} + \frac{r_{\pi 2}+R_S}{1+\beta_2}}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)\left(\frac{r_{\pi 2}+R_S}{1+\beta_2}\right)}$$

$$R_{out} = \boxed{\frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel \left(\frac{r_{\pi 2}+R_S}{1+\beta_2}\right)}$$

(d) Looking down from the emitter of $Q_1$ we see an equivalent resistance of $R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit:

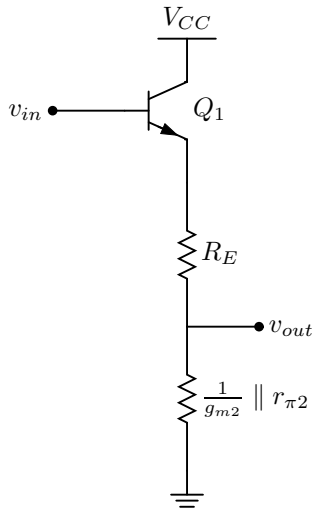



$$A_v = \boxed{\frac{R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)\left(R_E + \frac{1}{g_{m2}}\right)}$$

$$R_{out} = \boxed{\frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel \left(R_E + \frac{1}{g_{m2}}\right)}$$

(e) Looking into the emitter of $Q_2$ we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit:



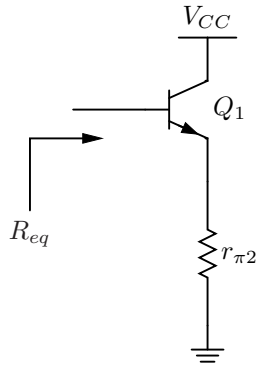

$$A_v = \frac{R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}} \cdot \frac{\frac{1}{g_{m2}} \parallel r_{\pi 2}}{R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

$$= \boxed{\frac{\frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + R_E + \frac{1}{g_{m2}} \parallel r_{\pi 2}}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1)\left(R_E + \frac{1}{g_{m2} \parallel r_{\pi 2}}\right)}$$

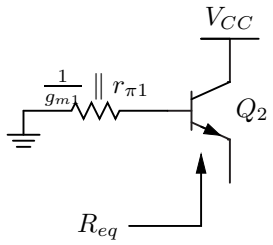
$$R_{out} = \boxed{\left(\frac{1}{g_{m1}} \parallel r_{\pi 1} + R_E\right) \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

5.69 (a) Looking into the base of $Q_2$ we see an equivalent resistance of $r_{\pi 2}$ (assuming the emitter of $Q_2$ is grounded), so we can draw the following equivalent circuit for finding the impedance at the base of $Q_1$:




$$R_{eq} = \boxed{r_{\pi 1} + (1 + \beta_1) r_{\pi 2}}$$

(b) Looking into the emitter of $Q_1$ we see an equivalent resistance of $\frac{1}{g_{m1}} \parallel r_{\pi 1}$ (assuming the base of $Q_1$ is grounded), so we can draw the following equivalent circuit for finding the impedance at the emitter of $Q_2$:




$$R_{eq} = \boxed{\frac{r_{\pi 2} + \frac{1}{g_{m1}} \parallel r_{\pi 1}}{1 + \beta_2}}$$

(c)

$$\begin{aligned} \frac{I_{C1} + I_{C2}}{I_{B1}} &= \frac{\beta_1 I_{B1} + \beta_2 (1 + \beta_1) I_{B1}}{I_{B1}} \\ &= \boxed{\beta_1 + \beta_2 (1 + \beta_1)} \end{aligned}$$

If we assume that $\beta_1, \beta_2 \gg 1$, then this simplifies to $\beta_1 \beta_2$, meaning a Darlington pair has a current gain approximately equal to the product of the current gains of the individual transistors.

5.70 (a)

$$R_{CS} = \boxed{r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel R_E)}$$

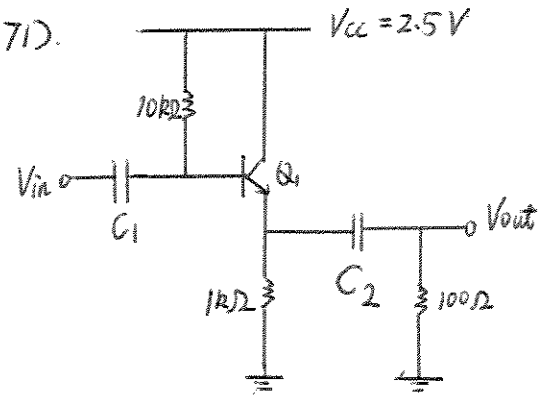
(b)

$$A_v = \boxed{\frac{r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel R_E)}{\frac{1}{g_{m1}} + r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel R_E)}}$$

$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1) [r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel R_E)]}$$

$$R_{out} = \boxed{\frac{1}{g_{m1}} \parallel r_{\pi 1} \parallel [r_{o2} + (1 + g_{m2} r_{o2}) (r_{\pi 2} \parallel R_E)]}$$

7D.

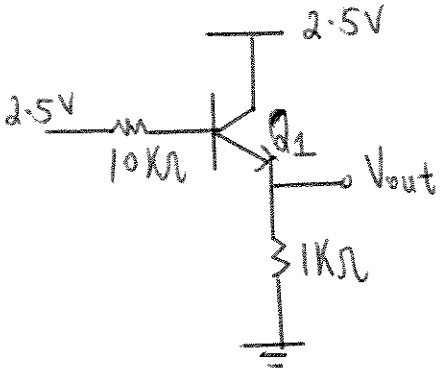



$I_s = 7 \times 10^{-16} A$

$\beta = 100$

$V_A = 5V$

DC Analysis: (Ignore $r_o$'s effect)




$$I_c = \beta \left( \frac{2.5 - \left(V_{BE} + \frac{I_c}{\alpha} 1K\Omega\right)}{10K\Omega} \right)$$

Rearrange

$$I_c = \frac{2.5 - V_{BE}}{\frac{10K\Omega}{\beta} + \frac{1K\Omega}{\alpha}}$$

Guess: $V_{BE} = 0.7V$, $I_c = 1.621mA$

check for $V_{BE}$: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.740V$, not 0.7, reiterate

$V_{BE} = 0.740V$, $I_c = 1.59mA$

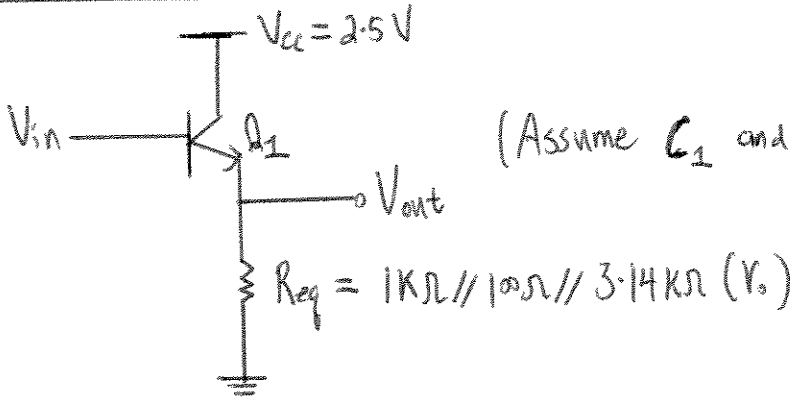
Check for $V_{BE}$: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.740V$, converged.

So $I_c = 1.59mA$, $g_m = 0.0612\left(\frac{1}{\Omega}\right)S$, $\frac{1}{g_m} = 16.34\Omega$,

$r_o = 3.14K\Omega$

71)

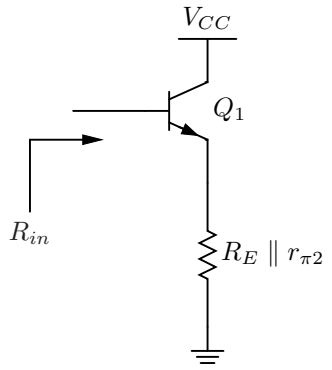
AC Analysis: (Include $r_o$)




(Assume $C_1$ and $C_2$ are large)

$$A_v = \frac{(1K\Omega // 100\Omega // 3.14K\Omega)}{16.34\Omega + (1K\Omega // 100\Omega / 3.14K\Omega)} = 0.84$$

5.72 (a) Looking into the base of $Q_2$ we see an equivalent resistance of $r_{\pi 2}$, so we can draw the following equivalent circuit for finding $R_{in}$:

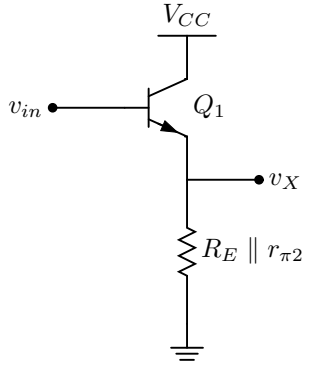



$$R_{in} = \boxed{r_{\pi 1} + (1 + \beta_1)(R_E \parallel r_{o1})}$$

Looking into the collector of $Q_2$ we see an equivalent resistance of $r_{o2}$. Thus,

$$R_{out} = \boxed{R_C \parallel r_{o2}}$$

(b) Looking into the base of $Q_2$ we see an equivalent resistance of $r_{\pi 2}$, so we can draw the following equivalent circuit for finding $v_X/v_{in}$:




$$\frac{v_X}{v_{in}} = \frac{R_E \parallel r_{\pi 2} \parallel r_{o1}}{\frac{1}{g_{m1}} + R_E \parallel r_{\pi 2} \parallel r_{o1}}$$

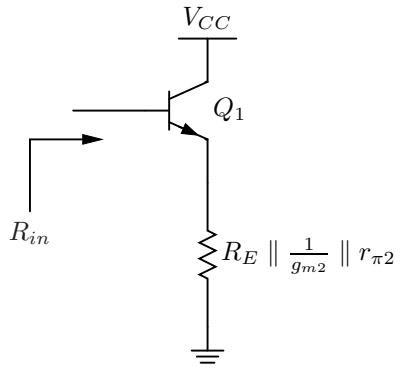
We can find $v_{out}/v_X$ by inspection.

$$\frac{v_{out}}{v_X} = -g_{m2}(R_C \parallel r_{o2})$$

$$A_v = \frac{v_X}{v_{in}} \cdot \frac{v_{out}}{v_X}$$

$$= \boxed{-g_{m2}(R_C \parallel r_{o2}) \frac{R_E \parallel r_{\pi 2} \parallel r_{o1}}{\frac{1}{g_{m1}} + R_E \parallel r_{\pi 2} \parallel r_{o1}}}$$

5.73 (a) Looking into the emitter of $Q_2$ we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding $R_{in}$:

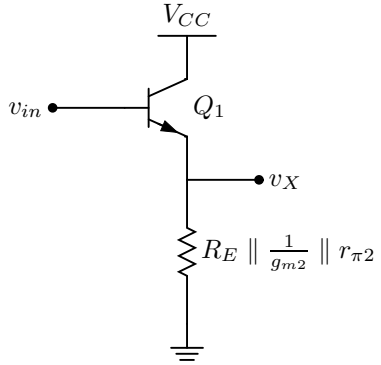



$$R_{in} = \boxed{r_{\pi 1} + (1+\beta_1)\left(R_E \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

Looking into the collector of $Q_2$, we see an equivalent resistance of $\infty$ (because $V_A = \infty$), so we have

$$R_{out} = \boxed{R_C}$$

(b) Looking into the emitter of $Q_2$ we see an equivalent resistance of $\frac{1}{g_{m2}} \parallel r_{\pi 2}$, so we can draw the following equivalent circuit for finding $v_X/v_{in}$:




$$\frac{v_X}{v_{in}} = \frac{R_E \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + E \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}$$

We can find $v_{out}/v_X$ by inspection.

$$\frac{v_{out}}{v_X} = g_{m2} R_C$$

$$A_v = \frac{v_X}{v_{in}} \cdot \frac{v_{out}}{v_X}$$

$$= \boxed{g_{m2} R_C \frac{R_E \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}} + E \parallel \frac{1}{g_{m2}} \parallel r_{\pi 2}}}$$

5.74

$$
\begin{aligned}
R_{out} &= R_C = \boxed{1\ \text{k}\Omega} \\
A_v &= -g_m R_C = -10 \\
g_m &= 10\ \text{mS} \\
I_C &= g_m V_T = 260\ \mu\text{A} \\
\frac{V_{CC} - V_{BE}}{R_B} &= I_B = \frac{I_C}{\beta} \\
R_B &= \beta \frac{V_{CC} - V_T \ln(I_C/I_S)}{I_C} \\
&= \boxed{694\ \text{k}\Omega} \\
R_{in} &= R_B \parallel r_\pi = 9.86\ \text{k}\Omega > 5\ \text{k}\Omega
\end{aligned}
$$

In sizing $C_B$, we must consider the effect a finite impedance in series with the input will have on the circuit parameters. Any series impedance will cause $R_{in}$ to increase and will not impact $R_{out}$. However, a series impedance can cause gain degradation. Thus, we must ensure that $|Z_B| = \left|\frac{1}{j\omega C_B}\right|$ does not degrade the gain significantly.

If we include $|Z_B|$ in the gain expression, we get:

$$A_v = -\frac{R_C}{\frac{1}{g_m} + \frac{(|Z_B|) \| R_B}{1+\beta}}$$

Thus, we want $\frac{1}{1+\beta}|Z_B| \ll \frac{1}{g_m}$ to ensure the gain is not significantly degraded.

$$
\begin{aligned}
\frac{1}{1+\beta}\left|\frac{1}{j\omega C_B}\right| &\ll \frac{1}{g_m} \\
\frac{1}{1+\beta}\frac{1}{2\pi f C_B} &= \frac{1}{10}\frac{1}{g_m} \\
C_B &= \boxed{788\ \text{nF}}
\end{aligned}
$$

5.75

$$R_{out} = R_C \leq 500\ \Omega$$

To maximize gain, we should maximize $R_C$.

$$\begin{aligned} R_C &= \boxed{500\ \Omega} \\ V_{CC} - I_C R_C &\geq V_{BE} - 400\ \text{mV} = V_T \ln(I_C/I_S) - 400\ \text{mV} \\ I_C &\leq 4.261\ \text{mA} \end{aligned}$$

To maximize gain, we should maximize $I_C$.

$$\begin{aligned} I_C &= \boxed{4.261\ \text{mA}} \\ I_B &= \frac{I_C}{\beta} = \frac{V_{CC} - V_{BE}}{R_B} \\ &= \frac{I_C}{\beta} = \frac{V_{CC} - V_T \ln(I_C/I_S)}{R_B} \\ R_B &= \boxed{40.613\ \text{k}\Omega} \end{aligned}$$

5.76

$$\begin{aligned} R_{out} = R_C &= \boxed{1\ \text{k}\Omega} \\ |A_v| &= g_m R_C \\ &= \frac{I_C R_C}{V_T} \\ &\geq 20 \\ I_C &\geq 520\ \mu\text{A} \end{aligned}$$

In order to maximize $R_{in} = R_B \parallel r_\pi$, we need to maximize $r_\pi$, meaning we should minimize $I_C$ (since $r_\pi = \frac{\beta V_T}{I_C}$).

$$\begin{aligned} I_C &= 520\ \mu\text{A} \\ I_B = \frac{I_C}{\beta} &= \frac{V_{CC} - V_{BE}}{R_B} \\ &= \frac{V_{CC} - V_T \ln(I_C/I_S)}{R_B} \\ R_B &= \boxed{343\ \text{k}\Omega} \end{aligned}$$

5.77

$$\begin{aligned}
R_{out} &= R_C = \boxed{2\ \text{k}\Omega} \\
A_v &= -g_m R_C \\
&= -\frac{I_C R_C}{V_T} \\
&= -15 \\
I_C &= 195\ \mu\text{A} \\
V_{BE} &= V_T \ln(I_C/I_S) = 689.2\ \text{mV} \\
V_{CE} &\geq V_{BE} - 400\ \text{mV} = 289.2\ \text{mV}
\end{aligned}$$

To minimize the supply voltage, we should minimize $V_{CE}$.

$$\begin{aligned}
V_{CE} &= 289.2\ \text{mV} \\
\frac{V_{CC} - V_{CE}}{R_C} &= I_C \\
V_{CC} &= 679.2\ \text{mV}
\end{aligned}$$

Note that this value of $V_{CC}$ is less than the required $V_{BE}$. This means that the value of $V_{CC}$ is constrained by $V_{BE}$, not $V_{CE}$. In theory, we could pick $V_{CC} = V_{BE}$, but in this case, we'd have to set $R_B = 0\ \Omega$, which would short the input to $V_{CC}$. Thus, let's pick a reasonable value for $R_B$, $R_B = \boxed{100\ \Omega}$.

$$\begin{aligned}
\frac{V_{CC} - V_{BE}}{R_B} &= I_B = \frac{I_C}{\beta} \\
V_{CC} &= \boxed{689.4\ \text{mV}}
\end{aligned}$$

5.78

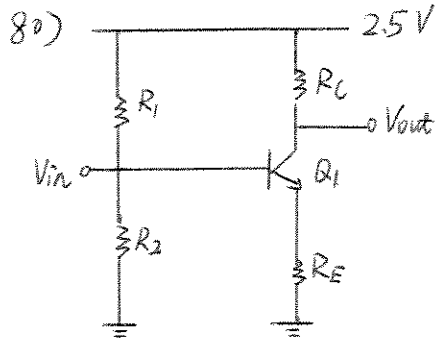
$$\begin{aligned}
|A_v| &= g_m R_C \\
&= \frac{I_C R_C}{V_T} \\
&= A_0 \\
R_{out} &= R_C \\
A_0 &= \frac{I_C R_{out}}{V_T} \\
I_C &= \frac{A_0 V_T}{R_{out}} \\
P &= I_C V_{CC} \\
&= \boxed{\frac{A_0 V_T}{R_{out}} V_{CC}}
\end{aligned}$$

Thus, we must trade off a small output resistance with low power consumption (i.e., as we decrease $R_{out}$, power consumption increases and vice-versa).

5.79

$$
\begin{aligned}
P &= (I_B + I_C)V_{CC} \\
&= \frac{1+\beta}{\beta} I_C V_{CC} \\
&= 1 \text{ mW} \\
I_C &= 396 \ \mu\text{A} \\
\frac{V_{CC} - V_{BE}}{R_B} &= I_B = \frac{I_C}{\beta} \\
R_B &= \beta \frac{V_{CC} - V_T \ln(I_C/I_S)}{I_C} \\
&= \boxed{453 \text{ k}\Omega} \\
A_v &= -g_m R_C \\
&= -\frac{I_C R_C}{V_T} \\
&= -20 \\
R_C &= \boxed{1.31 \text{ k}\Omega}
\end{aligned}
$$

80)




$A_V = 5$

$R_{out} = R_c = 500\Omega$

$R_E I_c \approx 300\,mV$

$$A_V = \frac{R_c I_c}{R_E I_c + V_T} = \frac{R_c I_c}{300+26} \Rightarrow \begin{matrix} R_c I_c = 1.63V \Rightarrow I_c = 3.26\,mA \\ R_E I_c \approx 300mV \Rightarrow R_E = 92\Omega \end{matrix}$$

$$R_1 = \frac{2.5 - (V_{BE} + 0.3)}{10\, I_B}, \quad V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.7624$$

$$10\, I_B = 0.326\,mA$$

$$R_1 = \frac{2.5 - (0.7624 + 0.3)}{0.326} = 4.41K$$

$$R_2 = \frac{(0.7624 + 0.3)}{(9)(0.0326)} = 3.62K$$

$V_{CE} = 2.5 - 1.63 - 0.3 = 0.57$, $V_{BE} = 0.7624$.

$Q_1$ is in soft saturation region, so active region characteristics still apply.

$R_c = 500\Omega$
$R_1 = 4.41K\Omega$
$R_2 = 3.62K\Omega$
$R_E = 92\Omega$

$\Rightarrow$ $A_V = 5$, $R_{out} = 500\Omega$

5.81

$$R_{out} = R_C \geq 1\ \text{k}\Omega$$

To maximize gain, we should maximize $R_{out}$.

$$\begin{aligned}
R_C &= \boxed{1\ \text{k}\Omega} \\
V_{CC} - I_C R_C - I_E R_E &= V_{CE} \geq V_{BE} - 400\ \text{mV} \\
V_{CC} - I_C R_C - 200\ \text{mV} &\geq V_T \ln(I_C/I_S) - 400\ \text{mV} \\
I_C &\leq 1.95\ \text{mA}
\end{aligned}$$

To maximize gain, we should maximize $I_C$.

$$\begin{aligned}
I_C &= 1.95\ \text{mA} \\
I_E R_E &= \frac{1+\beta}{\beta} I_C R_E = 200\ \text{mV} \\
R_E &= \boxed{101.5\ \Omega} \\
V_{CC} - 10 I_B R_1 - I_E R_E &= V_{BE} = V_T \ln(I_C/I_S) \\
R_1 &= \boxed{7.950\ \text{k}\Omega} \\
9 I_B R_2 - I_E R_E &= V_{BE} = V_T \ln(I_C/I_S) \\
R_2 &= \boxed{5.405\ \text{k}\Omega}
\end{aligned}$$

5.82

$$
\begin{aligned}
P &= (10I_B + I_C)\, V_{CC} \\
&= \left(10\frac{I_C}{\beta} + I_C\right) V_{CC} \\
&= 5 \text{ mW} \\
I_C &= 1.82 \text{ mA} \\
I_E R_E &= \frac{1+\beta}{\beta} I_C R_E = 200 \text{ mV} \\
R_E &= \boxed{109\ \Omega} \\
A_v &= -\frac{R_C}{\frac{1}{g_m} + R_E} \\
&= -\frac{R_C}{\frac{V_T}{I_C} + R_E} \\
&= -5 \\
R_C &= \boxed{616\ \Omega} \\
V_{CC} - 10I_B R_1 - 200 \text{ mV} &= V_{BE} = V_T \ln(I_C/I_S) \\
R_1 &= \boxed{8.54\ \text{k}\Omega} \\
9I_B R_2 - 200 \text{ mV} &= V_{BE} = V_T \ln(I_C/I_S) \\
R_2 &= \boxed{5.79\ \text{k}\Omega}
\end{aligned}
$$

5.83

$$R_{in} = \frac{1}{g_m} = 50\ \Omega \text{ (since } R_E \text{ doesn't affect } R_{in})$$
$$g_m = 20\ \text{mS}$$
$$I_C = g_m V_T = 520\ \mu\text{A}$$
$$I_E R_E = \frac{1+\beta}{\beta} I_C R_E = 260\ \text{mV}$$
$$R_E = \boxed{495\ \Omega}$$
$$A_v = g_m R_C = 20$$
$$R_C = \boxed{1\ \text{k}\Omega}$$
$$V_{CC} - 10 I_B R_1 - I_E R_E = V_{BE} = V_T \ln(I_C/I_S)$$
$$R_1 = \boxed{29.33\ \text{k}\Omega}$$
$$9 I_B R_2 - I_E R_E = V_{BE} = V_T \ln(I_C/I_S)$$
$$R_2 = \boxed{20.83\ \text{k}\Omega}$$

To pick $C_B$, we must consider its effect on $A_v$. If we assume the capacitor has an impedance $Z_B$ and $|Z_B| \ll R_1, R_2$, then we have:

$$A_v = \frac{R_C}{\frac{1}{g_m} + \frac{|Z_B|}{1+\beta}}$$

Thus, we should choose $\frac{1}{1+\beta}|Z_B| \ll \frac{1}{g_m}$.

$$\frac{1}{1+\beta}|Z_B| = \frac{1}{1+\beta}\frac{1}{2\pi f C_B} = \frac{1}{10}\frac{1}{g_m}$$
$$C_B = \boxed{1.58\ \mu\text{F}}$$

5.84

$$\begin{aligned}
R_{out} = R_C &= \boxed{500\ \Omega} \\
A_v &= g_m R_C = 8 \\
g_m &= 16\ \text{mS} \\
I_C &= g_m V_T = 416\ \mu\text{A} \\
I_E R_E &= \frac{1+\beta}{\beta} I_C R_E = 260\ \text{mV} \\
R_E &= \boxed{619\ \Omega} \\
V_{CC} - 10 I_B R_1 - I_E R_E &= V_{BE} = V_T \ln(I_C/I_S) \\
R_1 &= \boxed{36.806\ \text{k}\Omega} \\
9 I_B R_2 - I_E R_E &= V_{BE} = V_T \ln(I_C/I_S) \\
R_2 &= \boxed{25.878\ \text{k}\Omega}
\end{aligned}$$

To pick $C_B$, we must consider its effect on $A_v$. If we assume the capacitor has an impedance $Z_B$ and $|Z_B| \ll R_1, R_2$, then we have:

$$A_v = \frac{R_C}{\frac{1}{g_m} + \frac{|Z_B|}{1+\beta}}$$

Thus, we should choose $\frac{1}{1+\beta}|Z_B| \ll \frac{1}{g_m}$.

$$\begin{aligned}
\frac{1}{1+\beta}|Z_B| = \frac{1}{1+\beta}\frac{1}{2\pi f C_B} &= \frac{1}{10}\frac{1}{g_m} \\
C_B &= \boxed{1.26\ \mu\text{F}}
\end{aligned}$$

5.85

$$
\begin{aligned}
R_{out} &= R_C = \boxed{200\ \Omega} \\
A_v &= g_m R_C = \frac{I_C R_C}{V_T} = 20 \\
I_C &= 2.6\ \text{mA} \\
P &= V_{CC}\left(10 I_B + I_C\right) \\
&= V_{CC}\left(10\frac{I_C}{\beta} + I_C\right) \\
&= \boxed{7.15\ \text{mW}}
\end{aligned}
$$

5.86

$$\begin{aligned}
P &= (I_C + 10I_B)\, V_{CC} \\
&= \left(I_C + 10\frac{I_C}{\beta}\right) V_{CC} \\
&= 5 \text{ mW} \\
I_C &= 1.82 \text{ mA} \\
A_v &= g_m R_C \\
&= \frac{I_C R_C}{V_T} \\
&= 10 \\
R_C &= \boxed{143\ \Omega} \\
I_E R_E &= \frac{1+\beta}{\beta} I_C R_E = 260 \text{ mV} \\
R_E &= \boxed{141.6\ \Omega} \\
V_{CC} - 10I_B R_1 - I_E R_E &= V_{BE} = V_T \ln(I_C/I_S) \\
R_1 &= \boxed{8.210\ \text{k}\Omega} \\
9I_B R_2 - I_E R_E &= V_{BE} = V_T \ln(I_C/I_S) \\
R_2 &= \boxed{6.155\ \text{k}\Omega}
\end{aligned}$$

To pick $C_B$, we must consider its effect on $A_v$. If we assume the capacitor has an impedance $Z_B$ and $|Z_B| \ll R_1, R_2$, then we have:

$$A_v = \frac{R_C}{\frac{1}{g_m} + \frac{|Z_B|}{1+\beta}}$$

Thus, we should choose $\frac{1}{1+\beta}|Z_B| \ll \frac{1}{g_m}$.

$$\begin{aligned}
\frac{1}{1+\beta}|Z_B| &= \frac{1}{1+\beta}\frac{1}{2\pi f C_B} = \frac{1}{10}\frac{1}{g_m} \\
C_B &= \boxed{5.52\ \mu\text{F}}
\end{aligned}$$

5.87

$$\begin{aligned}
R_{in} &= \frac{1}{g_m} = 50\ \Omega \text{ (since } R_E \text{ doesn't affect } R_{in}) \\
g_m &= 20\ \text{mS} \\
I_C &= g_m V_T = 520\ \mu\text{A} \\
A_v &= g_m R_C = 20 \\
R_C &= \boxed{1\ \text{k}\Omega} \\
I_E R_E &= \frac{1+\beta}{\beta} I_C R_E = 260\ \text{mV} \\
R_E &= \boxed{495\ \Omega}
\end{aligned}$$

To minimize the supply voltage, we should allow $Q_1$ to operate in soft saturation, i.e., $V_{BC} = 400$ mV.

$$\begin{aligned}
V_{BE} &= V_T \ln(I_C/I_S) = 715\ \text{mV} \\
V_{CE} &= V_{BE} - 400\ \text{mV} = 315\ \text{mV} \\
V_{CC} - I_C R_C - I_E R_E &= V_{CE} \\
V_{CC} &= \boxed{1.095\ \text{V}} \\
V_{CC} - 10 I_B R_1 - I_E R_E &= V_{BE} \\
R_1 &= \boxed{2.308\ \text{k}\Omega} \\
9 I_B R_2 - I_E R_E &= V_{BE} \\
R_2 &= \boxed{20.827\ \text{k}\Omega}
\end{aligned}$$

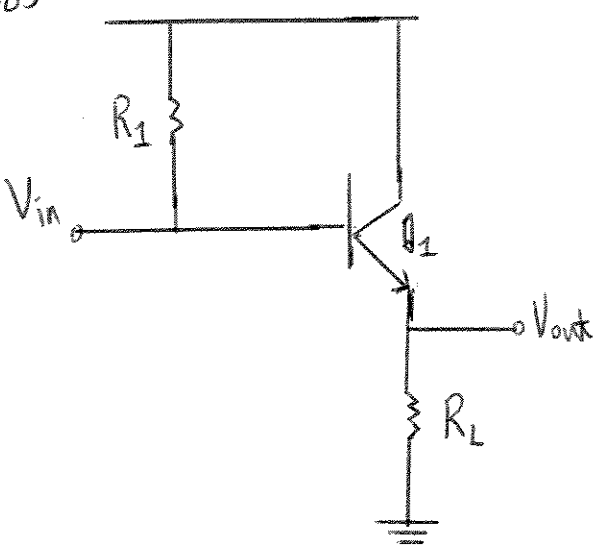
To pick $C_B$, we must consider its effect on $A_v$. If we assume the capacitor has an impedance $Z_B$ and $|Z_B| \ll R_1, R_2$, then we have:

$$A_v = \frac{R_C}{\frac{1}{g_m} + \frac{|Z_B|}{1+\beta}}$$

Thus, we should choose $\frac{1}{1+\beta}|Z_B| \ll \frac{1}{g_m}$.

$$\begin{aligned}
\frac{1}{1+\beta}|Z_B| = \frac{1}{1+\beta}\frac{1}{2\pi f C_B} &= \frac{1}{10}\frac{1}{g_m} \\
C_B &= \boxed{1.58\ \mu\text{F}}
\end{aligned}$$

88)




$A_v = 0.85$
$R_{in} > 10K\Omega$
$R_L = 200\Omega$

$$A_V = \frac{R_L}{R_L + \frac{1}{g_m}} = 0.85 \Rightarrow \frac{200}{200 + \frac{1}{g_m}} = 0.85$$

$$\Rightarrow 200 = 0.85\left(200 + \frac{1}{g_m}\right) \Rightarrow \frac{1}{g_m} = 35.294\Omega$$

$$\Rightarrow I_c = \frac{26\,mV}{35.294\Omega} = 0.737\,mA, \quad V_{BE} = V_T \ln\left(\frac{0.737}{6\times10^{-15}}\right) = 0.724V$$

$$R_{in} = R_1 \,//\, (r_\pi + (1+\beta)(200\Omega))$$

$$R_{in} = R_1 \,//\, 23.73K$$

$$R_{in} = \frac{R_1\, 23.73K}{R_1 + 23.73K} > 10K \Rightarrow R_1 > 17.28K \text{ (Input Impedance requirement)}$$

To support an $I_c$ of 0.737, $R_1$ must be determined.

88)

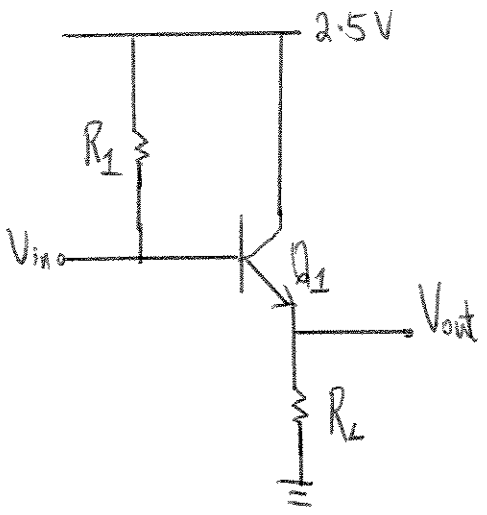
$$R_1 = \frac{2.5 - (0.724 + (0.737)(0.2)/0.99)}{0.737 / 100}$$

$$R_1 = 220.77\,K\Omega$$

$$R_1 = 220.77\,K\Omega \Rightarrow R_{in} = 220.77\,K\Omega \,//\, 23.73\,K\Omega$$
$$R_{in} = 21.43\,K\Omega > 10\,K\Omega$$

$$R_1 = 220.77\,K\Omega \quad \Rightarrow \quad A_v = 0.85$$
$$R_L = 200\,\Omega \qquad\qquad R_{in} = 21.43\,K\Omega$$

89)




Power = 5 mW

$A_v = 0.9$

$$A_v = \frac{R_L}{R_L + \frac{1}{g_m}} = 0.9 \Rightarrow R_L = 0.9\left(R_L + \frac{1}{g_m}\right)$$

$$R_L = 9\frac{1}{g_m}$$

$$\text{Power} = 2.5\left(I_c + \frac{I_c}{\beta}\right) \Rightarrow I_c = 1.98\,\text{mA}$$

$$\frac{1}{g_m} = \frac{V_T}{I_c} = \frac{26\,\text{mV}}{1.98\,\text{mA}} = 13.13\,\Omega$$

$$R_L = (9)(13.13) = 118.17\,\Omega.$$

This is the minimum load resistance, since anything lower will lower the voltage gain.

5.90 As stated in the hint, let's assume that $I_E R_E \gg V_T$. Given this assumption, we can assume that $R_E$ does not affect the gain.

$$\begin{aligned} I_E R_E &= 10 V_T = 260 \text{ mV} \\ A_v &= \frac{R_L}{\frac{1}{g_m} + R_L} = 0.8 \\ g_m &= 80 \text{ mS} \\ I_C &= g_m V_T = 2.08 \text{ mA} \\ \frac{1+\beta}{\beta} I_C R_E &= 260 \text{ mV} \\ R_E &= \boxed{124\ \Omega} \\ V_{CC} - I_B R_1 - I_E R_E &= V_{BE} = V_T \ln(I_C / I_S) \\ R_1 &= \boxed{71.6\ \text{k}\Omega} \end{aligned}$$

To pick $C_1$, we must consider its effect on $A_v$. If we assume the capacitor has an impedance $Z_1$ and $|Z_1| \ll R_1$, then we have:

$$A_v = \frac{R_E}{\frac{1}{g_m} + R_E + \frac{|Z_1|}{1+\beta}}$$

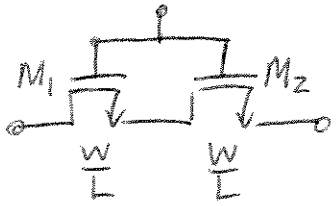
Thus, we should choose $\frac{1}{1+\beta}|Z_1| \ll \frac{1}{g_m}$.

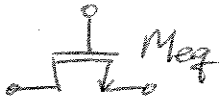
$$\begin{aligned} \frac{1}{1+\beta}|Z_1| &= \frac{1}{1+\beta}\frac{1}{2\pi f C_1} = \frac{1}{10}\frac{1}{g_m} \\ C_1 &= \boxed{12.6 \text{ pF}} \end{aligned}$$

To pick $C_2$, we must also consider its effect on $A_v$. Since the capacitor appears in series with $R_L$, we need to ensure that $|Z_2| \ll R_L$, assuming the capacitor has impedance $Z_2$.

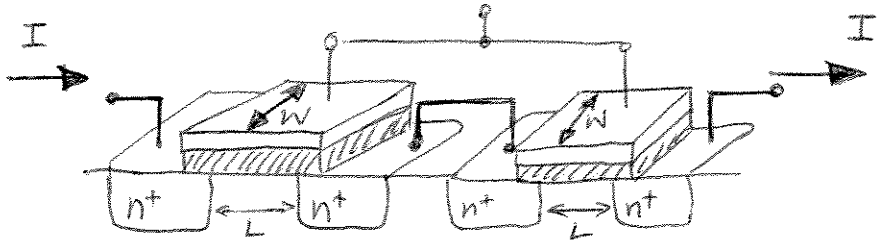
$$\begin{aligned} |Z_2| &= \frac{1}{2\pi f C_2} = \frac{1}{10} R_L \\ C_2 &= \boxed{318 \text{ pF}} \end{aligned}$$

1.

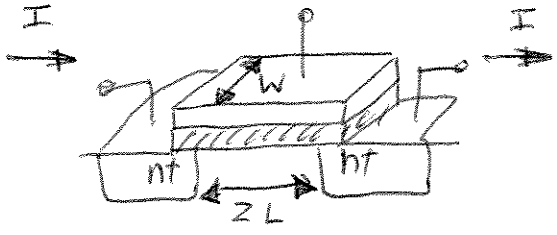



Intuitively, this is similar to having twice of the original channel length:



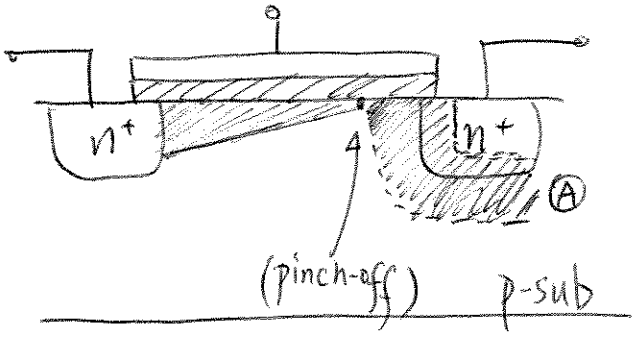

Since current flowing into either non-gate terminals must come out at the other terminal (KCL) and the intermediate node is equipotential, this is as if we have a Meq with width W & length 2L:




This approximation can simplify a lot of calculations.

2. A key point to remember: the charge density APPROACHES zero (not EQUALS) at pinch-off. In other words, Q is never exactly equal to zero (albeit very close.) Another way to view this phenomenon is by observing $I = Q \cdot v$: recognize that $v$ is finite. Since we get some finite value of I at pinch-off, we expect $Q \neq 0$.

Consider the following:




The shaded region, (A), represents a reverse-biased pn junction. Just as a diode, there exist minority profiles on p & n sides, which $\neq 0$.

Pinch-off implies that the depletion region created no longer has free carriers. The depletion still sweeps all electrons from inversion channel to drain.

3. Given: $C_{ox} = 10\ fF/\mu m^2$ $W = 5\mu m$ $L = 0.1\mu m$
$V_{GS} - V_{TH} = 1\ V$ $V_{DS} = 0$

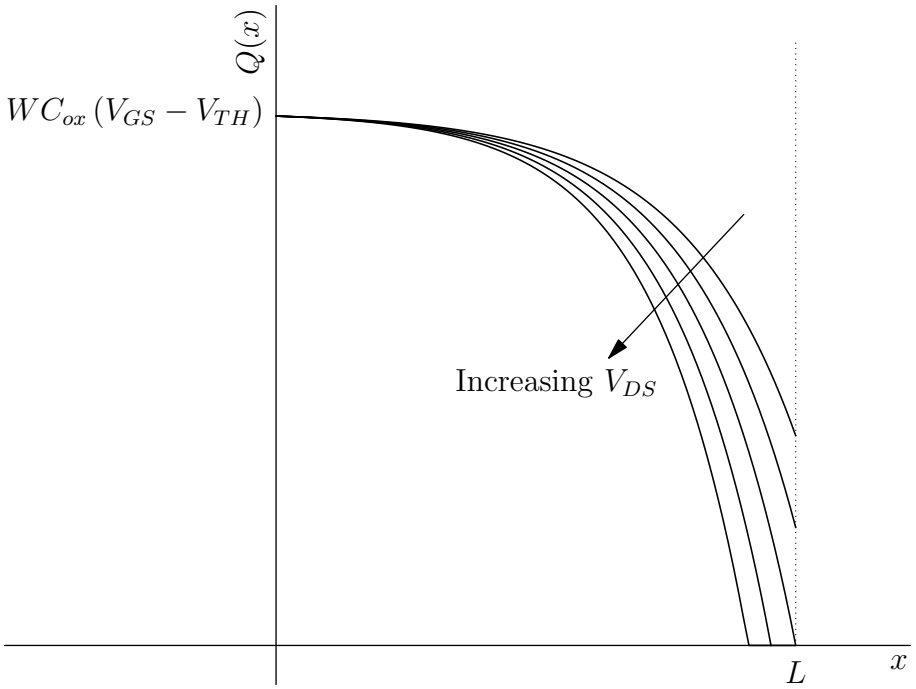
Find: total charge stored in channel, $Q_{tot}$

$$Q_{tot} = W C_{ox} (V_{GS} - V_{TH}) L$$

$$= (5\mu m)(10\ fF/\mu m^2)(1\ V)(0.1\mu m) = 5\ fC$$

6.4 (a)

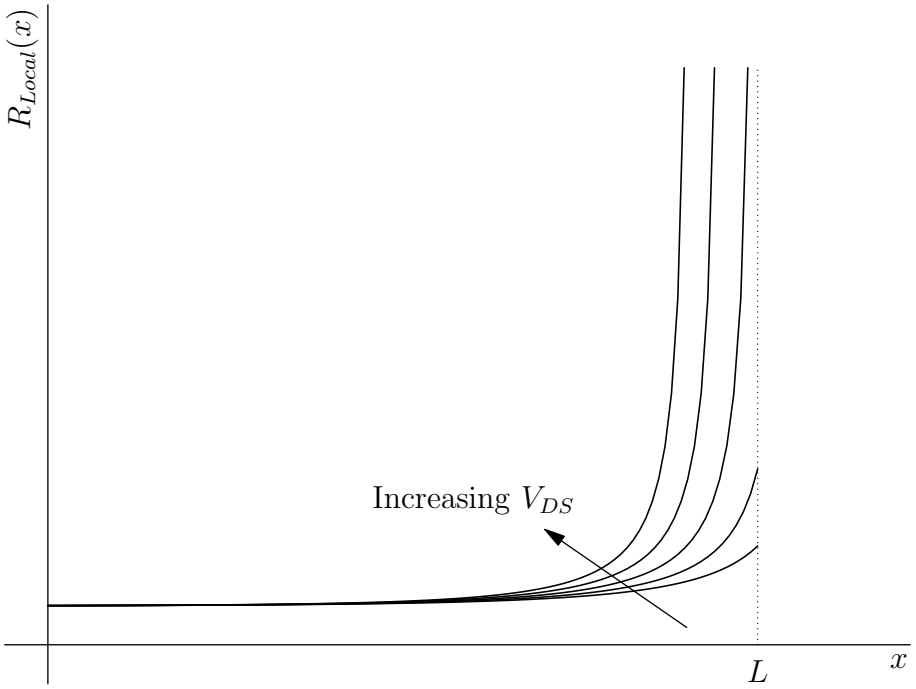
$$\begin{aligned} Q(x) &= WC_{ox}\left(V_{GS} - V(x) - V_{TH}\right) \\ &= WC_{ox}\left(V_{GS} - V_{TH}\right) - WC_{ox}V(x) \end{aligned}$$




The curve that intersects the axis at $x = L$ (i.e., the curve for which the channel begins to pinch off) corresponds to $V_{DS} = V_{GS} - V_{TH}$.

(b)

$$R_{Local}(x) \propto \frac{1}{\mu Q(x)}$$



Note that $R_{Local}$ diverges at $x = L$ when $V_{DS} = V_{GS} - V_{TH}$.

5. $I_D = WC_{ox}\left[V_{GS} - V(x) - V_{TH}\right]\mu_n \frac{dV(x)}{dx}$

Define: $A = \frac{I_D}{WC_{ox}\mu_n}$, $B = V_{GS} - V_{TH}$

$$\Rightarrow \quad A = (B - V)\frac{dV}{dx} = \frac{d}{dx}\left(BV - \frac{V^2}{2}\right)$$

Integrating $A = \frac{d}{dx}(BV - V^2/2)$ gives:

$$Ax = BV - V^2/2 \Rightarrow V^2 - 2BV + 2Ax = 0$$

Using quadratic formula:

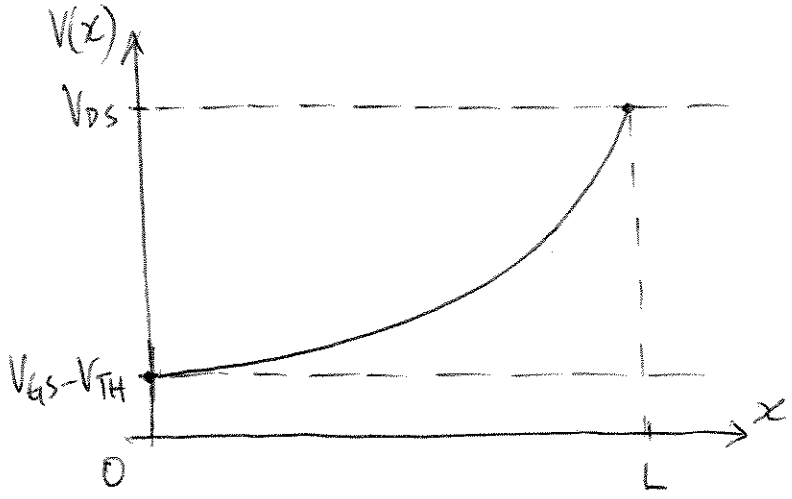
$$V_{+,-} = \frac{2B \pm \sqrt{4B^2 - 4\cdot 2A}}{2} = B \pm \sqrt{B^2 - 2Ax}$$

$$= B\left(1 \pm \sqrt{1 - 2\left(\frac{A}{B^2}\right)x}\right)$$

$$= (V_{GS} - V_{TH})\left\{1 \pm \sqrt{1 - \left[2 \cdot \frac{I_D}{WC_{ox}\mu_n (V_{GS} - V_{TH})^2}\right]x}\right\}$$

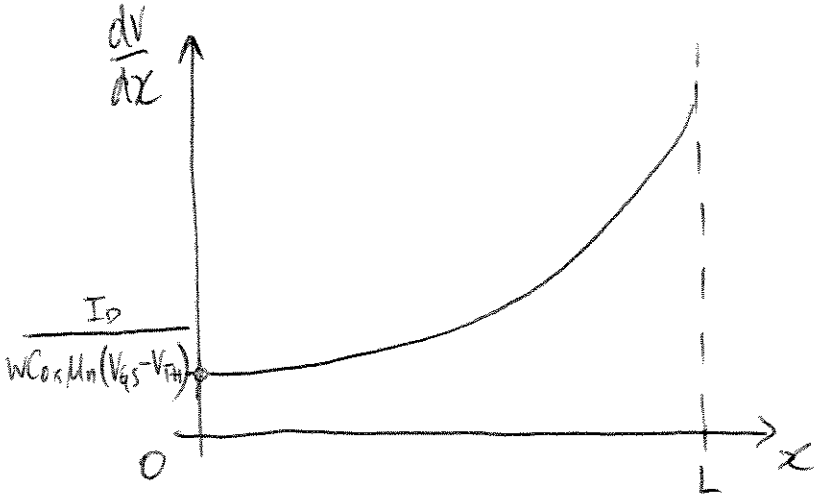
We know that $0 \le V(x) \le V_{GS} - V_{TH}$ (pinch-off), and the term inside the square root is $> 0$. Therefore, we take $V_-$ as the solution.

i.e. $$V(x) = (V_{GS}-V_{TH})\left\{1-\sqrt{1-\left[\frac{2I_D}{WC_{ox}\mu_n(V_{GS}-V_{TH})^2}\right]x}\right\}$$




$\because I_D \propto W$
$\Rightarrow V(x)$ is independent of $W$.

$$\frac{dV}{dx} = \frac{I_D}{WC_{ox}\mu_n(V_{GS}-V_{TH})}\cdot\left[1-\frac{2I_D\cdot x}{WC_{ox}\mu_n(V_{GS}-V_{TH})^2}\right]^{-\frac{1}{2}}$$



6. No.

By varying $V_{GS} - V_{TH}$ & $V_{DS}$, we can only obtain $\mu_n C_{ox} \frac{W}{L}$, but not $\mu_n C_{ox}$ & $\frac{W}{L}$ individually.

7. Given: NMOS $I_D = 1mA$ $V_{GS} - V_{TH} = 0.6\,V$

$I_D = 1.6mA$ $V_{GS} - V_{TH} = 0.8\,V$

(triode region) $\mu_n C_{ox} = 200\ \frac{\mu A}{V^2}$

Find $V_{DS}$ & $W/L$.

$$1\,mA = \mu_n C_{ox} \frac{W}{L} \left[(0.6)V_{DS} - V_{DS}^2/2\right] \quad \text{——} \ (1)$$

$$1.6\,mA = \mu_n C_{ox} \frac{W}{L} \left[(0.8)V_{DS} - V_{DS}^2/2\right] \quad \text{——} \ (2)$$

$$(2) \div (1): \quad 1.6 = \frac{0.8\,V_{DS} - V_{DS}^2/2}{0.6\,V_{DS} - V_{DS}^2/2} = \frac{1.6 - V_{DS}}{1.2 - V_{DS}}$$

$$\Rightarrow V_{DS} = \frac{1.6(0.2)}{0.6} \approx 0.533\ V$$

$$\Rightarrow \frac{W}{L} = \frac{I_D}{\mu_n C_{ox}\left[(V_{GS} - V_{TH})V_{DS} - V_{DS}^2/2\right]}$$

$$= \frac{1\,mA}{200\ \frac{\mu A}{V^2}\left[(0.6V)(0.533V) - (0.533V)^2/2\right]}$$

$$\approx 28.$$

8. $I_D = \frac{1}{2} \mu C_{ox} \frac{W}{L} [2(V_{GS} - V_{TH}) V_{DS} - V_{DS}^2]$

$$g_m \triangleq \frac{\partial I_D}{\partial V_{GS}} = \frac{1}{2} \mu C_{ox} \frac{W}{L} \cdot 2 V_{DS} = \mu C_{ox} \frac{W}{L} V_{DS}$$

$$g_m \big|_{V_{DS}=0} = 0.$$

Intuitively, when $V_{GS} > V_{TH}$, mobile charges (channel) become available. This determines the on-resistance. But since there is no $I_D$ ($\because V_{DS} = 0$), it does not matter if there is an incremental change in $V_{GS}$ (i.e. $\partial V_{GS}$). Since varying $V_{GS}$ gives no change in $I_D$, $g_m|_{V_{DS}=0} = 0$.

9. Given: $V_{DD} = 1.8\ V$ $V_{TH} = 0.4\ V$ $\frac{W}{L} = 20$ $\mu_n C_{ox} = 200\ \frac{\mu A}{V^2}$

Find minimum-on resistance.

$$R_{on} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{DD} - V_{TH})}$$

$$= \frac{1}{\left(200\ \frac{\mu A}{V^2}\right)(20)(1.8 - 0.4)V} = 179.\ \Omega$$

10. $$500 = \frac{1}{\mu_n C_{ox} \frac{W}{L} (1 - V_{TH})}$$

$$400 = \frac{1}{\mu_n C_{ox} \frac{W}{L} (1.5 - V_{TH})}$$

For the same NMOS, $\mu_n C_{ox}$ & $\frac{W}{L}$ are fixed

$$\Rightarrow 500(1 - V_{TH}) \stackrel{?}{=} 400(1.5 - V_{TH})$$

$$500(0.6) \neq 400(1.1)$$

$\therefore$ This is not possible.

11. $I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2\right]$

$$r_{DS,tri} \triangleq \left(\frac{\partial I_D}{\partial V_{DS}}\right)^{-1} = \left[\frac{\partial}{\partial V_{DS}}\left\{\frac{1}{2}\mu C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2\right]\right\}\right]^{-1}$$

$$= \left[\mu C_{ox}\frac{W}{L}(V_{GS}-V_{TH}) - \mu C_{ox}\frac{W}{L}V_{DS}\right]^{-1}$$

$$= \frac{1}{\mu C_{ox}\frac{W}{L}(V_{GS}-V_{TH}-V_{DS})}$$

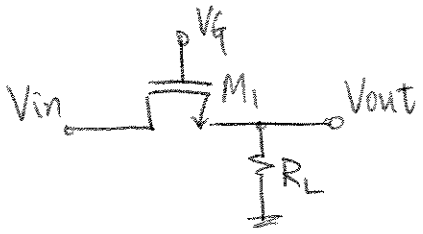
12. When MOS operates as a resistor,

$$R_{on} = \frac{1}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})}$$

$$\Rightarrow \tau = R_{on} C_{GS} = \frac{WLC_{ox}}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})} = \frac{L^2}{\mu (V_{GS} - V_{TH})}$$

To minimize the time constant,
1) use minimum channel length, and
2) maximize overdrive voltage.

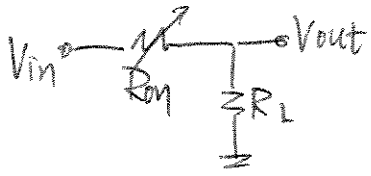
13.




Given $V_{in} \approx 0$
$V_G = 1.8\ V$
$R_L = 100\Omega$

Find $\frac{W}{L}$ such that signal output attenuates by only 5%.

$V_{in} \approx 0$ implies that we can approximate $M_1$ as a linear resistance controlled by $V_G$. Therefore, the equivalent circuit becomes a resistive divider:

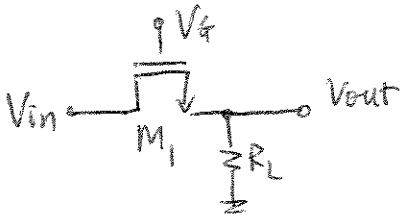



$$V_{out} = 0.95\, V_{in}$$
$$= \frac{R_L}{R_{on} + R_L} V_{in}$$
$$\Rightarrow R_{on} \cong 5.3\,\Omega$$

$$\therefore \frac{W}{L} = \frac{1}{\mu C_{ox}(V_{GS} - V_{TH}) R_{on}} \approx \frac{1}{200\frac{\mu A}{V^2}(1.8 - 0.4)(5.3\Omega)}$$

$$= 674.$$

14.




$V_o \sim$ few mV.

(a) $V_{in} = V_o \cos\omega t$ $\qquad V_{out} = 0.95(V_o \cos\omega t)$

$$V_{out} = \frac{R_L}{R_{ON}+R_L} V_{in} \quad \Rightarrow \frac{R_L}{R_{ON}+R_L} = 0.95$$

$$R_{ON} = \frac{R_L}{\left(\frac{0.95V_o}{1-0.95V_o}\right)} = \frac{1}{\mu_n C_{ox} \frac{W}{L}(V_G - V_{TH})}$$

$$\therefore \frac{W}{L} = \frac{0.95V_o/(1-0.95V_o)}{\mu_n C_{ox} R_L (V_G - V_{TH})}$$

(b) $V_{out} = 0.95\, V_{in} = 0.95(V_o \cos\omega t + 0.5)$

$\approx 0.95 \times 0.5 = 0.475$

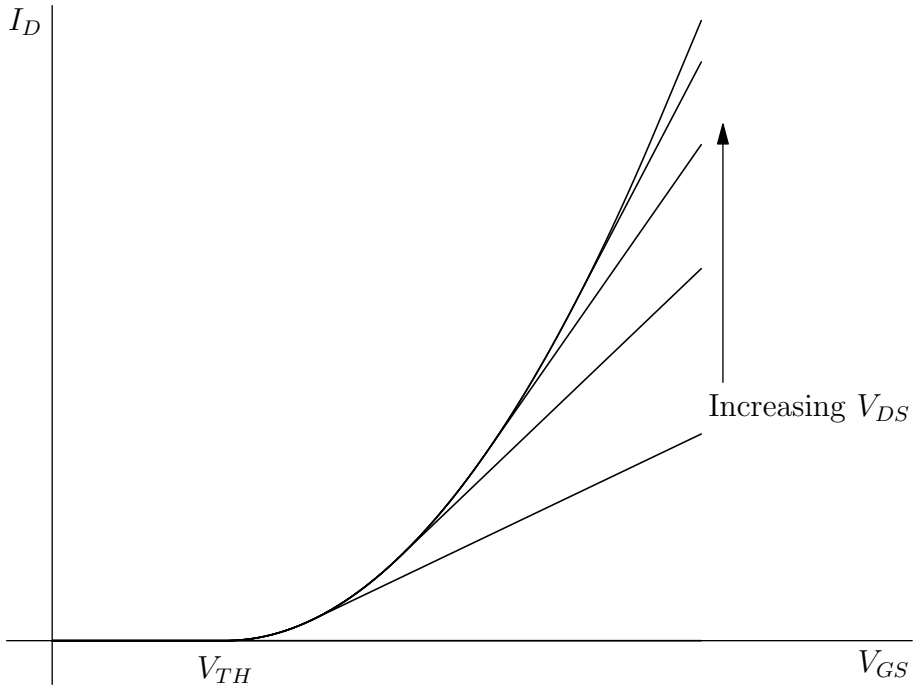
($\because V_o$ is relatively small)

$$\therefore R_{ON} = \frac{R_L}{0.9} = \frac{1}{\mu_n C_{ox} \frac{W}{L}(V_G - V_{TH})}$$

$$\Rightarrow \frac{W}{L} = \frac{0.9}{\mu_n C_{ox} R_L (V_G - V_{TH})}$$

Results show that if there is no DC voltage as input, the $R_{ON}$ varies with changing sinewave. With a DC bias voltage, $R_{ON}$ becomes more stable (independent of $V_o$).

6.15




Initially, when $V_{GS}$ is small, the transistor is in cutoff and no current flows. Once $V_{GS}$ increases beyond $V_{TH}$, the curves start following the square-law characteristic as the transistor enters saturation. However, once $V_{GS}$ increases past $V_{DS} + V_{TH}$ (i.e., when $V_{DS} < V_{GS} - V_{TH}$), the transistor goes into triode and the curves become linear. As we increase $V_{DS}$, the transistor stays in saturation up to larger values of $V_{GS}$, as expected.

16. The peak of the parabola signifies pinch-off (i.e. $V_{DS} = V_{GS} - V_{TH}$). This means that (with $\lambda = 0$) $I_D$ cannot be increased further by increasing $V_{DS}$. Since this curve must be continuous, the peak $I_D$ must originate from the peak of the parabola.

6.17

$$
\begin{aligned}
I_D &= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS}-V_{TH}\right)^{\alpha},\ \alpha<2 \\
g_m &\triangleq \frac{\partial I_D}{\partial V_{GS}} \\
&= \frac{\alpha}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS}-V_{TH}\right)^{\alpha-1} \\
&= \boxed{\frac{\alpha I_D}{V_{GS}-V_{TH}}}
\end{aligned}
$$

18. $I_D = W C_{ox} (V_{GS} - V_{TH}) v_{SAT}$

$$g_m \triangleq \frac{\partial I_D}{\partial V_{GS}} = W C_{ox} v_{SAT}$$

19. (a) OFF $\because V_{GS} = 0$

(b) SATURATION $\because V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

(c) TRIODE (LINEAR) $\because V_{GS} > V_{TH}$ &
$V_{DS} \ll \geq (V_{GS} - V_{TH})$

(d) TRIODE $\because V_{GS} > V_{TH}$ & $V_{DS} < V_{GS} - V_{TH}$
(REMEMBER: MOSFET is symmetric)

(e) TRIODE $\because V_{GS} > V_{TH}$ & $V_{DS} < V_{GS} - V_{TH}$

(f) OFF $\because V_{GS} = 0$

(g) SATURATION $\because V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

(h) SATURATION $\because V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

(i) SATURATION $\because V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

20. (a) OFF $\because$ $V_{GS} = 0$ $(V_{GS} < V_{TH})$

(b) OFF $\because$ $V_{GS} = 0$ $(V_{GS} < V_{TH})$

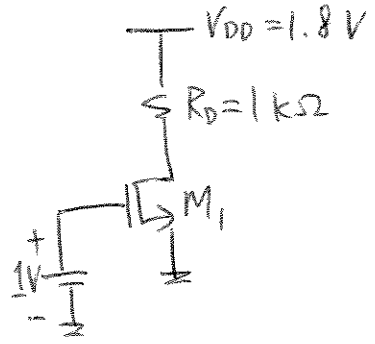
(c) TRIODE (LINEAR) $\because$ $V_{GS} > V_{TH}$ &
$V_{DS} \ll 2(V_{GS} - V_{TH})$

(d) SATURATION $\because$ $V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

6.21 Since they're being used as current sources, assume $M_1$ and $M_2$ are in saturation for this problem. To find the maximum allowable value of $\lambda$, we should evaluate $\lambda$ when $0.99I_{D2} = I_{D1}$ and $1.01I_{D2} = I_{D1}$, i.e., at the limits of the allowable values for the currents. However, note that for any valid $\lambda$ (remember, $\lambda$ should be non-negative), we know that $I_{D2} > I_{D1}$ (since $V_{DS2} > V_{DS1}$), so the case where $1.01I_{D2} = I_{D1}$ (which implies $I_{D2} < I_{D1}$) will produce an invalid value for $\lambda$ (you can check this yourself). Thus, we need only consider the case when $0.99I_{D2} = I_{D1}$.

$$\begin{aligned}
0.99I_{D2} &= 0.99\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_B - V_{TH}\right)^2\left(1 + \lambda V_{DS2}\right) \\
&= I_{D1} \\
&= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_B - V_{TH}\right)^2\left(1 + \lambda V_{DS1}\right) \\
0.99\left(1 + \lambda V_{DS2}\right) &= 1 + \lambda V_{DS1} \\
\lambda &= \boxed{0.02\ \mathrm{V}^{-1}}
\end{aligned}$$

22.




$\lambda = 0$, $V_{TH} = 0.4\,V$

$\mu_n C_{ox} = 200\,\frac{\mu A}{V^2}$

$M_1$ sits at the edge of saturation when $V_{DS} = V_{GS} - V_{TH}$.

$\Rightarrow V_{DS,edge} = (1 - 0.4)\,V = 0.6\,V$

By KCL, $I_{D1} = I_{R_D} = \frac{V_{DD} - V_{DS}}{R_D} = \frac{1.2\,V}{1\,k\Omega} = 1.2\,mA$

$\therefore I_{D1} = 1.2\,mA = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2$

$\Rightarrow \frac{W}{L} = \frac{2I_{D1}}{\mu_n C_{ox}(V_{GS} - V_{TH})^2} = \frac{2(1.2mA)}{\left(200\,\frac{\mu A}{V^2}\right)(1 - 0.4)^2 V^2}$

$\cong 33.$

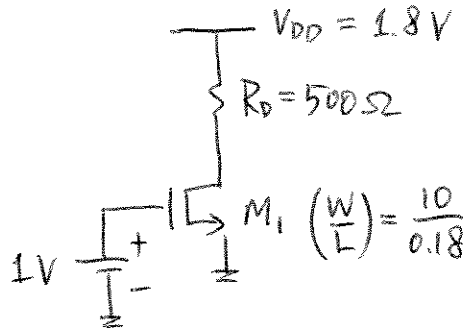
23. If gate oxide thickness, $t_{ox}$, doubles, the corresponding capacitance, $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$, is halved.

$\Rightarrow \mu_n C_{ox}$ is also halved
$\Rightarrow I_{D_1}$ is halved $\Rightarrow V_{DS}$ increases
$\Rightarrow M_1$ stays in saturation ($V_{DS} > V_{GS} - V_{TH}$)

$$I_{D_1} = \frac{1.2\,mA}{2} = 0.6\,mA$$

$$\Rightarrow V_{DS} = (1.8\,V) - (0.6\,mA)(1\,k\Omega) = 1.2\,V$$

24.




$\lambda = 0$

To avoid triode region, $V_{DS} \geq V_{GS} - V_{TH}$.

$\Rightarrow V_{DS} \geq 1 - 0.4 = 0.6\,V$

$$\Rightarrow I_{D_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2$$

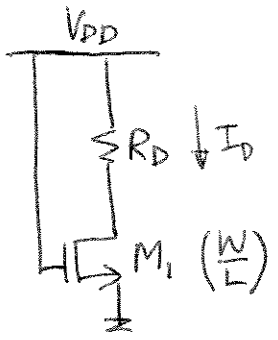
$$= \frac{1}{2}\left(200\,\frac{\mu A}{V^2}\right)\left(\frac{10}{0.18}\right)(0.6)^2 = 2\,mA$$

By KCL, $\dfrac{V_{DD} - V_{DS}}{R_D} = 2\,mA$

$\therefore V_{DD} = (2mA)(500\,\Omega) + 0.6\,V = 1.6\,V$

Minimum $V_{DD} = 1.6\,V$

25.



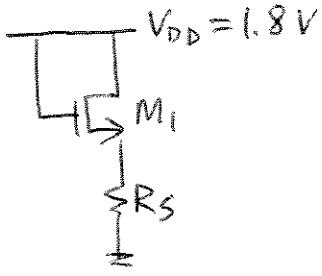

$\lambda = 0$

When $M_1$ operates at the edge of saturation, $V_{DS} = V_{GS} - V_{TH}$. Also, by KCL:

$$I_{R_D} = I_{D_1} \Rightarrow \frac{V_{DD} - (V_{DD} - V_{TH})}{R_D} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{DD} - V_{TH})^2$$

$$\therefore V_{TH} = R_D \cdot \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{DD} - V_{TH})^2}_{I_D}$$

26.




$\lambda = 0$

Find $\left(\frac{W}{L}\right)$ with bias current $= I_1$.

Since $V_{DS} = V_{GS}$ for $M_1$, this device always operates in saturation region (given $V_{GS} > V_{TH}$).

By KCL, $I_1 = I_{R_S}$; by Ohm's law, $V_S = I_1 R_S$

$$\Rightarrow \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{DD} - I_1 R_S - V_{TH})^2 = I_1$$

$$\therefore \frac{W}{L} = \frac{2 I_1}{\mu_n C_{ox} (V_{DD} - I_1 R_S - V_{TH})^2}$$

5.27

$$V_{DD} - I_D R_D = V_{GS} = V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox}\frac{W}{L}}}$$

$$\frac{2I_D}{\mu_n C_{ox}\frac{W}{L}} = (V_{DD} - V_{TH} - I_D R_D)^2$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[(V_{DD} - V_{TH})^2 - 2I_D R_D (V_{DD} - V_{TH}) + I_D^2 R_D^2\right]$$

We can rearrange this to the standard quadratic form as follows:

$$\left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}R_D^2\right) I_D^2 - \left(\mu_n C_{ox}\frac{W}{L}R_D (V_{DD} - V_{TH}) + 1\right) I_D + \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{DD} - V_{TH})^2 = 0$$

Applying the quadratic formula, we have:

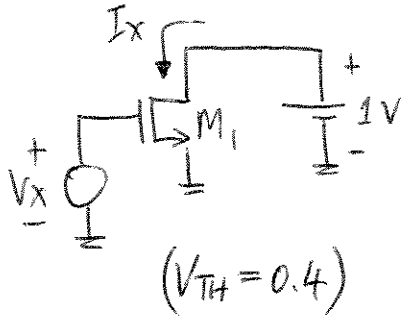
$$I_D = \frac{\left(\mu_n C_{ox}\frac{W}{L}R_D (V_{DD} - V_{TH}) + 1\right) \pm \sqrt{\left(\mu_n C_{ox}\frac{W}{L}R_D (V_{DD} - V_{TH}) + 1\right)^2 - 4\left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}R_D (V_{DD} - V_{TH})\right)^2}}{2\left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}R_D^2\right)}$$

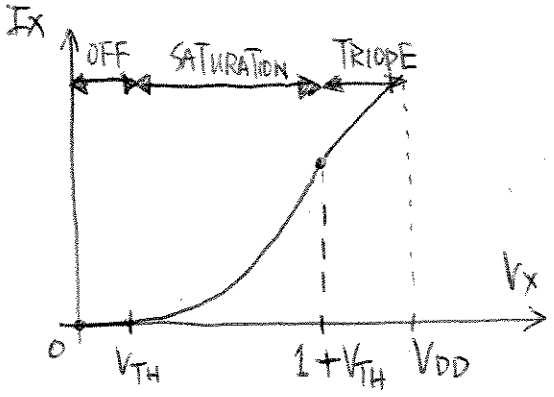
$$= \frac{\mu_n C_{ox}\frac{W}{L}R_D (V_{DD} - V_{TH}) + 1 \pm \sqrt{\left(\mu_n C_{ox}\frac{W}{L}R_D (V_{DD} - V_{TH}) + 1\right)^2 - \left(\mu_n C_{ox}\frac{W}{L}R_D (V_{DD} - V_{TH})\right)^2}}{\mu_n C_{ox}\frac{W}{L}R_D^2}$$

$$= \boxed{\frac{\mu_n C_{ox}\frac{W}{L}R_D (V_{DD} - V_{TH}) + 1 \pm \sqrt{1 + 2\mu_n C_{ox}\frac{W}{L}R_D (V_{DD} - V_{TH})}}{\mu_n C_{ox}\frac{W}{L}R_D^2}}$$

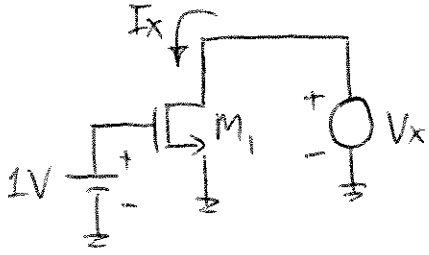
Note that mathematically, there are two possible solutions for $I_D$. However, since $M_1$ is diode-connected, we know it will either be in saturation or cutoff. Thus, we must reject the value of $I_D$ that does not match these conditions (for example, a negative value of $I_D$ would not match cutoff or saturation, so it would be rejected in favor of a positive value).

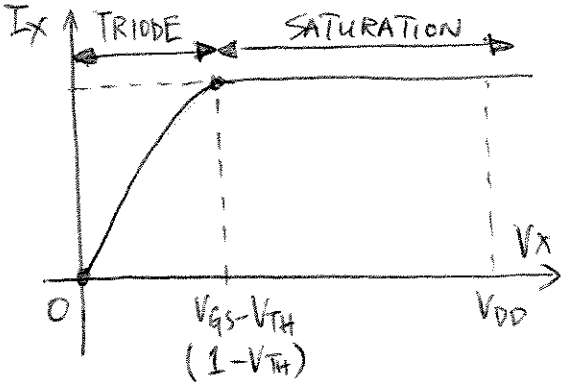
28. (a)


Ix
M1
1 V
Vx
(VTH = 0.4)

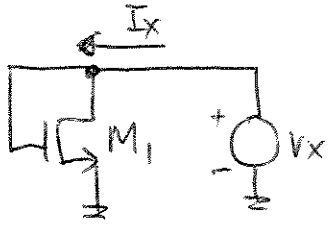

Ix
OFF
SATURATION
TRIODE
Vx
0
VTH
1+VTH
VDD

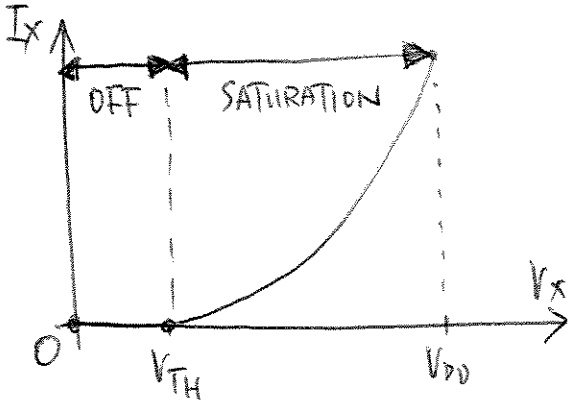
(b)


Ix
M1
Vx
1V

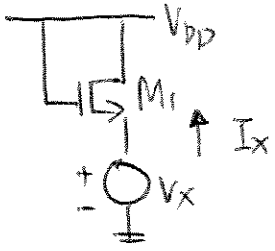

Ix
TRIODE
SATURATION
Vx
0
VGS-VTH
(1-VTH)
VDD

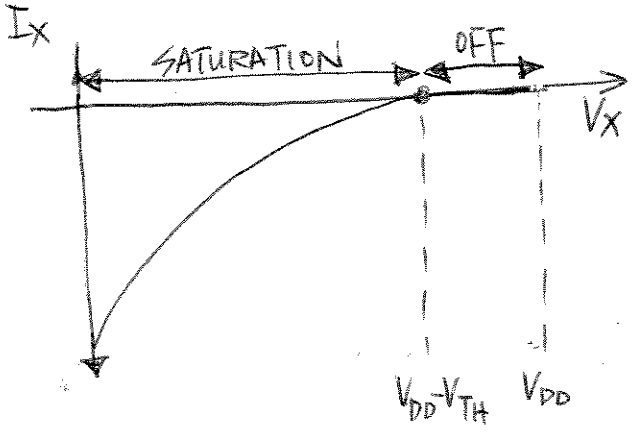
(c)


Ix
M1
Vx

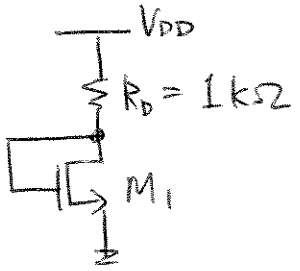

Ix
OFF
SATURATION
Vx
0
VTH
VDD

(d)


VDD
M1
Ix
Vx


Ix
SATURATION
OFF
Vx
VDD-VTH
VDD

29.




$\left(\frac{W}{L}\right) = \frac{10}{0.18}$ , $\lambda = 0.1\ V^{-1}$

Find $I_{D1}$

Since $M_1$ is diode-connected, it operates in saturation.

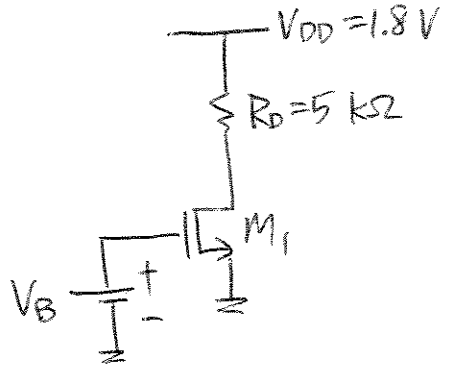
By KCL, $\frac{V_{DD} - V_G}{R_D} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_G - V_{TH})^2(1+\lambda V_G)$

One can solve this by (1) using a graphing calculator, (2) trial-and-error, (3) or iteratively finding $V_G$.

Using any method gives $V_G \approx 0.807\ V$

$\Rightarrow I_D = \frac{V_{DD} - V_G}{R_D} \approx 1\ mA$

30.




$\frac{W}{L}=\frac{20}{0.18}$, $\lambda=0.1\ V^{-1}$

At the edge of saturation,

$$I_{D_1}=\frac{V_{DD}-(V_B-V_{TH})}{R_D}=\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_B-V_{TH})^2(1+\lambda(V_B-V_{TH}))$$

This equation can be solved by using a graphing calculator, special programs, or iteratively.

Using any method gives $V_B \approx 0.57\ V$
$(I_D \approx 0.33\ mA)$

31. An NMOS device with $\lambda = 0$ must provide a transconductance of $1/50\ \frac{1}{\Omega}$.

(a) Given $I_D = 0.5\ mA$, find $W/L$.

$$g_m = \frac{1}{50} = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

$$\Rightarrow \frac{W}{L} = \frac{g_m^2}{2 \mu_n C_{ox} I_D} = \frac{\left(\frac{1}{50}\ \frac{1}{\Omega}\right)^2}{2\left(200\ \frac{\mu A}{V^2}\right)(0.5\ mA)} \approx 2000$$

(b) Given $V_{GS} - V_{TH} = 0.5\ V$, find $W/L$.

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$\Rightarrow \frac{W}{L} = \frac{g_m}{\mu_n C_{ox} (V_{GS} - V_{TH})} = \frac{\left(\frac{1}{50}\ \frac{1}{\Omega}\right)}{\left(200\ \frac{\mu A}{V^2}\right)(0.5\ V)} \approx 200$$

(c) Given $V_{GS} - V_{TH} = 0.5\ V$, find $I_D$.

$$\Rightarrow I_D = \frac{g_m (V_{GS} - V_{TH})}{2} = \frac{\left(\frac{1}{50}\ \frac{1}{\Omega}\right)(0.5\ V)}{2} \approx 5\ mA$$

32. (a) $g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}$ ($I_D$ constant)

Doubling $(W/L)$ implies a $\sqrt{2}$ times increase in $g_m$: $g_{m_{NEW}} = \sqrt{2\mu_n C_{ox}\left(2\frac{W}{L}\right) I_D} = \sqrt{2}\, g_m$.

(b) $g_m = \frac{2I_D}{V_{GS}-V_{TH}}$ ($I_D$ constant)

Doubling $(V_{GS}-V_{TH})$ decreases $g_m$ by half:

$g_{m_{NEW}} = \frac{2I_D}{2(V_{GS}-V_{TH})} = \frac{1}{2} g_m$

(c) $g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}$ ($W/L$ constant)

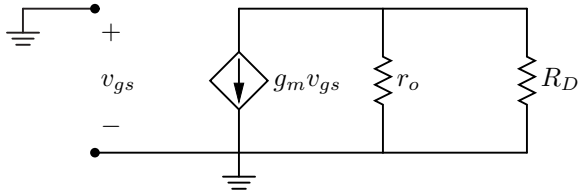
Doubling $I_D$ increases $g_m$ by $\sqrt{2}$ times.

(d) $g_m = \frac{2I_D}{V_{GS}-V_{TH}}$ ($V_{GS}-V_{TH}$ constant)

Doubling $I_D$ increases $g_m$ by 2 times.

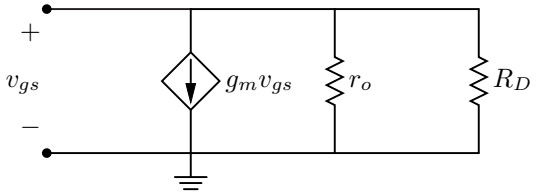
6.33 (a) Assume $M_1$ is operating in saturation.

$$V_{GS} = 1 \text{ V}$$
$$V_{DS} = V_{DD} - I_D R_D = V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2\left(1 + \lambda V_{DS}\right)R_D$$
$V_{DS} = 1.35 \text{ V} > V_{GS} - V_{TH}$, which verifies our assumption
$$I_D = 4.54 \text{ mA}$$
$$g_m = \mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right) = \boxed{13.333 \text{ mS}}$$
$$r_o = \frac{1}{\lambda I_D} = \boxed{2.203 \text{ k}\Omega}$$



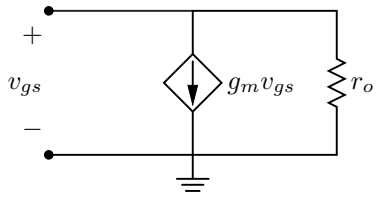

(b) Since $M_1$ is diode-connected, we know it is operating in saturation.

$$V_{GS} = V_{DS} = V_{DD} - I_D R_D = V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2\left(1 + \lambda V_{GS}\right)R_D$$
$$V_{GS} = V_{DS} = 0.546 \text{ V}$$
$$I_D = 251 \text{ μA}$$
$$g_m = \mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right) = \boxed{3.251 \text{ mS}}$$
$$r_o = \frac{1}{\lambda I_D} = \boxed{39.881 \text{ k}\Omega}$$



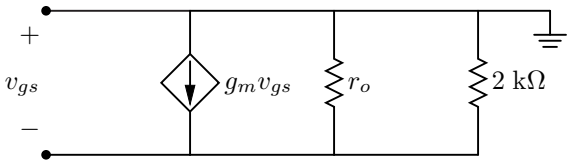

(c) Since $M_1$ is diode-connected, we know it is operating in saturation.

$$I_D = 1 \text{ mA}$$
$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = \boxed{6.667 \text{ mS}}$$
$$r_o = \frac{1}{\lambda I_D} = \boxed{10 \text{ k}\Omega}$$



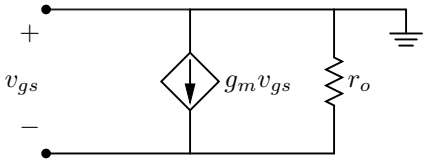

(d) Since $M_1$ is diode-connected, we know it is operating in saturation.

$$\begin{aligned}
V_{GS} &= V_{DS} \\
V_{DD} - V_{GS} &= I_D(2\text{ k}\Omega) = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2\left(1 + \lambda V_{GS}\right)(2\text{ k}\Omega) \\
V_{GS} &= V_{DS} = 0.623\text{ V} \\
I_D &= 588\ \mu\text{A} \\
g_m &= \mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right) = \boxed{4.961\text{ mS}} \\
r_o &= \frac{1}{\lambda I_D} = \boxed{16.996\text{ k}\Omega}
\end{aligned}$$



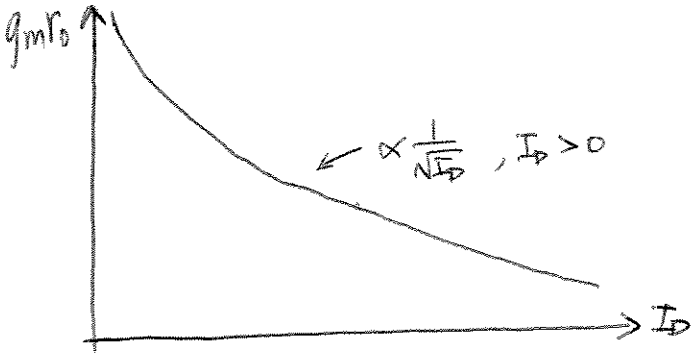

(e) Since $M_1$ is diode-connected, we know it is operating in saturation.

$$\begin{aligned}
I_D &= 0.5\text{ mA} \\
g_m &= \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = \boxed{4.714\text{ mS}} \\
r_o &= \frac{1}{\lambda I_D} = \boxed{20\text{ k}\Omega}
\end{aligned}$$



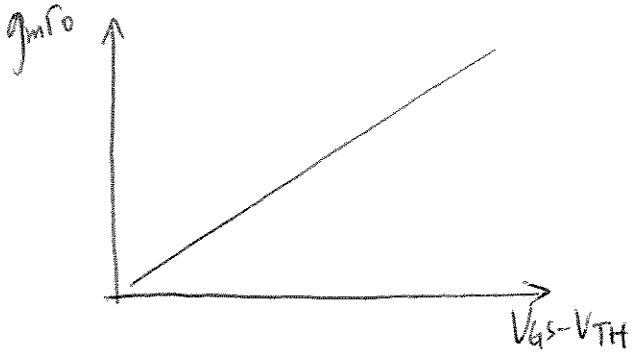
34. $g_m = \sqrt{2\mu C_{ox}\frac{W}{L} I_D}$ $\quad r_o = \left(\frac{\partial I_D}{\partial V_{DS}}\right)^{-1} = \frac{1}{\lambda I_D}$

$$g_m r_o = \frac{\sqrt{2\mu C_{ox}\left(W/L\right) I_D}}{\lambda I_D} = \frac{1}{\lambda}\sqrt{\frac{2\mu C_{ox}\left(W/L\right)}{I_D}}$$

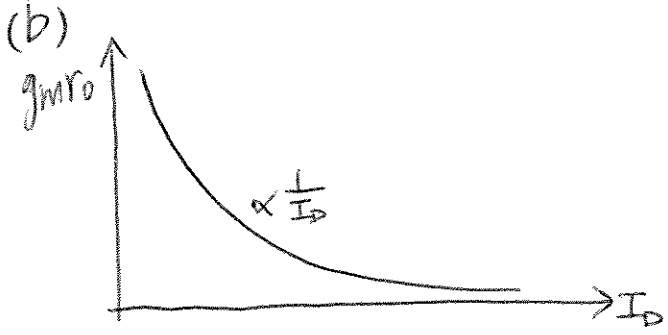


35 (a) $g_m = \mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$ $\qquad r_o = \frac{1}{\lambda I_D}$

$$g_m r_o = \frac{\mu C_{ox} (W/L)(V_{GS} - V_{TH})}{\lambda I_D}$$




(b)



36. Given NMOS with $\lambda = 0.1\ V^{-1}$ $g_m r_o = 20$ $V_{DS} = 1.5\ V$

determine $W/L$ if $I_D = 0.5\ mA$.

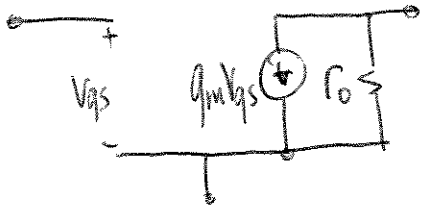
$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.1\ V^{-1})(0.5\ mA)} = 20\ k\Omega$$

$$\Rightarrow g_m = \frac{20}{20\ k\Omega} = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$

$$\therefore \frac{W}{L} = \left(\frac{20}{20\ k\Omega}\right)^2 \frac{1}{2\mu_n C_{ox} I_D}$$

$$= \left(\frac{1}{1\ k\Omega}\right)^2 \frac{1}{2\left(200\ \frac{\mu A}{V^2}\right)(0.5\ mA)} \approx 5.$$

37.




Given $\lambda = 0.2\ V^{-1}$

$g_m r_o = 20$

$V_{DS} = 1.5\ V$

$I_D = 0.5\ mA$

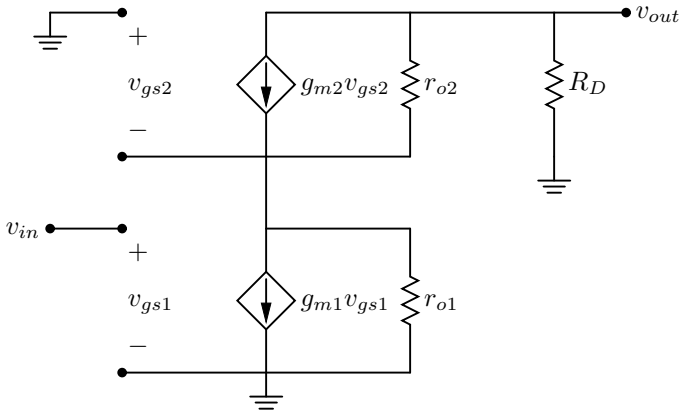
Calculate $\frac{W}{L}$.

$$g_m = \frac{20}{r_o} = 20 \cdot \lambda I_D = 20(0.2\ V^{-1})(0.5\ mA)$$
$$= 0.002\ 1/\Omega$$

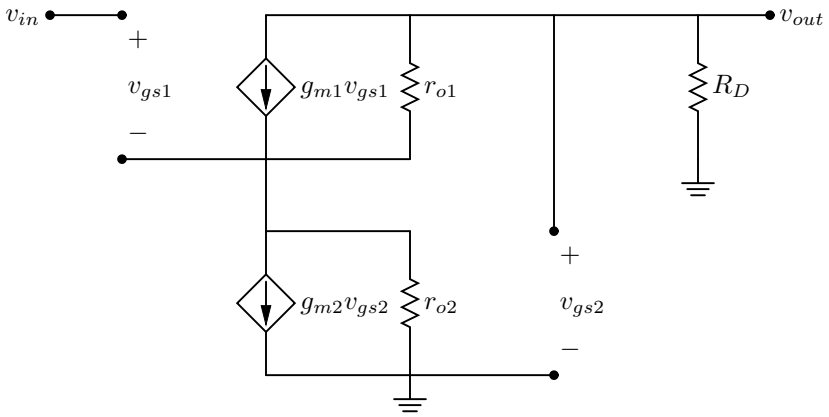
$$\Rightarrow g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

$$\therefore \frac{W}{L} = \frac{g_m^2}{2 \mu_n C_{ox} I_D} = \frac{(0.0002\ 1/\Omega)^2}{2\left(200 \frac{\mu A}{V^2}\right)(0.5\ mA)} = 20$$

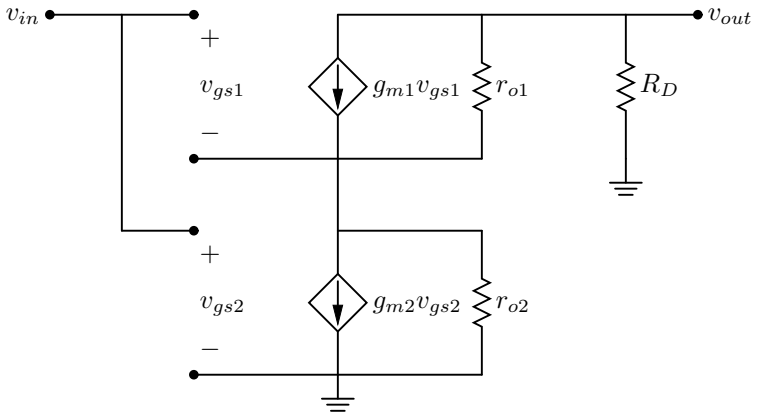
6.38 (a)


$v_{out}$
+
$v_{gs2}$
−
$g_{m2}v_{gs2}$
$r_{o2}$
$R_D$
$v_{in}$
+
$v_{gs1}$
−
$g_{m1}v_{gs1}$
$r_{o1}$

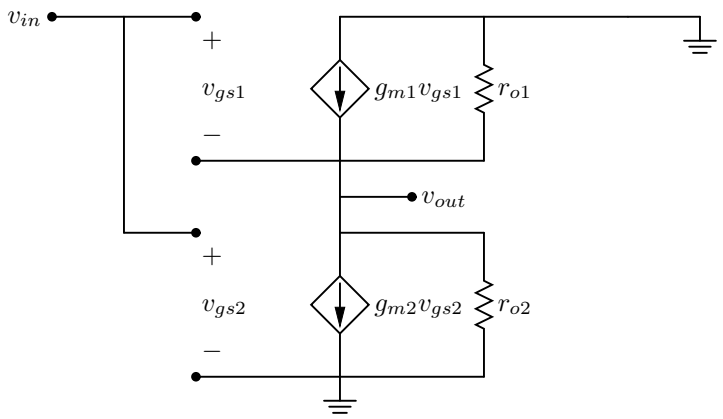
(b)


$v_{in}$
+
$v_{gs1}$
−
$g_{m1}v_{gs1}$
$r_{o1}$
$v_{out}$
$R_D$
+
$v_{gs2}$
−
$g_{m2}v_{gs2}$
$r_{o2}$

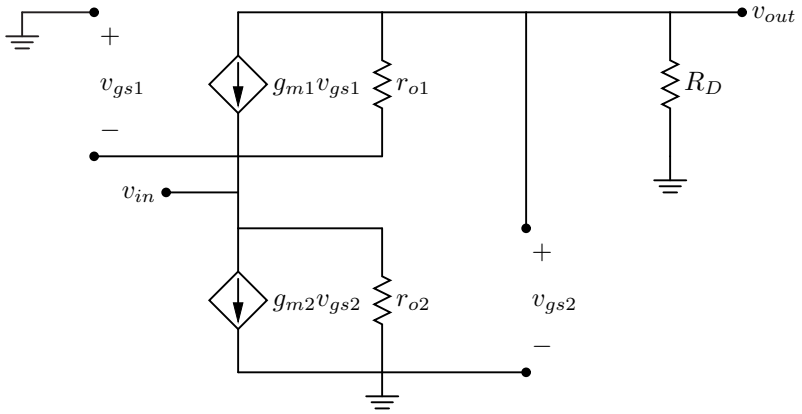
(c)


$v_{in}$
$v_{out}$
+
$v_{gs1}$
−
$g_{m1}v_{gs1}$
$r_{o1}$
$R_D$
+
$v_{gs2}$
−
$g_{m2}v_{gs2}$
$r_{o2}$

(d)


$v_{in}$
+
$v_{gs1}$
−
$g_{m1}v_{gs1}$
$r_{o1}$
$v_{out}$
+
$v_{gs2}$
−
$g_{m2}v_{gs2}$
$r_{o2}$

(e)


$v_{out}$
+
$v_{gs1}$
−
$g_{m1}v_{gs1}$
$r_{o1}$
$R_D$
$v_{in}$
+
$v_{gs2}$
−
$g_{m2}v_{gs2}$
$r_{o2}$

39. (a) OFF $\because |V_{SG}| = 0$

(b) OFF $\because |V_{SG}| < |V_{TH}| = 0.4\,V$

(c) SATURATION $\because |V_{SD}| > |V_{SG}| - |V_{TH}|$

(d) OFF $\because V_{SG} < |V_{TH}|$

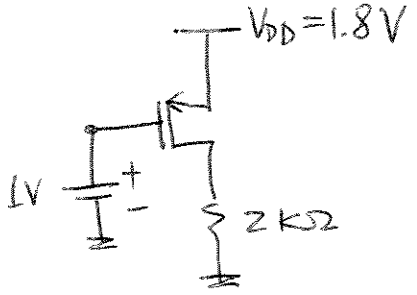
40. (a) SATURATION $\because$ $V_{SD} > V_{SG} - |V_{TH}|$

(b) LINEAR (RESISTIVE) $\because$ $V_{SG} > |V_{TH}|$
$V_{SD} \ll 2(V_{SG} - |V_{TH}|)$

(c) (EDGE OF) SATURATION $\because$ $V_{SG} > |V_{TH}|$
$V_{SD} = V_{SG} - |V_{TH}|$

(d) TRIODE $\because$ $V_{SG} > |V_{TH}|$
$V_{SD} < V_{SG} - |V_{TH}|$

41.




$\lambda = 0$

At the edge of saturation, $V_{SD} = V_{SG} - |V_{TH}|$
$\Rightarrow V_D = 1.4\,V.$

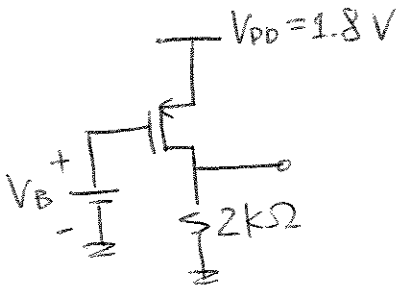
By KCL, $I_{D_1} = I_R$

$$\Rightarrow \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{SG}-|V_{TH}|)^2 = \frac{V_D}{2\,k\Omega}$$

$$\therefore \frac{W}{L} = \frac{V_D}{2\,k\Omega}\cdot\frac{2}{\mu_p C_{ox}(V_{SG}-|V_{TH}|)^2}$$

$$= \frac{1.4\,V}{2\,k\Omega}\cdot\frac{2}{100\,\frac{\mu A}{V^2}(0.8\,V-0.4V)^2} \approx 87.5$$

42.




$\lambda = 0$

When $V_B = 1\,V$, $W/L = 87.5$

When $V_B = 0.8\,V$,

$$I_D = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{SG} - |V_{TH}|)^2$$

$$= \frac{1}{2}\left(100\,\frac{\mu A}{V^2}\right)(87.5)(1-0.4)^2 V^2 \approx 1.6\,mA$$

$\Rightarrow V_D = I_D(2k\Omega) \approx 3.2\,V$, which exceeds the supply voltage!

$\therefore$ PMOS goes into triode:
($\because I_D$ is too large)

By KCL,

$$\frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[(V_{SG} - |V_{TH}|)\cdot 2V_{SD} - V_{SD}^2\right] = (V_{DD} - V_{SD})/2k\Omega$$

Solving this equation numerically (or trial-and-error) gives $V_{SD} \approx 0.18\ V$

$$\Rightarrow I_D = \frac{V_{DD} - V_{SD}}{2k\Omega} = \frac{(1.8 - 0.18)V}{2k\Omega} \approx 0.81\ mA$$

6.43 (a) Assume $M_1$ is operating in triode (since $|V_{GS}| = 1.8$ V is large).

$$|V_{GS}| = \boxed{1.8 \text{ V}}$$

$$V_{DD} - |V_{DS}| = |I_D|\,(500\ \Omega) = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[2\left(|V_{GS}| - |V_{TH}|\right)|V_{DS}| - |V_{DS}|^2\right](500\ \Omega)$$

$$|V_{DS}| = \boxed{0.418 \text{ V}} < |V_{GS}| - |V_{TH}|\text{, which verifies our assumption}$$

$$|I_D| = \boxed{2.764 \text{ mA}}$$

(b) Since $M_1$ is diode-connected, we know it is operating in saturation.

$$|V_{GS}| = |V_{DS}|$$

$$V_{DD} - |V_{GS}| = |I_D|(1\text{ k}\Omega) = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left(|V_{GS}| - |V_{TH}|\right)^2(1\text{ k}\Omega)$$

$$|V_{GS}| = |V_{DS}| = \boxed{0.952 \text{ V}}$$

$$|I_D| = \boxed{848\ \mu\text{A}}$$

(c) Since $M_1$ is diode-connected, we know it is operating in saturation.

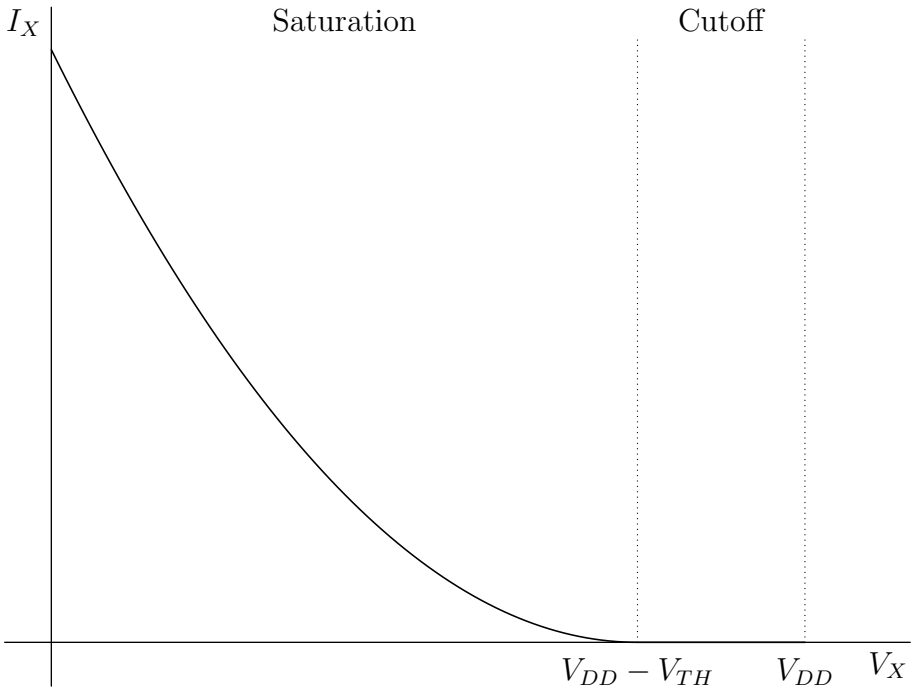
$$|V_{GS}| = |V_{DS}|$$

$$|V_{GS}| = V_{DD} - |I_D|(1\text{ k}\Omega) = V_{DD} - |I_D|(1\text{ k}\Omega) = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left(|V_{GS}| - |V_{TH}|\right)^2(1\text{ k}\Omega)$$

$$|V_{GS}| = |V_{GS}| = \boxed{0.952 \text{ V}}$$

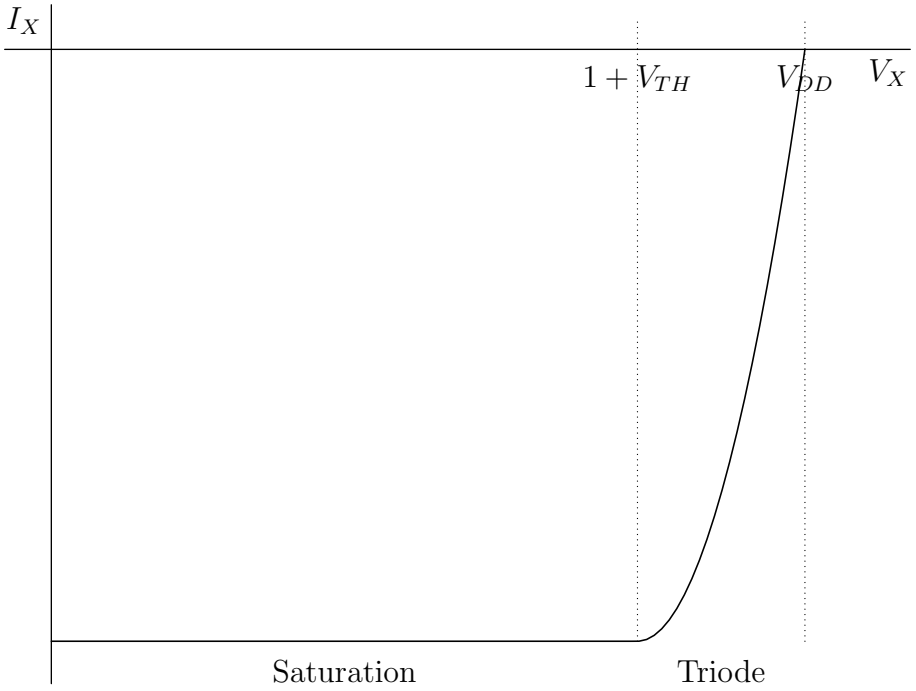
$$|I_D| = \boxed{848\ \mu\text{A}}$$

6.44 (a)



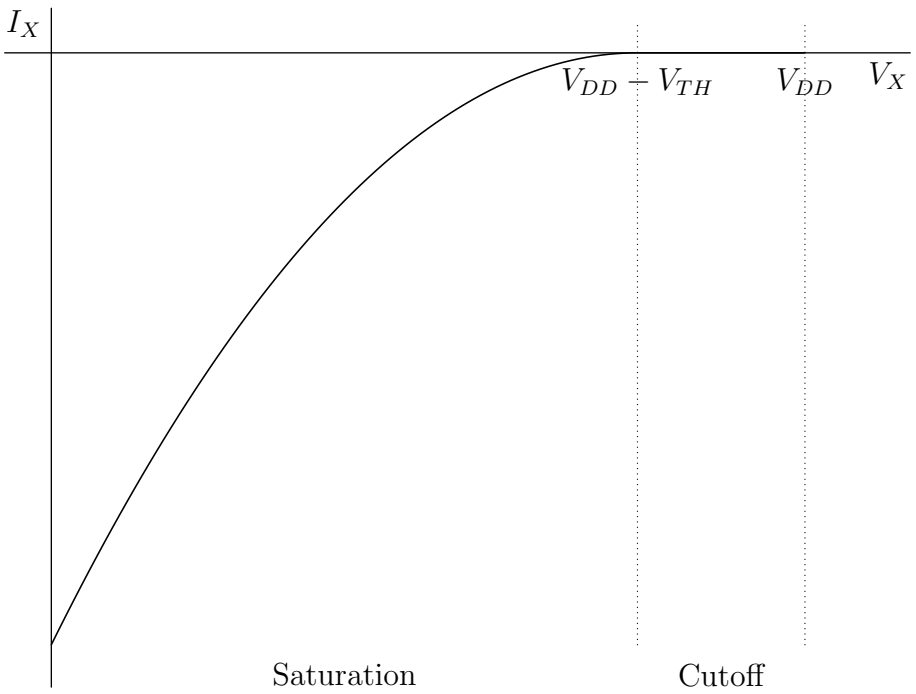

$M_1$ goes from saturation to cutoff when $V_X = V_{DD} - V_{TH} = 1.4$ V.

(b)



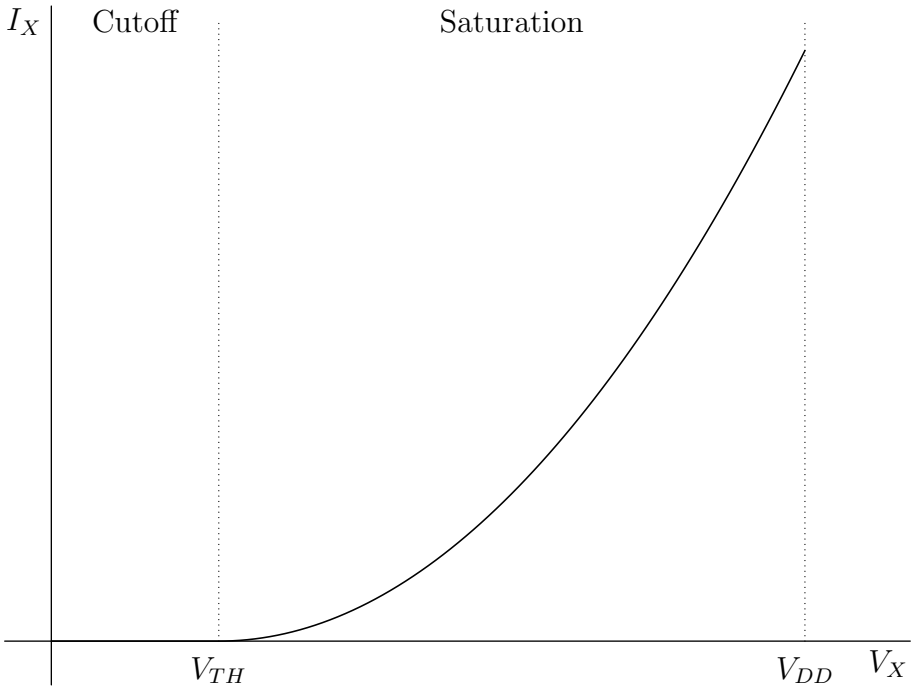

$M_1$ goes from saturation to triode when $V_X = 1 + V_{TH} = 1.4$ V.

(c)



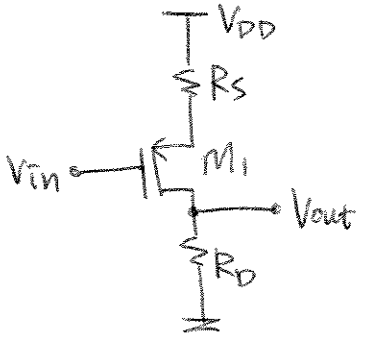

$M_1$ goes from saturation to cutoff when $V_X = V_{DD} - V_{TH} = 1.4\,\text{V}$.

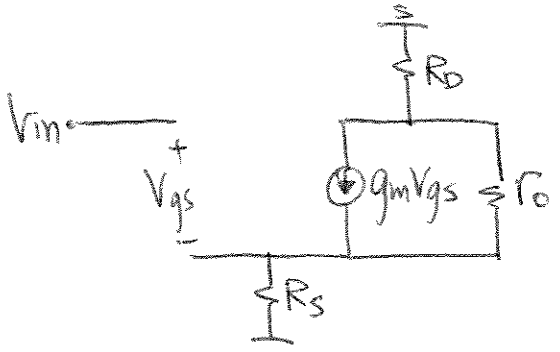
(d)



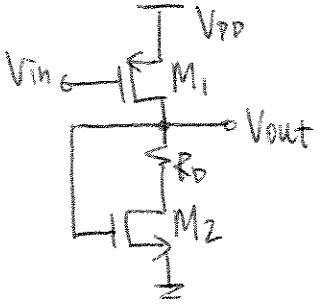

$M_1$ goes from cutoff to saturation when $V_X = V_{TH} = 0.4\,\text{V}$.

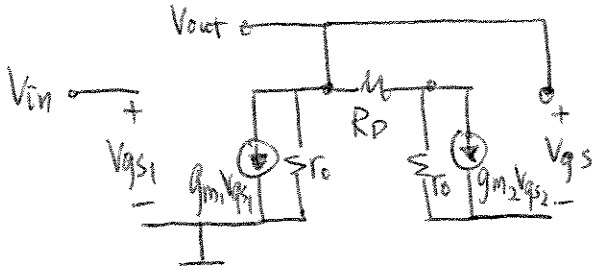
45. (a)


VDD
RS
Vin
M1
Vout
RD

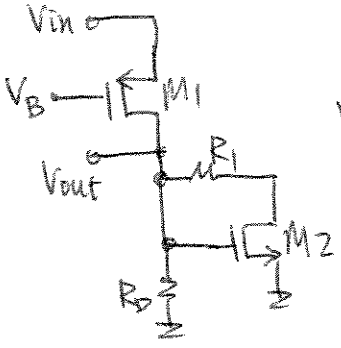

RD
Vin
+
Vgs
−
gmVgs
ro
RS

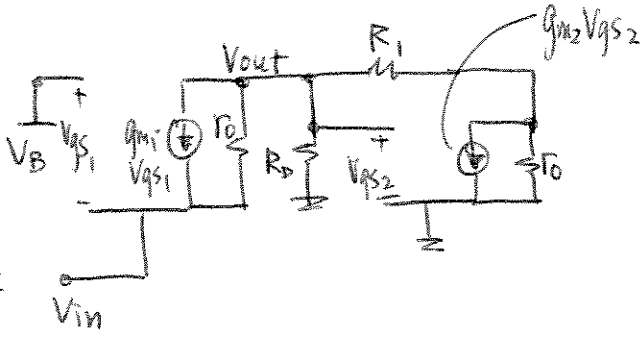
(b)


VDD
Vin
M1
Vout
RD
M2

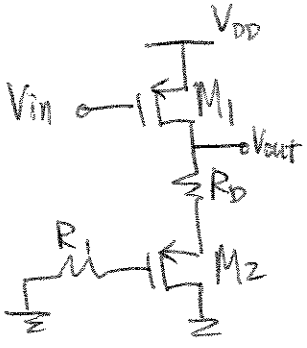

Vout
Vin
+
Vgs1
−
gm1Vgs1
ro
RD
ro
gm2Vgs2
+
Vgs
−

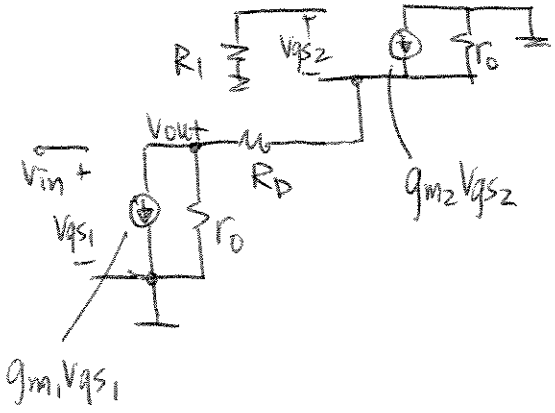
(c)


Vin
VB
M1
Vout
R1
M2
RD

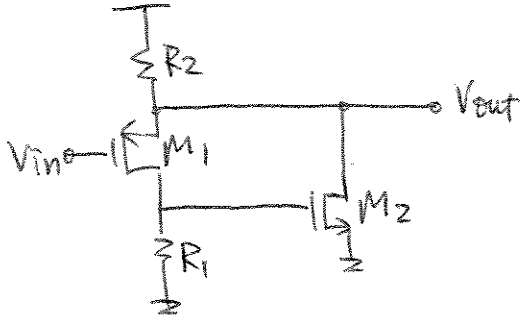

gm2Vgs2
Vout
R1
VB
+
Vgs1
−
gm1
Vgs1
ro
RD
+
Vgs2
−
ro
Vin

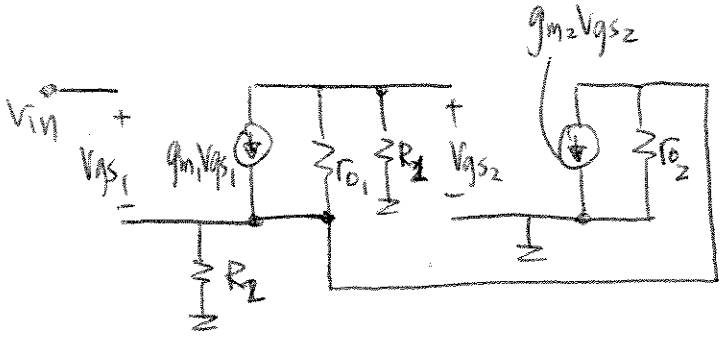
(d)


VDD
Vin
M1
Vout
RD
R1
M2

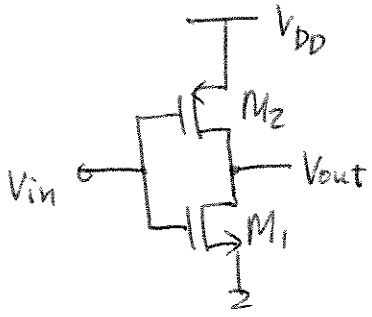

R1
Vgs2
−
ro
Vout
Vin
+
RD
gm2Vgs2
Vgs1
−
ro
gm1Vgs1

(e)


R2
Vout
Vin
M1
M2
R1

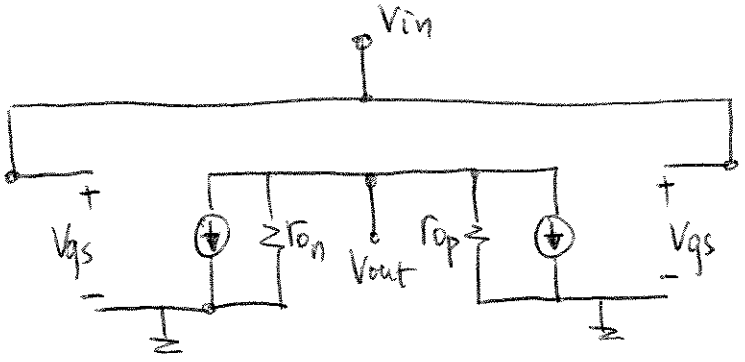

gm2Vgs2
Vin
+
Vgs1
−
gm1Vgs1
ro1
R1
+
Vgs2
−
ro2
R2

46.



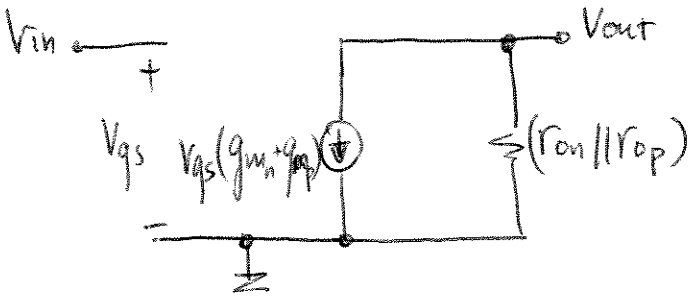

Assume $\lambda_n$ & $\lambda_p$.

(a)




They are in "parallel" because from the small-signal model, both their respective SOURCE and DRAIN nodes are the same.

(b) Assuming both $M_1$ & $M_2$ are in saturation, we can combine $r_o$'s & $g_m$'s:




$$\therefore \quad \frac{V_{out}}{V_{in}} = -(g_{mn} + g_{mp})(r_{on} // r_{op})$$

7.1

$$
\begin{aligned}
V_{GS} &= V_{DD} = 1.8 \text{ V} \\
V_{DS} &> V_{GS} - V_{TH} \text{ (in order for } M_1 \text{ to operate in saturation)} \\
V_{DS} &= V_{DD} - I_D(1 \text{ k}\Omega) \\
&= V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2 (1 \text{ k}\Omega) \\
&> V_{GS} - V_{TH} \\
\frac{W}{L} &< \boxed{2.04}
\end{aligned}
$$

② To get $I_{DS} = 1\ mA$,

$$\frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH})^2 = 1 \times 10^{-3}\ A.$$

$$\frac{1}{2}(200 \times 10^{-6})\left(\frac{20}{0.18}\right)_1 (V_{GS} - V_{TH})^2 = 10^{-3}$$

$$(V_{GS} - V_{TH})^2 = 0.09$$

$$V_{GS} - V_{TH} = 0.3,$$

$$\text{ie. } V_{GS} = 0.7.,$$

$$\text{Since } V_{GS} = \frac{R_2}{R_1 + R_2} \times 1.8$$

$$0.7 = \frac{R_2}{R_1 + R_2} \times 1.8$$

$$0.7\ R_1 = \cdot\ R_2,$$

$$\therefore \frac{R_1}{R_2} = \frac{11}{7}. \quad \text{——— ①}$$

To get input impedance $\geq$ 20k.

$$R_1 // R_2 \geq 20\ k\Omega. \quad \text{——— ②}$$

By inspection, setting $R_1 = 55\ k\Omega$ and $R_2 = 35\ k\Omega$ will satisfy both ① and ②.

7.3

$$
\begin{aligned}
V_{GS} &= V_{DD} - I_D(100\ \Omega) \\
V_{DS} &= V_{DD} - I_D(1\ \text{k}\Omega + 100\ \Omega) \\
&> V_{GS} - V_{TH} \text{ (in order for } M_1 \text{ to operate in saturation)} \\
V_{DD} - I_D(1\ \text{k}\Omega + 100\ \Omega) &> V_{DD} - I_D(100\ \Omega) - V_{TH} \\
I_D(1\ \text{k}\Omega + 100\ \Omega) &< I_D(100\ \Omega) + V_{TH} \\
I_D(1\ \text{k}\Omega) &< V_{TH} \\
I_D &< 400\ \mu\text{A}
\end{aligned}
$$

Since $g_m$ increases with $I_D$, we should pick the maximum $I_D$ to determine the maximum transconductance that $M_1$ can provide.

$$
\begin{aligned}
I_{D,max} &= 400\ \mu\text{A} \\
g_{m,max} &= \frac{2I_{D,max}}{V_{GS} - V_{TH}} \\
&= \frac{2I_{D,max}}{V_{DD} - I_{D,max}(100\ \Omega) - V_{TH}} \\
&= \boxed{0.588\ \text{mS}}
\end{aligned}
$$

④ a) $\because \quad V_{RS} = 200\,mV,$

$\therefore \quad I_{DS} R_S = 200\,mV$

$$I_{DS} = \frac{0.2}{100}$$

$$I_{DS} = 2\,mA.$$

For $M_1$ to stay in saturation,

$$V_{DS} \geq V_{GS} - V_{TH}.$$

$$\because \quad V_{DS} = V_D - V_S$$

$$= [1.8 - (2\times10^{-3}\times500)] - 0.2$$

$$= 0.6,$$

$$\therefore \quad V_{GS} - V_{TH} \leq 0.6,$$

Since $\quad I_{DS} = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2,$

$\left(\frac{W}{L}\right)$ is min. when $(V_{GS} - V_{TH})$ is max,

$\therefore$ min $\left(\frac{W}{L}\right)_1$ is when $(V_{GS} - V_{TH}) = 0.6V,$

$$2\times10^{-3} = \frac{1}{2}(200\times10^{-6})\left(\frac{W}{L}\right)_1 (0.6)^2$$

$$\therefore \text{ min. } \left(\frac{W}{L}\right)_1 \simeq 56$$

b/ With $(V_{GS} - V_{TH}) = 0.6$,

$$V_{GS} = 1,$$

$$\therefore V_G = 1 + V_S$$

$$V_G = 1.2V,$$

ie. $1.8 \times \frac{R_2}{R_1 + R_2} = 1.2V,$

$$\frac{R_2}{R_1} = 2 \qquad \text{——①}$$

Input impedance $= R_2 // R_1$,

i'e. $R_2 // R_1 \geq 30k\Omega$ ——②

Set $R_1 = 50k\Omega$ and $R_2 = 100k\Omega$ will satisfy both ① & ②.

7.5

$$
\begin{aligned}
I_{D1} &= 0.5 \text{ mA} \\
V_{GS} &= V_{TH} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} \\
&= 0.612 \text{ V} \\
V_{GS} &= \frac{1}{10} I_{D1} R_2 \\
R_2 &= \boxed{12.243 \text{ k}\Omega} \\
V_{GS} &= V_{DD} - \frac{1}{10} I_{D1} R_1 - \frac{11}{10} I_{D1} R_S \\
R_1 &= \boxed{21.557 \text{ k}\Omega}
\end{aligned}
$$

7.6

$$
\begin{aligned}
I_D &= 1 \text{ mA} \\
g_m &= \frac{2I_D}{V_{GS} - V_{TH}} = \frac{1}{100} \\
V_{GS} &= 0.6 \text{ V} \\
V_{GS} &= V_{DD} - I_D R_D \\
R_D &= \boxed{1.2 \text{ k}\Omega}
\end{aligned}
$$

(7) $I_{DS} = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2$

$0.5 \times 10^{-3} = (100 \times 10^{-6})\left(\frac{50}{0.18}\right)(V_{GS} - V_{TH})^2$

$\therefore V_{GS} = 0.534\,V$

$\therefore R_2 = \frac{0.534}{0.05 \times 10^{-3}}$

$R_2 = \underline{10.68\,k\Omega}$

$\because V_{D1} = 1.8 - (1.1 \times I_{DS} \times 2\,k\Omega) = 0.1\,I_{DS}(R_1 + R_2)$

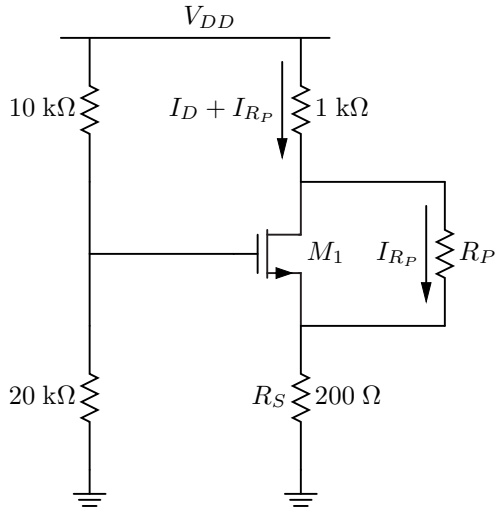
$\therefore 14\,k\Omega = R_1 + 10.68\,k\Omega.$

$\therefore R_1 = \underline{3320\,\Omega}$

7.8 First, let's analyze the circuit excluding $R_P$.

$$\begin{aligned}
V_G &= \frac{20\ \text{k}\Omega}{10\ \text{k}\Omega + 20\ \text{k}\Omega} V_{DD} = 1.2\ \text{V} \\
V_{GS} &= V_G - I_D R_S = V_{DS} = V_{DD} - I_D(1\ \text{k}\Omega + 200\ \Omega) \\
I_D &= 600\ \mu\text{A} \\
V_{GS} &= 1.08\ \text{V} \\
\frac{W}{L} &= \frac{2I_D}{\mu_n C_{ox} \left(V_{GS} - V_{TH}\right)^2} = 12.9758 \approx \boxed{13}
\end{aligned}$$

Now, let's analyze the circuit with $R_P$.

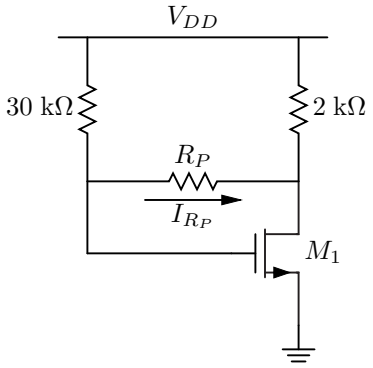



$$\begin{aligned}
V_G &= 1.2\ \text{V} \\
I_D + I_{R_P} &= \frac{V_{DD} - V_{DS}}{1\ \text{k}\Omega + 200\ \Omega} \\
V_{GS} &= V_G - \left(I_D + I_{R_P}\right) R_S = V_{DS} + V_{TH} \\
V_G - \frac{V_{DD} - V_{DS}}{1\ \text{k}\Omega + 200\ \Omega} R_S &= V_{DS} + V_{TH} \\
V_{DS} &= 0.6\ \text{V} \\
V_{GS} &= 1\ \text{V} \\
I_D &= \frac{1}{2}\mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_{TH}\right)^2 \\
&= 467\ \mu\text{A} \\
I_D + I_{R_P} &= I_D + \frac{V_{DS}}{R_P} = \frac{V_{DD} - V_{DS}}{1\ \text{k}\Omega + 200\ \Omega} \\
R_P &= \boxed{1.126\ \text{k}\Omega}
\end{aligned}$$

7.9 First, let's analyze the circuit excluding $R_P$.

$$V_{GS} = V_{DD} = 1.8 \text{ V}$$
$$V_{DS} = V_{DD} - I_D(2 \text{ k}\Omega) = V_{GS} - 100 \text{ mV}$$
$$V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2 (2 \text{ k}\Omega) = V_{GS} - 100 \text{ mV}$$
$$\frac{W}{L} = \boxed{0.255}$$

Now, let's analyze the circuit with $R_P$.




$$V_{GS} = V_{DD} - I_{R_P}(30 \text{ k}\Omega)$$
$$I_{R_P} = \frac{V_{GS} - V_{DS}}{R_P} = \frac{50 \text{ mV}}{R_P}$$
$$V_{GS} = V_{DD} - (I_D - I_{R_P})(2 \text{ k}\Omega) + 50 \text{ mV}$$
$$V_{DD} - I_{R_P}(30 \text{ k}\Omega) = V_{DD} - \left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2 - I_{R_P}\right)(2 \text{ k}\Omega) + 50 \text{ mV}$$
$$V_{DD} - I_{R_P}(30 \text{ k}\Omega) = V_{DD} - \left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{DD} - I_{R_P}(30 \text{ k}\Omega) - V_{TH}\right)^2 - I_{R_P}\right)(2 \text{ k}\Omega) + 50 \text{ mV}$$
$$I_{R_P} = 1.380 \text{ μA}$$
$$R_P = \frac{50 \text{ mV}}{I_{R_P}} = \boxed{36.222 \text{ k}\Omega}$$

(10) For $M_1$,

$$I_x = \frac{1}{2}(200\times100^{-6})\left(\frac{W_1}{0.25}\right)(0.8-0.4)^2 \times (1+0.1(0.8))$$

$$10^{-3} = 0.16\times10^{-4}\left(\frac{W_1}{0.25}\right)(1.08)$$

$$\therefore W_1 \approx 14.5\,\mu //$$

For $M_2$,

$$0.5\times10^{-3} = 0.16\times10^{-4}\left(\frac{W_2}{0.25}\right)(1.08)$$

$$\therefore W_2 = 7.25\,\mu //$$

Output resistance $= r_o$

$$= \frac{1}{\lambda}\times\frac{1}{I_D}$$

$$\therefore r_{o1} = \left(\frac{1}{0.1}\right)\left(\frac{1}{10^{-3}}\right)$$

$$= 10\,k\Omega //$$

$$r_{o2} = \left(\frac{1}{0.1}\right)\left(\frac{1}{0.5\times10^{-3}}\right)$$

$$= 20\,k\Omega //$$

(11) $R_{on} = \frac{1}{\lambda}\left(\frac{1}{I_D}\right)$

$= \frac{1}{0.5\times10^{-3}\,\lambda} = 20\,k\Omega$

$\therefore \lambda = 0.1\ V^{-1}$ //

7.12 Since we're not given $V_{DS}$ for the transistors, let's assume $\lambda = 0$ for large-signal calculations. Let's also assume the transistors operate in saturation, since they're being used as current sources.

$$I_X = \frac{1}{2}\mu_n C_{ox}\frac{W_1}{L_1}\left(V_{B1} - V_{TH}\right)^2 = 0.5 \text{ mA}$$

$$W_1 = \boxed{3.47 \text{ µm}}$$

$$I_Y = \frac{1}{2}\mu_n C_{ox}\frac{W_2}{L_2}\left(V_{B2} - V_{TH}\right)^2 = 0.5 \text{ mA}$$

$$W_2 = \boxed{1.95 \text{ µm}}$$

$$R_{out1} = r_{o1} = \frac{1}{\lambda I_X} = 20 \text{ k}\Omega$$

$$R_{out2} = r_{o2} = \frac{1}{\lambda I_Y} = 20 \text{ k}\Omega$$

Since $I_X = I_Y$ and $\lambda$ is the same for each current source, the output resistances of the current sources are the same.

7.13 Looking into the source of $M_1$ we see a resistance of $\frac{1}{g_m}$. Including $\lambda$ in our analysis, we have

$$\begin{aligned}\frac{1}{g_m} &= \frac{1}{\mu_p C_{ox}\frac{W}{L}\left(V_X - V_{B1} - |V_{TH}|\right)\left(1+\lambda V_X\right)} \\ &= \boxed{372\ \Omega}\end{aligned}$$

(14)

$$I_X = \frac{1}{2}(100\times10^{-6})\left(\frac{20}{0.25}\right)(1-1.8+0.4)^2$$

$$= 0.64\,mA$$

$$I_Y = \frac{1}{2}(100\times10^{-6})\left(2\times\frac{20}{0.25}\right)(1-1.8+0.4)^2$$

$$= 1.28\,mA$$

$\because r_o \propto \frac{1}{I}$

and $I_Y = 2I_X$

$\therefore r_{out,M_1} = 2\,r_{out,M_2}$

(15) $|I_{DS1}| = |I_{DS2}|$,

$$\frac{1}{2}(200\times10^{-6})\left(\frac{10}{0.18}\right)(V_B-0.4)^2(1+0.1\times0.9)$$

$$= \frac{1}{2}(100\times10^{-6})(1.8-V_B-0.4)^2(1+0.1\times0.9)$$

$$\times\left(\frac{30}{0.18}\right)$$

$$2(V_B-0.4)^2 = 3(1.4-V_B)^2$$

$$\sqrt{\frac{2}{3}}(V_B-0.4) = (1.4-V_B)$$

$$1.816\,V_B = 1.7264$$

$$V_B = 0.95 //$$

(16) a) For $M_1$,

$$I_{D1} = \frac{1}{2}(200\times10^{-6})\left(\frac{5}{0.18}\right)(V_B - 0.4)^2 (1+0.1\times0.9)$$

$$\therefore V_B \approx 0.806\ V$$

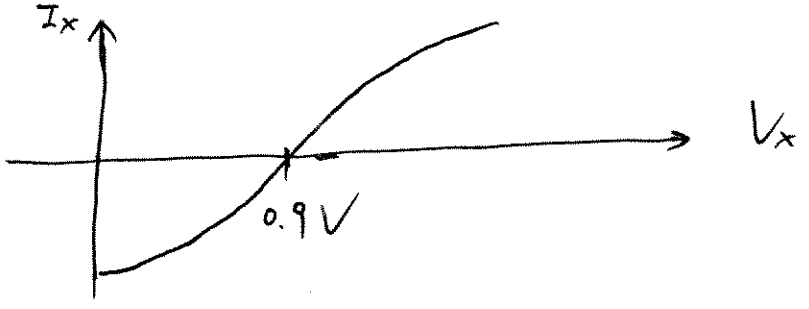
b) There are 3 regions of operation:

For $V_x < V_B - V_{TH1}$, $M_1$ is in triode.
and $|I_{DS2}| > |I_{DS1}|$

For $|V_x - V_{DD}| > |V_B - V_{DD} - V_{TH2}|$, $M_2$ is in triode
and $I_{DS1} > |I_{DS2}|$

For $V_B - V_{TH} < V_x$ and $|V_x - V_{DD}| < |V_B - V - V_{TH2}|$
$M_1$ and $M_2$ are in saturation.
and $I_{DS1} = |I_{DS2}| = 0.5\ mA$ at $V_x = 0.9\ V$

In all cases, $I_x = I_{DS1} - |I_{DS2}|$



7.17 (a) Assume $M_1$ is operating in saturation.

$$I_D = 0.5 \text{ mA}$$

$$V_{GS} = V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}}$$

$$= \boxed{0.573 \text{ V}}$$

$V_{DS} = V_{DD} - I_D R_D = 0.8 \text{ volt} > V_{GS} - V_{TH}$, verifying that $M_1$ is in saturation

(b)

$$A_v = -g_m R_D$$

$$= -\frac{2I_D}{V_{GS} - V_{TH}} R_D$$

$$= \boxed{-11.55}$$

7.18 (a) Assume $M_1$ is operating in saturation.

$$I_D = 0.25 \text{ mA}$$

$$V_{GS} = V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox}\frac{W}{L}}}$$

$$= \boxed{0.55 \text{ V}}$$

$V_{DS} = V_{DD} - I_D R_D = 1.3 \text{ V} > V_{GS} - V_{TH}$, verifying that $M_1$ is in saturation

(b)

$$V_{GS} = 0.55 \text{ V}$$

$$V_{DS} > V_{GS} - V_{TH} \text{ (to ensure } M_1 \text{ remains in saturation)}$$

$$V_{DD} - I_D R_D > V_{GS} - V_{TH}$$

$$V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2 R_D > V_{GS} - V_{TH}$$

$$\frac{W}{L} < \frac{2\left(V_{DD} - V_{GS} + V_{TH}\right)}{\mu_n C_{ox}\left(V_{GS} - V_{TH}\right)^2 R_D}$$

$$= 366.67$$

$$= 3.3\frac{20}{0.18}$$

Thus, $W/L$ can increase by a factor of $\boxed{3.3}$ while $M_1$ remains in saturation.

$$A_v = -g_m R_D$$

$$= -\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right) R_D$$

$$A_{v,max} = -\mu_n C_{ox}\left(\frac{W}{L}\right)_{max}\left(V_{GS} - V_{TH}\right) R_D$$

$$= \boxed{-22}$$

7.19

$$
\begin{aligned}
P &= V_{DD} I_D < 1 \text{ mW} \\
I_D &< 556 \text{ µA} \\
A_v &= -g_m R_D \\
&= -\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} R_D \\
&= -5 \\
\frac{W}{L} &< \frac{20}{0.18} \\
R_D &> \boxed{1.006 \text{ k}\Omega}
\end{aligned}
$$

7.20 (a)

$$\begin{aligned} I_{D1} &= I_{D2} = 0.5 \text{ mA} \\ A_v &= -g_{m1} \left( r_{o1} \parallel r_{o2} \right) \\ &= -\sqrt{2\mu_n C_{ox} \left( \frac{W}{L} \right)_1 I_{D1}} \left( \frac{1}{\lambda_1 I_{D1}} \parallel \frac{1}{\lambda_2 I_{D2}} \right) \\ &= -10 \\ \left( \frac{W}{L} \right)_1 &= \boxed{7.8125} \end{aligned}$$

(b)

$$\begin{aligned} V_{DD} - V_B &= V_{TH} + \sqrt{\frac{2 \left| I_{D2} \right|}{\mu_p C_{ox} \left( \frac{W}{L} \right)_2}} \\ V_B &= \boxed{1.1 \text{ V}} \end{aligned}$$

(21) $|A_v| = g_{m_1} (r_{o1} // r_{o2})$

$$g_{m1} = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{20}{0.18}\right) \times (0.001)}$$

(Since $V_{ds_1}$ is not given, assume (if $\lambda_1 V_{DS_1}$) has minimal effect on $g_{m_1}$)

$$= 6.67 \text{ mS} \qquad (S = \Omega^{-1})$$

$$r_{o1} = \frac{1}{\lambda_1 \times I_{D1}}$$

$$= \frac{1}{0.1 \times 1mA}$$

$$= 10 \text{ k}\Omega.$$

$$r_{o_2} \simeq \infty$$

($\because \lambda_2 << \lambda_1$)

$$\therefore |A_v| = 6.67 \times 10^{-3} \times 10^3 \times 10$$

$$= 66.7 //$$

7.22 (a) If $I_{D1}$ and $I_{D2}$ remain constant while $W$ and $L$ double, then $g_{m1} \propto \sqrt{(W/L)_1 \, I_{D1}}$ will not change (since it depends only on the ratio $W/L$), $r_{o1} \propto \frac{1}{I_{D1}}$ will not change, and $r_{o2} \propto \frac{1}{I_{D2}}$ will not change. Thus, $A_v = -g_{m1} \left( r_{o1} \parallel r_{o2} \right)$ will not change.

(b) If $I_{D1}$, $I_{D2}$, $W$, and $L$ double, then $g_{m1} \propto \sqrt{(W/L)_1 \, I_{D1}}$ will increase by a factor of $\sqrt{2}$, $r_{o1} \propto \frac{1}{I_{D1}}$ will halve, and $r_{o2} \propto \frac{1}{I_{D2}}$ will halve. This means that $r_{o1} \parallel r_{o2}$ will halve as well, meaning $A_v = -g_{m1} \left( r_{o1} \parallel r_{o2} \right)$ will decrease by a factor of $\sqrt{2}$.

(23). To get higher voltage gain,

(a) is preferred.

For the same dimensions of transistors and same bias current,

(a) has a high "$g_m$" than (b).

$\therefore g_{m_1} > g_{m_2}$.

(since $\mu_n C_{ox} > \mu_p C_{ox}$)

while $(r_{o1} \| r_{o2})$ is the same for both cases.

(24) $A_V = g_{m_2} (r_{o1} // r_{o2})$

$$r_{o1} = \frac{1}{0.15 \times 0.5mA}$$

$$= 13.3\ k\Omega.$$

$$r_{o2} = \frac{1}{0.05 \times 0.5mA}$$

$$= 40\ k\Omega.$$

$\therefore r_{o1} // r_{o2} = 10\ k\Omega.$

$$\therefore 15 = \left[\sqrt{2 \times (100 \times 10^{-6}) \left(\frac{W}{L}\right)_2 0.5mA}\right] \cdot (10\ k\Omega)$$

$$\left(\frac{W}{L}\right)_2 = 22.5 //$$

(25) From Eq (7.57),

$$3 = \sqrt{\frac{20/0.18}{(w/L)_2}}$$

$$\therefore (w/L)_2 \approx 12.3 //.$$

7.26 (a)

$$I_{D1} = I_{D2} = 0.5 \text{ mA}$$

$$V_{GS1} = V_{TH} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}}$$

$$= 0.7 \text{ V}$$

$$V_{DS1} = V_{GS1} - V_{TH} \text{ (in order of } M_1 \text{ to operate at the edge of saturation)}$$

$$= V_{DD} - V_{GS2}$$

$$V_{GS2} = V_{DD} - V_{GS1} + V_{TH} = V_{TH} + \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}}$$

$$\left(\frac{W}{L}\right)_2 = \boxed{4.13}$$

(b)

$$A_v = -\frac{g_{m1}}{g_{m2}}$$

$$= -\frac{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}}}{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_2 I_{D2}}}$$

$$= -\sqrt{\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}}$$

$$= \boxed{-3.667}$$

(c) Since $(W/L)_1$ is fixed, we must minimize $(W/L)_2$ in order to maximize the magnitude of the gain (based on the expression derived in part (b)). If we pick the size of $M_2$ so that $M_1$ operates at the edge of saturation, then if $M_2$ were to be any smaller, $V_{GS2}$ would have to be larger (given the same $I_{D2}$), driving $M_1$ into triode. Thus, $(W/L)_2$ is its smallest possible value (without driving $M_1$ into saturation) when $M_1$ is at the edge of saturation, meaning the gain is largest in magnitude with this choice of $(W/L)_2$.

7.27 (a)

$$\begin{aligned} A_v &= -\frac{g_{m1}}{g_{m2}} \\ &= -\frac{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}}}{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_2 I_{D2}}} \\ &= -\sqrt{\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}} \\ &= -5 \\ \left(\frac{W}{L}\right)_1 &= \boxed{277.78} \end{aligned}$$

(b)

$$\begin{aligned} V_{DS1} &> V_{GS1} - V_{TH} \text{ (to ensure } M_1 \text{ is in saturation)} \\ V_{DD} - V_{GS2} &> V_{GS1} - V_{TH} \\ V_{DD} - V_{TH} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}} &> \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}} \\ I_{D1} = I_{D2} &< \boxed{1.512 \text{ mA}} \end{aligned}$$

7.28 For this problem, recall that looking into the drain of a transistor with a grounded gate and source we see a resistance of $r_o$, and looking into either terminal of a diode-connected transistor we see a resistance of $\frac{1}{g_m} \parallel r_o$.

(a)

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{o2}\right)}$$

(b)

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m3}} \parallel r_{o3}\right)}$$

(c)

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m3}} \parallel r_{o3}\right)}$$

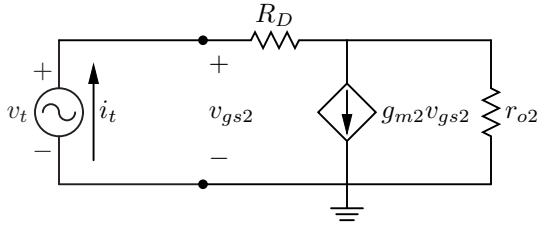
(d)

$$A_v = \boxed{-g_{m2}\left(r_{o2} \parallel r_{o1} \parallel \frac{1}{g_{m3}} \parallel r_{o3}\right)}$$

(e)

$$A_v = \boxed{-g_{m2}\left(r_{o2} \parallel r_{o1} \parallel \frac{1}{g_{m3}} \parallel r_{o3}\right)}$$

(f) Let's draw a small-signal model to find the equivalent resistance seen looking up from the output.




$$\begin{aligned}
i_t &= g_{m2}v_{gs2} + \frac{v_t - i_t R_D}{r_{o2}} \\
v_{gs2} &= v_t \\
i_t &= g_{m2}v_t + \frac{v_t - i_t R_D}{r_{o2}} \\
i_t\left(1 + \frac{R_D}{r_{o2}}\right) &= v_t\left(g_{m2} + \frac{1}{r_{o2}}\right) \\
\frac{v_t}{i_t} &= \frac{1 + \frac{R_D}{r_{o2}}}{g_{m2} + \frac{1}{r_{o2}}} = \frac{r_{o2} + R_D}{1 + g_{m2}r_{o2}} \\
A_v &= \boxed{-g_{m1}\left(r_{o1} \parallel \frac{r_{o2} + R_D}{1 + g_{m2}r_{o2}}\right)}
\end{aligned}$$

7.30 (a) Assume $M_1$ is operating in saturation.

$$\begin{aligned}
I_D &= 1 \text{ mA} \\
I_D R_S &= 200 \text{ mV} \\
R_S &= 200\ \Omega \\
A_v &= -\frac{R_D}{\frac{1}{g_m} + R_S} \\
&= -\frac{R_D}{\frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}} + R_S} \\
&= -4 \\
\frac{W}{L} &= \boxed{1000} \\
V_{GS} &= V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}} \\
&= 0.5 \text{ V} \\
V_{DS} &= V_{DD} - I_D R_D - I_D R_S \\
&= 0.6 \text{ V} > V_{GS} - V_{TH}, \text{ verifying that } M_1 \text{ is in saturation}
\end{aligned}$$

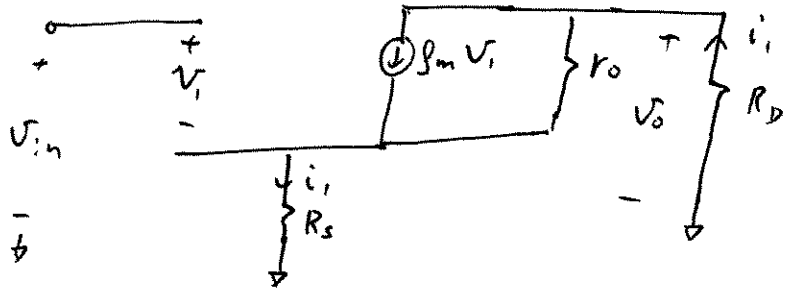
$\boxed{\text{Yes}}$, the transistor operates in saturation.

(b) Assume $M_1$ is operating in saturation.

$$\begin{aligned}
\frac{W}{L} &= \frac{50}{0.18} \\
R_S &= 200\ \Omega \\
A_v &= -\frac{R_D}{\frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}} + R_S} \\
&= -4 \\
R_D &= \boxed{1.179 \text{ k}\Omega} \\
V_{GS} &= V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}} \\
&= 0.590 \text{ V} \\
V_{DS} &= V_{DD} - I_D R_D - I_D R_S \\
&= 0.421 \text{ V} > V_{GS} - V_{TH}, \text{ verifying that } M_1 \text{ is in saturation}
\end{aligned}$$

$\boxed{\text{Yes}}$, the transistor operates in saturation.

(31) The small signal model is:




$$V_o = -i_1 R_D \qquad \text{——— (1)}$$

$$i_1 = g_m V_1 + \frac{V_o - V_1}{r_o}$$

$$= \frac{g_m r_o - 1) V_1 + V_o}{r_o}$$

$$i_1 \approx g_m V_1 + \frac{V_o}{r_o}$$

$$\therefore -\frac{V_o}{R_D} = g_m V_1 + \frac{V_o}{r_o} \qquad \text{——— (2)}$$

$$V_{in} = V_1 + i_1 R_s$$

$$\therefore V_1 = V_{in} + \frac{V_o}{R_D} R_s \qquad \text{——— (3)}$$

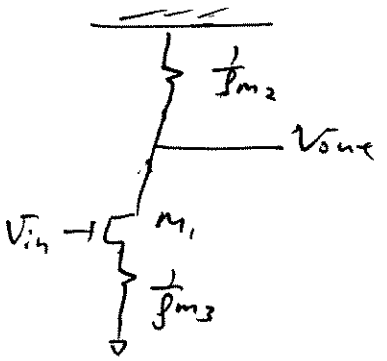
(2) combines with (3):

$$-\frac{V_o}{R_D} = g_m V_{in} + g_m V_o \frac{R_s}{R_D} + \frac{V_o}{r_o}$$

$$-V_o \left[ \frac{1}{R_D} + g_m \frac{R_s}{R_D} + \frac{1}{r_o} \right] = g_m V_{in}$$

$$\therefore \text{Volt. gain} = \frac{V_o}{V_{in}} = -\left[ \frac{g_m}{r_o + g_m R_s r_o + R_D} \right] (r_o R_D) \quad //$$

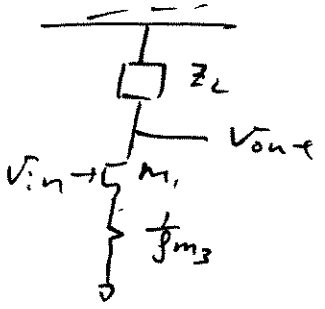
(32) a/ Equivalent circuit is:




$$\therefore A_v = - \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m3}}}$$

b/ Similar to Prob. 28 (f),

Equivalent circuit is:

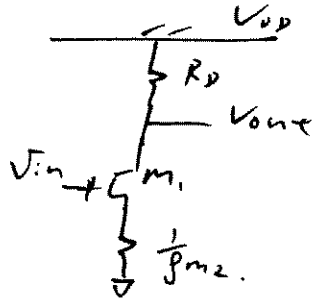



From Prob. 28 (f),

$$Z_L = \frac{1}{g_{m2}} \quad (\text{as } r_{o2} \to \infty)$$

$$\therefore A_v = - \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m3}}}$$

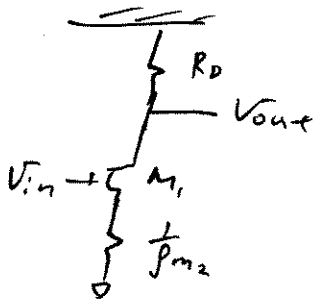
c/ Equivalent circuit is:




$$\therefore A_v = - \frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

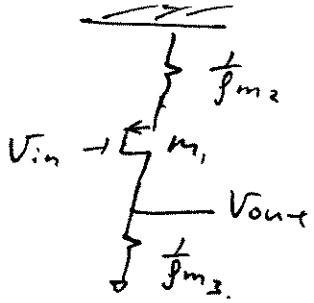
(d). Equivalent circuit is

$$A_V = -\frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$




(e) Equivalent circuit is

$$A_V = \frac{\frac{1}{g_{m3}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$



(33) a/ From Eg. (7.71),

$$R_{out} = (1 + g_{m_1} r_{o_1}) \frac{1}{g_{m_2}} + r_{o_1}$$

b/ From Eg. (7.71),

$$R_{out} = (1 + g_{m_1} r_{o_1}) \frac{1}{g_{m_2}} + r_{o_1}$$

c/ From Eg. (7.71),

$$R_{out} = (1 + g_{m_2} r_{o_2})(r_{o_1} \,//\, \frac{1}{g_{m_3}}) + r_{o_2}$$

d/ From Eg. (7.71),

$$R_{out} = (1 + g_{m_1} r_{o_1})(r_{o_2} \,//\, \frac{1}{g_{m_3}}) + r_{o_1}$$

(34) To find $\left(\frac{W}{L}\right)_1$

$$10^{-3} = \frac{1}{2}(200\times10^{-6})\left(\frac{W}{L}\right)(1-0.4)^2 \times (1+0.1V_{DS})_1$$

where $V_{DS} = 1.8 - 1k\Omega \times 1mA$

$$= 0.8V_1$$

$$\therefore \left(\frac{W}{L}\right) \simeq 25.7 //$$

Voltage gain, $(A_v) = -g_{m_1}(r_{o_1} // R_D)$

$$g_{m_1} = \sqrt{2(200\times10^{-6})(25.7)\times10^{-3} \times (1+0.1\times0.8)}$$

$$= 3.33\ mS.$$

$$r_{o1} = \frac{1}{0.1 \times 10^{-3}}$$

$$= 10k\Omega.$$

$$\therefore A_v = (-3.33\times10^{-3})(10k\Omega // 1k\Omega)$$

$$\simeq -3.03 //$$

(35) With $\lambda = 0$,

$$10^{-3} = \frac{1}{2}(200 \times 10^{-6})\left(\frac{W}{L}\right)(1 - 0.4)^2$$

$$\therefore \left(\frac{W}{L}\right) \approx 27.8 //$$

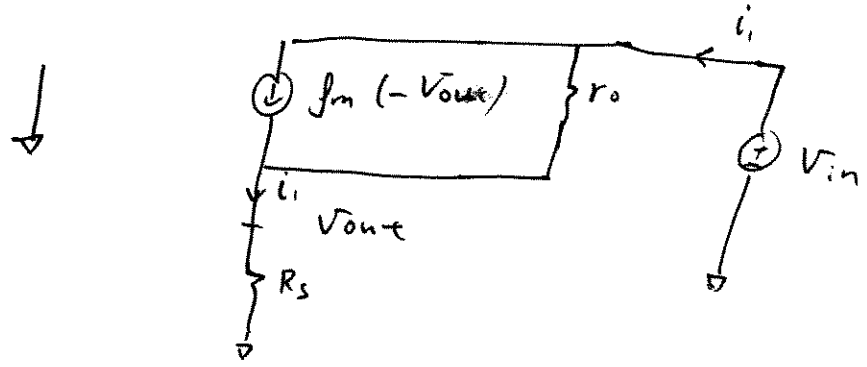
$$A_V = -g_m R_D$$

$$= -\sqrt{2(200 \times 10^{-6})(27.8) \times 10^{-3}} \times 1000$$

$$= -3.33 //$$

Without $r_o$, gain increases due mainly to increase in load resistance.

(36) The small-signal circuit is:




$$i_1 = \frac{V_{out}}{R_s} \qquad \text{——— ①}$$

$$i_1 = g_m(-V_{out}) + \frac{V_{in} - V_{out}}{r_o} \qquad \text{——— ②}$$

$$\therefore \frac{V_{out}}{R_s} = -g_m V_{out} + \frac{V_{in}}{r_o} - \frac{V_{out}}{r_o}$$

$$V_{out}\left(\frac{1}{R_s} + g_m + \frac{1}{r_o}\right) = \frac{V_{in}}{r_o}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{r_o}\left(\frac{R_s\, r_o}{r_o + g_m r_o R_s + R_s}\right)$$

$$= \frac{R_s}{g_m r_o R_s + r_o + R_s}$$

Since $(g_m r_o R_s + r_o) > 0$, the voltage gain $< 1$.

This is expected: Any variation in $V_{in}$ causes minimal change in the bias current.
∵ $V_{out}$ is determined largely by the amount of bias current ( ∵ $V_{out}$ is set by $V_{GS1}$ )
∴ There is almost no variation in $V_{out}$. (ie. $\frac{V_{out}}{V_{in}} \ll 1$)

(37) a/ $|\text{Voltage gain}| = g_m R_D$

$= 5$

$\therefore g_m = \frac{5}{500}$

$= 10\,mS.$

$= \sqrt{2(200\times10^{-6})\left(\frac{W}{L}\right)\times 10^{-3}}$

$\therefore \frac{W}{L} = 250$ //

b/ $V_D = 1.8 - 500\times 10^{-3}$

$= 1.3\,V$

To obtain $V_{DS} \geq V_{GS} - V_{TH} + 0.2$,

$V_D \geq V_G - 0.2$

$\therefore V_G \leq 1.5$

Also, $I_{R_1+R_2} = 0.1\times10^{-3}A.$

$\therefore R_1+R_2 = \frac{1.8}{0.1\times10^{-3}}$

$= 18\,k\Omega.$

choose $R_2 = 15\,k\Omega$ & $R_1 = 3\,k\Omega$

c) With twice of $(W/L)_1$, $M_1$ will go further away from triode. As $(W/L)_1$ doubles, & $I_{bias}$ is fixed by the current source, $V_{GS}$ is forced to decrease (so $M_1$ will have same $I_{DS}$). Thus, $(V_{GS} - V_{TH})$ decreases, and $V_{OS}$ can be allowed to drop more before $M_1$ goes into triode.

Gain will be increased by $\sqrt{2}$, because gain $\propto g_m$, and $g_m \propto \sqrt{W/L}$.

(38) a/ $V_G = 1.8V$.

$\therefore V_{D,min} = 1.8 - 0.4$ (for $M_1$ stays in saturation)

$= 1.4V$

$\therefore R_{D,max} = \frac{1.4V}{1mA}$

$= 1.4k\Omega$ //

b/ $|\text{Voltage gain}| = g_m R_D$

$= 5.$

$\therefore g_m = \frac{5}{R_D}$

$= 3.57mS.$

$= \sqrt{2 \times (200\times10^{-6}) (\frac{W}{L}) \times 10^{-3}}$

$\therefore (\frac{W}{L}) = 31.9$ //

(39) To get $R_{in} = 50\Omega$,

$$\frac{1}{g_m} = 50\Omega$$

$$\therefore g_m = 20\ mS.$$

volt gain $(A_v) = g_m R_D$

$$= 4,$$

$$\therefore R_D = \frac{4}{0.02}$$

$$R_D = 200\Omega //$$

$$\therefore g_m = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{W}{L}\right) \times 0.5 \times 10^{-3}}$$

$$\therefore \left(\frac{W}{L}\right) = 2000 //$$

7.42 (a)

$$\begin{aligned} R_{out} &= R_D = 500\ \Omega \\ V_G &= V_{DD} \\ V_D &> V_G - V_{TH} \text{ (in order for } M_1 \text{ to operate in saturation)} \\ V_{DD} - I_D R_D &> V_{DD} - V_{TH} \\ I_D &< \boxed{0.8\ \text{mA}} \end{aligned}$$

(b)

$$\begin{aligned} I_D &= 0.8\ \text{mA} \\ R_{in} &= \frac{1}{g_m} \\ &= \frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}} \\ &= 50\ \Omega \\ \frac{W}{L} &= \boxed{1250} \end{aligned}$$

(c)

$$\begin{aligned} A_v &= g_m R_D \\ g_m &= \frac{1}{50}\ \text{S} \\ R_D &= 500\ \Omega \\ A_v &= \boxed{10} \end{aligned}$$

7.43 (a)

$$\begin{aligned} I_D &= I_1 = 1 \text{ mA} \\ V_G &= V_{DD} \\ V_D &= V_G - V_{TH} + 100 \text{ mV} \\ V_{DD} - I_D R_D &= V_G - V_{TH} + 100 \text{ mV} \\ R_D &= \boxed{300 \ \Omega} \end{aligned}$$

(b)

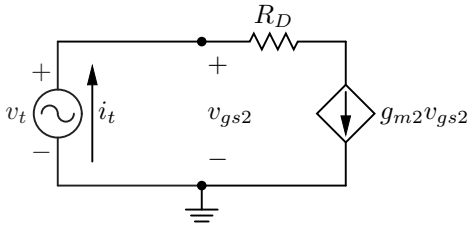
$$\begin{aligned} R_D &= 300 \ \Omega \\ A_v &= g_m R_D \\ &= \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} R_D \\ &= 5 \\ \frac{W}{L} &= \boxed{694.4} \end{aligned}$$

7.44 For this problem, recall that looking into the drain of a transistor with a grounded gate and source we see a resistance of $r_o$, and looking into either terminal of a diode-connected transistor we see a resistance of $\frac{1}{g_m} \parallel r_o$.

(a) Referring to Eq. (7.109) with $R_D = \frac{1}{g_{m2}}$ and $g_m = g_{m1}$, we have

$$A_v = \boxed{\frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + R_S}}$$

(b) Let's draw a small-signal model to find the equivalent resistance seen looking up from the output.




$$\begin{aligned} i_t &= g_{m2}v_{gs2} \\ v_{gs2} &= v_t \\ i_t &= g_{m2}v_t \\ \frac{v_t}{i_t} &= \frac{1}{g_{m2}} \\ A_v &= \boxed{\frac{g_{m1}}{g_{m2}}} \end{aligned}$$

(c) Referring to Eq. (7.119) with $R_D = \frac{1}{g_{m2}}$, $R_3 = R_1$, and $g_m = g_{m1}$, we have

$$A_v = \boxed{\frac{R_1 \parallel \frac{1}{g_{m1}}}{R_S + R_1 \parallel \frac{1}{g_{m1}}} \frac{g_{m1}}{g_{m2}}}$$

(d)

$$A_v = \boxed{g_{m1}\left(R_D + \frac{1}{g_{m2}} \parallel r_{o3}\right)}$$

(e)

$$A_v = \boxed{g_{m1}\left(R_D + \frac{1}{g_{m2}}\right)}$$

7.45 (a)

$$\frac{v_X}{v_{in}} = -g_{m1}\left(R_{D1} \parallel \frac{1}{g_{m2}}\right)$$
$$\frac{v_{out}}{v_X} = g_{m2}R_{D2}$$
$$\frac{v_{out}}{v_{in}} = \frac{v_X}{v_{in}}\frac{v_{out}}{v_X}$$
$$= \boxed{-g_{m1}g_{m2}R_{D2}\left(R_{D1} \parallel \frac{1}{g_{m2}}\right)}$$

(b)

$$\lim_{R_{D1}\to\infty} -g_{m1}g_{m2}R_{D2}\left(R_{D1} \parallel \frac{1}{g_{m2}}\right) = \boxed{-g_{m1}R_{D2}}$$

This makes sense because the common-source stage acts as a transconductance amplifier with a transconductance of $g_{m1}$. The common-gate stage acts as a current buffer with a current gain of 1. Thus, the current $g_{m1}v_{in}$ flows through $R_{D2}$, meaning $v_{out} = -g_{m1}v_{in}R_{D2}$, so that $\frac{v_{out}}{v_{in}} = -g_{m1}R_{D2}$.
This type of amplifier (with $R_{D1} = \infty$) is known as a cascode and will be studied in detail in Chapter 9.

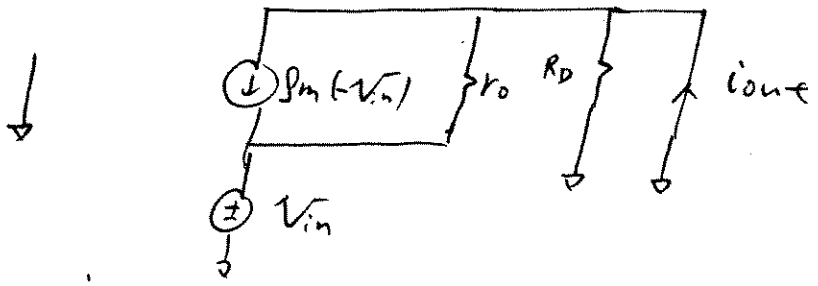
7.40

$$
\begin{aligned}
I_D &= 0.5 \text{ mA} \\
R_{in} &= \frac{1}{g_m} \\
&= \frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}} \\
&= 50\ \Omega \\
\frac{W}{L} &= \boxed{2000} \\
V_D &> V_G - V_{TH} \text{ (in order for } M_1 \text{ to operate in saturation)} \\
V_{DD} - I_D R_D &> V_b - V_{TH} \\
R_D &< 2.4 \text{ k}\Omega
\end{aligned}
$$

Since $|A_v| \propto R_D$, we need to maximize $R_D$ in order to maximize the gain. Thus, we should pick $R_D = \boxed{2.4 \text{ k}\Omega}$. This corresponds to a voltage gain of $A_v = -g_m R_D = -48$.

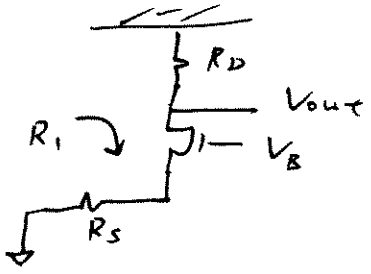
(41) Voltage gain ($A_v$) = $G_m$ $R_{out}$,
where $G_m$ and $R_{out}$ are the transconductance and output resistance of the circuit respectively.

To find $G_m$:




$$G_m = \frac{i_{out}}{V_{in}} = g_m + \frac{1}{r_o}$$
$$\simeq g_m \quad (\because g_m r_o >> 1)$$

To find $R_{out}$:




$$R_{out} = R_D // R_1$$
$$= R_D // [(1 + g_m r_o) R_S + r_o]$$
(from Eq. (7.110))
$$\simeq R_D // (g_m r_o R_S + r_o) \quad (\because g_m r_o >> 1)$$
$$= \frac{g_m r_o R_S R_D + r_o R_D}{R_D + g_m r_o R_S + r_o}$$

$$\therefore \text{Voltage gain} = g_m \left[ \frac{g_m r_o R_D R_S + r_o R_D}{R_D + g_m r_o R_S + r_o} \right]$$

7.42 (a)

$$\begin{aligned} R_{out} &= R_D = 500\ \Omega \\ V_G &= V_{DD} \\ V_D &> V_G - V_{TH} \text{ (in order for } M_1 \text{ to operate in saturation)} \\ V_{DD} - I_D R_D &> V_{DD} - V_{TH} \\ I_D &< \boxed{0.8\ \text{mA}} \end{aligned}$$

(b)

$$\begin{aligned} I_D &= 0.8\ \text{mA} \\ R_{in} &= \frac{1}{g_m} \\ &= \frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}} \\ &= 50\ \Omega \\ \frac{W}{L} &= \boxed{1250} \end{aligned}$$

(c)

$$\begin{aligned} A_v &= g_m R_D \\ g_m &= \frac{1}{50}\ \text{S} \\ R_D &= 500\ \Omega \\ A_v &= \boxed{10} \end{aligned}$$

7.43 (a)

$$\begin{aligned} I_D &= I_1 = 1 \text{ mA} \\ V_G &= V_{DD} \\ V_D &= V_G - V_{TH} + 100 \text{ mV} \\ V_{DD} - I_D R_D &= V_G - V_{TH} + 100 \text{ mV} \\ R_D &= \boxed{300 \ \Omega} \end{aligned}$$

(b)

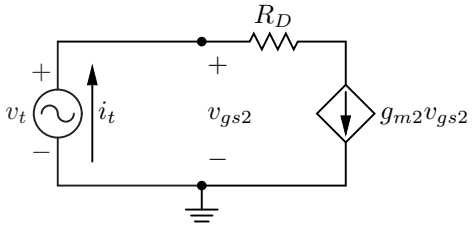
$$\begin{aligned} R_D &= 300 \ \Omega \\ A_v &= g_m R_D \\ &= \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} R_D \\ &= 5 \\ \frac{W}{L} &= \boxed{694.4} \end{aligned}$$

7.44 For this problem, recall that looking into the drain of a transistor with a grounded gate and source we see a resistance of $r_o$, and looking into either terminal of a diode-connected transistor we see a resistance of $\frac{1}{g_m} \parallel r_o$.

(a) Referring to Eq. (7.109) with $R_D = \frac{1}{g_{m2}}$ and $g_m = g_{m1}$, we have

$$A_v = \boxed{\frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + R_S}}$$

(b) Let's draw a small-signal model to find the equivalent resistance seen looking up from the output.




$$\begin{aligned} i_t &= g_{m2}v_{gs2} \\ v_{gs2} &= v_t \\ i_t &= g_{m2}v_t \\ \frac{v_t}{i_t} &= \frac{1}{g_{m2}} \\ A_v &= \boxed{\frac{g_{m1}}{g_{m2}}} \end{aligned}$$

(c) Referring to Eq. (7.119) with $R_D = \frac{1}{g_{m2}}$, $R_3 = R_1$, and $g_m = g_{m1}$, we have

$$A_v = \boxed{\frac{R_1 \parallel \frac{1}{g_{m1}}}{R_S + R_1 \parallel \frac{1}{g_{m1}}} \frac{g_{m1}}{g_{m2}}}$$

(d)

$$A_v = \boxed{g_{m1}\left(R_D + \frac{1}{g_{m2}} \parallel r_{o3}\right)}$$

(e)

$$A_v = \boxed{g_{m1}\left(R_D + \frac{1}{g_{m2}}\right)}$$

7.45 (a)

$$\frac{v_X}{v_{in}} = -g_{m1}\left(R_{D1} \parallel \frac{1}{g_{m2}}\right)$$
$$\frac{v_{out}}{v_X} = g_{m2}R_{D2}$$
$$\frac{v_{out}}{v_{in}} = \frac{v_X}{v_{in}}\frac{v_{out}}{v_X}$$
$$= \boxed{-g_{m1}g_{m2}R_{D2}\left(R_{D1} \parallel \frac{1}{g_{m2}}\right)}$$

(b)

$$\lim_{R_{D1}\to\infty} -g_{m1}g_{m2}R_{D2}\left(R_{D1} \parallel \frac{1}{g_{m2}}\right) = \boxed{-g_{m1}R_{D2}}$$

This makes sense because the common-source stage acts as a transconductance amplifier with a transconductance of $g_{m1}$. The common-gate stage acts as a current buffer with a current gain of 1. Thus, the current $g_{m1}v_{in}$ flows through $R_{D2}$, meaning $v_{out} = -g_{m1}v_{in}R_{D2}$, so that $\frac{v_{out}}{v_{in}} = -g_{m1}R_{D2}$.
This type of amplifier (with $R_{D1} = \infty$) is known as a cascode and will be studied in detail in Chapter 9.

(46) $\frac{V_x}{V_{in}} = \left(R_{D1} \,//\, \frac{1}{g_{m2}}\right) g_{m1}$

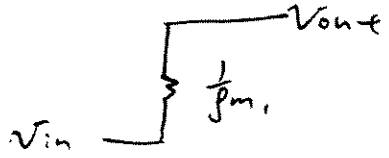
$\frac{V_{out}}{V_x} = g_{m2} R_{D2}$.

$\therefore \frac{V_{out}}{V_{in}} = g_{m1} g_{m2} R_{D2} \left(R_{D1} \,//\, \frac{1}{g_{m2}}\right)$

Similar to prob. (45), voltage gain approaches that of cascode stage as $R_{D1}$ approaches infinity. The gain is $g_{m1} R_{D2}$.

(4-7) With $\lambda = 0$, $M_1$ appears as a diode-connected device.

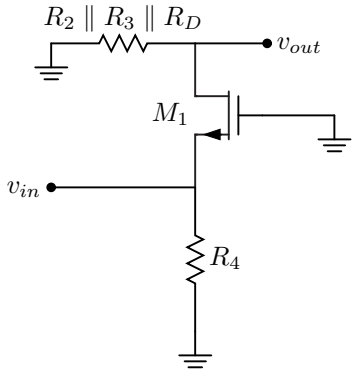
$\therefore$ the circuit becomes:




ie. $\frac{V_{out}}{V_{in}} = 1$ //

This is not a common-gate (CG) amplifier, because the gate is not fixed. (ie. gate is not at an "a.c. ground")

7.48 For small-signal analysis, we can short the capacitors, producing the following equivalent circuit.




$$A_v = \boxed{g_m \left( R_2 \parallel R_3 \parallel R_D \right)}$$

7.49

$$V_{GS} = V_{DS}$$

$$V_{GS} = V_{DD} - I_D R_S = V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2\left(1 + \lambda V_{GS}\right)R_S$$

$$V_{GS} = V_{DS} = 0.7036 \text{ V}$$

$$I_D = 1.096 \text{ mA}$$

$$A_v = \frac{r_o \parallel R_S}{\frac{1}{g_m} + r_o \parallel R_S}$$

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D} = 6.981 \text{ mS}$$

$$r_o = \frac{1}{\lambda I_D} = 9.121 \text{ k}\Omega$$

$$A_v = \boxed{0.8628}$$

7.50

$$
\begin{aligned}
A_v &= \frac{R_S}{\frac{1}{g_m} + R_S} \\
&= \frac{R_S}{\frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})} + R_S} \\
&= 0.8 \\
V_{GS} &= 0.64 \text{ V} \\
I_D &= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 \\
&= 960 \text{ μA} \\
V_G &= V_{GS} + V_S = V_{GS} + I_D R_S \\
&= \boxed{1.12 \text{ V}}
\end{aligned}
$$

(51)

$$A_v = \frac{R_s}{\frac{1}{g_m} + R_s}$$

$$= 0.8$$

$$0.8 = \frac{500}{\frac{1}{g_m} + 500}$$

$$\therefore g_m = 8\,mS.$$

$$\therefore I_{ds} = \frac{1}{2}\beta (V_{GS} - V_t)^2,$$

where $\beta = \left(\frac{W}{L}\right) \mu_n C_{ox}$

and $g_m = \beta (V_{GS} - V_t)$.

$$\therefore I_{ds} = \frac{1}{2} g_m (V_{GS} - V_t)$$

$$= \frac{1}{2} g_m (1.8 - I_{ds}(500) - 0.4)$$

$$I_{ds} = 4 \times 10^{-3} (1.4 - 500\, I_{ds})$$

$$\therefore I_{ds} = 1.87\,mA.$$

$$\therefore g_m = \sqrt{2 (200 \times 10^{-6}) \frac{W}{L} \times 1.87 \times 10^{-3}}$$

$$\therefore \frac{W}{L} \approx 85.7 //$$

(52) To get $R_{out} = 100\,\Omega$,

$$\frac{1}{g_m} = 100$$

$$\therefore g_m = 10\ mS.$$

$$\because \quad I_{ds} = \frac{1}{2}\beta (V_{GS} - V_{TH})^2.$$

where $\beta = \mu_n C_{ox} \frac{W}{L}$.

and $g_m = \beta (V_{GS} - V_{TH})$

$$\therefore I_{ds} = \frac{1}{2} g_m (V_{GS} - V_{TH})$$

$$= \frac{1}{2} (10 \times 10^{-3})(0.9 - 0.4)$$

$$\therefore I_{ds} = 2.5\ mA.$$

$$\because g_m = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{W}{L}\right) (2.5 \times 10^{-3})}$$

$$\therefore \left(\frac{W}{L}\right) = 100 //$$

(53) To see $R_{out} = 50\Omega$,

$$\frac{1}{g_m} = 50\Omega$$

$$\therefore g_m = 20\,mS.$$

$$\text{Power } (P) = 1.8 \times I_{DS}$$

$$= 2 \times 10^{-3}\,W.$$

$$\therefore I_{DS} = 1.11\,mA.$$

$$\because g_m = \sqrt{2 \times (200 \times 10^{-6})\left(\frac{W}{L}\right)(1.11\,mA)}$$

$$\therefore \frac{W}{L} = 900$$ //

(54)

$$A_v = \frac{R_L}{\frac{1}{g_m} + R_L}$$

$$\therefore 0.8 = \frac{50}{\frac{1}{g_m} + 50}$$

$$g_m = 80\ mS$$

$$\text{Power } (P) = 1.8 \times I_{DS}$$

$$= 3\ mW$$

$$\therefore I_{DS} = 1.67\ mA$$

$$g_m = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{W}{L}\right) (1.67 \times 10^{-3})}$$

$$\therefore \left(\frac{W}{L}\right) = 800$$

7.55 For this problem, recall that looking into the drain of a transistor with a grounded gate and source we see a resistance of $r_o$, and looking into either terminal of a diode-connected transistor we see a resistance of $\frac{1}{g_m} \parallel r_o$.

(a)

$$A_v = \boxed{\frac{r_{o1} \parallel (R_S + r_{o2})}{\frac{1}{g_{m1}} + r_{o1} \parallel (R_S + r_{o2})}}$$

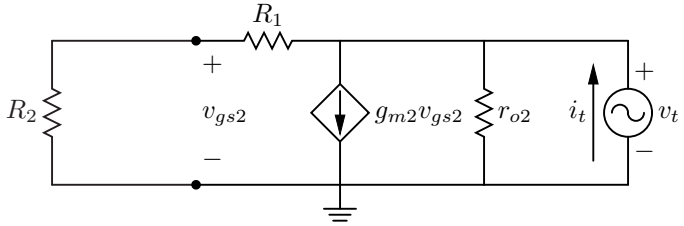
(b) Looking down from the output we see an equivalent resistance of $r_{o2} + (1 + g_{m2} r_{o2}) R_S$ by Eq. (7.110).

$$A_v = \boxed{\frac{r_{o1} \parallel [r_{o2} + (1 + g_{m2} r_{o2}) R_S]}{\frac{1}{g_{m1}} + r_{o1} \parallel [r_{o2} + (1 + g_{m2} r_{o2}) R_S]}}$$

(c)

$$A_v = \boxed{\frac{r_{o1} \parallel \frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + r_{o1} \parallel \frac{1}{g_{m2}}}}$$

(d) Let's draw a small-signal model to find the equivalent resistance seen looking down from the output.




$$\begin{aligned}
i_t &= \frac{v_t}{R_1 + R_2} + g_{m2} v_{gs2} + \frac{v_t}{r_{o2}} \\
v_{gs2} &= \frac{R_2}{R_1 + R_2} v_t \\
i_t &= \frac{v_t}{R_1 + R_2} + g_{m2} \frac{R_2}{R_1 + R_2} v_t + \frac{v_t}{r_{o2}} \\
i_t &= v_t \left( \frac{1}{R_1 + R_2} + \frac{g_{m2} R_2}{R_1 + R_2} + \frac{1}{r_{o2}} \right) \\
\frac{v_t}{i_t} &= (R_1 + R_2) \parallel \left( \frac{R_1 + R_2}{g_{m2} R_2} \right) \parallel r_{o2} \\
A_v &= \boxed{\frac{r_{o1} \parallel (R_1 + R_2) \parallel \left( \frac{R_1 + R_2}{g_{m2} R_2} \right) \parallel r_{o2}}{\frac{1}{g_{m1}} + r_{o1} \parallel (R_1 + R_2) \parallel \left( \frac{R_1 + R_2}{g_{m2} R_2} \right) \parallel r_{o2}}}
\end{aligned}$$

(e)

$$A_v = \boxed{\frac{r_{o2} \parallel r_{o3} \parallel \frac{1}{g_{m1}}}{\frac{1}{g_{m2}} + r_{o2} \parallel r_{o3} \parallel \frac{1}{g_{m1}}}}$$

(f) Looking up from the output we see an equivalent resistance of $r_{o2} + (1 + g_{m2} r_{o2})\, r_{o3}$ by Eq. (7.110).

$$A_v = \boxed{\frac{r_{o1} \parallel [r_{o2} + (1 + g_{m2} r_{o2})\, r_{o3}]}{\frac{1}{g_{m1}} + r_{o1} \parallel [r_{o2} + (1 + g_{m2} r_{o2})\, r_{o3}]}}$$

(56) $\frac{V_x}{V_{in}} = \frac{g_{m_2}}{\frac{1}{g_{m_1}} + g_{m_2}}$.

$$\frac{V_{out}}{V_x} = g_{m_2} R_D$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{R_D}{\frac{1}{g_{m_1}} + \frac{1}{g_{m_2}}}$$ //

b) if $g_{m_1} = g_{m_2}$.

$$\frac{V_{out}}{V_{in}} = \frac{g_{m_1} R_D}{2}$$ //

(57)

$\therefore R_{out} = 1k\Omega.$

$\therefore R_D = 1k\Omega.$

$\therefore A_V = 5$

$= g_{m1} R_D$

$\therefore g_{m1} (1000) = 5$

$g_{m1} = 5mS.$

$\therefore M_1$ is 00 mV away from triode,

$V_D = (V_G - V_{TH}) + 0.1.$

$V_D = (1.8 - 0.4) + 0.1$

$V_D = 1.5V$

$\therefore I_{DS} = \frac{1.8 - 1.5}{R_D} = \frac{0.3}{R_D}$

$= 0.3mA$

$\therefore g_{m1} = \sqrt{2 \times (200 \times 10^{-6}) (\frac{W}{L}) I_{DS1}}$

$\therefore (\frac{W}{L}) \approx 208$

$\therefore R_D = 1k\Omega$ , $R_G = 10k\Omega$ , $(\frac{W}{L}) = 208$

7.58

$$
\begin{aligned}
P &= V_{DD} I_D = 2 \text{ mW} \\
I_D &= 1.11 \text{ mA} \\
R_D I_D &= 1 \text{ V} \\
R_D &= 900 \ \Omega \\
A_v &= -g_m R_D \\
&= -\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} R_D \\
&= -5 \\
\frac{W}{L} &= \boxed{69.44}
\end{aligned}
$$

(59) $|A_v| = g_m R_L$.

$\therefore$ To achieve maximum gain, use maximum $R_L$

i.e. set $R_D = 500\,\Omega$.

For maximum $g_m$, use maximum $I_{DS}$.

(while keeping $M_1$ in saturation),

i.e. $V_D \geq V_G - V_{TH}$

$1.8 - (I_{DS})(500) \geq 1.8 - 0.4$,

$\therefore \quad I_{DS} \leq \frac{0.4}{500}$

$I_{DS,max} = 0.8\,mA$.

Note: Setting a large $R_D$ in this case would force $I_{DS,max}$ to be lower (in order to keep $M_1$ in saturation).

But since $A_v \propto R_D$, while $A_v \propto \sqrt{I_{DS}}$, sacrificing $I_{DS}$ to get higher $R_D$ would yield a higher gain.

7.60 Let's let $R_1$ and $R_2$ consume exactly 5 % of the power budget (which means the branch containing $R_D$, $M_1$, and $R_S$ will consume 95 % of the power budget). Let's also assume $V_{ov} = V_{GS} - V_{TH} = 300$ mV exactly.

$$
\begin{aligned}
I_D V_{DD} &= 0.95(2\text{ mW}) \\
I_D &= 1.056\text{ mA} \\
I_D R_S &= 200\text{ mV} \\
R_S &= \boxed{189.5\ \Omega} \\
V_{ov} &= V_{GS} - V_{TH} = 300\text{ mV} \\
I_D &= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{ov}^2 \\
\frac{W}{L} &= \boxed{117.3} \\
A_v &= -\frac{R_D}{\frac{1}{g_m} + R_S} \\
&= -\frac{R_D}{\frac{1}{\sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}} + R_S} \\
&= -4 \\
R_D &= \boxed{1.326\text{ k}\Omega} \\
\frac{V_{DD}^2}{R_1 + R_2} &= 0.05(2\text{ mW}) \\
R_1 + R_2 &= \frac{V_{DD}^2}{0.1\text{ mW}} \\
V_G &= V_{GS} + I_D R_S = V_{ov} + V_{TH} + I_D R_S = 0.9\text{ V} \\
V_G &= \frac{R_2}{R_1 + R_2}V_{DD} \\
&= \frac{R_2}{\frac{V_{DD}^2}{0.1\text{ mW}}} = 0.9\text{ V} \\
R_2 &= \boxed{29.16\text{ k}\Omega} \\
R_1 &= \boxed{3.24\text{ k}\Omega}
\end{aligned}
$$

7.61 Let's let $R_1$ and $R_2$ consume exactly 5 % of the power budget (which means the branch containing $R_D$, $M_1$, and $R_S$ will consume 95 % of the power budget).

$$
\begin{aligned}
R_D &= 200\ \Omega \\
I_D V_{DD} &= 0.95(6\text{ mW}) \\
I_D &= 3.167\text{ mA} \\
I_D R_S &= V_{ov} = V_{GS} - V_{TH} \\
R_S &= \frac{V_{ov}}{I_D} \\
g_m &= \frac{2I_D}{V_{ov}} \\
A_v &= -\frac{R_D}{\frac{1}{g_m} + R_S} \\
&= -\frac{R_D}{\frac{V_{ov}}{2I_D} + \frac{V_{ov}}{I_D}} \\
&= -5 \\
V_{ov} &= 84.44\text{ mV} \\
R_S &= \boxed{26.67\ \Omega} \\
\frac{W}{L} &= \frac{2I_D}{\mu_n C_{ox} V_{ov}^2} = \boxed{4441} \\
\frac{V_{DD}^2}{R_1 + R_2} &= 0.05(6\text{ mW}) \\
R_1 + R_2 &= \frac{V_{DD}^2}{0.3\text{ mW}} \\
V_G &= V_{GS} + I_D R_S = V_{ov} + V_{TH} + I_D R_S = 0.5689\text{ V} \\
V_G &= \frac{R_2}{R_1 + R_2} V_{DD} \\
&= \frac{R_2}{\frac{V_{DD}^2}{0.3\text{ mW}}} = 0.5689\text{ V} \\
R_2 &= \boxed{6.144\text{ k}\Omega} \\
R_1 &= \boxed{4.656\text{ k}\Omega}
\end{aligned}
$$

7.62

$$
\begin{aligned}
R_{in} &= R_1 = \boxed{20\ \text{k}\Omega} \\
P &= V_{DD} I_D = 2\ \text{mW} \\
I_D &= 1.11\ \text{mA} \\
V_{DS} &= V_{GS} - V_{TH} + 200\ \text{mV} \\
V_{DD} - I_D R_D &= V_{DD} - V_{TH} + 200\ \text{mV} \\
R_D &= 180\ \Omega \\
A_v &= -g_m R_D \\
&= -\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} R_D \\
&= -6 \\
\frac{W}{L} &= \boxed{2500} \\
V_{GS} &= V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}} \\
&= 0.467\ \text{V} \\
V_{GS} &= V_{DD} - I_D R_S \\
R_S &= \boxed{1.2\ \text{k}\Omega} \\
\frac{1}{2\pi f C_1} &\ll R_1 \\
\frac{1}{2\pi f C_1} &= \frac{1}{10} R_1 \\
f &= 1\ \text{MHz} \\
C_1 &= \boxed{79.6\ \text{pF}} \\
\frac{1}{2\pi f C_S} \parallel R_S &\ll \frac{1}{g_m} \\
\frac{1}{2\pi f C_S} &= \frac{1}{10} \frac{1}{g_m} \\
g_m &= \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = 33.33\ \text{mS} \\
C_S &= \boxed{52.9\ \text{nF}}
\end{aligned}
$$

(63) Power ($P$) = 2mW.

$\therefore I_{DS1} = |I_{DS2}| = \frac{2\,mW}{1.8\,V} = 1.11\,mA.$

$r_{o1} = r_{o2} = \frac{1}{\lambda I_{DS}}$

$= \frac{1}{0.1 \times 1.11 \times 10^{-3}}$

$= 9000\,\Omega.$

Gain ($A_V$) = $g_{m1}(r_{o1} // r_{o2}) = 20$,

$g_{m1}\left(\frac{9000}{2}\right) = 20.$

$\therefore g_{m1} = 4.44\ mS.$

Set $V_{DS1}$ (i.e. $V_{out}$) = 1.2V

(which is $< 1.5V$)

$\therefore V_{IN} = V_{GS1} \leq 1.2 + V_{TH}.$

(for $M_1$ to stay in saturation)

Set $V_{GS1} = 1.2V$

$\therefore g_{m1} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TH})$

$\left(\frac{W}{L}\right)_1 = 27.75$

For $M_2$, $\therefore M_2$ must be in saturation for $V_{out} \leq 1.5V$.

$\therefore V_{DD} - V_B \leq V_{DD} - 1.5V + V_{TH}.$

$\therefore V_B \geq 1.1V$

Set $V_B = 1.2V$

$$|I_{DS2}| = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (|V_{GS_2}| - V_{TH})^2 (1 + \lambda |V_{DS_2}|)$$

$$1.11 \times 10^{-3} = \frac{1}{2} \times 100 \times 10^{-6} \left(\frac{W}{L}\right)_2 (0.6 - 0.2)^2 (1 + 0.1 \times (1.8 - 1.5))$$

(assuming $V_{out} = 1.5V$)

$$\therefore \left(\frac{W}{L}\right)_2 \approx 135$$

$$\therefore \left(\frac{W}{L}\right)_1 = 27.75 \qquad \left(\frac{W}{L}\right)_2 = 135$$

$V_{IN} = 1.2 \qquad V_b = 1.1$

$I_{DS1} = I_{DS2} = 1.11mA$

7.64 (a)

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel R_G \parallel r_{o2}\right)}$$

(b)

$$\begin{aligned}
P &= V_{DD} I_{D1} = 3 \text{ mW} \\
I_{D1} &= |I_{D2}| = 1.67 \text{ mA} \\
|V_{GS2}| &= |V_{DS2}| = V_{DS} = \frac{V_{DD}}{2} \\
|I_{D2}| &= \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(|V_{GS2}| - |V_{TH}|\right)^2 \left(1 + \lambda_p |V_{DS2}|\right) \\
\left(\frac{W}{L}\right)_2 &= \boxed{113} \\
A_v &= -g_{m1}\left(r_{o1} \parallel R_G \parallel r_{o2}\right) \\
R_G &= 10\left(r_{o1} \parallel r_{o2}\right) \\
r_{o1} &= \frac{1}{\lambda_n I_{D1}} = 6 \text{ k}\Omega \\
r_{o2} &= \frac{1}{\lambda_p |I_{D2}|} = 3 \text{ k}\Omega \\
R_G &= 10\left(r_{o1} \parallel r_{o2}\right) = \boxed{20 \text{ k}\Omega} \\
A_v &= -\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}}\left(r_{o1} \parallel R_G \parallel r_{o2}\right) \\
&= -15 \\
\left(\frac{W}{L}\right)_1 &= \boxed{102.1} \\
V_{IN} &= V_{GS1} = V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left(1 + \lambda_n V_{DS1}\right)}} \\
&= \boxed{0.787 \text{ V}}
\end{aligned}$$

(65) a) Impedance looking into drain of $M_2$

$$= (1 + g_{m2} r_{o2}) R_S + r_{o2}$$

$$\approx 10 r_{o1}$$

Assume $g_{m2} r_{o2} \gg 1$.

$\therefore g_{m2} r_{o2} R_S + r_{o2} \approx 10 r_{o1}$

$\because r_{o1} = r_{o2}$ ($\lambda_1 = \lambda_2$ and $I_{DS1} = |I_{DS2}|$)

$\therefore g_{m2} R_S + 1 = 10$

$$g_{m2} R_S = 9 \quad \text{——— (1)}$$

Given $V_B = 1V$,

set $|V_{GS2}| = 0.6V$ (i.e. $V_{GS2} - V_{TH} = 0.2V$)

$\therefore V_{S2} = 1.6V$

$\therefore V_{RS} = 1.8V - 1.6V = 0.2V$

$\because$ Power $= 2mW$

$$I_{DS1} = |I_{DS2}| = \frac{2mW}{1.8V} = 1.11mA.$$

$$\therefore R_S = \frac{V_{RS}}{1.11 \times 10^{-3}} \approx 180\Omega //$$

From (1), $g_{m2} = \frac{9}{180} = 50\,mS.$

$\because g_{m2} = \left(\frac{W}{L}\right)_2 (100 \times 10^{-6}) (V_{GS2} - V_{TH})$

$\therefore \left(\frac{W}{L}\right)_2 = 2500 //$

b). $|\text{gain } (A_v)| = g_{m_1}(r_{o_1} // 10r_{o_1})$

$$30 = g_{m_1}(0.909\, r_{o_1})$$

$$r_{o_1} = \frac{1}{0.1 \times 1.11 \times 10^{-3}}$$

$$= 909\ \Omega$$

$$\therefore g_{m_1} = 3.66\ mS.$$

$$\therefore g_{m_1} = \sqrt{2(\mu_n C_{ox})\left(\frac{W}{L}\right)_1 \times I_{DS_1}}$$

$$\therefore \left(\frac{W}{L}\right)_1 \approx 30.2$$ //

7.66

$$P = V_{DD} I_{D1} = 1 \text{ mW}$$

$$I_{D1} = |I_{D2}| = 556 \text{ μA}$$

$$V_{ov1} = V_{GS1} - V_{TH} = \sqrt{2I_D}\mu_n C_{ox} \left(\frac{W}{L}\right)_1 = 200 \text{ mV}$$

$$\left(\frac{W}{L}\right)_1 = \boxed{138.9}$$

$$\begin{aligned} A_v &= -\frac{g_{m1}}{g_{m2}} \\ &= -\frac{\sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D1}}}{\sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_2 |I_{D2}|}} \\ &= -\sqrt{\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}} \\ &= -4 \end{aligned}$$

$$\left(\frac{W}{L}\right)_2 = \boxed{8.68}$$

$$V_{IN} = V_{GS1} = V_{ov1} + V_{TH} = \boxed{0.6 \text{ V}}$$

7.67

$$P = V_{DD} I_D = 3 \text{ mW}$$

$$I_D = I_1 = \boxed{1.67 \text{ mA}}$$

$$R_{in} = \frac{1}{g_m} = \frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}} = 50\ \Omega$$

$$\frac{W}{L} = \boxed{600}$$

$$A_v = g_m R_D = \frac{1}{50\ \Omega} R_D = 5$$

$$R_D = \boxed{250\ \Omega}$$

7.68

$$
\begin{aligned}
P &= V_{DD} I_D = 2 \text{ mW} \\
I_D &= 1.11 \text{ mA} \\
V_D &= V_G - V_{TH} + 100 \text{ mV} \\
V_{DD} - I_D R_D &= V_G - V_{TH} + 100 \text{ mV} \\
V_G &= V_{DD} \\
A_v &= g_m R_D = \frac{2I_D}{V_{GS} - V_{TH}} R_D = 4 \\
R_D &= A_v \frac{V_{GS} - V_{TH}}{2I_D} \\
V_{DD} - I_D A_v \frac{V_{GS} - V_{TH}}{2I_D} &= V_{DD} - V_{TH} + 100 \text{ mV} \\
V_{GS} &= 0.55 \text{ V} \\
R_D &= \boxed{270 \ \Omega} \\
V_S &= V_{DD} - V_{GS} = I_D R_S \\
R_S &= \boxed{1.125 \text{ k}\Omega} \\
\frac{W}{L} &= \frac{2I_D}{\mu_n C_{ox} \left(V_{GS} - V_{TH}\right)^2} = \boxed{493.8}
\end{aligned}
$$

(69) Power $= 5\,mW$

$\therefore I_{DS1} = \frac{5\times10^{-3}}{1.8} = 2.78\,mA$

Gain $(A_V) = g_m R_D = 5$

$V_{G1} = V_{OUT} = 1.8 - IR_D$

$V_{S1} = IR_S$

Let $R_S = \frac{10}{g_{m1}}$

$\therefore V_{S1} = \frac{10I}{g_{m1}}$

$\therefore V_{GS1} = 1.8 - IR_D - \frac{10I}{g_{m1}}$

$\because I_{DS} = \frac{1}{2} g_m (V_{GS} - V_{TH})$

$2.78\times10^{-3} = \frac{g_m}{2}\left(1.8 - 2.78\times10^{-3} R_D - \frac{2.78\times10^{-2}}{g_m}\right)$

$= 0.9 g_m - 1.39\times10^{-3} g_m R_D - 1.39\times10^{-2}$

$\because g_m R_D = A_V = 5$

$2.78\times10^{-3} = 0.9 g_m - 6.95\times10^{-3} - 1.39\times10^{-2}$

$\therefore g_m \approx 26.3\,mS$

and $R_D = \frac{5}{26.3\times10^{-3}} \approx 190\,\Omega$

$R_S = \frac{10}{26.3\times10^{-3}} = 380\,\Omega$

$\because g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_{DS1}} \Rightarrow \left(\frac{W}{L}\right) \approx 622$

(70) $\because R_S \approx \frac{1}{g_m}$,

$\therefore R_{in} \approx \frac{1}{g_m} = 50\,\Omega$

i.e. $g_m = 20\,mS$. //

$$|\text{gain } (A_v)| = \frac{g_m R_D}{1 + g_m R_S} = 4$$

$$g_m R_D = 4 + 4 g_m R_S$$

$$R_D = \frac{4 + 0.08 R_S}{0.02} = 200 + 4R_S \quad \text{—— ②}$$

$\therefore R_S I_D + V_{GS} - V_{TH} + 0.25 = 1.8 - I_D R_D$ (KVL)

and $I_D = \frac{1}{2} g_m (V_{GS} - V_{TH})$

i.e. $V_{GS} - V_{TH} = 100\, I_D$

$\therefore R_S I_D + 100 I_D + 0.25 = 1.8 - I_D R_D$

From ①:

$R_S I_D + 100 I_D + 0.25 = 1.8 - 200 I_D - 4 I_D R_S$

$5 R_S I_D + 300 I_D = 1.55$

Set $R_S = \frac{10}{g_m} = 500\,\Omega$

$\therefore 2500 I_D + 300 I_D = 1.55$

$\therefore I_D = 0.554\,mA$ //

$\because I_D = \frac{1}{2} g_m (V_{GS} - V_{TH})$

$0.554 \times 10^{-3} = \frac{1}{2} \times 20 \times 10^{-3} (V_{GS} - 0.4)$

$\therefore V_{GS} = 0.455\,V$

To find $(\frac{W}{L})$:

$$g_m = \sqrt{2 (\frac{W}{L}) \mu_n C_{ox} I_{DS}}$$

$$\therefore (\frac{W}{L}) \approx 1805$$

To find $R_D$:

$$\therefore R_D = 200 + 4R_S \quad \text{(from (1))}$$

$$R_D = 2200$$

To find $R_1$ and $R_2$,

$$\therefore R_1 + R_2 = 20\,k\Omega$$

and $V_{GS} = V_G - I_D R_S = 0.455\,V$

i.e $V_G = 0.732\,V$

$$V_G = \frac{R_1}{R_1 + R_2} \times V_{DD}$$

$$\therefore R_1 = 813\,\Omega$$

$$R_2 \approx 11.9\,k\Omega.$$

$\therefore R_1 = 813\,\Omega$, $R_2 = 11.9\,k\Omega$, $R_D = 2200\,\Omega$, $R_S = 500\,\Omega$

$(\frac{W}{L}) = 1805$, $I_{DS} = 0.554\,mA$.

(71) $R_{in} = R_G = 10\,k\Omega$ //

Power $= 2\,mW$

$\therefore I_{DS} = \frac{2\,mW}{1.8\,V} = 1.11\,mA$ //

$A_v = \frac{R_S}{\frac{1}{g_m} + R_S} = 0.8$

$\therefore R_S = \frac{4}{g_m}$ ——— (1)

$\because V_{out} = \frac{V_{DD}}{2} = I_{DS} R_S$

$I_{DS} R_S = 0.9$ ——— (2)

$\because V_G = 1.8\,V$ and $V_S = 0.9$

$\therefore V_{GS} = 0.9\,V$

From (2), $\because I_{DS} = 1.11\,mA$

$R_S = \frac{0.9\,V}{1.11\,mA} \approx 810\,\Omega$ //

From (1), $g_m = \frac{4}{810\,\Omega} \approx 4.94\,mS$

$\because g_m = \left(\frac{W}{L}\right)(\mu_n C_{ox})(V_{GS} - V_{TH})$

$\frac{W}{L} \approx 49.4$ //

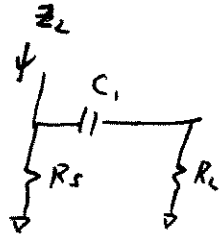
(72) $R_{in} = R_G = 20k\Omega$ //

$\therefore$ Power $= 3mW$

$\therefore I_{DS} = \frac{3mW}{1.8V} = 1.67mA$

$V_{x,at\ DC} = I_{DS} R_S = 0.9V$

$\therefore R_S = 540\Omega$




Load impedance, $Z_L = R_S // (\frac{1}{sC_1} + R_L)$
(at 100 MHz)

$= 540 // (\frac{1}{2\pi \times 10^8 C_1} + 50)$

Voltage gain $(A_V) = \frac{Z_L}{\frac{1}{g_m} + Z_L}$

$g_m = \frac{2 I_{DS}}{V_{GS} - V_{TH}}$

$= \frac{2 \times 1.67 \times 10^{-3}}{(1.8 - 0.9) - 0.4}$

$= 6.67mS.$

$\therefore A_V = \frac{Z_L}{+Z_L} = 0.8$

$Z_L = 120 + Z_L (0.8)$

$\therefore Z_L = 150$

$$\therefore 150 = 540 \,//\, \left(\frac{1}{2\pi\times10^{8} C_1} + 50\right)$$

$$= 540 \,//\, \left[\frac{1 + 50(2\pi\times10^{8} C_1)}{2\pi\times10^{8} C_1}\right]$$

$$= \frac{540 \times \dfrac{1+50(2\pi\times10^{8} C_1)}{2\pi\times10^{8} C_1}}{540 + \dfrac{1+50(2\pi\times10^{8} C_1)}{2\pi\times10^{8} C_1}}$$

$$\therefore \frac{1+50(2\pi\times10^{8} C_1)}{2\pi\times10^{8} C_1} \approx 208$$

$$\therefore C_1 \approx 10.1\,pF //$$

To find $(\frac{W}{L})$:

$$\because g_m = \left(\frac{W}{L}\right) \mu_n C_{ox} (V_{GS} - V_{TH})$$

$$\frac{W}{L} = 66.7 //$$

$$\therefore \frac{W}{L} = 66.7 \;,\; C_1 \approx 10.1\,pF \;,\; R_S = 540\,\Omega.$$

7.73

$$
\begin{aligned}
P &= V_{DD} I_{D1} = 3 \text{ mW} \\
I_{D1} &= I_{D2} = 1.67 \text{ mA} \\
A_v &= \frac{r_{o1} \parallel r_{o2}}{\frac{1}{g_{m1}} + r_{o1} \parallel r_{o2}} \\
&= \frac{r_{o1} \parallel r_{o2}}{\frac{1}{\sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D1}}} + r_{o1} \parallel r_{o2}} \\
&= 0.9 \\
r_{o1} &= r_{o2} = \frac{1}{\lambda I_{D1}} = 6 \text{ k}\Omega \\
\left(\frac{W}{L}\right)_1 &= \boxed{13.5}
\end{aligned}
$$

Let $V_{ov2} = V_{GS2} - V_{TH} = 0.3$ V. Let's assume that $V_{OUT} = V_{DS2} = V_{ov2}$.

$$
\begin{aligned}
V_{GS2} &= V_b = V_{ov2} + V_{TH} = \boxed{0.7 \text{ V}} \\
\left(\frac{W}{L}\right)_2 &= \frac{2I_{D2}}{\mu_n C_{ox} \left(V_{GS2} - V_{TH}\right)^2 \left(1 + \lambda V_{DS2}\right)} \\
&= \boxed{161} \\
V_{GS1} &= V_{TH} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left(1 + \lambda V_{DS1}\right)}} \\
V_{DS1} &= V_{DD} - V_{DS2} = 1.5 \text{ V} \\
V_{GS1} &= 1.44 \text{ V} \\
V_{IN} &= V_{GS1} + V_{DS2} = \boxed{1.74 \text{ V}}
\end{aligned}
$$